Calculation and fitting of Nucleon form factors from lattice QCD

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Introduction

- Lattice QCD allows for the determination of nucleon structure quantities such as the electromagnetic form factors G_E, G_M, the axial form factor G_A and the induced pseudo-scalar form factor G_P. These observables are related to the matrix elements of the vector or axial current and can be extracted from three- and two-point correlation functions.

- The excited state contamination in correlation functions might lead to a deviation from the fitted value when only the plateau and summation methods are adopted. To take excited state contribution into account is a possible cure to the problem that some lattice calculations have failed so far to reproduce the results extracted from experiment.

- New analysis methods and fit-ansatz are investigated and tested for different nucleon form factors, lattice sizes and pion masses. In addition an alternative method to extract the required observables from matrix element is under study.

Extraction and Analysis

- Form factors can be determined by calculating a ratio,

$$R_{\gamma_\mu}(\vec{q}, t, t_s) = \frac{C_{3,\gamma_\mu}(\vec{q}, t, t_s)}{C_2(0, t_s)} \sqrt{\frac{C_2(\vec{q}, t_s - t) C_2(0, t) C_2(0, t_s)}{C_2(0, t_s - t) C_2(\vec{q}, t) C_2(\vec{q}, t_s)}}, \tag{1}$$

using two-point correlation function and three-point function with the vector or axial current. The correlation functions are projected to definite momentum transfer $\vec{q}$ via the Fourier transformation so that the momentum dependent form factors can be extracted from the ratio

$$\begin{aligned} \mathrm{Im}[R_{\gamma_5\gamma_i}(\vec{q})] &= \frac{1}{\sqrt{2E(m+E)}}\left((m+E)G_A(q^2)\delta_{3i} - \frac{q_3 q_i}{2m} G_P(q^2)\right) \quad i = 1,2,3 \\ \mathrm{Re}[R_{\gamma_0}(\vec{q})] &= \sqrt{\frac{m+E}{2E}}\, G_E(q^2) \\ \mathrm{Re}[R_{\gamma_i}(\vec{q})] &= \varepsilon_{ij} q_j \frac{1}{\sqrt{2E(E+m)}} G_M(q^2) \quad i = 1,2 \end{aligned} \tag{2}$$

Considering the contribution from the first excited state to $C_3(t, t_s)$ and $C_2(t)$ at non-vanishing momentum transfer, the ratio can be expressed approximately as

$$R(t, t_s) = G_X + c_1 e^{-m_\pi t} + c_2 e^{-2m_\pi(t_s - t)}, \tag{3}$$

where the energy gap between the ground and the first excited states are set to the rest mass of one or two pions. The form factors G_X and coefficients c_1, c_2 may be determined from the fits involving the data points of single source-sink seperation t_s (individual excited fit) and multiple t_s (simultaneous excited fit), as shown in Fig. 1 and Fig. 2.

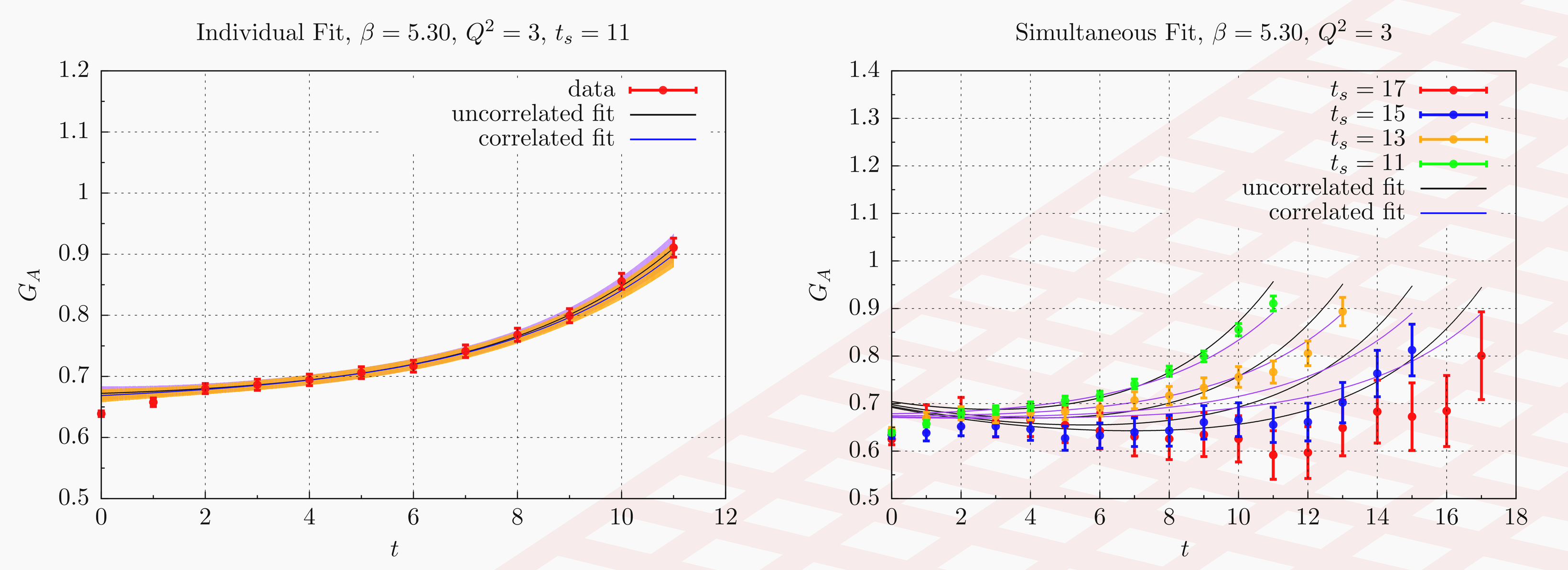

Figure 1: Two kinds of excited state fits for the axial form factor G_A at the momentum transfer of $Q^2 = 3$. Left: uncorrelated and correlated fits for the data of $t_s = 11$. Right: fits for four $t_s = 11, 13, 15, 17$ simultaneously.

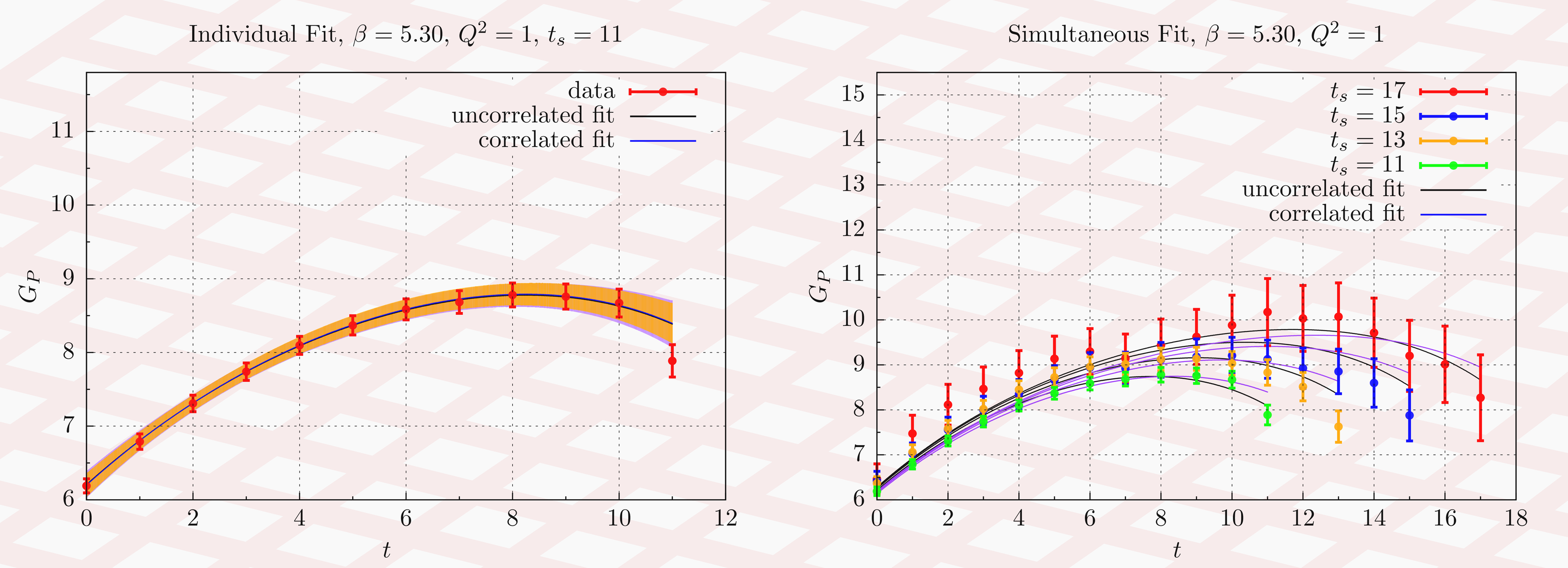

Figure 2: The individual fit and the simultaneous fit for the induced pseudo-scalar form factor G_P at $Q^2 = 1$. Left: uncorrelated and correlated fits for the data of $t_s = 11$. Right: fits for four $t_s = 11, 13, 15, 17$ simultaneously.

- We explore an alternative method to extract the nucleon form factors by computing a ratio of three-point function and the t, t_s, $\vec{q}$ dependent factors,

$$\widetilde{R}_{\gamma_\mu}(\vec{q}, t, t_s) \equiv \frac{C_{3,\gamma_\mu}(\vec{q}, t, t_s)}{Z_B(0) Z_B(-\vec{q})^* e^{-E_q t} e^{-m(t_s - t)}} \tag{4}$$

where the ratio $\widetilde{R}_{\gamma_\mu}(\vec{q}, t, t_s)$ can be used to extract the form factors in a similar way like in eq. (2). To determin the factors $Z_B(0)$, $Z_B(-\vec{q})$, E_q a linear fit to the function of nucleon two-point function is implemented

$$\log C_2(\vec{p}, t) = \log\left(|Z_B(\vec{p})|^2(1 + m/E_p)\right) - E_p t \tag{5}$$

In the extraction of form factors the identical fit-ansatz in eq. (3) is used to get rid of the excited state contribution. We show some examples of analysis in Fig. 3 and Fig. 4

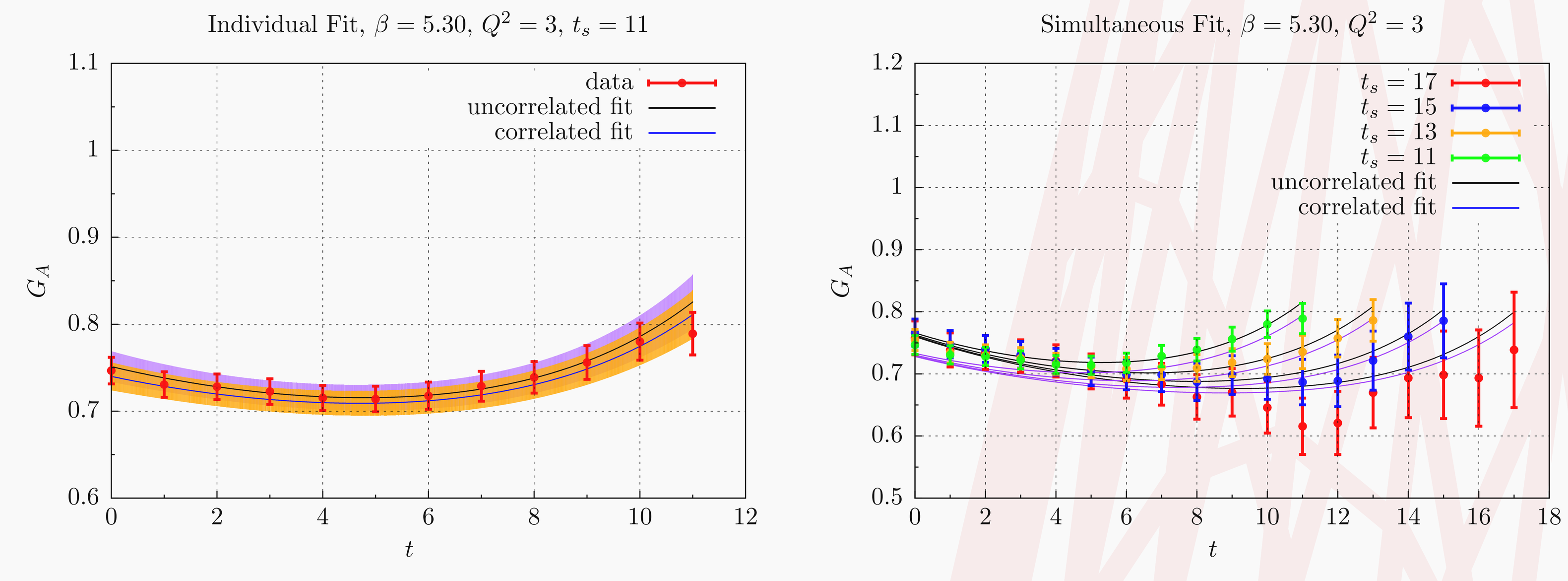

Figure 3: Excited fits to the ratio calculated in an alternative method. In these plots the axial form factor G_A at $Q^2 = 3$ is extracted. Left: individual fit for the data of $t_s = 11$. Right: simultaneous fit for $t_s = 11, 13, 15, 17$.

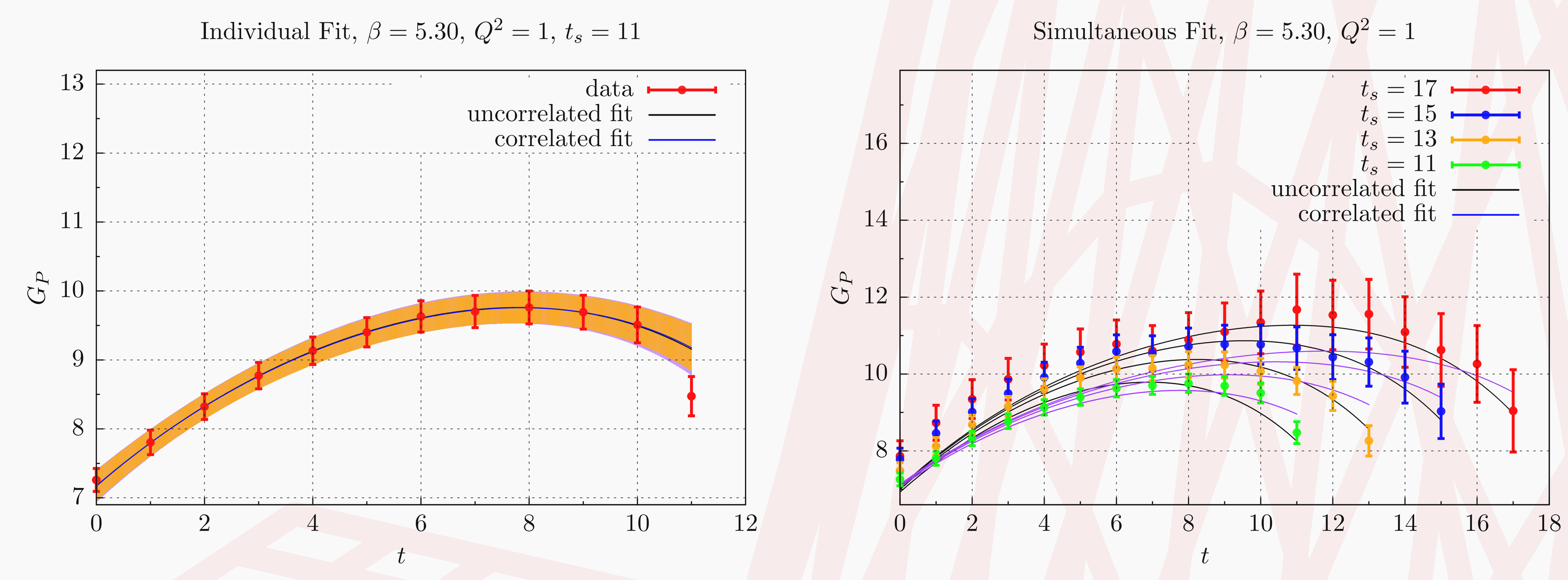

Figure 4: Excited fits for G_P with the ratio $\widetilde{R}_{\gamma_\mu}$ at $Q^2 = 1$. Left: individual fit for the data of $t_s = 11$. Right: simultaneous fit for $t_s = 11, 13, 15, 17$.

- To compare the analysis results we fit the G_A in different methods at $Q^2 = 3$, on one of the lattice ensembles, as shown in Tab. 1.

Extraction method	uncorrelated fit	χ^2_{red}	correlated fit	χ^2_{red}
Ind. fit to t_s=11 with R	0.682(23)	0.301	0.689(20)	0.915
Ind. fit to t_s=11 with $\widetilde{R}$	0.644(22)	0.020	0.657(20)	0.934
Ind. fit to t_s=13 with R	0.671(36)	0.076	0.681(33)	0.424
Ind. fit to t_s=13 with $\widetilde{R}$	0.654(34)	0.007	0.670(30)	0.573
Sim. fit to t_s=11,13, 15,17 with R	0.621(39)	0.926	0.682(17)	1.072
Sim. fit to t_s=11,13, 15,17 with $\widetilde{R}$	0.631(42)	0.215	0.643(17)	0.790

Table 1: Comparison of excited fit methods for the extraction of G_A at $Q^2 = 3$. Here the Ind. fit refers to the individual fit while Sim. fit refers to the simultaneous fit. The ratio in eq. (1) and the one in eq. (4) are calculated respectively in an ensemble with $\beta = 5.30$.

Conclusion & Outlook

- The excited state fits are necessary to be included as complementary methods in the analysis of nucleon form factors in lattice QCD.
- The form factors can also be extracted from the ratio proportional to the three-point function. In the analysis with this ratio a three-parameters fit-ansatz can describe the data well with reasonable Chi-squares.
- Further investigations for the theoretical assumption of excited state contribution is also necessary to improve the analysis.

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