Electroweak probes in the presence of resonances on the lattice

Andria Agadjanov[1], V. Bernard[2], Ulf-G. Meißner[1], A. Rusetsky[1]

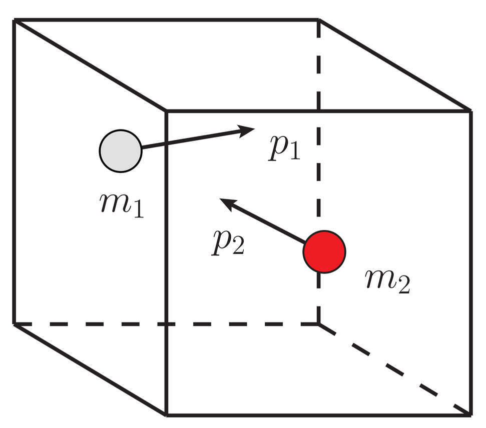
[1]Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), [2]IPN Orsay

1. Motivation

- The pion photoproduction $\gamma^* N \to \pi N$ near the $\Delta(1232)$ resonance region. The $\Delta N \gamma^*$ transition is experimentally most accessible one to reveal a possible hadron deformation.
- The rare B-meson decays $B \to K^* \gamma^*$ and $B \to K^* l^+ l^-$ with $K^*(892) \to K\pi$. These processes are forbidden at tree level and thus sensitive to physics beyond the Standard Model.
- The resonances are treated as stable particles in current lattice QCD simulations.
- A proper theoretical framework for the lattice extraction of the corresponding amplitudes and form factors at lower pion masses is needed.

2. Theoretical background

- Lattice QCD provides an *ab initio* method to find the properties of resonances.
- Lattice simulations are done in a finite space $\Rightarrow$ impossible to prepare asymptotic states $\Rightarrow$ no resonances on the lattice.
- Lüscher approach: resonance parameters (its mass and width) are found by determination of the scattering phase shift from the finite volume two particle energy spectrum.
- Example: a scalar resonance.



$\cot\delta_0(s) = w_{00}(\eta)$ (Lüscher equation),

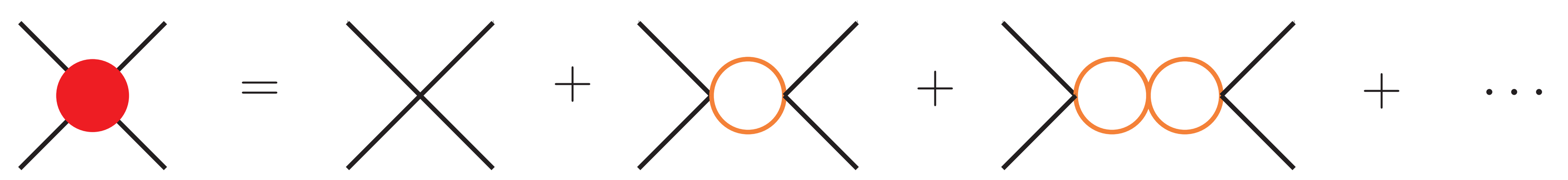
$\eta = \frac{pL}{2\pi}, \quad p^2 = \lambda(s, m_1^2, m_2^2)/4s,$

$p_R \cot\delta_0(s_R) = -\frac{1}{a_0} + \frac{1}{2} r_0 p_R^2 + \cdots = -ip_R$

▷ Lattice data $\Rightarrow$ Lüscher equation $\Rightarrow a_0, r_0 \Rightarrow$ resonance pole p_R

3. The framework: non-relativistic EFT

- Very useful theoretical tool for generalization of the Lüscher approach to the case of 3-point functions, involving resonances.
  - ▷ The total number of heavy particles is conserved.
  - ▷ The theory can be formulated in a manifestly Lorentz-invariant way.
  - ▷ The theory is matched to the full QFT (e.g., ChPT).
- The pole structure of the T-matrix emerges as a sum of bubble-chain diagrams:

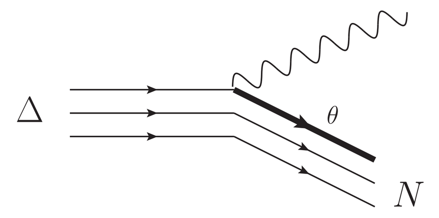


$$T \propto 1 + cJ + cJ^2 + \cdots = \frac{1}{1 - cJ}$$

- Previous work [1]: the case of the *scalar* resonance form factor (analog: $\Delta\Delta\gamma^*$) in the external *scalar* field.
- We modify it in two aspects:
  - ▷ inclusion of *spin*;
  - ▷ generalization to the *transition* form factors.

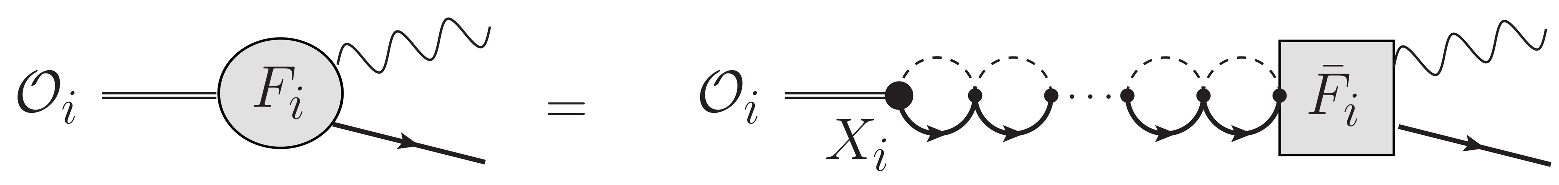
4. Results for the $\Delta N \gamma^*$ transition

- Kinematics: the Δ is at rest $\mathbf{P} = 0$, the nucleon 3-momentum $\mathbf{Q}$ along 3-axis. To perform the fit (*see below*): vary p, while $|\mathbf{Q}|$ fixed.
  - ▷ twisted boundary conditions;



$\hookrightarrow$ already applied for the nucleon form factor

  - ▷ asymmetric boxes $L \times L \times L'$.



  - ▷ The current matrix elements in a finite volume $F_i = F_i(p_n, |\mathbf{Q}|)$, i=1, 2, 3 $\to G_M, G_E, G_C$ form factors, are measured on the lattice.
  - ▷ The $\bar{F}_i(p_n, |\mathbf{Q}|)$ are volume-independent irreducible amplitudes.
- Real energy axis:

$$\mathcal{A}_i(p_n, |\mathbf{Q}|) = e^{i\delta(p_n)} V^{1/2} \left(\frac{1}{|\delta'(p_n) + \phi'(\eta_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2} |F_i(p_n, |\mathbf{Q}|)|,$$

where $\mathcal{A}_i = \mathcal{A}_i(p_n, |\mathbf{Q}|)$ are $\gamma^* N \to \pi N$ multipole amplitudes.

▷ *The narrow width approximation*:

$$|\mathrm{Im}\, \mathcal{A}_i(p_A, |\mathbf{Q}|)| = \sqrt{\frac{8\pi}{p_A \Gamma}} |F_i^A(p_A, |\mathbf{Q}|)|,$$

where $F_i^A(p_A, |\mathbf{Q}|)$ are the $\Delta N \gamma^*$ matrix elements.

- Complex energy plane:

$$F_i^R(p_R, |\mathbf{Q}|) = Z_R^{1/2} \bar{F}_i(p_R, |\mathbf{Q}|), \quad Z_R = \left(\frac{p_R}{8\pi E_R} \right)^2 \left(\frac{16\pi p_R^3 E_R^3}{w_{1R} w_{2R} (2p_R h'(p_R^2) + 3ip_R^2)} \right).$$

▷ *The narrow width approximation*:

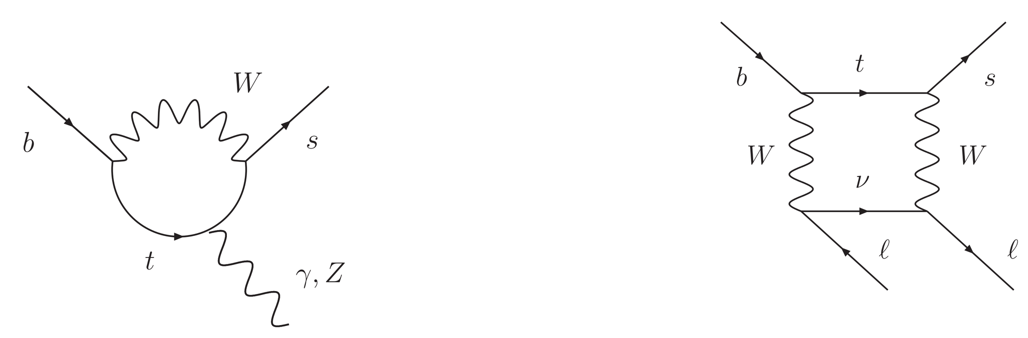
$$F_i^R(p_R, |\mathbf{Q}|) \to F_i^A(p_A, |\mathbf{Q}|) \quad \text{as} \quad p_R \to p_A\,!$$

▷ Prescription on the lattice:

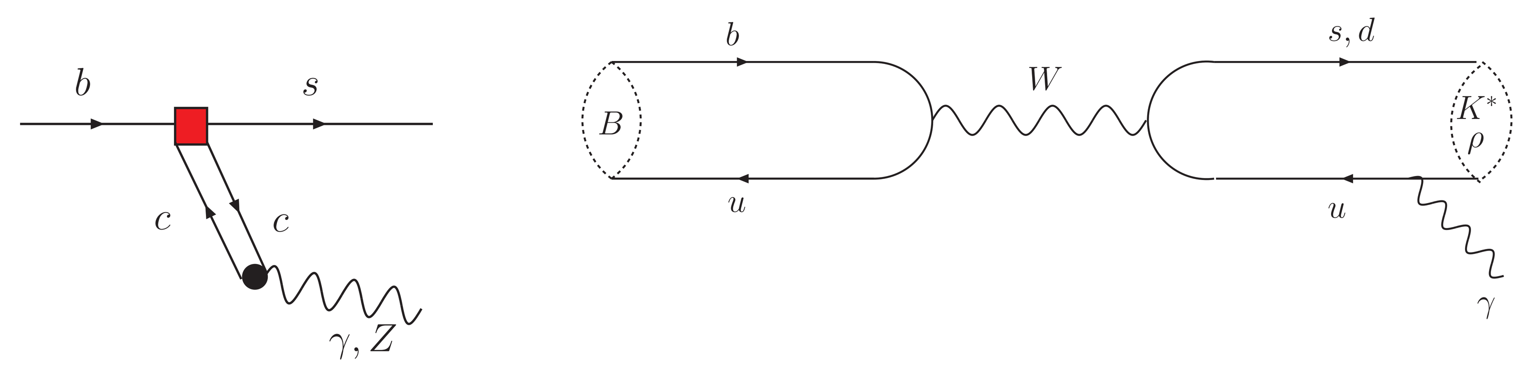
$$F_i^R(p_R, |\mathbf{Q}|) = i\, p_R^{-3} Z_R^{1/2} (A_i(|\mathbf{Q}|) + p_R^2 B_i(|\mathbf{Q}|) + \cdots),$$

where $A_i(|\mathbf{Q}|), \ldots$ are fitted to latice data.

5. Preliminary study of $B \to K^*$ transitions



- The $b \to s$ transitions proceed through the loops in SM.



Long distance contributions

- If $q^2 > m_{c\bar{c}}^2$, then since $V_{ub} V_{us}^* << V_{tb} V_{ts}^* \Rightarrow$ we can apply our framework!
- Outlook: complete the work on $B \to K^*$.

References

[1] V. Bernard et al., JHEP **1209** (2012) 023.
[2] A. Agadjanov et al., arXiv:1405.3476 [hep-lat].
[3] C. Alexandrou et al., Phys. Rev. D **77** (2008) 085012.
[4] R. Horgan et al., Phys. Rev. D **89** (2014) 094501.