Baryon χPT

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$\boxed{\text{unproofed notes}}$

The first step in connecting NP to QCD is to understand the low-energy EFT of QCD, Chiral Perturbation Theory. (χPT). This Theory explicitly includes the π (K) degrees of freedom in the most general Lagrangian which is invariant under the (approximate) chiral symmetry of QCD.

The next step is to understand how to encode the interactions of the π with "matter fields" such as baryons, heavy mesons, heavy baryons, etc.

The third step is then to understand how a system of 2 nucleons can be described in an EFT framework.

Etc.

You have been learning about χPT in other lectures. Today, I will provide an introduction to χPT for single baryons. This will allow us to systematically understand the light quark mass corrections to baryon observables, just as in χPT, a perturbative expansion in m_q, encoded in π-loop corrections.

Recall, the pion fields ~~are~~ can be encoded in the Σ field

$$\Sigma(x) = \exp\{2i\phi(x)/f\} \qquad SU(2): \phi(x) = \begin{pmatrix} \frac{\pi^0(x)}{\sqrt{2}} & \pi^+(x) \\ \pi^-(x) & -\frac{\pi^0(x)}{\sqrt{2}} \end{pmatrix}$$

Under a general chiral rotation

$$\Sigma \to L \Sigma R^\dagger, \quad L = e^{i\theta_a^L t_a}, \quad R = e^{i\theta_a^R t_a} \qquad SU(2): \; t_a = \frac{\sigma_a}{2}$$

In contrast, the baryon field transforms under the vector subgroup, $SU(2)_V$. This is just isospin

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad N \to U N$$

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To include these matter fields in a χ invariant Lagrangian, it is useful to work with the ξ field, defined as

$$\xi^2 = \Sigma \quad \Rightarrow \quad \xi = \exp\{i\phi/f\}$$

Under a general χ transformation, this field transforms as

$$\xi \to L \xi \mathcal{U}^\dagger = \mathcal{U} \xi R^\dagger$$ This defines $\mathcal{U}(x)$

In the case of a vector transformation

$$L = R = V$$
$$\Rightarrow \mathcal{U} = V$$

Otherwise, $\mathcal{U}$ is a complex, x-dependent transformation

The choice of nucleon field is not unique. Any choice which transforms the same under the vector subgroup is valid. eg.

$$N' = \xi N \quad , \quad \xi N \to L \xi \mathcal{U}^\dagger \mathcal{U} N$$
$$= L \xi N$$

under $SU(2)_V$, $L = R = \mathcal{U}$

This is to be expected. Any Lagrangian which includes all operators which are invariant under all the symmetries, must in the end give the same S-matrix elements. The difference between these representations is just a field redefinition which does not effect on-shell matrix elements. — Weinberg 1979

So - the Lagrangian should be built from the most general set of operators built from N, ξ

It is convenient to introduce 2 vector fields

$$A_\mu = \frac{i}{2}\left(\xi\partial_\mu\xi^+ - \xi^+\partial_\mu\xi\right) = \frac{\partial_\mu\phi}{f} + O(\phi^3)$$

$$V_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^+ + \xi^+\partial_\mu\xi\right) = \frac{1}{2f^2}\left[\phi, \partial_\mu\phi\right] + O(\phi^4)$$

Under chiral transformations

$$A_\mu \to \mathcal{U} A_\mu \mathcal{U}^+$$

$$V_\mu \to \mathcal{U} V_\mu \mathcal{U}^+ - \partial_\mu \mathcal{U}\, \mathcal{U}^+$$

The point of this is to "dress" the nucleon w/ ξ fields so that it is invariant under the full $SU(2)_L \otimes SU(2)_R$ symmetry.

So now, let's construct operators in the Lagrangian.

We will use Young Tableau method.

In SU(2), the basic building block

$$\boxed{2} = \text{Iso-doublet}$$

We can contract two fundamental elements

$$\square \otimes \square = \begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array} \oplus \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$$

$$\begin{array}{|c|c|}\hline 2 & 3 \\ \hline\end{array} : \frac{2 \cdot 3}{2} = 3 \text{ state : Iso-vector}$$

$$\begin{array}{|c|}\hline 2 \\ \hline 1 \\ \hline\end{array} : \frac{2 \cdot 1}{2} = 1 \text{ state: iso-singlet}$$

In SU(2), the anti-state and the state are the same. Contrast this with SU(3) $\boxed{3}$ = triplet

$$\square \times \square = \begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array} + \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$$

$$6 \oplus \bar{3}$$

In SU(3), the anti-state is a $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$

So a singlet is $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array}$

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Back to SU(2)

$\square$ = doublet, anti-doublet

$\square\square$ = triplet

$\begin{array}{c}\square\\[-4pt]\square\end{array}$ = singlet

The Lagrangian will be formed by all objects which can be combined into singlets: all operators in the Lagrangian must be invariant under the symmetry transformations - else they would not represent the symmetry we are respecting.

- How many quark mass operators can we add?

Recall the QCD $\mathcal{L}$,

$$\mathcal{L}_{QCD} \supset \bar{\psi}_q m_q \psi_q$$

The quark is an iso-doublet

The anti-quark is an anti-iso-doublet

So our spurion analysis lets us temporarily pretend m_q transforms in such a way to keep the term in $\mathcal{L}_{QCD}$ invariant under $SU(2)_L \otimes SU(2)_R$

$m_q \to U m_q U^\dagger$ where $U \in SU(2)_L \otimes SU(2)_R$

This means the quark mass can be thought of

as built from $\underset{\substack{\uparrow \\ \text{doublet}}}{\square} \otimes \underset{\substack{\uparrow \\ \text{anti-doublet}}}{\square}$

$m_q = \square \otimes \square = \square\square \oplus \begin{smallmatrix}\square\\\square\end{smallmatrix}$ The quark mass has a piece

which transforms as an iso-triplet
and a piece which transforms as
an iso-singlet.

The nucleon is also an iso-doublet $\square$
anti-nucleon anti-iso-doublet $\square$

So, the quark mass operators in $\mathcal{L}$ will be formed from

all singlets in the product

$$\bar{N} m_q N = \square \otimes \Big(\square\square \oplus \underset{\substack{\uparrow \\ \text{singlet}}}{\begin{smallmatrix}\square\\\square\end{smallmatrix}}\Big) \otimes \square$$

$$= \square \otimes (\square\square) \otimes \square \oplus \underbrace{\square \otimes \underset{\substack{\uparrow \\ \text{singlet}}}{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \otimes \square}_{(\square \otimes \square)\begin{smallmatrix}\square\\\square\end{smallmatrix}}$$

$\square \otimes \square\square \otimes \square$

$= \square \otimes \left(\square\square\square \oplus \begin{smallmatrix}\square\square\\ \square\end{smallmatrix} \right)$

singlet $\begin{smallmatrix}\square\square\\ \square\end{smallmatrix} = \square$

$= \square \otimes \left(\square\square\square \oplus \square \right)$

$\square \times \square = \square\square \oplus \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

So, only the singlet $\bar{N} \times N$ piece contributes with singlet m_q

$\begin{smallmatrix}2 & 3 & 4\end{smallmatrix}$: $\dfrac{2 \cdot 3 \cdot 4}{2 \cdot 3} = 4$ states

$\square \times \square\square\square \not\supset \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

So this term gives us $\underline{1}$ contribution also.

What is a singlet for $\bar{N} N$? $\bar{N}$ = row vector

N = column vector

So the singlet is simply the dot product in isospin space

$$\bar{N} N = (\bar{p} p + \bar{n} n) \qquad \left(N = \begin{pmatrix} p \\ n \end{pmatrix} \right)$$

What about the singlet contribution from m_q ?

m_q is a matrix, in isospin space. How do we make a singlet (scalar) with a matrix?

$$\mathrm{tr}(m_q) = \text{scalar}$$

So, we see the SU(2) Lagrangian for the Nucleon has 2 independent quark mass operators.

$$\mathcal{L}_{m_q} = \bar{N} \otimes m_q \otimes N = 2\alpha_N \bar{N} m_q N + 2\sigma_N \bar{N} N \, tr\, m_q$$

Well, this is almost right. Recall, we need to use our spurion field for m_q. m_q is only invariant under the vector subgroup, so we need to "dress" it with ξ fields so that it becomes invariant under the full $SU(2)_L \otimes SU(2)_R$ transformations

The two spurions we can form are

$$\mathcal{M}_{\pm} = \frac{1}{2}\left(\xi m_q^{\dagger} \xi \pm \xi^{\dagger} m_q \xi^{\dagger}\right)$$

where I have promoted m_q to be in an arbitrary representation of $SU(2)_{L\times R}$ hence the (†)

$\mathcal{M}_+ \to$ even number of π fields after expanding in π/f

$\mathcal{M}_- \to$ odd number of π fields after expanding

A standard way to write the $\mathcal{L}$

$$\mathcal{L}_{m_q} = 2\alpha_N \bar{N} M_+ N + 2\sigma_N \bar{N} N \,\mathrm{tr}\, M_+$$

Notice, $\mathrm{tr} M_+ = \frac{1}{2} \mathrm{tr}(\xi m_q^\dagger \xi + \xi^\dagger m_q \xi^\dagger)$
$= \frac{1}{2} \mathrm{tr}(\Sigma m_q^\dagger + m_q \Sigma^\dagger)$

This is almost the operator from the π Lagrangian. It is just missing the B-condensate factor.

Also, notice, since we have the quark mass in $\mathcal{L}$, which is not a ~~sca~~ renormalization scale, scheme independent quantity, our coefficients must actually be scale/scheme dependent.

This is undesireable. But, we are free to re-write $\mathcal{L}$, as long as it still preserves the symmetry.

1: $m_q \to 2Bm_q$ We might as well keep the coupling of B & m_q from meson Lagrangian. We know this combination is scale/scheme independent.

But now, our coefficients must ~~be~~ have dimension $= -1$

2) This is also undesireable. Knowing factors of (4π) emerge from loops so that we always get $f \to 4\pi f$ when computing loop corrections, let us chose to pull out a factor of $4\pi f$ so that our coefficients become dimensionless. Moreover – Naive Dimensional Analysis arguments will then suggest these coefficients should naturally be $O(1)$

$$\mathcal{L}_{m_q} = \frac{\alpha_N}{4\pi f} \bar{N} \chi_+ N + \frac{\sigma_N}{4\pi f} \bar{N} N \, tr(\chi_+)$$

$$\chi_\pm = \frac{1}{2}\left(\xi (2Bm_q)^\dagger \xi \pm \xi^\dagger (2Bm_q) \xi^\dagger \right)$$

HW

— If you are not familiar with a Young Tableau, learn about it (google answers seem good)

— consider SU(3) flavor, then our baryons form an octet

$$B = \begin{array}{|c|c|}\hline 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} : \frac{4 \cdot 3 \cdot 2}{3} = 8 \text{ states}, \quad \underset{2}{N}, \underset{1}{\Lambda}, \underset{3}{\Sigma}, \underset{2}{\Xi}$$

— the quark mass operator is still formed from

$$q \otimes \bar{q}$$

— what is a $\bar{q}$ = ? $\quad \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array} = \bar{q}$

$\begin{array}{|c|}\hline \\ \hline\end{array} = q$

— what is a $\bar{B}$ = ? $\bar{B}$ is defined as the tableau that will give $\bar{B} \otimes B > \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline\end{array}$ scalar

$$\bar{B} \otimes \begin{array}{|c|c|}\hline & \\ \hline \\ \cline{1-1}\end{array} > \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline\end{array}$$

— using $\bar{B}$, B and m_q, how many quark mass operators can you make?

Now that we have our quark mass operators, we can add them to the standard kinetic operator

$$\mathcal{L} = \bar{N}[i\not{D} - m_N]N + \frac{\alpha_N}{4\pi f}\bar{N}\chi_+ N + \frac{\sigma_N}{4\pi f}\bar{N}N \, tr\, \chi_+$$

$$D_\mu N = \partial_\mu N + V_\mu N$$

$$\cancel{D_{\mu i}{}^j N_j = \partial_\mu N \delta_i{}^j}$$

$$D_{\mu,i}{}^j N_j = \delta_i{}^j \partial_\mu N_j + V_{\mu,i}{}^j N_j$$

But before we can go any further, we must recognize a problem w/this $\mathcal{L}$. The nucleon mass is not a small scale. $m_N \sim \Lambda_\chi = 4\pi f$

Time derivatives acting on N will bring down large powers of m_N. Inside loop graphs, we will have $\frac{i\partial_t N}{\Lambda_\chi} = \frac{m_N}{\Lambda_\chi} N$

which is an $O(1)$ contribution. We do not have any power counting to tell us which contributions are most relevant!!

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But we know we are not interested in describing dynamics near the nucleon mass scale, because this is in the regime where χPT has broken down. What we want is a theory that describes the "soft pion" dynamics of the nucleon. In this limit, the nucleon will certainly be well approximated by a non-relativistic description. Jenkins & Manohar formulated "Heavy Baryon χPT", modelled after the Heavy Quark Effective Field Theory.

For a non-relativistic nucleon, it is convenient to parameterize the nucleon momentum

$$P_\mu = m_N \underbrace{v_\mu}_{\text{nucleon velocity}} + \underbrace{k_\mu}_{\text{soft momentum}}$$

We then ~~[illegible]~~ use the velocity dependent field

$$N_v(x) = e^{im_N v\cdot x} \frac{1+\not{v}}{2} N(x)$$

In the nucleon rest frame, $v_\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$\not{v} = \gamma_0$$

$$\frac{1+\gamma_0}{2} = \text{particle projector}$$

So this picks out the non-relativistic nucleon dof.

$$\partial_\mu N_v = \partial_\mu \left(e^{im_N v\cdot x} \frac{1+\not{v}}{2} N \right)$$

$$= i m_N v_\mu N_v + e^{im_N v\cdot x} \frac{1+\not{v}}{2} \partial_\mu N$$

$$= i m_N v_\mu N_v - i p_\mu N_v$$

$$= -i(p_\mu - m_N v_\mu) N_v \qquad ; \quad \cancel{} \; p_\mu = m_N v_\mu + k_\mu$$

$$= -i k_\mu N_v$$

The derivative on this NR velocity field only brings down a power of the soft momentum.

This type of phase is exactly how you reduce Klein-Gordon or Dirac to their non-relativistic limits

If we look at the original kinetic term

$$N(x) = e^{-im_N v\cdot x}\left(N_{v+}^{(x)} + N_{v-}^{(x)}\right)$$

$$N_{v\pm} = \frac{1 \pm \not{v}}{2} N(x)\, e^{im_N v\cdot x}$$

$$\bar{N}\left[i\not{D} - m_N\right]N \longrightarrow \bar{N}_v i v\cdot D N_v + O\left(\frac{1}{m_N}\right)$$

This ~~rotatn~~ transformation systematically moves all dependence on the large nucleon mass into a power series in $1/m_N$.

One way to think of it is we are expanding about the energy scale m_N, so the anti-nucleon will take $2m_N$ to excite. This is a heavy state that can be integrated out of the theory, and so it's effects will always come from $\longrightarrow = \frac{i}{\not{p} - 2m_N} \sim \frac{-i}{2m_N} + O\left(\frac{p}{3m_N^2}\right)$

This is just a qualitative understanding. In HQET, we can match exactly to QCD, whereas in the case of the nucleon, the starting point of a relativistic

Nucleon Lagrangian is not self consistent, so it is not a good starting point. We can not match the non-relativistic $\mathcal{L}$ to it. But we can match to it qualitatively to understand some normalizations.

For example

$$\langle N | \bar{q} \gamma_\mu \gamma_5 \tau q | N \rangle \to g_A \quad \text{the nucleon axial charge}$$

$$\mathcal{L}_{Rel.} \supset g_A \bar{N} \gamma_\mu \gamma_5 A^\mu N \,, \quad A_\mu \ \text{axial vector current}$$

$$\downarrow$$

$$2 g_A \bar{N}_v \, S \cdot A \, N_v$$

S_μ = Pauli-Lubanski Spin vector satisfies

$$S_\mu S^\mu N_v = \frac{1-d}{4} N_v$$

$$v \cdot S = 0$$

So now we can write down the leading, nontrivial Lagrangian for Nucleons & π's

$$\mathcal{L} = \bar{N}_v \, i v\cdot D N_v - \frac{\alpha_N}{4\pi f} \bar{N} X_+ N - \frac{\sigma_N}{4\pi f} \bar{N} N \, \mathrm{tr}\, X_+ + 2 g_A \bar{N}_v S_\mu A^\mu N_v$$

this is only the axial charge at LO. We should call it $g_{\pi NN}$ but in the literature, it is almost always called g_A.

Note, the nucleon propagator is trivial

$$G_N(k) = \frac{i P_+}{v\cdot k + i\varepsilon} \qquad P_+ = \frac{1+\not{v}}{2}$$

The nucleon is a static source. If we are not careful, when we probe the nucleon w/external currents, we will screw up the singularity structure. For static properties, there is no problem, but for quantities where the nucleon recoil may be important, we may have to re-sum the leading kinetic corrections to the nucleon propagator.

What is $m_N = ?$

It is not the nucleon mass. That recieves chiral corrections from the m_q operators.

$$m_N = \lim_{m_q \to 0} m_N(m_q)$$

the nucleon mass in the chiral limit.

We can now compute the self energy corrections.

The leading corrections are:

$$\begin{aligned} \mathrm{tr}\, X_+ &= \tfrac{1}{2}\, \mathrm{tr}\left(2Bm_q(\Sigma + \Sigma^\dagger)\right) \\ &= \tfrac{1}{2} \cdot 2 \cdot \mathrm{tr}\, 2Bm_q \\ &= 2B(m_u + m_d) \\ &= 2\, m_\pi^2 \end{aligned}$$

$$\begin{aligned} \bar{N} X_+ N &= \bar{N}\, \tfrac{1}{2}\, 2Bm_q(\Sigma + \Sigma^\dagger) N \\ &= \bar{N}\, 2Bm_q N \\ &= \bar{p}p \cdot 2Bm_u \\ &\quad + \bar{n}n\, 2Bm_d \end{aligned}$$

$$\cancel{m_p \neq M_0 - \frac{\alpha}{4\pi f}}$$

$$m_p = M_0 - \alpha \cdot \frac{2B\delta}{4\pi f} + (\alpha + 2\sigma)\frac{2B\hat{m}}{4\pi f}$$

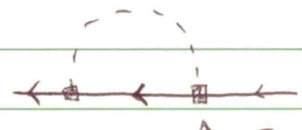
$$\hat{m} = \tfrac{1}{2}(m_u + m_d)$$
$$\delta = \tfrac{1}{2}(m_d - m_u)$$

$$m_n = M_0 + \alpha\, \frac{2B\delta}{4\pi f} + (\alpha + 2\sigma)\, \frac{2B\hat{m}}{4\pi f}$$

These corrections are "not interesting"

We could have predicted them from a Taylor expansion in m_q.

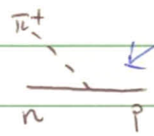
The first "interesting" correction comes from the leading loop graph



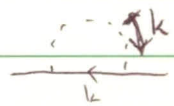
$$A_\mu = \frac{\partial_\mu \phi}{f} + O(\partial\phi^3) \qquad \phi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}$$

$$\bar{N}\phi N = -\bar{n}\frac{\pi^0}{\sqrt{2}}n + \bar{p}\frac{\pi^0}{\sqrt{2}}p + \underbrace{\bar{n}\pi^- p} + \bar{p}\pi^+ n$$

$$\bar{N}\frac{\partial_\mu\phi}{f}N \supset$$



and the pion is derivatively coupled to the nucleon as must be the case, since it is a pseudo-Nambu Goldstone boson



$$-i\delta\Sigma = \left(\frac{ig_A}{f}\right)^2 \mu^{d-2}\int \frac{d^dk}{(2\pi)^d} S^\mu(+ik_\mu)\frac{i}{k\cdot v+i\varepsilon}\frac{i}{k^2-m_\pi^2+i\varepsilon}S^\nu(-ik_\nu)\underbrace{\sum_c C_{NN\pi}^2}_{\text{sum over Clebsch gordon coefficients for interactions}}$$

$$= -(i)^6\frac{g_A^2}{f^2}\mu^{d-2}\int\frac{d^dk}{(2\pi)^d}\frac{S^\mu S^\nu k_\mu k_\nu}{[k\cdot v+i\varepsilon][k^2-m_\pi^2+i\varepsilon]}$$

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What is the behavior of this integral?

You can see I have already set up the calculation for dim-reg. We are interested in chiral corrections so we want a regularization scheme that respects chiral symmetry. Only dim-reg does this.

What is the dimension of our integral?

$$\left[\int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{[k\cdot v + i\varepsilon][k^2 - m_\pi^2 + i\varepsilon]} \right] = d + 2 - 3 = d - 1$$

So, as we take $d \to 4$, this integral has dimension 3.

By the "black magic" of dim reg, we know the integral must be proportional to m_π^3

After performing the integral, we find

$$\delta \Sigma_{\text{ }} = -3\pi g_A^2 \frac{m_\pi^3}{(4\pi f)^2}$$

This is the leading, non-analytic in m_q contribution to the nucleon mass. There are a few important things to note

The coefficient is "unaturally large"

$$-3\pi g_A^2 , \qquad g_A = 1.27$$
$$3\pi \approx 10$$

these factors

So this is a "large" correction since it is enhanced by

It's sign is fixed and negative.

This is the first "interesting" correction.

$$m_\pi^2 \sim m_q$$
$$m_\pi^3 \sim m_q^{3/2}$$

This is non-analytic in the light quark mass (like chiral logs for π's) and so it can not be represented by a finite number of local operators.

This term, in some sense, is the real prediction from χPT. We would like to find numerical evidence for this behavior in our LQCD calculations

This correction is the same for the proton and neutron.

There are a few other important points to make:

The parameterization of the nucleon momentum has ambiguity

$$p_\mu = m_N v_\mu + k_\mu \qquad p^2 = m_N^2 + 2m_N v\cdot k + k^2$$

$$\Rightarrow 2m_N v\cdot k + k^2 = 0$$

$$v \rightarrow v + \frac{\varepsilon}{m_N} \qquad k \rightarrow k - \varepsilon$$

$$p_\mu \rightarrow m_N\left(v_\mu + \frac{\varepsilon_\mu}{m_N}\right) + k_\mu - \varepsilon_\mu$$

$$= p_\mu$$

This shift in the momentum leaves the total momentum invariant. If we demand the S-matrix elements be invariant under this transformation, this will exactly fix certain coefficients of higher order operators w/respect to lower dimension operators. This is known as Reparameterization Invariance. It is a fancy way of describing the enforcement of Lorentz invariance order by order in the $1/m_N$ expansion.

For example, the kinetic operators are

$$\mathcal{L}_\partial = \bar{N} i v\cdot\partial N - \bar{N}\frac{\partial^2}{2m_N}N + c\bar{N}\frac{(v\cdot\partial)^2}{2m_N}N + O\left(\frac{1}{m_N^2}\right)$$

The coefficient of the first operator is exactly constrained to be -1

The coefficient of the second operator is not constrained, but we have freedom to chose $c=+1$, with a suitable field redefinition so

$$\mathcal{L}_\partial = \bar{N} i v\cdot\partial N - \bar{N}\frac{\partial_\perp^2}{2m_N}N, \qquad \partial_\perp^2 = \partial^2 - (v\cdot\partial)^2$$

This is an obvious choice, because then our nucleon propagator becomes

$$G(k) = \frac{i}{v\cdot k - \frac{\vec{k}^2}{2m_N} + i\epsilon}$$

Which is precisely the expected leading kinetic correction in the NR expansion.

- The delta resonance is very close in mass to the nucleon and easily excitable.

The delta should be included as a dynamic dof. It slightly complicates the algebra, and the loop integrals are slightly more complicated.

- It introduces a new small scale
$\Delta \equiv m_\Delta - m_N$
which does not vanish in the chiral limit. So we must determine a good power counting.

Often, one finds $m_\pi \sim \Delta$ or $m_\pi \ll \Delta$

In LQCD calculations, it is common to have $m_\pi > \Delta$, so the Δ is a stable dof.

- Also, in the large N_c limit, $\Delta \to 0$, and so we also have a rigorous QFT way to include the Δ, even if $N_c = 3$ is quantitatively far from $N_c = \infty$