Non-perturbative renormalization of the axial current in $N_\mathrm{f}=3$ lattice QCD with Wilson fermions and tree-level improved gauge action

ALPHA Collaboration

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Motivation

Axial current

$$A_\mu^a(x) = \bar\psi(x)\,\frac{1}{2}\tau^a\,\gamma_\mu\gamma_5\,\psi(x)$$

Applications

- PCAC masses
- decay constants F_PS (in particular for scale setting with f_K)
- matching of HQET currents (our next project)
- ...

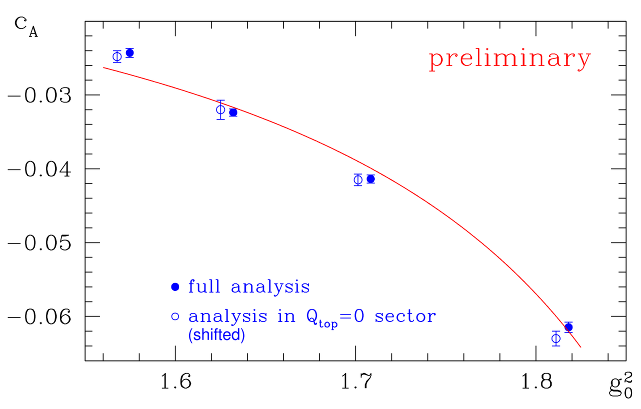
Improvement: $(A_\mathrm{I})_\mu^a(x) = A_\mu^a(x) + a c_\mathrm{A}\cdot\tilde\partial_\mu P^a(x)$

Renormalization: $(A_\mathrm{R})_\mu^a(x) = Z_\mathrm{A}\cdot(1+b_\mathrm{A} a m_\mathrm{q})\cdot(A_\mathrm{I})_\mu^a(x)$

- improvement coefficient c_A determined in a previous project (see below)
- renormalization coefficient Z_A is leading term, sensitive to errors
- Goal: non-perturbative determination of Z_A

Previous Project: Determination of c_A

- improvement condition based on PCAC mass
- PCAC mass is evaluated with two different external operators and c_A is adjusted so that the results are equal
- external operators have wave functions corresponding to $\eta^{(0)}$ and $\eta^{(1)}$ (approximate ground and first excited state, see section "Wave Functions")
- to be published, see [1]

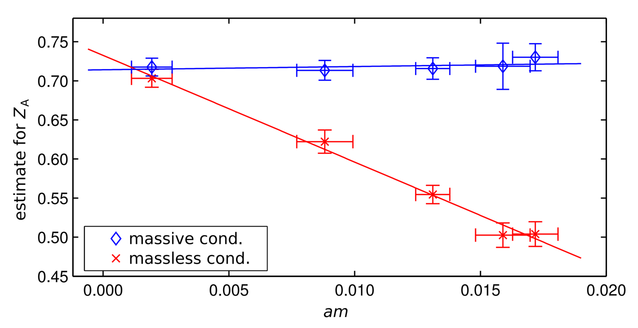


Renormalization Condition for Z_A

- taken from $N_\mathrm{f}=2$ case[2, 3]
- renormalization condition based on continuum chiral Ward identity, similar to PCAC
- insertions of two axial currents A_0 and external sources O_ext

$$\begin{aligned}&\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\,\epsilon^{abc}\left\langle A_0^a(x)\,A_0^b(y)\,O_\mathrm{ext}^c\right\rangle\\ -2m&\int \mathrm{d}^3\mathbf{x}\,\mathrm{d}^3\mathbf{y}\,\epsilon^{abc}\int_{y_0}^{x_0}\mathrm{d}x_0'\left\langle P^a(x_0',\mathbf{x})\,A_0^b(y)\,O_\mathrm{ext}^c\right\rangle\\ &= \mathrm{i}\int \mathrm{d}^3\mathbf{y}\,\langle V_0^c(y)\,O_\mathrm{ext}^c\rangle\end{aligned} \qquad (1)$$

- RHS due to variation of second A_0 insertion
- non-vanishing PCAC mass is explicitly taken into account to facilitate extrapolation to $m=0$
- plot on the right shows chiral extrapolation at $\beta=5.2$, taken from $N_\mathrm{f}=2$, with and without mass term

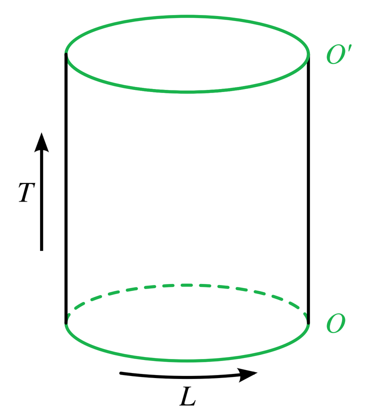


Schrödinger functional

- periodic in space, Dirichlet boundary conditions in time
- boundary fields ζ, ζ' are used to build source operators

Dimensions

$T/L = 3/2 \qquad L\approx 1.2\,\mathrm{fm}$



- trade-off between large infrared cutoff (small T) and small $\mathcal{O}(a^2)$ effects (large L)
- in [3], big $\mathcal{O}(a^2)$ ambiguities were observed at $N_\mathrm{f}=2$, $L=0.8\,\mathrm{fm}$

Source Operator

- pseudoscalar sources with wave functions (see below) at $x_0=0$ and $x_0=T$

$$O_\mathrm{ext}^c = -\frac{1}{6L^6}\,\epsilon^{cde}\,O'^d\,O^e$$

$$O^e = \sum_{\mathbf{uv}}\bar\zeta(\mathbf{u})\,\frac{1}{2}\tau^e\,\gamma_5\,\omega(\mathbf{u}-\mathbf{v})\,\zeta(\mathbf{v})$$

Wave Functions

choose WF ω_π that couples almost exclusively to the ground state

- periodic basis functions ω_1, ω_2, ω_3

$$\bar\omega_1(r) = \mathrm{e}^{-r/r_0} \qquad \bar\omega_2(r) = r\cdot\mathrm{e}^{-r/r_0} \qquad \bar\omega_3 = \mathrm{e}^{-r/(2r_0)}$$

$$\omega_i(x) = N_i\sum_{\mathbf{n}\in\mathbb{Z}^3}\bar\omega_i(|x-\mathbf{n}L|)$$

(r_0: some physical length scale; N_i: normalization)

- determine eigenvalues $\lambda^{(0)}>\lambda^{(1)}>\lambda^{(2)}$ and eigenvectors $\eta^{(0)}$, $\eta^{(1)}$, $\eta^{(2)}$ of 3×3 matrix $F_1(\omega_i,\omega_j)$ (boundary-boundary correlator with ω_i at $x_0=0$ and ω_j at $x_0=T$)
- our result:

$$\eta^{(0)} = (0.53176, 0.59773, 0.59996)$$

- approximate ω_π by

$$\omega_\pi \approx \sum_i \eta_i^{(0)}\omega_i$$

Correlators

basic Ward identity (1) in terms of renormalized and improved Schrödinger-functional correlation functions:

$$Z_\mathrm{A}^2\cdot\left[F_\mathrm{AA}^\mathrm{I}(x_0,y_0) - 2m\cdot\tilde F_\mathrm{PA}^\mathrm{I}(x_0,y_0)\right] = F_1$$

$$\Rightarrow \quad Z_\mathrm{A}(g_0^2) = \lim_{m\to 0}\sqrt{F_1}\left[F_\mathrm{AA}^\mathrm{I}(x_0,y_0) - 2m\cdot\tilde F_\mathrm{PA}^\mathrm{I}(x_0,y_0)\right]^{-1/2}$$

general form of correlators involved:

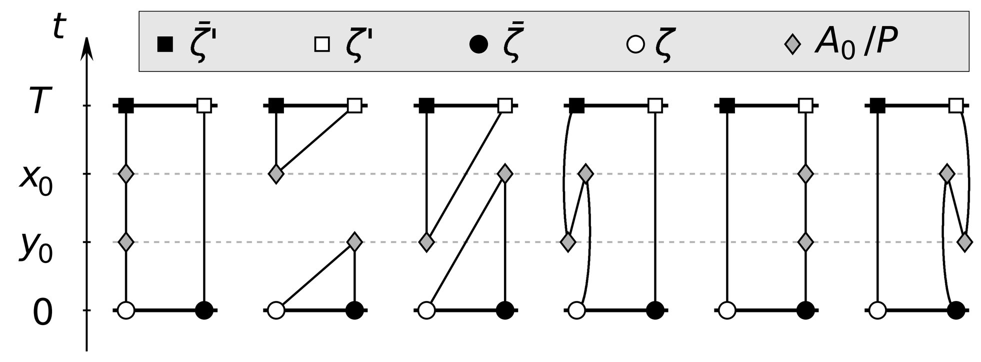
$$f_{XY}(x_0,y_0) = -\frac{a^6}{6L^6}\sum_{\mathbf{x},\mathbf{y}}\varepsilon^{abc}\varepsilon^{cde}\left\langle O'^d\cdot X^a\cdot Y^b\cdot O^e\right\rangle$$

with insertions X, Y of

$$A_0^a(x_0), \qquad \partial_0 P^a(x_0), \qquad \tilde P^a(x,y_0) = \sum_{t=y_0}^{x_0} w(t)\cdot P^a(t,\mathbf{x})$$

($\partial_0 P^a$ needed for improving A_0^a)

Connected and Disconnected Contributions:



- standard choice: $x_0 = 2/3\cdot T$ and $y_0 = 1/3\cdot T$
- implemented in SFCF code and checked against old results
- alternative definition $Z_\mathrm{A,con}$ with connected contributions only

Simulation Parameters and Preliminary Results

- re-use of configurations from c_A determination (generated with openQCD code[4])
- $N_\mathrm{f}=3$ and tree-level-improved (Lüscher–Weisz) action
- $\theta=0$, vanishing background field
- β tuned to keep L constant ($\approx 1.2\,\mathrm{fm}$)
- κ tuned towards vanishing (PCAC) quark mass

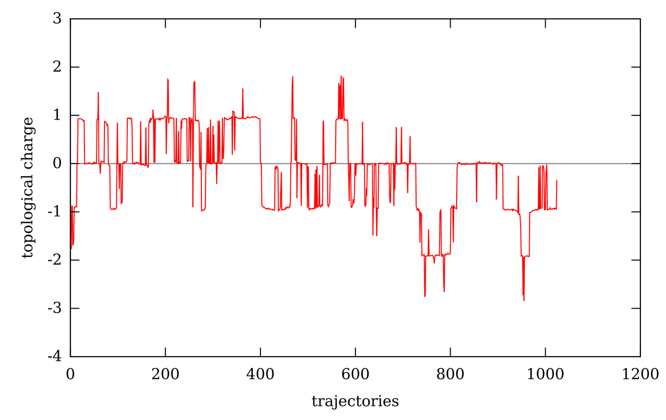
Preliminary Results:

L/a	T/a	β	κ	am_PCAC	$Z_\mathrm{A,con}$	Z_A
12	17	3.3	0.13652	$-0.00096(71)$	0.82(2)	0.66(3)
12	17	3.3	0.13660	$-0.0086(6)$	0.83(2)	0.65(3)
16	23	3.512	0.13700	$+0.0064(2)$	0.78(1)	0.77(1)
16	23	3.512	0.13703	$+0.0056(3)$	0.78(1)	0.77(2)
16	23	3.512	0.13710	$+0.0024(2)$	0.78(1)	0.75(2)
20	29	3.676	0.13680	$+0.0139(2)$	–	–
20	29	3.676	0.13700	$+0.0066(1)$	0.79(1)	0.79(2)
24	35	3.810	0.13712	$-0.00269(8)$	0.80(2)	0.79(1)

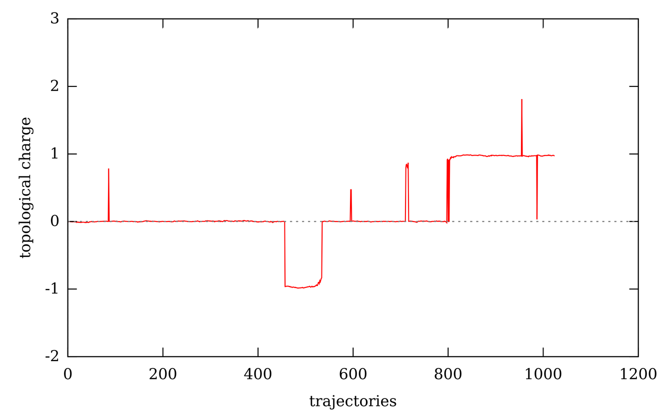
- some configurations still have to be analyzed
- $Z_\mathrm{A,con}$ not yet conclusive (need more statistics)
- Z_A: no strong mass dependence observed

Topology Freezing

- at finer lattices, topological charge Q gets stuck
- impact on results will be checked by alternative analysis taking only configurations with $Q=0$ into account (for c_A we got compatible results indicating no strong dependence on Q)



$L/a = 16, \beta = 3.512$



$L/a = 24, \beta = 3.81$

Outlook

- some measurements still to be done
- maybe new simulations at smaller masses
- analysis to be crosschecked
- determination of Z_V

References

Literatur

[1] J. Bulava, M. Della Morte, J. Heitger, and C. Wittemeier, *Determination of c_A in three-flavour lattice QCD with Wilson fermions and tree-level improved gauge action*, PoS **LATTICE2013** (2013) 311, [arXiv:1312.3591].

[2] M. Della Morte, R. Hoffmann, F. Knechtli, R. Sommer, and U. Wolff, *Non-perturbative renormalization of the axial current with dynamical Wilson fermions*, JHEP **0507** (2005) 007, [hep-lat/0505026].

[3] M. Della Morte, R. Sommer, and S. Takeda, *On cutoff effects in lattice QCD from short to long distances*, Phys.Lett. **B672** (2009) 407–412, [arXiv:0807.1120].

[4] M. Lüscher and S. Schaefer, *Lattice QCD with open boundary conditions and twisted-mass reweighting*, Comput.Phys.Commun. **184** (2013) 519–528, [arXiv:1206.2809].