The deconfinement phase transition in SU(2) lattice gauge theory

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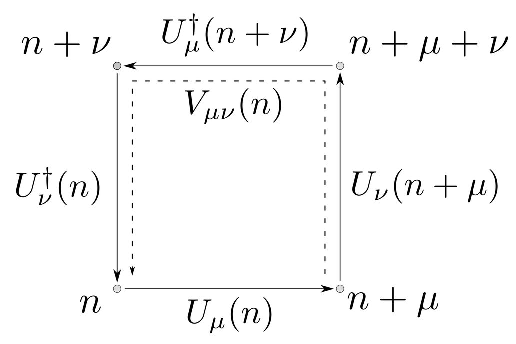
1 Lattice action for $SU(2)$ gauge theories

- Lattice gauge transporters: **Link variables** $U_\mu(n) = \exp\left[iaA_\mu(n)\right] \in SU(2)$
- **Wilson's plaquette action**

$$S_G[U] = \frac{\beta}{2}\sum_{n\in\Lambda}\sum_{\mu<\nu} \mathrm{tr}\left[1 - P_{\mu\nu}(n)\right]$$

for the plaquette $P_{\mu\nu} = U_\mu V_{\mu\nu}$ with the staple $V_{\mu\nu}$



- Wilson's ansatz provides correct continuum limit
- Note that this formulation is not unique and can be expanded to higher order in the lattice spacing such that the action takes into account more involved loops.

2 Update algorithms

- **Heatbath**:
  Sample update matrices $\chi = x_0\mathbb{1} + i\vec{x}\cdot\vec{\sigma} \in SU(2)$ along equilibrium distribution [1]

$$\int d\chi P[\chi] \propto \int_{-1}^{1} dx_0 \left(1 - x_0^2\right)^{1/2} e^{\beta k x_0} \int d^2\Omega_\chi$$

Update links with the sum of staples $V_\mu = \sum_{\nu>\mu} V_{\mu\nu}$

$$\boxed{U_\mu \mapsto U'_\mu = \frac{1}{k}\chi V^\dagger_\mu \quad \text{where} \quad k = \sqrt{\det V_\mu}}$$

- **Overrelaxation**:
  leaves the action invariant but speeds up the movement through configuration space

$$\boxed{U_\mu \mapsto U'_\mu = S^\dagger_\mu U^\dagger S^\dagger_\mu \quad \text{with} \quad S_\mu = \frac{1}{k} V_\mu}$$

3 Implementation in OpenCL

- Open **C**omputing **L**anguage:
  - based on C99, compiled at runtime
  - allows platform-independent computations → wide variety of devices accessible
- Currently implemented:
  - Several update algorithms: Metropolis, Heatbath, HMC, Overrelaxation
  - Preparations for dynamical fermions: Leapfrog integrator, Conjugate gradient inverter
- Planned features:
  - Include dynamical (staggered) fermions

4 The plaquette action

- Investigate the **average plaquette** as a measure for the plaquette action
- Observe increase around the deconfinement transition

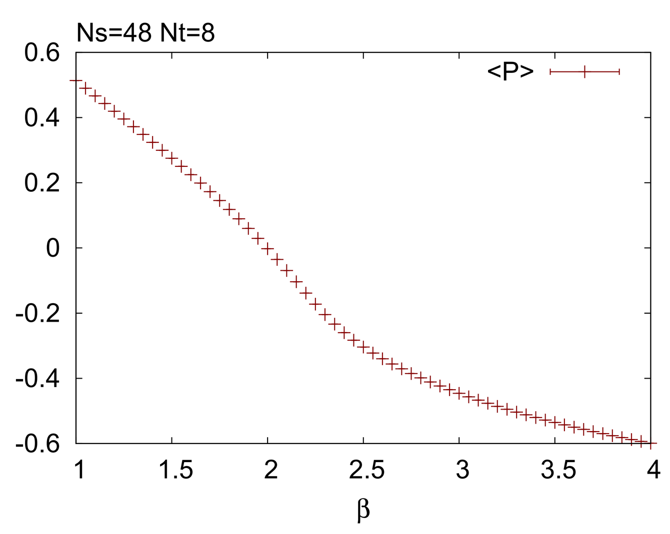


Figure 1: The average plaquette over β as a measure for the action.

5 $\mathbb{Z}(2)$ symmetry breaking: Deconfinement transition

With the present code it is possible to investigate the deconfinement transition in pure gauge theory utilizing Polyakov loops as true order parameter for centre symmetry breaking.

5.1 Polyakov loops on the lattice

- Polyakov loop = closed path of links around the torus in time direction [2]

$$L(n) = \prod_{n_\tau=0}^{N_\tau-1} U_4(\vec{n}, n_\tau) \equiv \text{static quark}$$

- Polyakov loop expectation value is a measure for the energy difference with and without static quark

$$\left\langle \left|\mathrm{tr}\, L\right| \right\rangle = \frac{1}{Z}\int \mathscr{D}U \frac{1}{V}\left|\sum_n \mathrm{Tr}\left[L(n)\right]\right| e^{-S_G[U]} = e^{-F_q/T}$$

$$\begin{aligned}\text{Confinement: } F_q = \infty &\Rightarrow \left\langle\left|\mathrm{tr}\, L\right|\right\rangle = 0\\ \text{Deconfinement: } F_q < \infty &\Rightarrow \left\langle\left|\mathrm{tr}\, L\right|\right\rangle \neq 0\end{aligned}$$

5.2 Centre symmetry breaking

- Topologically non-trivial gauge transformation:

$$\begin{aligned} g(\tau + N_\tau, x) = hg(\tau, x) \quad \text{with} \quad h \in \mathbb{Z}(2)\\ \Rightarrow U^g_\mu(\tau + N_\tau, x) = hU^g_\mu(\tau, x)h^\dagger\end{aligned}$$

- Polyakov loop transforms then like

$$\mathrm{tr}\, L^g = h\, \mathrm{tr}\, L$$

Spontaneous breaking of centre symmetry
≡
deconfinement phase transition
order parameter: $\left\langle\left|\mathrm{tr}\, L\right|\right\rangle$

5.3 Critical coupling

- Susceptibility

$$\chi_L \propto \left\langle\left|\mathrm{tr}\, L\right|^2\right\rangle - \left\langle\left|\mathrm{tr}\, L\right|\right\rangle^2$$

is maximal at the critical coupling β_c

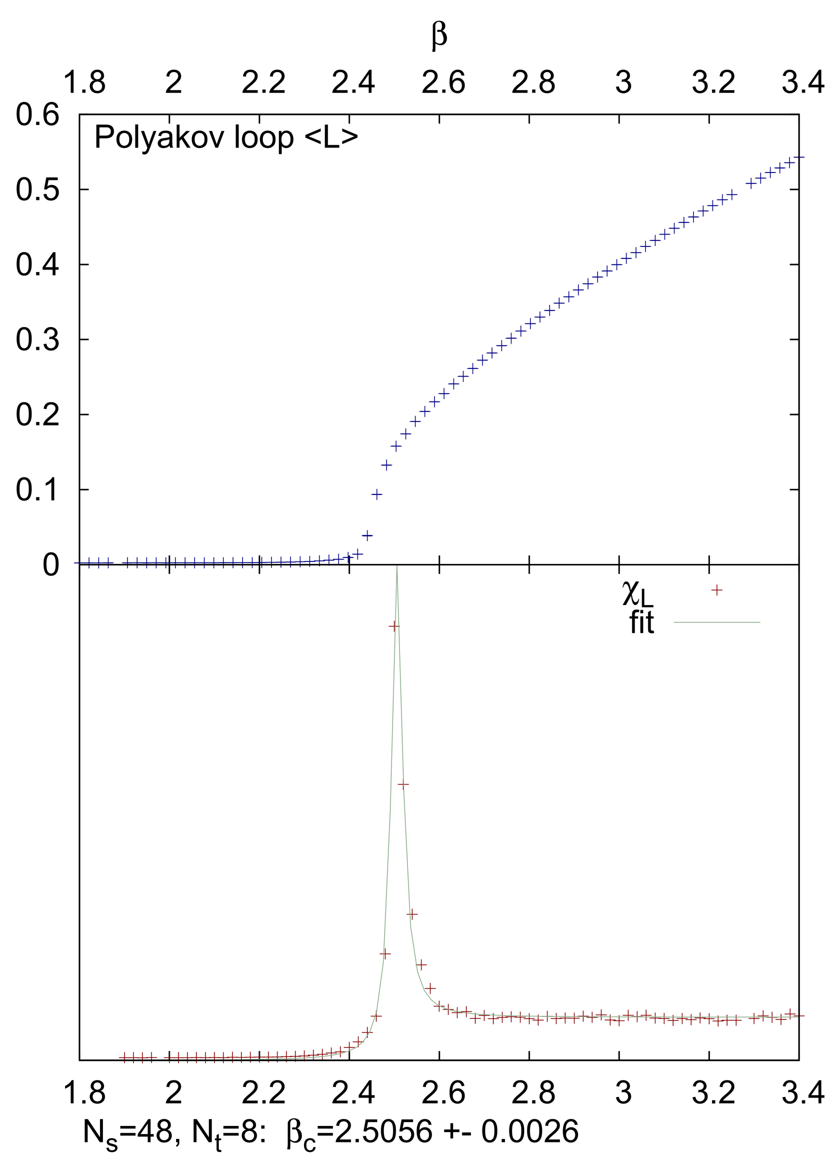


Figure 2: The Polyakov loop over β indicates the deconfinement phase transition for increasing temperature. Note the crossover transition, that is due to finite volume. The critical coupling was obtained from the maximum of a fit to the susceptibility χ_L

5.4 Polyakov loop distributions

- Local Polyakov loop distributions for increasing temperature (decreasing N_τ) [3]
- Deconfinement transition reflected by asymmetrical distribution
  ⇒ Average moves away from 0 (the confined phase) to the deconfined phase, where in principle both directions are possible

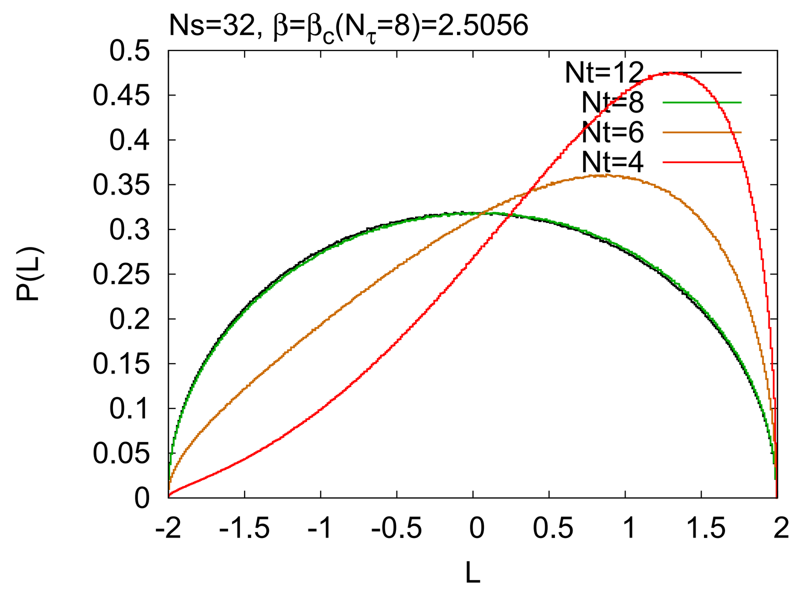


Figure 3: The local distribution of the Polyakov loop shows a clear asymmetry for increasing temperature.

6 Outlook: Including dynamical fermions

- Current works aim at a full **Hybrid Monte Carlo** simulation
  - Staggered Dirac operator:

$$D_{n,m} = \sum_\mu \frac{\eta_\mu(n)}{2a}\left(U_\mu(n)\delta_{m,n+\mu} - U_{-\mu}(n)\delta_{m,n-\nu}\right) + m\delta_{n,m}$$

  - Evolve the whole lattice configuration along a **fictitious time dimension** → **Global** Monte Carlo update
  - Integrate the **equations of motion**:

$$\frac{\partial}{\partial\tau}U_\mu(n) = i\pi_\mu(n)U_\mu(n) \quad \text{and} \quad \frac{\partial}{\partial\tau}\pi_\mu(n) = F_\mu(n)\Big|_{TA}$$

  - **Molecular Dynamics force term** for staggered fermions:

$$\begin{aligned}F_\mu(n) = &-\frac{\beta}{N_c}U_\mu(n)\sum_{\nu>\mu}V_{\mu\nu}(n)\\ &-\frac{\eta_\mu(n)}{a}U_\mu(n)\left[\rho(D\rho)^\dagger - (D\rho)\rho^\dagger\right]_{n+\mu}\end{aligned}$$

with $\rho = \left(D^\dagger D\right)^{-1}\phi$

- **Centre symmetry**:
  Fermi statistics ⇒ **Anti-periodic boundary conditions** for fermions, the winding gauge transformation then reads

$$\boxed{\begin{aligned}\psi^g(\tau + N_\tau, x) = -h\psi(\tau, x) \Rightarrow h = 1\\ \Rightarrow \text{Centre symmetry expl. } \textbf{broken}\end{aligned}}$$

- Pair production & string breaking are screening confinement
  ⇒ F_q is **finite**
  ⇒ Polyakov loop is not a true order parameter anymore
  ⇒ smooth **crossover**

References

[1] Michael Creutz. Monte Carlo study of quantized SU(2) gauge theory. *Physical Review D*, 21(8), April 1980.

[2] Owe Philipsen. Lattice QCD at non-zero temperature and baryon density. pages 273–330, 2010.

[3] Dominik Smith, Adrian Dumitru, Robert Pisarski, and Lorenz von Smekal. Effective potential for SU(2) Polyakov loops and Wilson loop eigenvalues. *Phys.Rev.*, D88(5):054020, 2013.