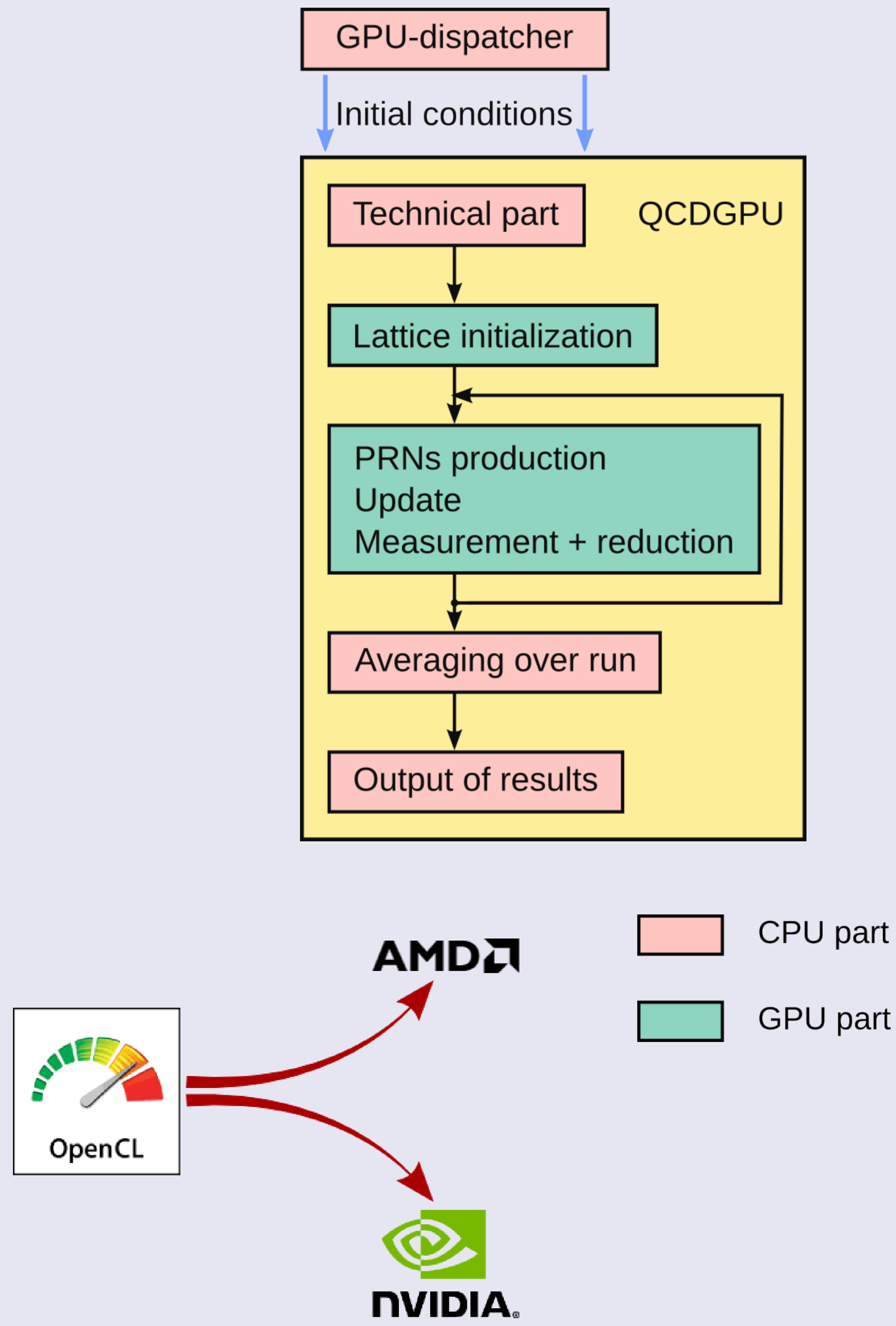
Spontaneous vacuum magnetization in pure SU(3) LGT

The spontaneous generation of chromomagnetic fields in 3+1 SU(3) gluodynamics at finite temperature is investigated on a lattice. The general method to investigate this phenomenon is based on the studying of the vacuum energy of the system dependence on the applied external chromomagnetic field. The external chromomagnetic fields corresponding to 3-rd and 8-th gauge group generators are introduced simultaneously on a lattice through "twisted" boundary conditions. It is obtained that at deconfinement phase the spontaneous vacuum magnetization takes place: the field corresponding third Gell-Mann matrix is generated within 2σ CL.



Introduction of external chromomagnetic field

External chromomagnetic field is supposed to be constant and directed along z axis, $\vec{H}^e = (0,0,H^e_z) = \overrightarrow{\mathrm{const}}$, $H^e = H^e_a T_a$, T_a are generators of the SU(N) group. It is introduced as additional flux through plaquettes.
It is known that

$$U_{\mu\nu}(n) = e^{ia^2 F_{\mu\nu}(n)}.$$

In case of external flux in z direction, supposing $A'_y = A_y + xH^e_z$, it transforms into

$$U'_{xy}(n) \approx \exp\left\{ia^2(\partial_x A'_y(n) - \partial_y A_x(n) + i[A_x(n), A'_y(n)]\right\} = \exp\left\{ia^2\left(\partial_x A_y - \partial_y A_x + i[A_x, A_y] + \partial_x A^e_y + i[A_x, A^e_y]\right)\right\}$$

Suppose the external field not strong, to neglect the commutators with the external field. So, last equation may be rewritten as follows

$$U'_{xy}(n) = U_{xy}(n) e^{ia^2 H^e_z}.$$

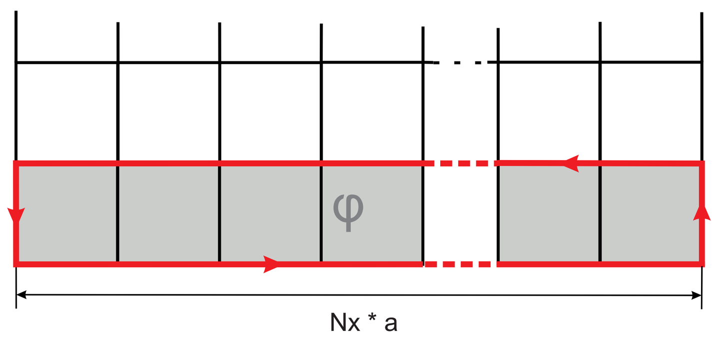
This is implemented through redefinition of the one of links forming plaquette:

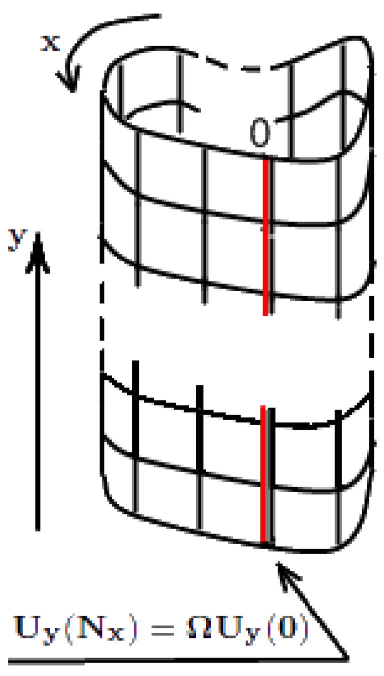
$$U'_y(n+\hat{x}) = U_y(n+\hat{x}) e^{ia^2 H^e_z(n)}.$$

It is possible to perform such "twist" only on the edge of the lattice:

$$U'_y(N_x, y, z, t) = U_y(0, y, z, t) e^{i\varphi}, \qquad \varphi = a^2 N_x H^e_z.$$

This corresponds to the flux through $1 \times N_x$ stripe of plaquettes in (xy) plane (see figure below).
This transformation is made for every y, so finally flux goes through the whole (xy) plane.

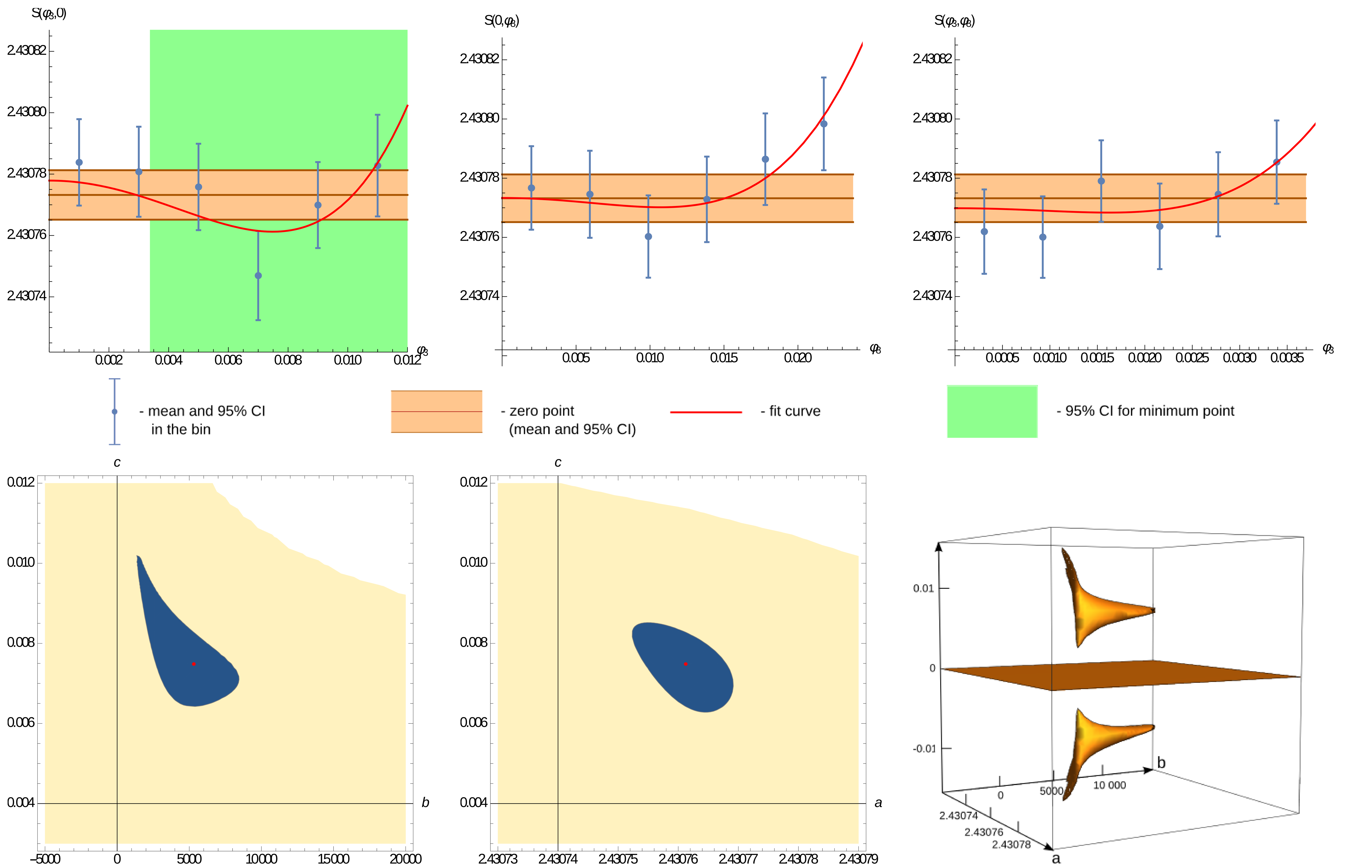




Spontaneous field generation

The spontaneous vacuum magnetization at high temperature is one of the posiible machanisms to produce magnetic fields in early Universe.
The magnetized vacuum means that for it the state with non-zero chromomagnetic field is energetically preferable.
The main idea of current approach is to find the global minimum of the action as function of two variables, φ_3 and φ_8.

Because of reproducing of the whole surface $S(\varphi_3, \varphi_8)$ is very time consuming, only some sections of this surface are investigated: $\varphi_8 = 0$, $\varphi_3 = 0$, and $\varphi_8 = 6.17\varphi_3$. Such choice of the directions in (φ_3, φ_8) space is connected with investigation in [V. V. Skalozub, A. V. Strelchenko, arXiv:hep-ph/0208071].
The presented results are obtained on 4×16^3 lattice at $\beta = 6$. This corresponds to temperature $T \sim 240$ MeV.



It is obtained within 2σ accuracy that the spontaneous chromomagnetic field generation takes place along direction $\varphi_8 = 0$ (the left top plot). The data are collected into bins and fitted by χ^2 method with 4-th power parabola $f(\varphi) = a + b(\varphi^2 - \varphi^2_{min})^2$. The estimate of field flux in the minimum: $\varphi_{3_{min}} = (7.484^{+11.27}_{-4.106}) \times 10^{-3}$. This corresponds to field strength $H_3 \approx 487.5$ MeV^2. In the bottom plots the surface obtained from confidence intervals for fitted parameters (the right plot) and its sections by planes $a = a_{min}$ and $b = b_{min}$ are shown. It can be seen that this surface doesn't cross the plane $\varphi_{min} = 0$, so the trivial minimum of the action is excluded at 95% CL.
In other investigated directions such minima are non-distinguishable.
So, there is the global minimum of the S surface in point with $H_3 \approx 487.5$ and $H_8 = 0$ and the vacuum of SU(3) gluodynamics can be spontaneously magnetized.

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