Comparison of gradient flow with other smearing techniques applied to $N_f = 2+1+1$-flavour QCD at nonzero temperature

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Abstract

We compare gradient Wilson flow with overimproved stout-link smearing and cooling approaches by fixing the average plaquette action in each method accordingly to the flow stopping condition. Then, as it shown for topological susceptibility, the results from all approaches are practically equivalent. Also, the temperature dependence for static quark potential extracted from Polyakov loop correlators is presented.

Basic concepts

Overimproved cooling
[M. Garcia Perez *et. al*, Nucl. Phys. **B 413**, 535 (1994)]

- sequential update of lattice links $U_\mu(x)$, minimizes local action
- Cabibbo–Marinari projection

$$S(\varepsilon) = \sum_{x,\mu,\nu} \frac{4-\varepsilon}{3}\, Re\,\mathrm{Tr}\left(1 - \Box_{\mu\nu}(x)\right) + \sum_{x,\mu,\nu} \frac{\varepsilon - 1}{48}\, Re\,\mathrm{Tr}\left(1 - \Box^{2\times2}_{\mu\nu}(x)\right)$$

$\varepsilon = -1$ — overimprovement

Overimproved stout–link smearing
[P.J. Moran and D.B. Leinweber, Phys. Rev. **D 77**, 094501 (2008)]

$$U_\mu(x) \to \exp\Big(-\rho_{sm}\frac{\delta S(\varepsilon)}{\delta U_\mu(x)}\Big) U_\mu(x)$$

- no $SU(3)$ projection is required; sequential update of lattice links

ρ_{sm} — smearing parameter ("time step")

Gradient Wilson flow
[M. Lüscher, Commun. Math. Phys. **293**, 899; JHEP 1008, **071** (2010)]

$$\dot V_\mu(x,\tau) = -g_0^2[\partial_{x,\mu} S_W(V(\tau))] V_\mu(x,\tau) \qquad V_\mu(x,0) = U_\mu(x)$$

$$S_W = S\big|_{\varepsilon=1} = \sum_{x,\mu,\nu} Re\,\mathrm{Tr}\left(1 - \Box_{\mu\nu}(x)\right)$$

- moves the gauge configurations along the gradient of the action

Direct correspondence to "ordinary" Wilson cooling:

$$N^0_{cool} \simeq 3\tau$$

[C. Bonati, M. D'Elia, Phys. Rev. **D 89**, 105005 (2014)]

Gauge configurations

Provided by tmfT Collaboration: first set at high pion mass

- twisted mass $N_f = 2+1+1$, $m_\pi \approx 400$ MeV, and $T_c \approx 197$ MeV
- $\beta = 1.90$, 1.95, and 2.10
- $a \approx 0.06 - 0.09$ fm
- $N_s = 24 - 32$, $N_\tau = 4 - 16$

Flow time and number of steps

$\beta = 1.90$			
$N_s^3 \times N_\tau$	N_{cool}	$N_{stout\ link}$	τ
$24^3 \times 4$	5	13	1.15
$24^3 \times 6$	4	10	0.88
$24^3 \times 8$	3	9	0.78
$24^3 \times 10$	3	8	0.74
$24^3 \times 12$	3	8	0.72
$32^3 \times 14$	3	8	0.71

$\beta = 1.95$, $N_s = 32$			
N_τ	N_{cool}	$N_{stout\ link}$	τ
6	5	12	1.09
8	4	10	0.93
10	4	10	0.89
12	4	10	0.86
14	4	9	0.85
16	4	9	0.84

Wilson flow stopping criteria:

$$t^2\langle -\frac{1}{2}\mathrm{Tr}\{F_{\mu\nu}F_{\mu\nu}\}\rangle = 0.3 \qquad t = \tau a^2$$

Flow time and number of cooling or smearing steps relation:

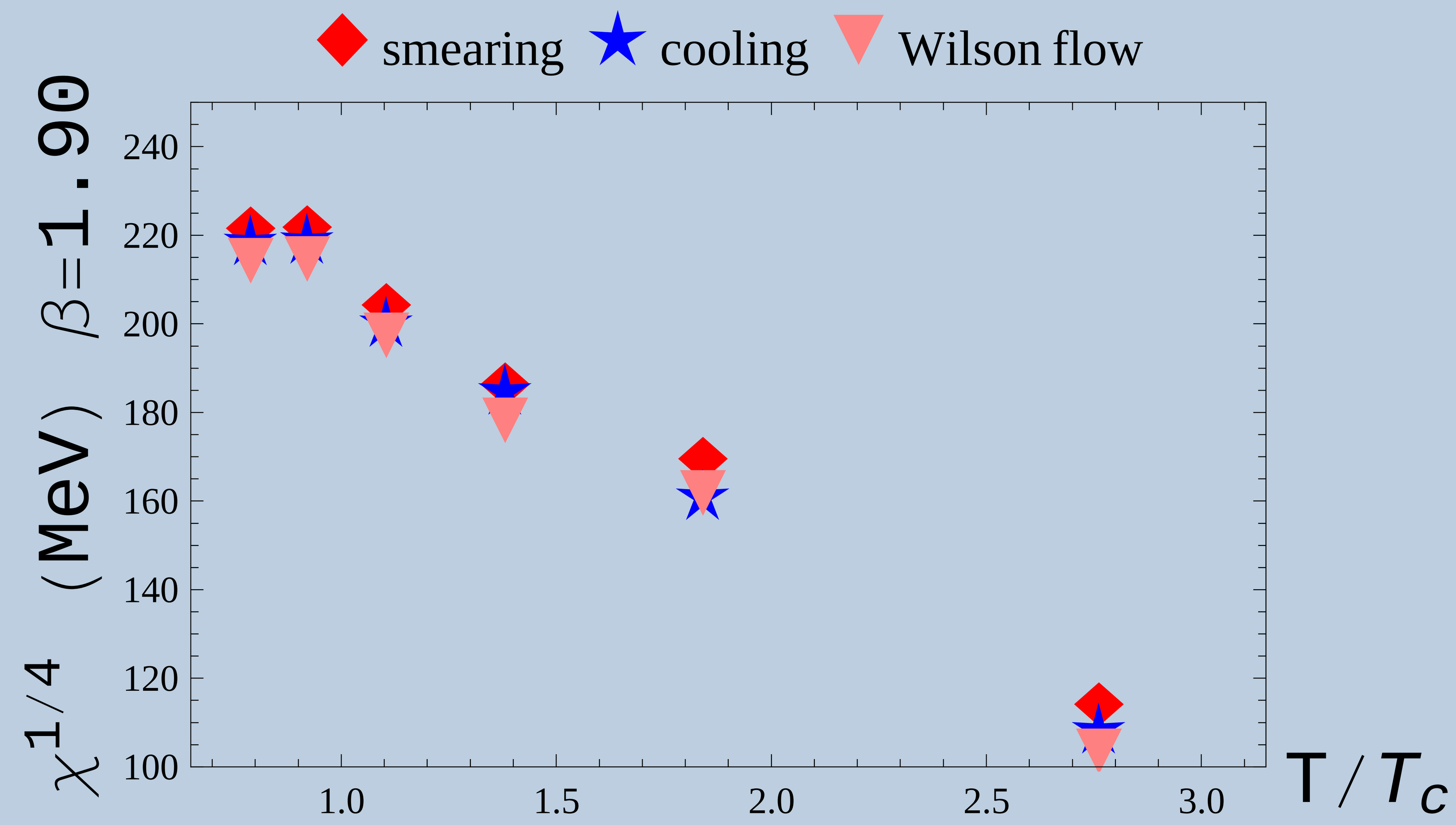
$$N_{cool} \simeq 4\,\tau \qquad N_{stout\ link} \simeq 11\,\tau$$

Topological susceptibility

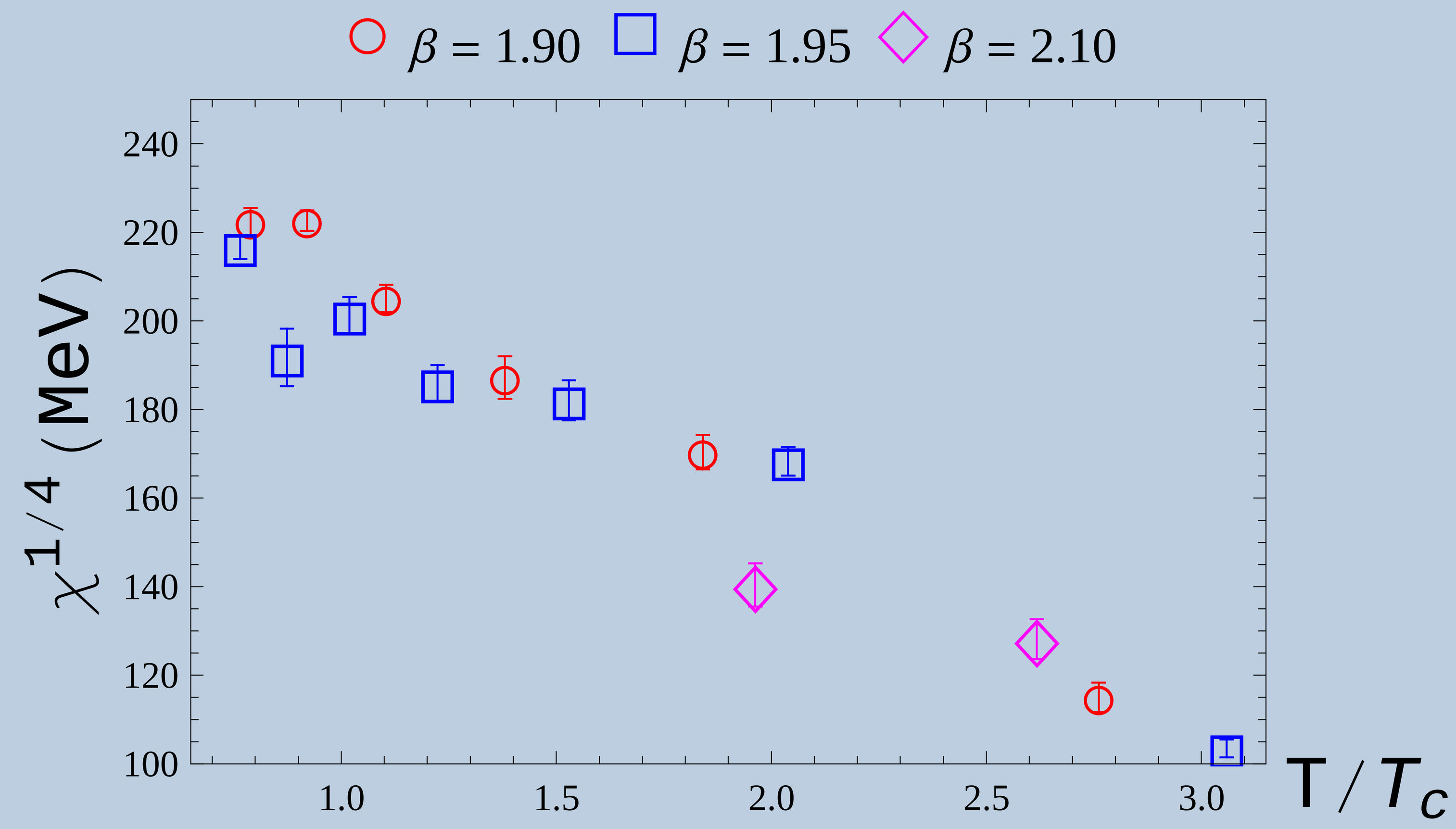
$q(x) = -\dfrac{1}{32\pi^2}\mathrm{Tr}\{F_{\mu\nu}\tilde F_{\mu\nu}\}$ — topological density; $\quad Q = a^4 \sum_x q(x)$

$\chi = \dfrac{a^{-4}}{N_s^3 N_\tau}\langle Q^2\rangle = a^4 \sum_x \langle q(0) q(x)\rangle$ — susceptibility

Comparison of the methods at one lattice scale:



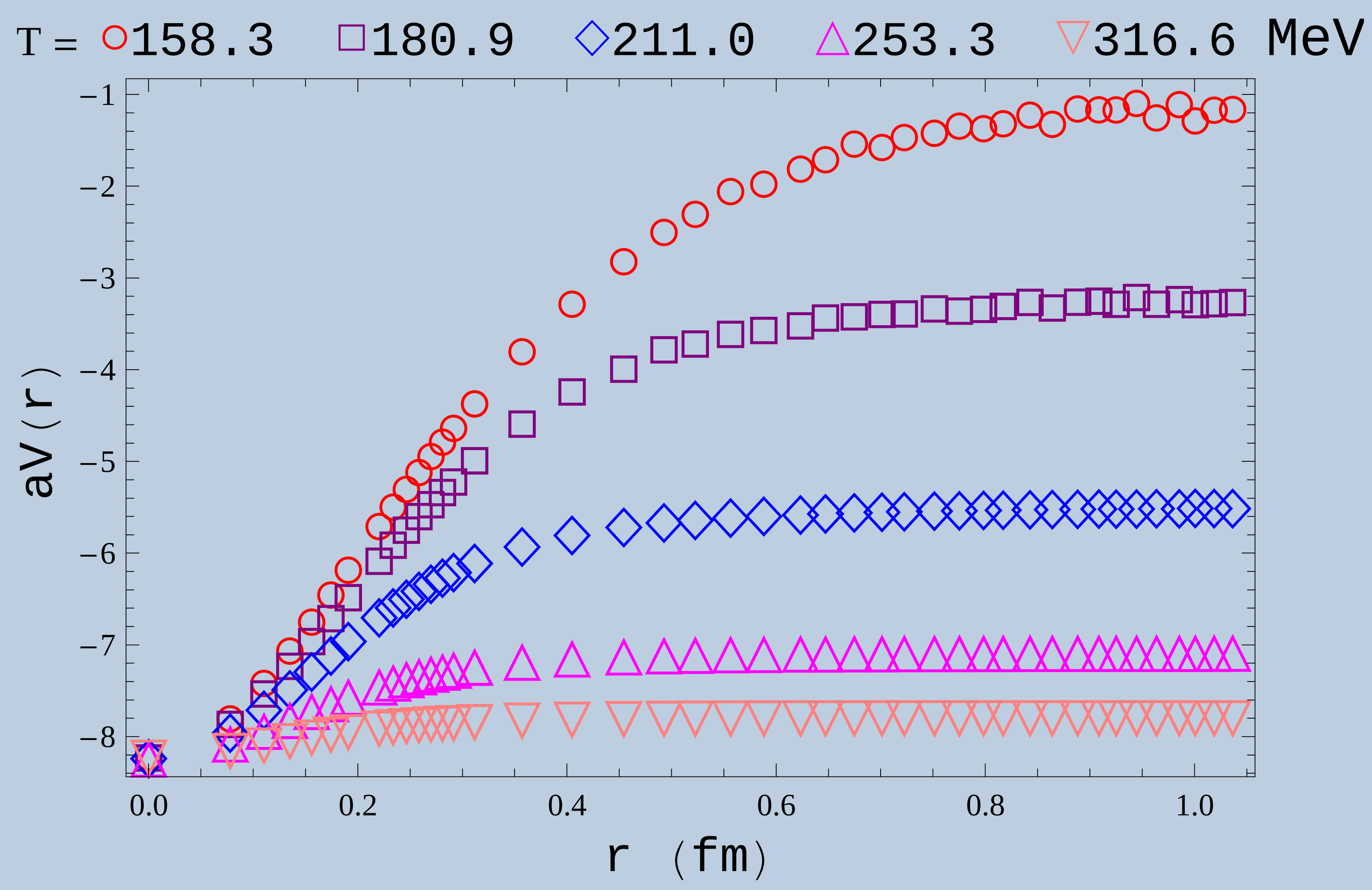
Continuum limit? A first scaling test:



Another application: Polyakov loop correlators

$$P_L(\mathbf{x}) = \mathrm{Tr}\prod_{t=0}^{N_\tau - 1} U_4(t,\mathbf{x}) \text{ — Polyakov loop}$$

$$aV(r) = -\frac{1}{N_\tau}\ln\langle P_L(\mathbf{x})P_L(\mathbf{y})\rangle \text{ — static quark potential}$$



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