QCD Thermodynamics

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- Lecture I: QCD at finite temperature and density, continuum and lattice
- Lecture II: Applications of lattice thermodynamics
- Lecture III: Generalities on the phase diagram, the finite T transition, chemical potential
- Lecture IV: Towards the QCD phase diagram at finite density

Literature

- O.P., "Lattice QCD at non-zero temperature and density",
  Les Houches lecture notes 2009, arxiv:1009.4089

- O.P., "The QCD equation of state from the lattice"
  Prog. Part. Nucl. Phys. 70 (2013) 55, arxiv:1207.5999

  Proper references to covered material in those articles

Textbooks:

- Gale, Kapusta, "Finite temperature field theory: principles and applications"
- Montvay, Münster, "Quantum fields on a lattice"
- Gattringer, Lang, "Quantum chromodynamics on the lattice"

Units for these lectures

- Natural units: $\hbar = 1, c = 1$
- Einstein sum convention: $a_i b_i \equiv \sum_i a_i b_i$

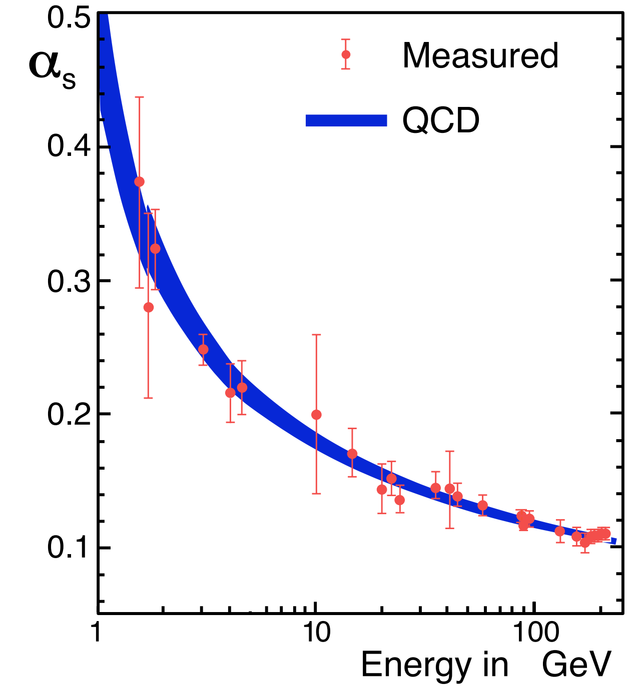
Hopefully only in a few instances (not intentional):

- Small circle approximation: $\pi = 1$
- Supra-natural approximations: $-1 = 1, i = 1$

Lecture I: QCD at finite temperature and density

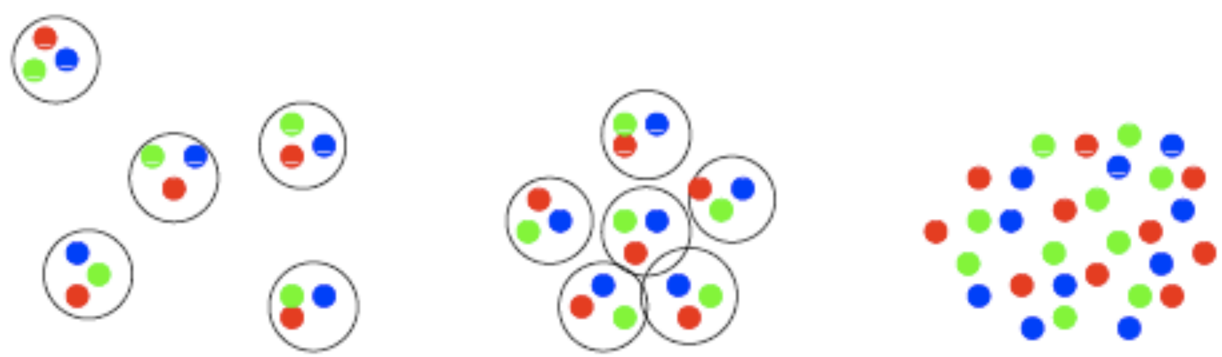
- Motivation: Why thermal QCD?
- The continuum formulation
- Differences and limitations of perturbation theory compared to T=0
- The lattice formulation

Why thermal QCD?



asymptotic freedom $\alpha_s(p \to \infty) \to 0$

$$T, \mu_B$$



Hadrongas

Quark-Gluon-Plasma

Chiral symmetry: broken (nearly) restored

Order parameters: $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle$

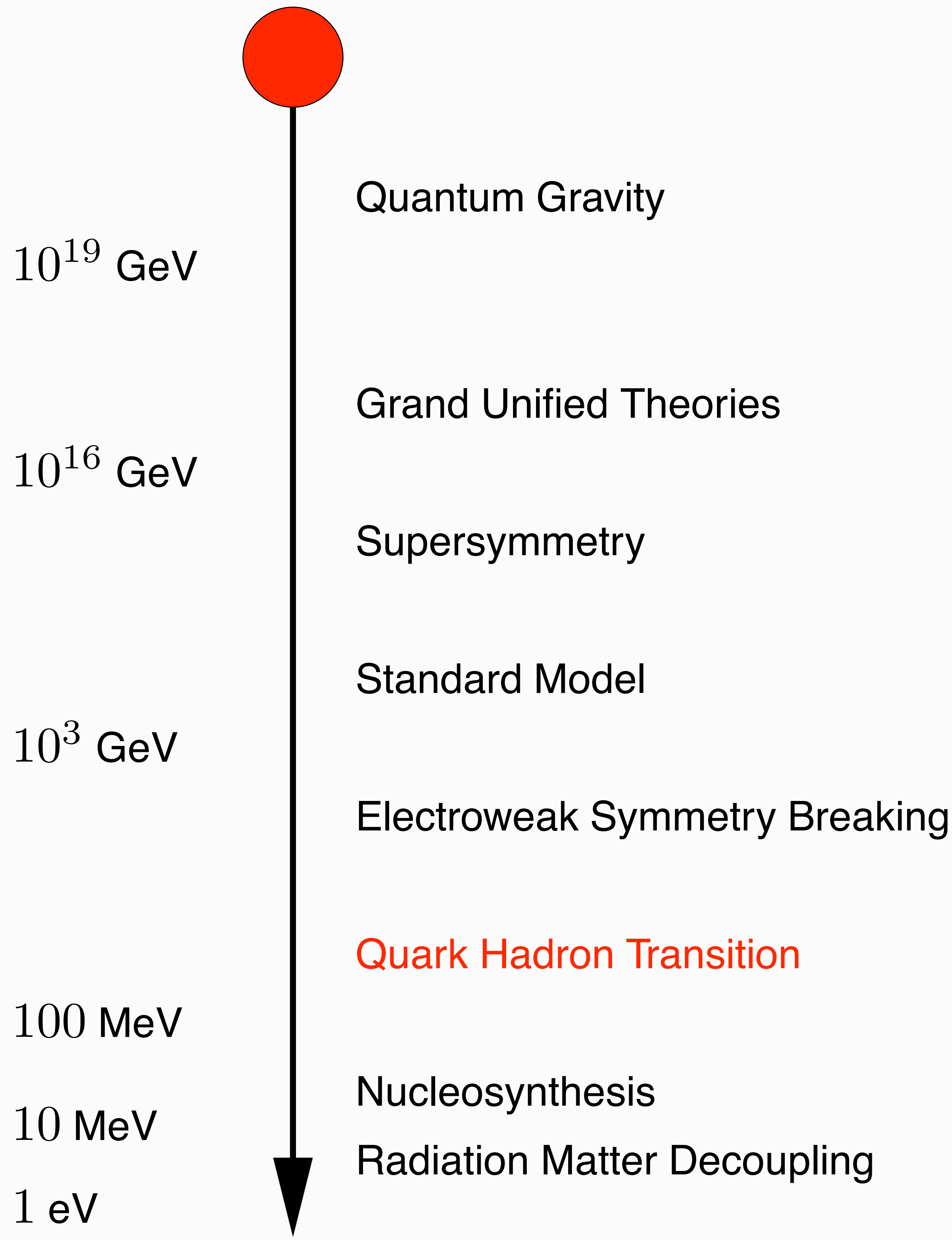
chiral condensate , Cooper pairs

Thermal QCD in nature

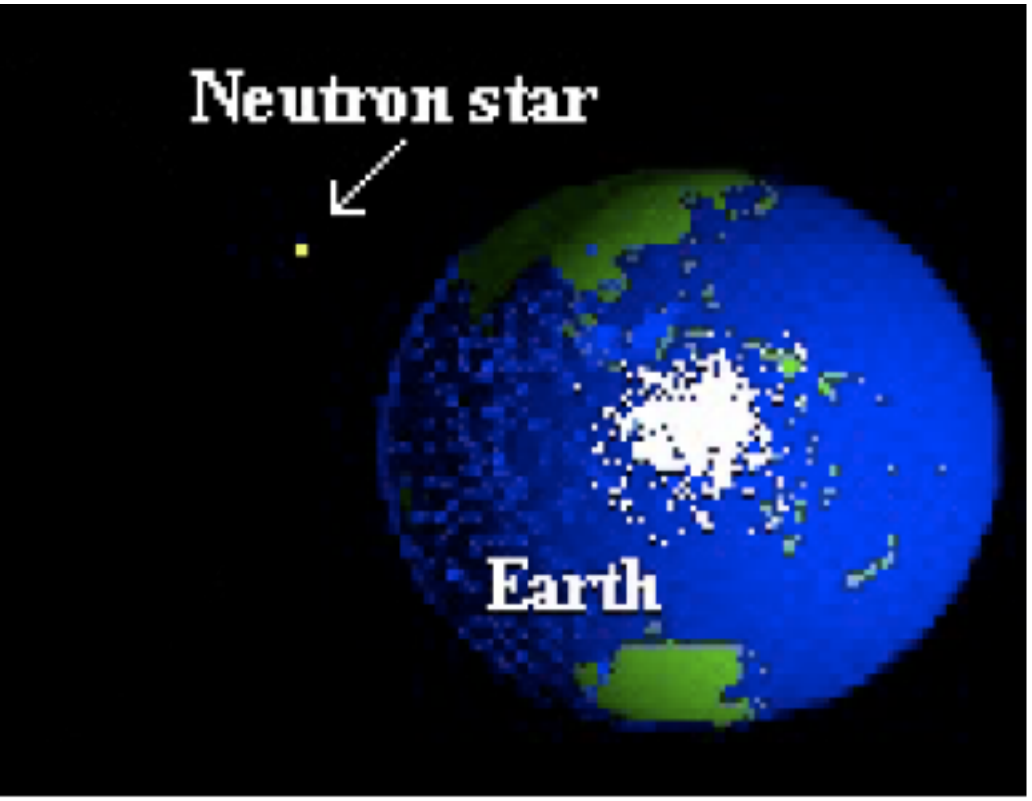
Physics of early universe:

non-abelian plasma physics
$(\mu_B \approx 0)$

➡ QCD is prototype

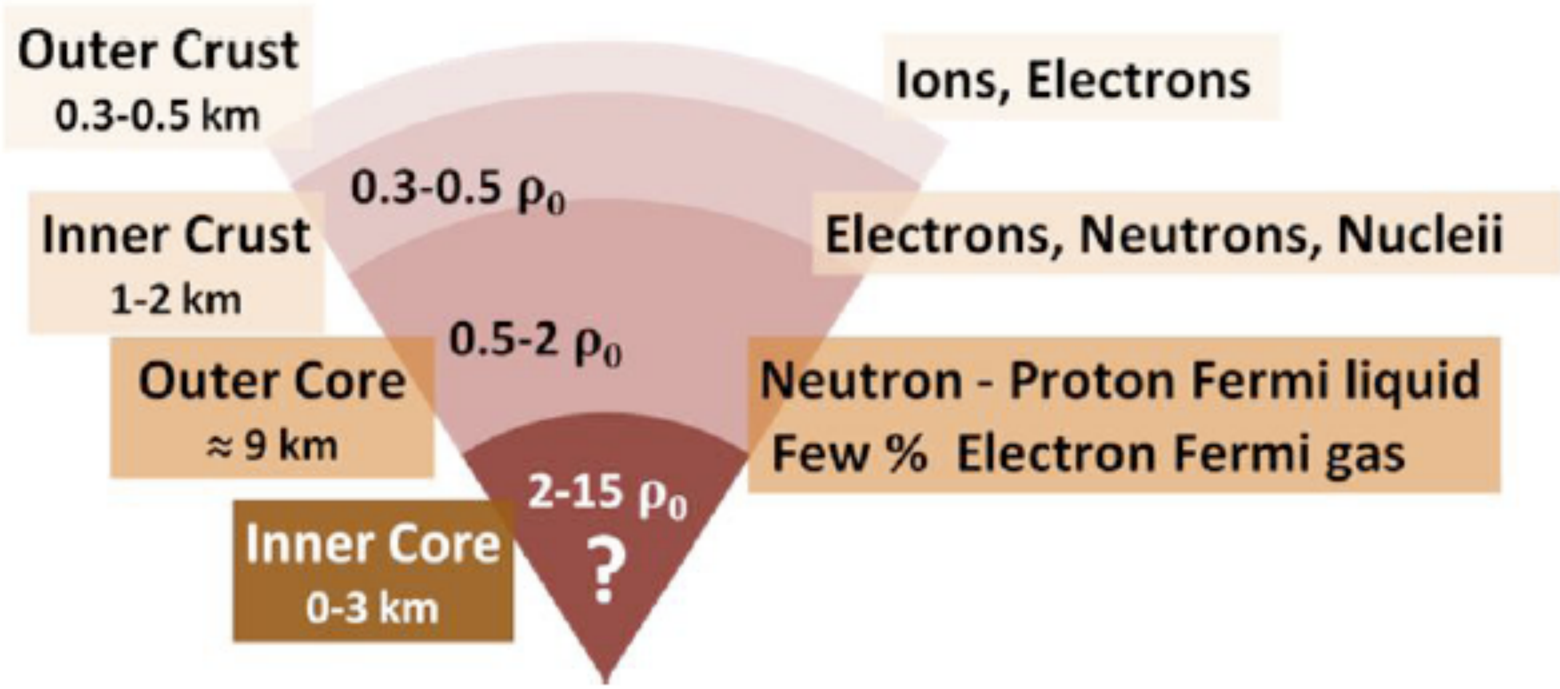


What are compact stars made of?

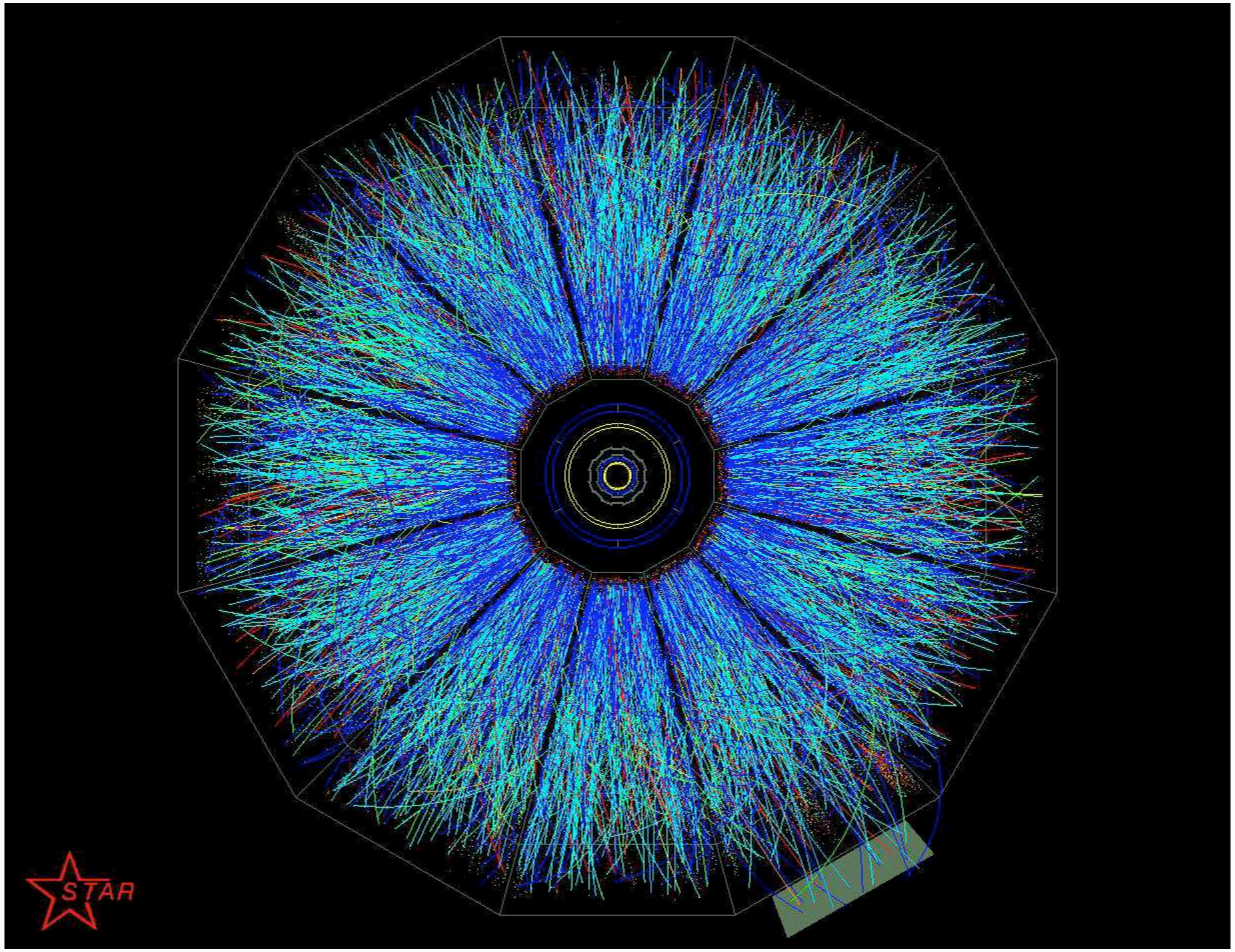


Radius ~ 10-12 km
Mass ~ 1.2-2.2 x Solar Mass

ρ_0 : nuclear density

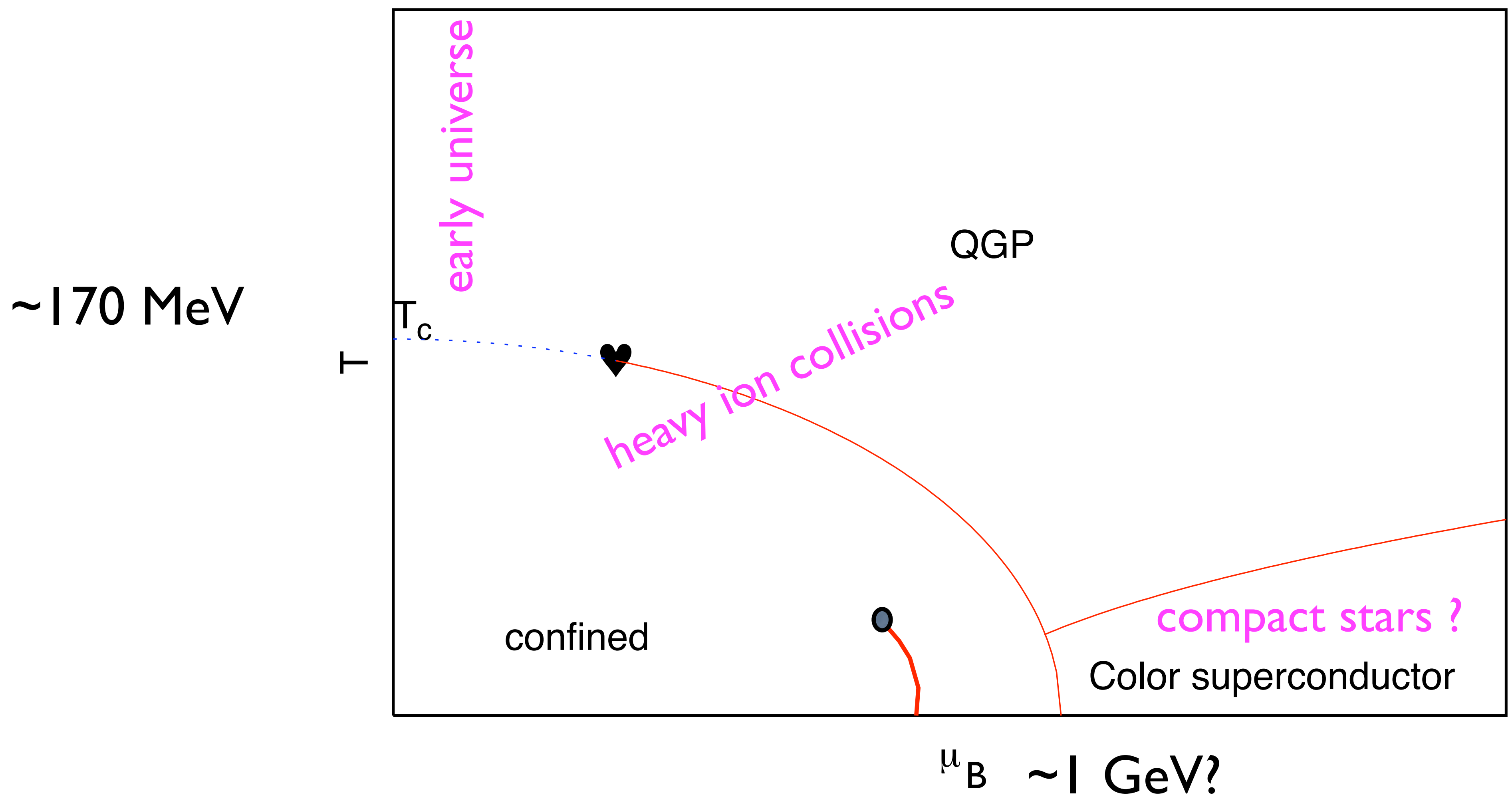


Thermal QCD in experiment



heavy ion collision experiments at RHIC, LHC, GSI....

QCD phase diagram: theorist's view (science fiction)

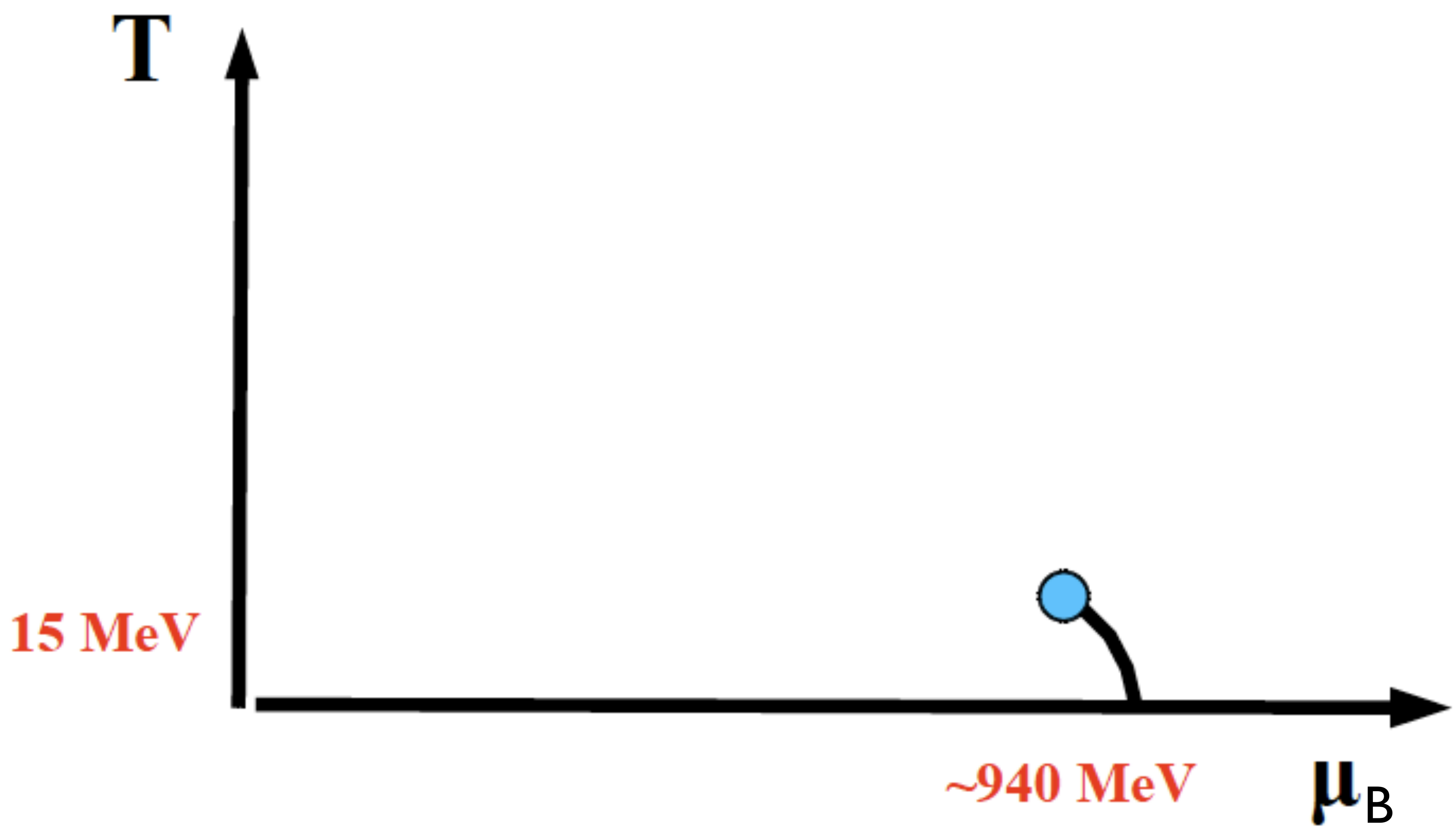


Until 2001: no finite density lattice calculations, sign problem!

Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...)

Check this from first principles QCD!

The QCD phase diagram established by experiment:



Nuclear liquid gas transition with critical end point

Statistical mechanics reminder

System of particles in volume V with conserved number operators, $N_i, i = 1, 2, \ldots$
in thermal contact with heatbath at temperature T

Canonical ensemble: exchange of energy with bath, particle number fixed

Grand canonical ensemble: exchange of energy and particles with the bath

Density matrix,
Partition function:

$$\rho = \mathrm{e}^{-\frac{1}{T}(H - \mu_i N_i)} , \qquad Z = \hat{\mathrm{Tr}}\rho , \quad \hat{\mathrm{Tr}}(\ldots) = \sum_n \langle n|(\ldots)|n\rangle$$

Thermodynamics:

$$F = -T \ln Z ,$$

$$p = \frac{\partial(T \ln Z)}{\partial V} ,$$

$$S = \frac{\partial(T \ln Z)}{\partial T} ,$$

$$\bar{N}_i = \frac{\partial(T \ln Z)}{\partial \mu_i} ,$$

$$E = -pV + TS + \mu_i \bar{N}_i$$

Densities:

$$f = \frac{F}{V}, \quad p = -f, \quad s = \frac{S}{V}, \quad n_i = \frac{\bar{N}_i}{V}, \quad \epsilon = \frac{E}{V}$$

QCD at finite temperature and density

Grand canonical partition function

$$Z(V,T,\mu;g,N_f,m_f) = \mathrm{Tr}(\mathrm{e}^{-(\mathrm{H}-\mu\mathrm{Q})/\mathrm{T}}) = \int \mathrm{DA}\,\mathrm{D}\bar{\psi}\,\mathrm{D}\psi\;\mathrm{e}^{-\mathrm{S_g}[\mathrm{A}_\mu]}\mathrm{e}^{-\mathrm{S_f}[\bar{\psi},\psi.\mathrm{A}_\mu]}$$

Action

$$S_g[A_\mu] = \int_0^{1/T} d\tau \int_V d^3x\, \frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}(x)F_{\mu\nu}(x),$$

$$S_f[\bar{\psi},\psi,A_\mu] = \int_0^{1/T} d\tau \int_V d^3x \sum_{f=1}^{N_f} \bar{\psi}_f(x)\left(\gamma_\mu D_\mu + m_f - \mu_f\gamma_0\right)\psi_f(x)$$

$$A_\mu(\tau,\mathbf{x}) = A_\mu(\tau+\frac{1}{T},\mathbf{x}), \qquad \psi_f(\tau,\mathbf{x}) = -\psi_f(\tau+\frac{1}{T},\mathbf{x})$$

quark number $N_q^f = \bar{\psi}_f\gamma_0\psi_f$

Parameters

$$g^2, m_u \sim 3\mathrm{MeV}, m_d \sim 6\mathrm{MeV}, m_s \sim 120\mathrm{MeV}, V, T, \mu = \mu_B/3$$

$N_f = 2+1$ sufficient up to T~300-400 MeV

Difference to T=0: compact, periodic time direction!

Fourier expansion of the fields: discrete Matsubara frequencies

$$A_\mu(\tau,\mathbf{x}) = \frac{1}{\sqrt{VT}} \sum_{n=-\infty}^{\infty} \sum_{\mathbf{p}} \mathrm{e}^{i(\omega_n\tau + \mathbf{p}\cdot\mathbf{x})} A_{\mu,n}(p)\ , \quad \omega_n = 2n\pi T\ , \qquad p_i = (2\pi n_i)/L$$

$$\psi(\tau,\mathbf{x}) = \frac{1}{\sqrt{V}} \sum_{n=-\infty}^{\infty} \sum_{\mathbf{p}} \mathrm{e}^{i(\omega_n\tau + \mathbf{p}\cdot\mathbf{x})} \psi_n(p)\ , \quad \omega_n = (2n+1)\pi T$$

Thermodynamic limit:

$$\frac{1}{V} \sum_{n_1,n_2,n_3} \stackrel{V\to\infty}{\longrightarrow} \int \frac{d^3p}{(2\pi)^3}$$

Modified Feynman rules:

Inverse (bosonic) free propagator:

$$\Delta^{-1} = p^2 + m^2 = \omega_n^2 + \mathbf{p}^2 + m^2 = (2n\pi T)^2 + \mathbf{p}^2 + m^2$$

Loop integration:

$$\sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3}$$

Perturbation theory at finite T

Split action into free (Gaussian) and interacting part, expand in interactions

$$Z = N \int D\phi \, \mathrm{e}^{-(S_0+S_i)} = N \int D\phi \, \mathrm{e}^{-S_0} \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} S_i^l$$

$$\ln Z = \ln Z_0 + \ln Z_i = \ln \left(N \int D\phi \, \mathrm{e}^{-S_0} \right) + \ln \left(1 + \sum_{l=1}^{\infty} \frac{(-1)^l}{l!} \frac{\int D\phi \, \mathrm{e}^{-S_0} S_i^l}{\int D\phi \, \mathrm{e}^{-S_0}} \right)$$

Renormalisation: Whatever renormalisation is necessary and sufficient at T=0 is also necessary and sufficient at finite temperature and density

UV behaviour: microscopic physics, depends on details of interactions

T, μ : macroscopic parameters, affect IR behaviour of the theory

Ideal gases from the Gaussian path integral

Important (sometimes unrealistic) model systems to (mis-)guide intuition

Real scalar field:

$$S_0 = \int_0^{\frac{1}{T}} d\tau \int d^3x \, \frac{1}{2}\phi(x)(-\partial_\mu\partial_\mu + m^2)\phi(x)$$

Fourier space:

$$S_0 = \frac{1}{2T^2}\sum_{n=-\infty}^{\infty}\sum_{\mathbf{p}}(\omega_n^2+\omega^2)\phi_n(p)\phi_n^*(p)$$

$$\omega = \sqrt{\mathbf{p}^2+m^2} \qquad \phi_n^*(p) = \phi_{-n}(-p)$$

$$Z_0 = N\prod_{\mathbf{p}}\int d\phi_0 \exp\left[-\frac{1}{2T^2}(\omega_0^2+\omega^2)\phi_0^2(p)\right]$$

$$\times \prod_{n>0}\int d\phi_n \, d\phi_n^* \; \exp\left[-\frac{1}{2T^2}(\omega_n^2+\omega^2)\phi_n(p)\phi_n^*(p)\right]$$

$$= N\prod_{\mathbf{p}}(2\pi)^{1/2}\left(\frac{\omega_0^2+\omega^2}{T^2}\right)^{-\frac{1}{2}}\prod_{n>0}\int d|\phi_n|\,|\phi_n|\exp\left[-\frac{1}{2T^2}(\omega_n^2+\omega^2)|\phi_n|^2\right]$$

$$= N\prod_{\mathbf{p}}(2\pi)^{1/2}\left(\frac{\omega_0^2+\omega^2}{T^2}\right)^{-\frac{1}{2}}\prod_{n>0}\left(\frac{\omega_n^2+\omega^2}{T^2}\right)^{-1} = N'\prod_{n=-\infty}^{\infty}\prod_{\mathbf{p}}\left(\frac{\omega_n^2+\omega^2}{T^2}\right)^{-\frac{1}{2}} = N'(\det\Delta^{-1})^{-1/2}$$

Note: T-independent constants may be dropped (no contribution to thermodynamics)

$$\ln Z_0 = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \sum_{\mathbf{p}} \ln \frac{\omega_n^2 + \omega^2}{T^2}$$

For Matsubara sum:

$$\ln \left[(2\pi n)^2 + \frac{\omega^2}{T^2} \right] = \int_1^{\omega^2/T^2} \frac{d\theta^2}{\theta^2 + (2\pi n)^2} + \ln(1 + (2\pi n)^2)$$

$$\sum_{n=-\infty}^{\infty} \frac{1}{n^2 + (\frac{\theta}{2\pi})^2} = \frac{2\pi^2}{\theta} \left(1 + \frac{2}{e^\theta - 1} \right)$$

$$\begin{aligned} \ln Z_0 &= -\sum_{\mathbf{p}} \int_1^{\omega/T} d\theta \left(\frac{1}{2} + \frac{1}{e^\theta - 1} \right) + \text{T-indep.} \\ &\stackrel{V\to\infty}{\longrightarrow} V \int \frac{d^3p}{(2\pi)^3} \left[\frac{-\omega}{2T} - \ln\left(1 - e^{-\frac{\omega}{T}}\right) \right] . \end{aligned}$$

Vacuum energy, pressure:

$$E_0 = -\partial_{\frac{1}{T}} \ln Z_0 = \frac{V}{2} \int \frac{d^3p}{(2\pi)^3} \omega \qquad p_0 = T\partial_V \ln Z_0 = -\frac{E_0}{V}$$

divergent, zero point energy!

Renormalisation: $$p_{\text{phys}}(T) = p(T) - p(T=0)$$

Final result: $$\ln Z_0 = -V \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - \mathrm{e}^{-\frac{\omega}{T}}\right)$$

m=0: $$p = \frac{\pi^2}{90} T^4$$

Fermion fields (Grassmann!): $$\ln Z_0 = 2V \int \frac{d^3p}{(2\pi)^3} \left[\ln\left(1 + \mathrm{e}^{-\frac{\omega-\mu}{T}}\right) + \ln\left(1 + \mathrm{e}^{-\frac{\omega+\mu}{T}}\right)\right]$$

two spin components

quarks and anti-quarks

m=0: $$p = \frac{7}{8}\frac{\pi^2}{90} T^4$$

General one-particle (field) expression: $$\ln Z_i^1(V,T) = \eta V \nu_i \int \frac{d^3p}{(2\pi)^3} \ln(1 + \eta\, \mathrm{e}^{-(\omega_i - \mu_i)/T})$$

$\eta = -1$ for bosons $\quad \nu_i$: spin and internal d.o.f

$\eta = 1$ for fermions

Ideal gases in QCD

Free gas of quarks and gluons: valid at infinite temperature, weak coupling limit

$$\frac{p}{T^4} = \left(2(N^2-1) + 4NN_f\frac{7}{8}\right)\frac{\pi^2}{90}$$

quark colours

2 gluon spin states   gluon colours   quark, anti-quark, each two spin states

Hadron resonance gas: at this point a model; later: strong coupling limit of full QCD

Quark-interactions "hidden" in hadrons; hadrons interact weakly

$$\ln Z(V,T) \approx \sum_i \ln Z_i^1(V,T) \qquad i = \pi, \rho, K, p, n, \ldots$$

IR-structure: divergences and mass scales

Inverse (bosonic) free propagator:

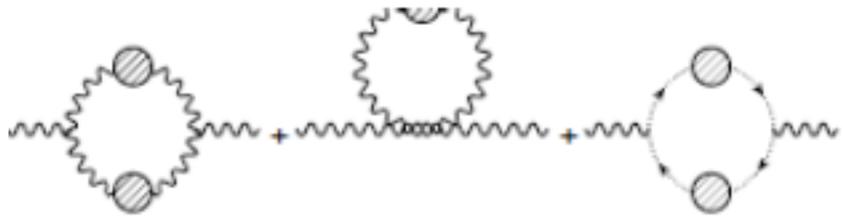
$$p^2 + m^2 = \omega_n^2 + \mathbf{p}^2 + m^2 = (2n\pi T)^2 + \mathbf{p}^2 + m^2$$

↑

effective thermal mass ~T

n=0 mode: propagator of a 3d theory, divergent for m=0!

Corrections:



$$m_E^{LO} = \left(\frac{N}{3} + \frac{N_f}{6}\right)^{1/2} gT$$

electric or Debye screening mass $\langle A_0(\mathbf{x}) A_0(\mathbf{y}) \rangle$

$$m_M^{LO} = 0, m_M \sim g^2 T \quad \text{from 2-loop}$$

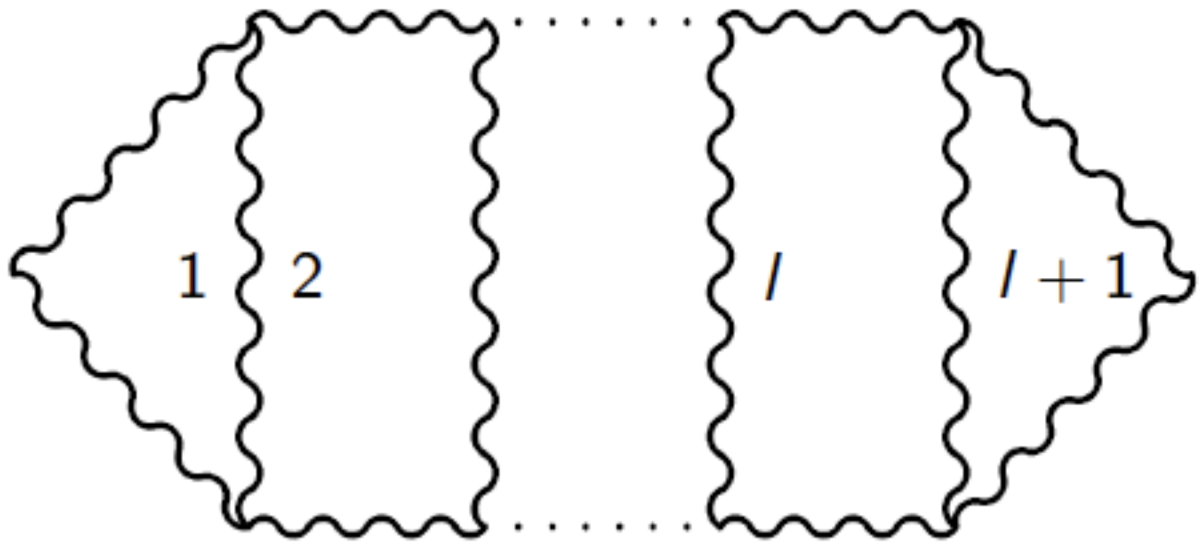
magnetic screening mass $\langle A_i(\mathbf{x}) A_i(\mathbf{y}) \rangle$

0-mode sector of 4d QCD at finite T contains 3d Yang-Mills theory with $g_3^2 \sim g^2 T$

Confining! Doom for perturbation theory....

The Linde problem of finite T QCD / 3d YM

- $(l+1)$-loop diagram:contribution to pressure



contribution from
Matsubara 0-mode:

$$\boxed{P \sim g^{2l} \left(T \int d^3p\right)^{l+1} p^{2l}(p^2+m^2)^{-3l}}$$

$$\begin{array}{rll} g^{2l} & \textit{for} & l=1,2 \\ g^6 T^4 \ln(T/m) & \textit{for} & l=3 \\ g^6 T^4 (g^2 T/m)^{l-3} & \textit{for} & l>3 \end{array}$$

magnetic mass $m_{mag} \sim g^2 T \Rightarrow$ **all loops** $(l>3)$ contribute to g^6

even for weak coupling!

Same problem for all observables!
Only the order to which it occurrs is different:

E.g. for magnetic mass already at leading order (2-loop)

Perturbation theory at finite temperature works only
up to a finite, observable-dependent order,
no matter how weak the coupling!

Salvation comes as a lattice...

The lattice formulation at zero density

Hypercubic lattice: $N_s^3 \times N_\tau$, Lattice spacing a, Wilson's YM action:

$$S_g[U] = \sum_x \sum_{1 \leq \mu < \nu \leq 4} \beta \left(1 - \frac{1}{N} \mathrm{ReTr} U_p\right)$$

Plaquette: $U_p = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^\dagger(x + a\hat{\nu}) U_\nu^\dagger(x)$ Lattice gauge coupling: $\beta = \dfrac{2N}{g^2}$

Periodic boundary conditions: $U_\mu(\tau, \mathrm{x}) = U_\mu(\tau + N_\tau, \mathrm{x}), U_\mu(\tau, \mathrm{x}) = U_\mu(\tau, \mathrm{x} + N_s)$

Transfer matrix formalism

Provides connection between path integral and Hamiltonian formalism

Rewrite action as sum over time slices:

$$S_g = \sum_\tau L[U_i(\tau+1), U_0(\tau), U_i(\tau)],$$

$$L[U_i(\tau+1), U_0(\tau), U_i(\tau)] = \frac{1}{2}L_1[U_i(\tau+1)] + \frac{1}{2}L_1[U_i(\tau)] + L_2[U_i(\tau+1), U_0(\tau), U_i(\tau)]$$

$$L_1[U_i(\tau)] = -\frac{\beta}{N}\sum_{p(\tau)} \mathrm{ReTr}U_p,$$ spatial plaquettes within one time slice

$$L_2[U_i(\tau+1), U_0(\tau), U_i(\tau)] = -\frac{\beta}{N}\sum_{p(\tau,\tau+1)} \mathrm{ReTr}U_p,$$ temporal plaquettes connecting slices

Transfer matrix: operator acting on square-integrable functions $\psi[U]$

Matrix elements: $T[U_i(\tau+1), U_i(\tau)] \; = \int DU_0(\tau)\; \exp - L[U_i(\tau+1), U_0(\tau), U_i(\tau)]$

Translation of states by one time-slice: $|\psi[U_i(\tau+1,\mathbf{x})]\rangle = T\,|\psi[U_i(\tau,\mathbf{x})]\rangle$

Identify: $T = e^{-aH}$

Rewrite partition function exactly:

$$Z = \int \prod_\tau \left(DU_i(\tau,\mathrm{x})\, T[U_i(\tau+1),U_i(\tau)]\right) = \hat{\mathrm{T}}\mathrm{r}(T^{N_\tau}) = \hat{\mathrm{T}}\mathrm{r}(\mathrm{e}^{-N_\tau aH})$$

Identify: $\frac{1}{T} \equiv aN_\tau$ $\qquad H|n\rangle = E_n|n\rangle$

complete set of energy eigenstates

Thermal expectation value: $\langle O\rangle = Z^{-1}\hat{\mathrm{T}}\mathrm{r}(\mathrm{e}^{-\frac{H}{T}}O) = Z^{-1}\sum_n \langle n|T^{N_\tau}O|n\rangle = \frac{\sum_n \langle n|O|n\rangle\,\mathrm{e}^{-aN_\tau E_n}}{\sum_n \mathrm{e}^{-aN_\tau E_n}}$

Thermodynamic limit: $N_s \to \infty$ but keep T finite

Vacuum expectation value: $\langle 0|O|0\rangle = \lim\limits_{N_\tau\to\infty} \frac{\sum_n \langle n|O|n\rangle\,\mathrm{e}^{-aN_\tau(E_n-E_0)}}{\sum_n \mathrm{e}^{-aN_\tau(E_n-E_0)}}$

The space-wise transfer matrix

- Hamiltonian translates in time;
  Spectrum: particle masses, from exp. decay of correlators in time

- May also define a Hamiltonian translating the system in space;
  Spectrum: screening masses, from exp. decay of correlators in space

$$T[U(z+1),U(z)] \equiv \mathrm{e}^{-aH_z}, \quad Z = \mathrm{Tr}(\mathrm{e}^{-aN_z H_z})$$ $$U(z)\ :\ \{U_\mu(z)|\mu \neq 3\}$$

Vacuum physics: $N_{x,y,z,\tau} \to \infty$ H, H_z spectra identical

Thermal physics: $N_{x,y,z} \to \infty$ and keep N_τ

H_z acts on states defined on $N_{x,y,\tau}$ lattice;

spectrum of theory on torus with one side squeezed

Finite T physics = finite size effect of the shortened time direction!

Adding fermions

Pick a suitable fermion action:

$$S_f = \sum_{x,y} \bar{\psi}(x) M_{xy}(m_f)\, \psi(y)$$

Full QCD partition function:

$$Z(N_s, N_\tau; \beta, m_f) = \int DU \prod_f \det M(m_f)\, \mathrm{e}^{-S_g[U]} ,$$

$$U_\mu(\tau, \mathbf{x}) = U_\mu(\tau + N_\tau, \mathbf{x}),$$

$$\psi(\tau, \mathbf{x}) = -\psi(\tau + N_\tau, \mathbf{x}) .$$

Wilson fermions:

$$S_f^W = \frac{1}{2a} \sum_{x,\mu,f} a^4\, \bar{\psi}_f(x)[(\gamma_\mu - r)U_\mu(x)\psi_f(x+\hat{\mu}) - (\gamma_\mu + r)U_\mu^\dagger(x-\hat{\mu})\psi_f(x-\hat{\mu})]$$

$$+ (m + 4\frac{r}{a}) \sum_{x,f} a^4\, \bar{\psi}_f(x)\psi_f(x)$$

pick your poison

- Wilson fermions
  add irrelevant ops. (going away in CL) to make doublers very massive
  breaks chiral symmetry for non-zero a

- staggered (Kogut-Susskind) fermions
  distribute spinor components on different sites, reduces to 4 flavours
  take 4th root of determinant to get to one flavour, keeps reduced chiral symm.
  non-local operation, have to take CL before chiral limit, mixing of spin, flavour

- domain wall fermions
  introduce 5th dimension, fermions massive in that dim. and chiral in the other
  expensive

- overlap fermions
  non-local formulation with modified chiral symmetry even for finite a
  orders of magnitude more expensive than Wilson

Continuum limit

$$\frac{1}{T} \equiv aN_\tau$$

Fixed scale approach:

- For a given lattice spacing, N_τ controls temperature;
- Allows only discrete temperatures, too large for many applications;
- Continuum limit requires series of lattice spacings

Fixed N_τ approach:

- For a given N_τ, vary the lattice spacing via $\beta(a)$;
- Allows continuous temperatures, but each T value has different cut-off!
- Continuum limit requires series of N_τ

Lines of constant physics and setting the scale

- Compute observable for series of a, N_τ , $\langle O\rangle(\beta, m_f)$
- Tune bare parameters such that for each lattice spacing renormalised parameters are constant $m_f^R(am_{u,d}(\beta), am_s(\beta), \beta) = \text{const}$.
- More practical: keep physical quantities constant $O_i^{\text{phys}}(am_{u,d}(\beta), am_s(\beta), \beta) \;=\; \text{const}$
- Non-trivial because of cut-off effects: Different for different quantities and actions $O^{\text{phys}}(a) = O^{\text{phys}} + c_1 a + c_2 a^2 + \ldots$

Perturbative relation for $\beta(a)$: only good very close to continuum limit

Λ_{QCD} on lattice:

$$a\Lambda_L = \left(\frac{6b_0}{\beta}\right)^{-b_1/2b_0^2} \mathrm{e}^{-\frac{\beta}{12b_0}},$$

$$b_0 = \frac{1}{16\pi^2}\left(11 - \frac{2}{3}N_f\right), \quad b_1 = \left(\frac{1}{16\pi^2}\right)^2\left[102 - \left(10 + \frac{2}{3}\right)N_f\right]$$

Non-perturbatively: Express computed quantity in units of another known quantity

E.g. for the critical temperature of a phase transition:

$$\frac{T_c}{m_H} = \frac{1}{a_c m_H N_\tau} = \frac{1}{a(\beta_c) m_H N_\tau}$$

Compute hadron mass at the critical lattice spacing: $a^{-1} = \frac{m_H[\mathrm{MeV}]}{(am_H)(\beta_c)}$

N.B.: Only possible when operating at physical quark masses!

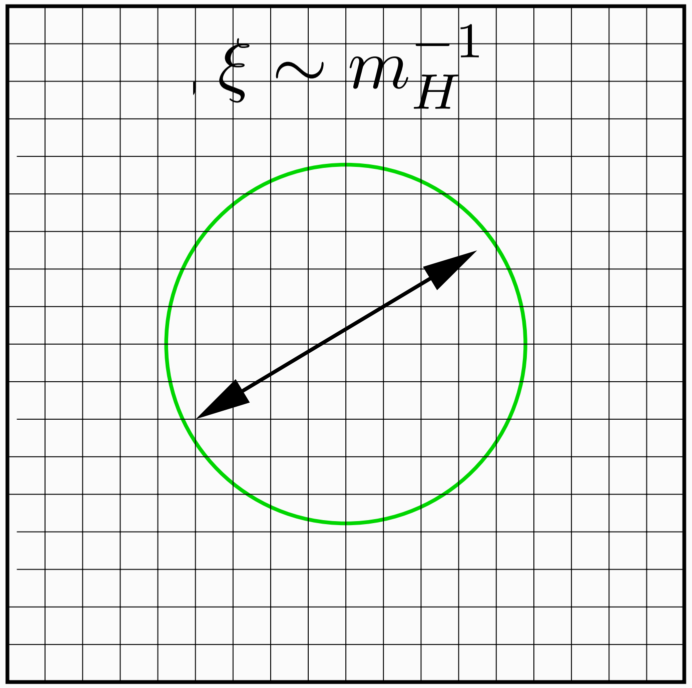
For unphysical quark masses:
(out of computational limitations or interest in certain limits, mass dependence etc.)

Take quantity that depends only weakly on quark masses:
String tension, Sommer scale

$$\frac{T}{\sqrt{\sigma}} = \frac{1}{a\sqrt{\sigma}N_\tau}, \quad \sigma \approx 425 \text{ MeV}; \quad Tr_0 = \frac{r_0}{aN_\tau}, \quad r^2 \frac{dV(r)}{dr} = 1.65$$

Requirements for and constraints from the lattice:

correlation length ξ : lightest gauge invariant (hadronic?) mass scale



$a \ll \xi \ll aL$!

scale of interest: $T_c \sim 200\mathrm{MeV} \sim (1\mathrm{fm})^{-1}$

feasible lattices: $32^3 \times 4, 16^3 \times 8$

$T = \frac{1}{aN_t}$ ➡ $N_\tau = 4, 8, 12$ implies $a \approx 0.25, 0.125, 0.083$ fm

➡ $aL \sim 1.5 - 3\mathrm{fm}$

low T (confined) phase: $m_\pi \gtrsim 250\mathrm{MeV}$ lighter just beginning...

high T (deconfined) phase: $m_\pi \sim T, \xi \sim 1/T$ ➡ $\frac{1}{N_t} \ll 1 \ll \frac{L}{N_t}$

➡ $T \lesssim 5T_c$

Summary Lecture I

- Perturbation theory of finite T QCD in continuum has infrared problems
- Long wavelength modes of finite T QCD are always confining, even at high T
- Finite T on the lattice is a finite size effect
- For simulations with fixed Nt discretisation errors are T-dependent