ROTATING QCD MATTER

Dubna International Advanced School of Theoretical Physics

Helmholtz International Summer School

Lattice QCD, Hadron Structure and Hadronic Matter

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Lecture 2

- Rotation and gravity
- Gravity and matrix elements
- Equivalence Principle and Spin
- Extension of Equivalence Principle (validity separately for quarks and gluons)
- EP for Spin-1 hadrons
- Rotation and lattice

Rotation and gravity

- Rotation – non-inertial frame
- May be described by deviation of metric from Minkowski
- Non-diagonal component of metric relevant $$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega \\ 0 & 1 & 0 & -x\Omega \\ 0 & 0 & 1 & 0 \\ y\Omega & -x\Omega & 0 & 1 + r^2\Omega^2 \end{pmatrix}$$
- Used to derive lattice action in rotating frame (Yamamotom Hirono)

PRL **111,** 081601 (2013)

Interactions with gravity (inertia) and matrix elements

- Link between hadronic physics and gravity – matrix element of quark/gluon energy momentum tensors
- Smallness is only in the size of Newton constant – matrix element is not suppressed
- Low momentum transfer limit – complementary way to describe the interaction with external classical field

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2}\sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu \qquad \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M\phi(q)$$

- Mass as charge – equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’); rederived from conservarion laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- - not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)
- Spin – unique probe of generic non-Riemannian gravity (Puetzfeld, Obukhov’09) – violation of PNEP

Gravitational Formfactors (second moments of GPDs)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws (Ji SR) - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius ”attraction vs repulsion” – “Regge” t-dependence from small x

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\rm Gr}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

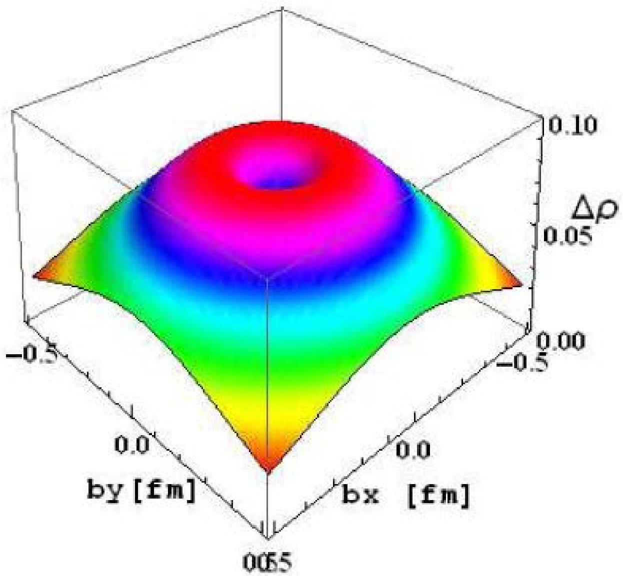

FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Gravitomagnetism (rotating frames and sources)

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

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Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

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(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Non-stationary (weak approximation to Kerr) – Obukhov, Silenko, OT '09,11,13

Gravity vs accelerated frame for spin and helicity

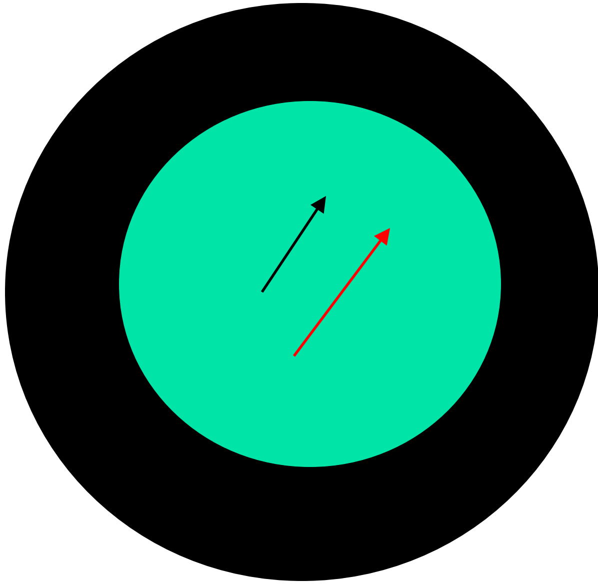
- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim \mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99)

Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $<P+q|T^{mn}|P-q> = P^{\{m}<P+q|J^{n\}}|P-q>/e$ up to the terms linear in q
- Special role of g=2 for any spin (asymptotic freedom for vector bosonsбcancellation of leading divergencies,…)
- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of “quantum” nature?!

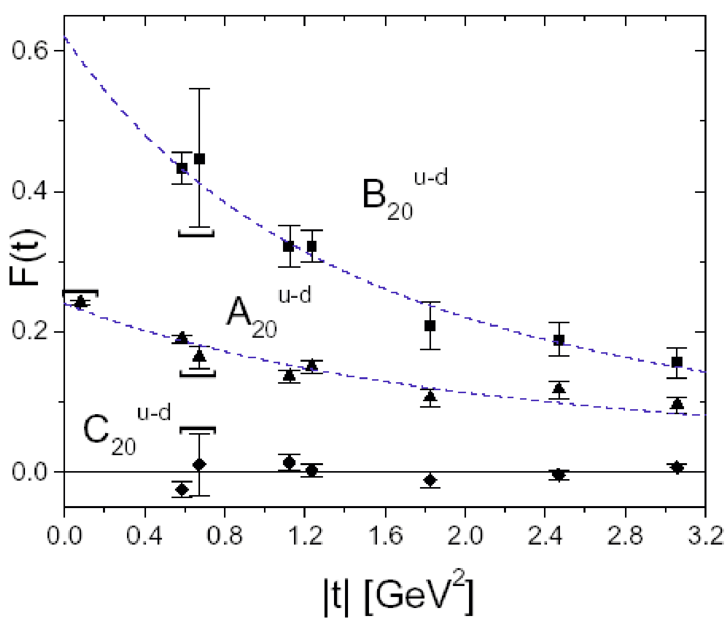
Cosmological implications of PNEP

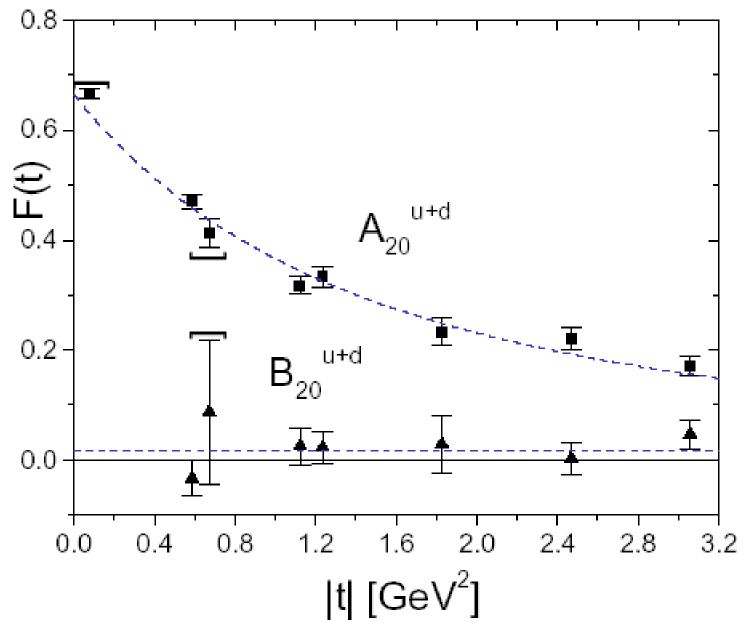
- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Generalization of Equivalence principle

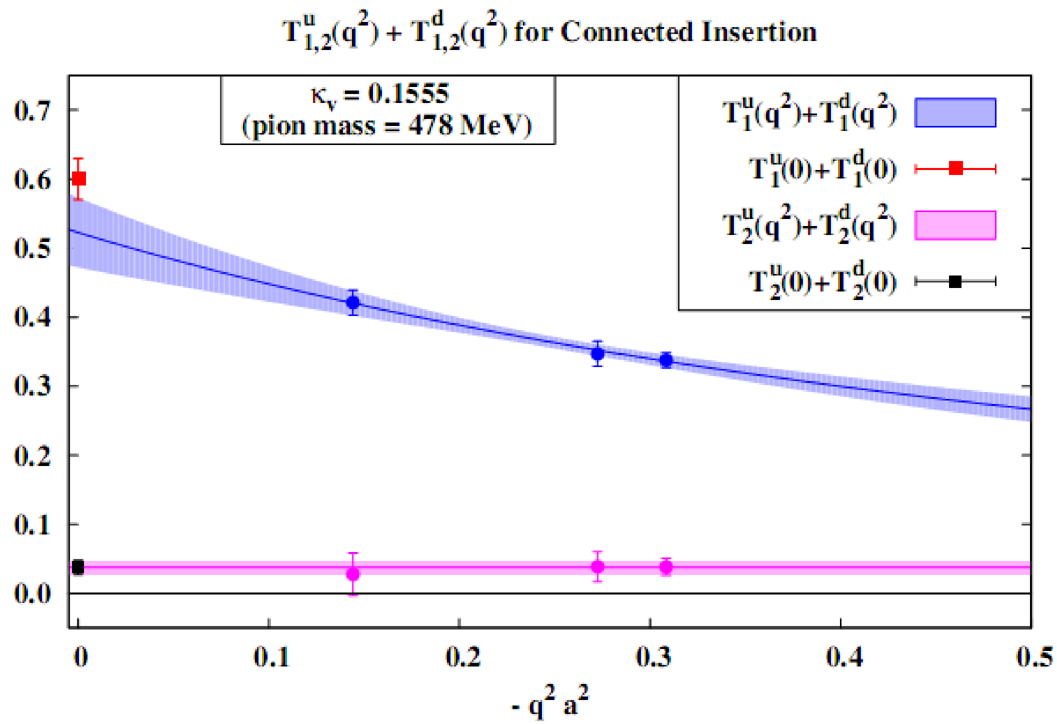
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. **arXiv:1312.4816**)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Supported by generic smallness of E (isoscalar AMM)

Vector mesons and ExEP

- J=1/2 -> J=1. QCD SR (Samsonov) calculation of Rho’s AMM gives g close to 2.

- Maybe because of similarity of moments
- $g-2=<E(x)>$; $B=<xE(x)>$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP: Gluons momentum fraction sizable

ExEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

Another (new!) manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2S^{\nu\nu_1}P^{\nu_2}...P\nu_n\int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2S^{\mu\nu}\delta_1(\mu^2)$$

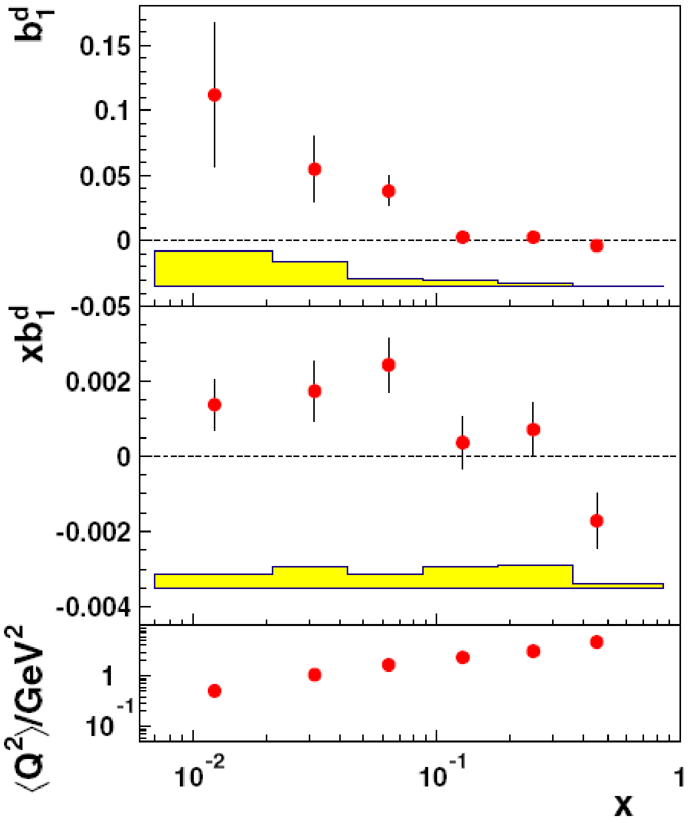
$$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2)$$ =0 for EEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence:

$$\int_0^1 C_i^T(x)dx = 0$$



EP and Sivers function

- Qualitatively similar to OAM and Anomalous Magnetic Moment (Brodsky,Burkardt,Schmidt)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $f_T^{(1)}(x) \sim xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle
- Broken by lensing function: imposing the relation -constraints for it?!

ExEP and Sivers function for deuteron

- ExEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (cf small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T_B^{\mu\rho} - x^\rho T_B^{\mu\nu}$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu})\partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha}\langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$

Relocalization and EEP (OT'98)

- Evolution of orbital and TOTAL angular momentum (at LO) – related due to non-trivial relations between DGLAP kernels

$$\frac{d}{dt}\begin{pmatrix} J_q \\ J_G \end{pmatrix} = \frac{\alpha(t)}{2\pi}\begin{pmatrix} \int_0^1 dx(x-1)P_{qq}(x) & 2n_f\int_0^1 dx x P_{qG}(x) \\ \int_0^1 dx x P_{Gq}(x) & -2n_f\int_0^1 dx x P_{qG}(x) \end{pmatrix}\begin{pmatrix} J_q \\ J_G \end{pmatrix}$$

- RI: $P_{ij}^{JJ(1)} = P_{ij}^{LL(1)}$ $\quad \int_0^1 dx x \Delta P_{Gq}(x) = \frac{1}{2}\int_0^1 dx x P_{Gq}(x)$
- EEP: $P^{LL(1)} = P^{TT(2)}$

CONCLUSIONS -II

- Rotation may be studies as a specific gravity field
- Gravity couplng to spin - manifested in hadron structure
- Some of evidences for validity of EP for quarks and gluons separately
- Lattice studies on the way