

Connecting Nuclear Physics to QCD with the lattice

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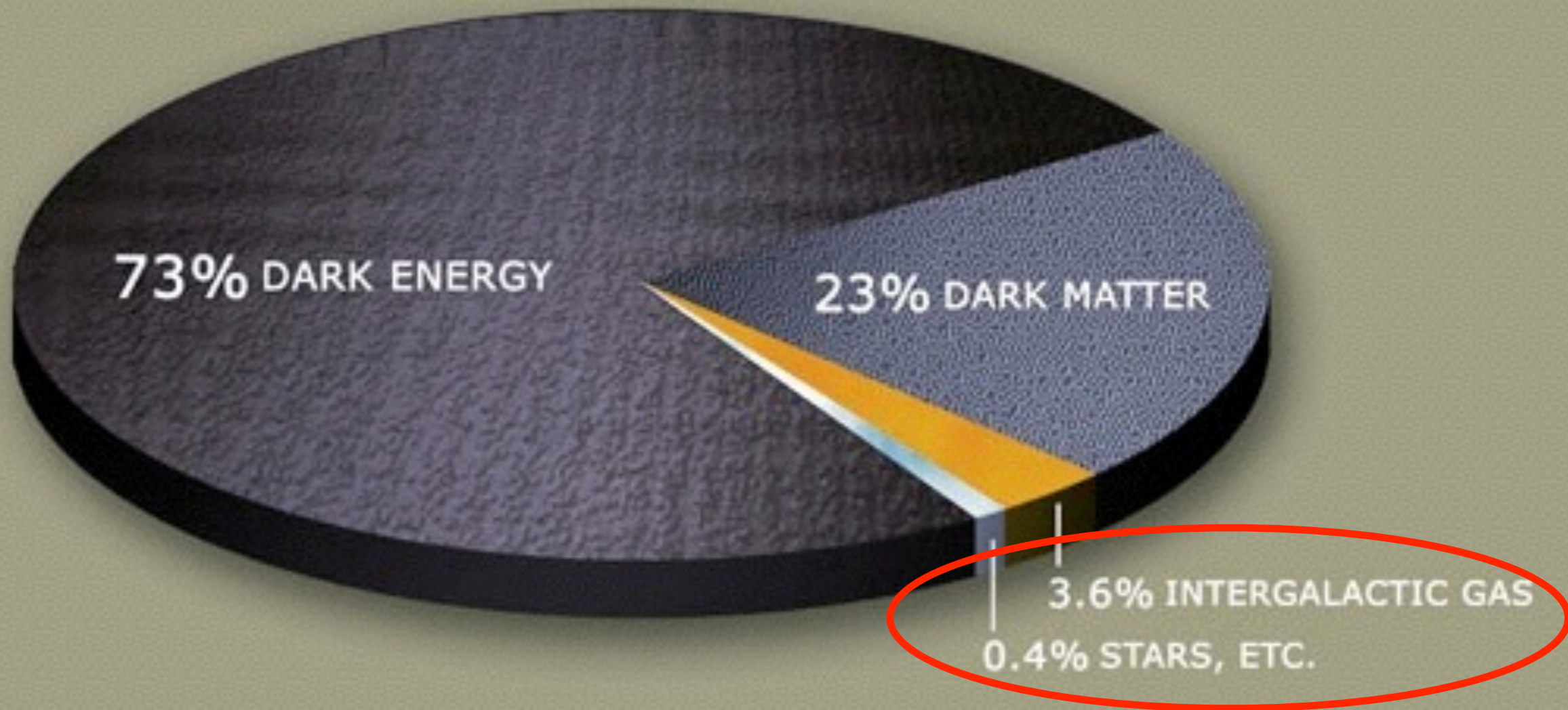
Helmholtz International Summer School

Dubna, Russia, **August 25-September 6, 2014**

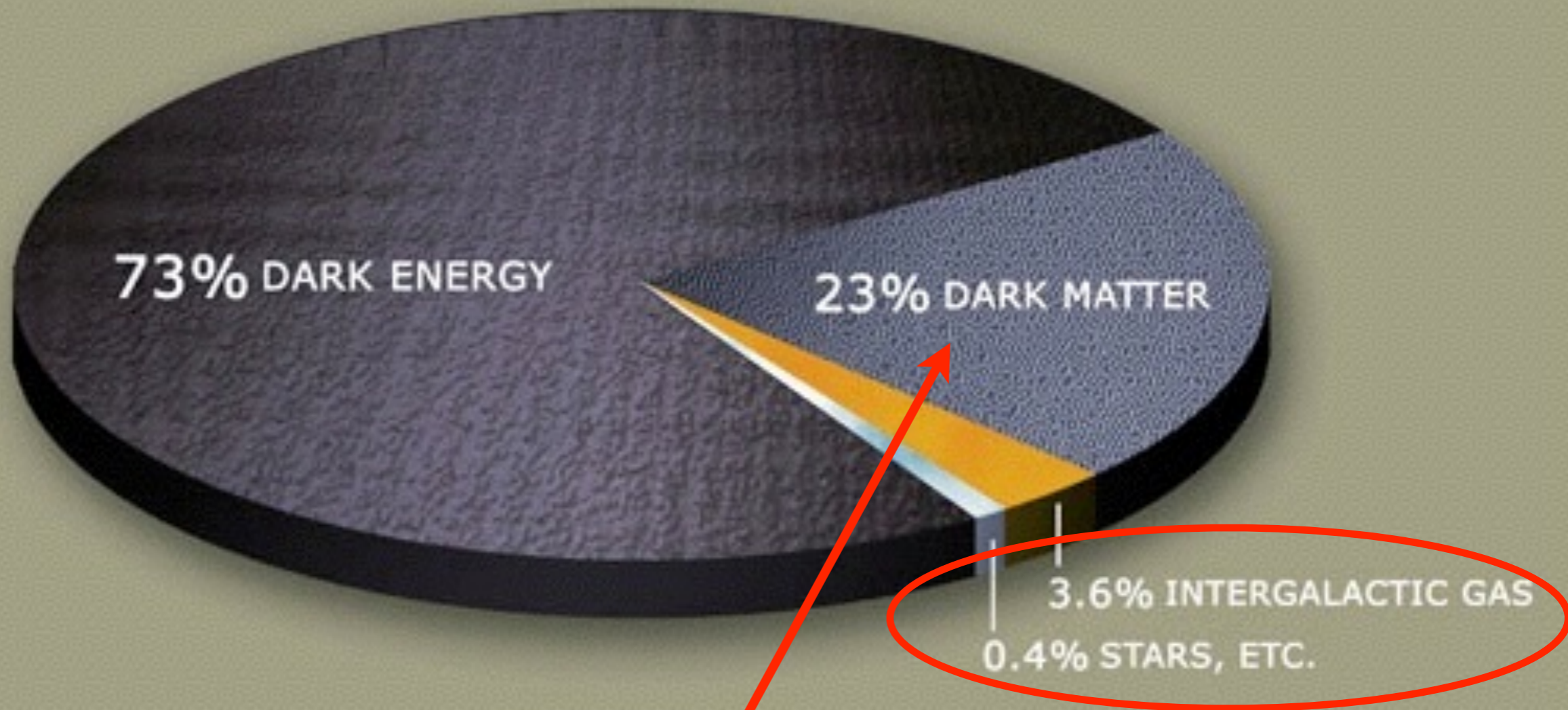
Lattice QCD, Hadron Structure and Hadronic Matter



Energy Budget of the Universe

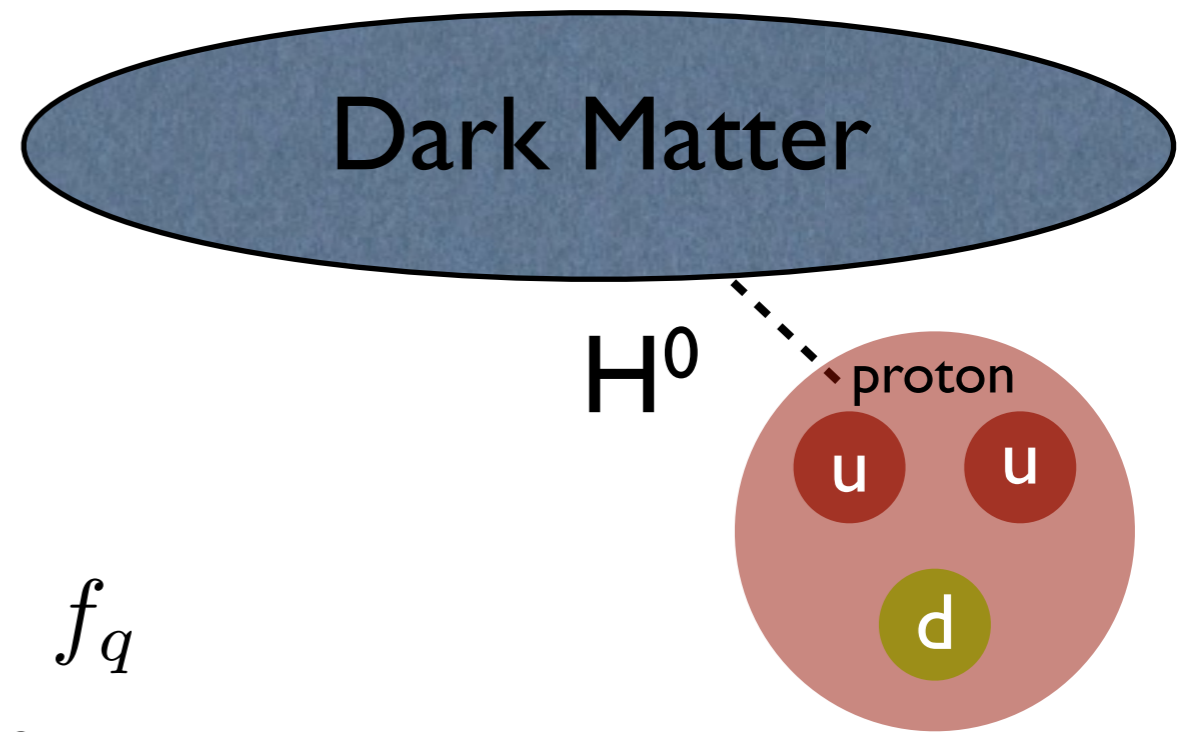


Energy Budget of the Universe



quark mass dependence of nucleon mass helps us understand sensitivities of direct dark matter detection

If Dark Matter couples to the scalar current of the nucleon (eg via Higgs) **Spin Independent cross section**



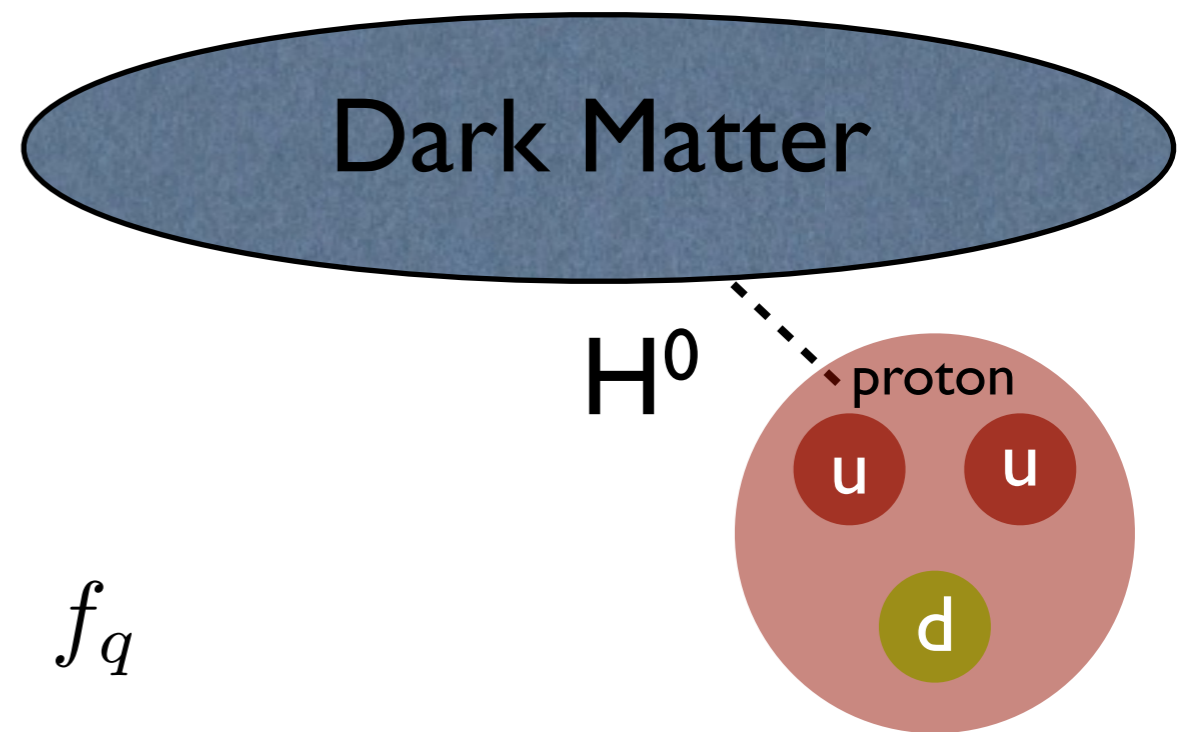
$$\sigma \propto |f|^2 \quad f = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q$$

$$f_q \equiv \frac{\langle N | m_q \bar{q}q | N \rangle}{m_N}$$

see eg. [Cheung, Hall, Pinner, Ruderman](#)
arXiv:1211.4873

with enhancement of A^2 for nucleus (Xenon)

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scalar current difficult to measure experimentally

see eg. [Cheung, Hall, Pinner, Ruderman](#)
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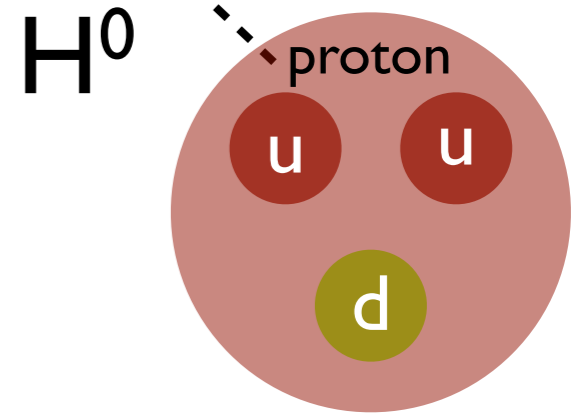
with enhancement of A^2 for nucleus (Xenon)

$f_{u,d}$ estimated from pion-nucleon scattering

f_s uncertainty dominates estimates of cross section

[Ellis, Olive, Savage](#)
Phys.Rev. D77 (2008)

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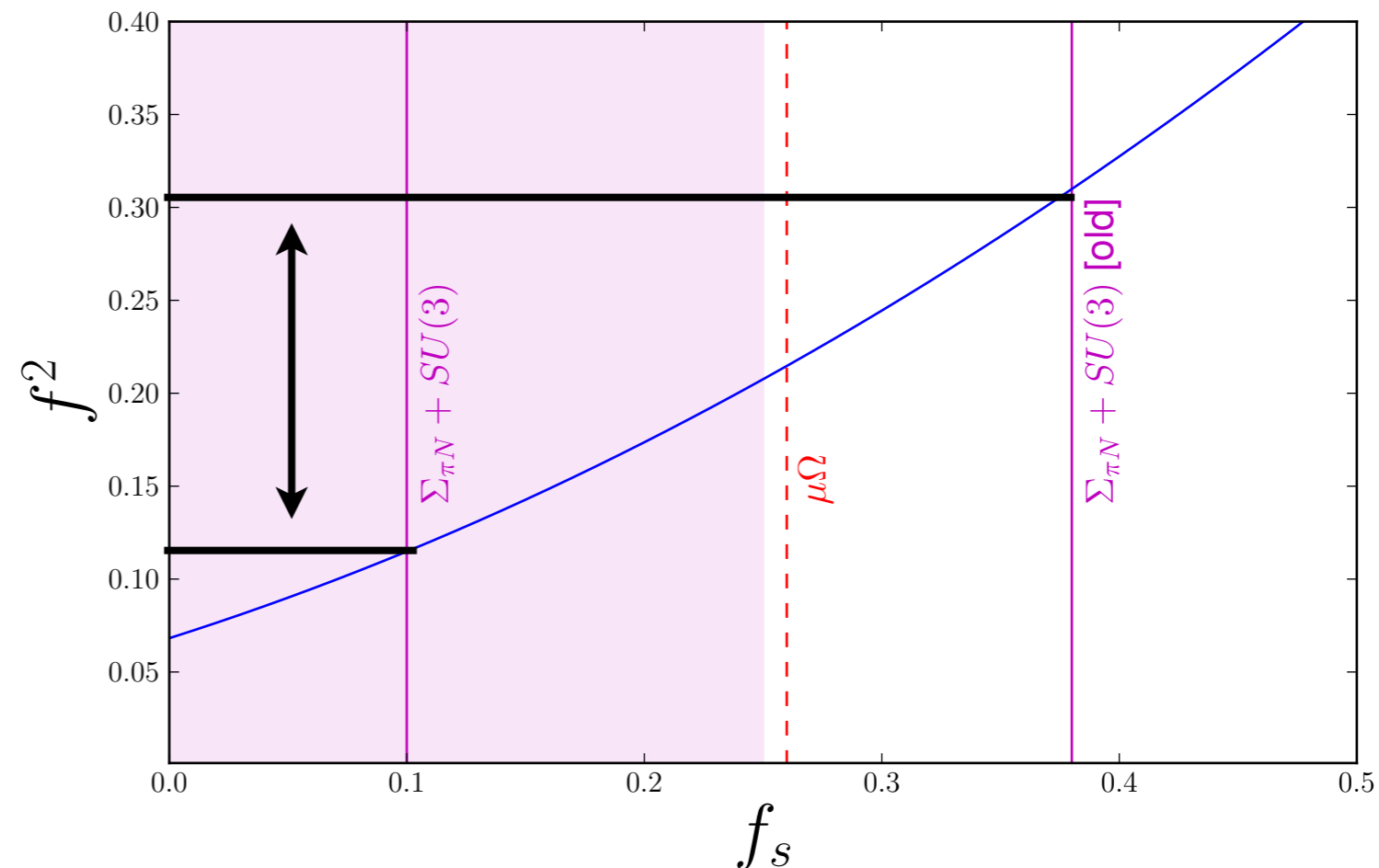


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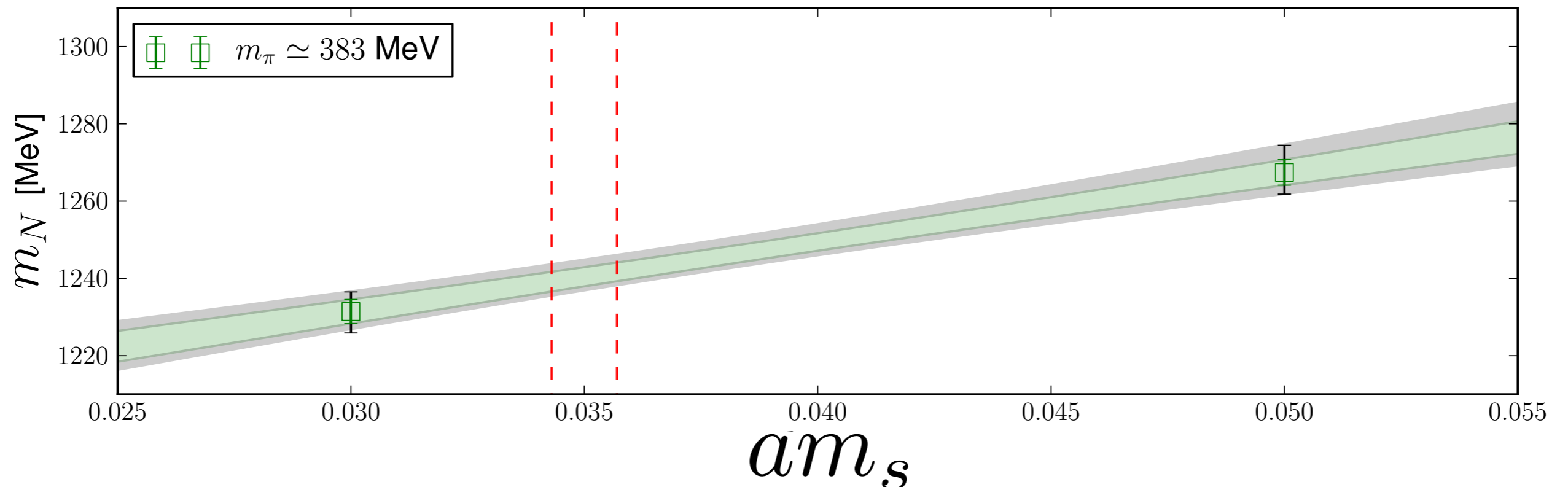
figure adapted from arXiv:1211.4873
thanks to J. Ruderman and collaborators



Lattice QCD perfect tool to compute strange content of nucleon $m_s \langle N | \bar{s}s | N \rangle$

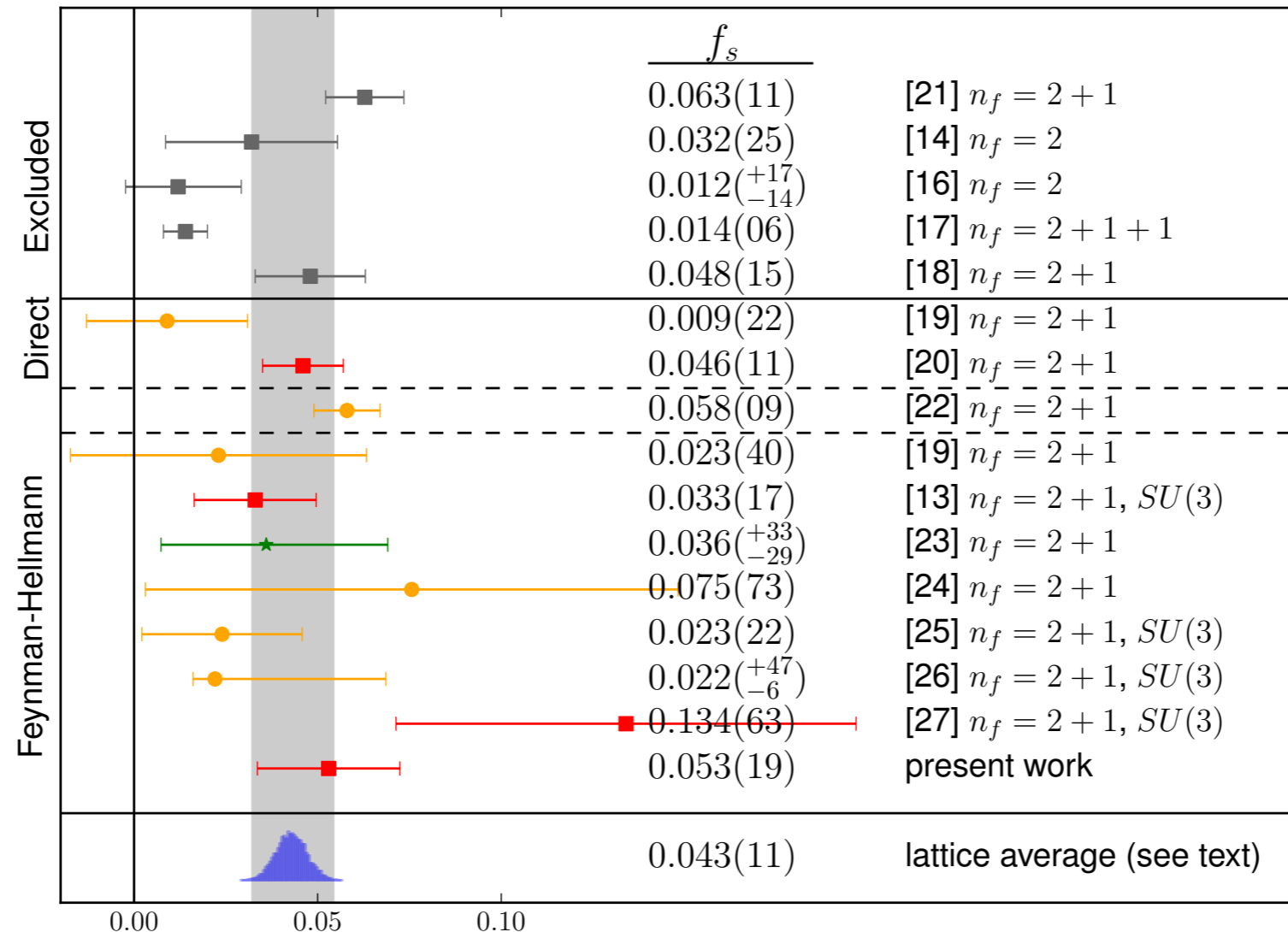
Feynman-Hellmann Theorem $m_q \langle N | \bar{q}q | N \rangle = m_q \frac{\partial}{\partial m_q} m_N$

am_s^{phys}



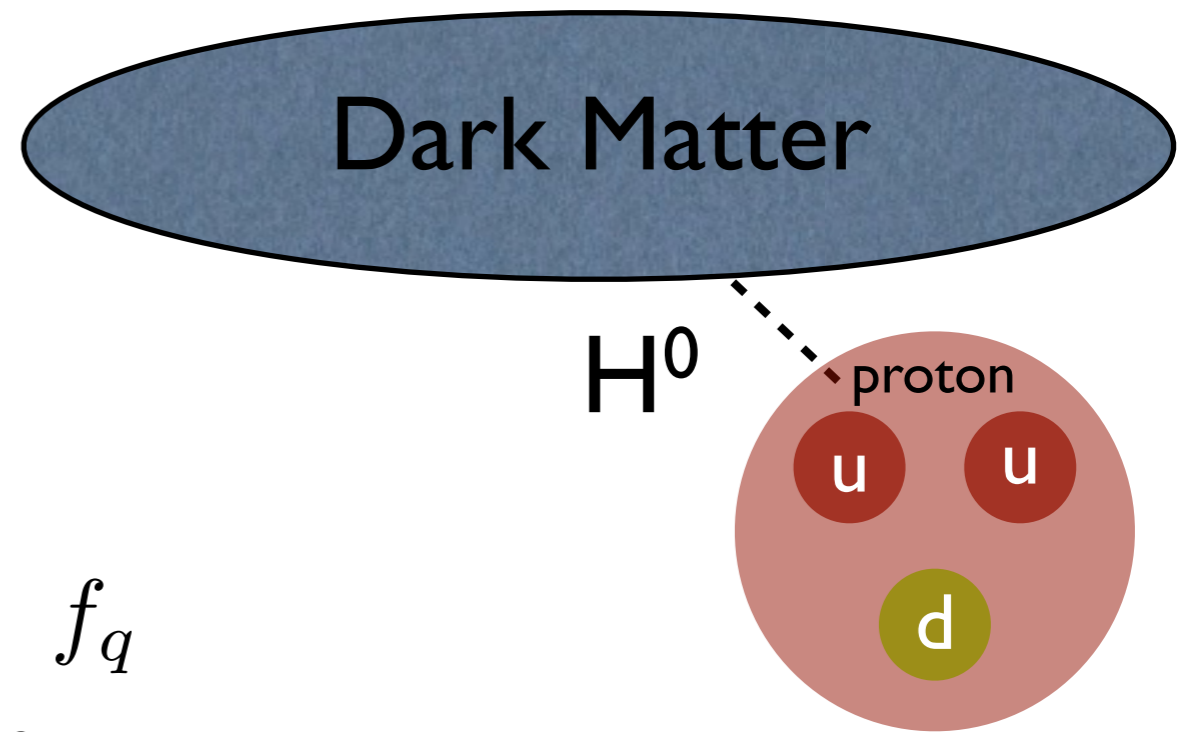
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$$\hat{f}_s = m_s \langle N | \bar{s}s | N \rangle / m_N$$

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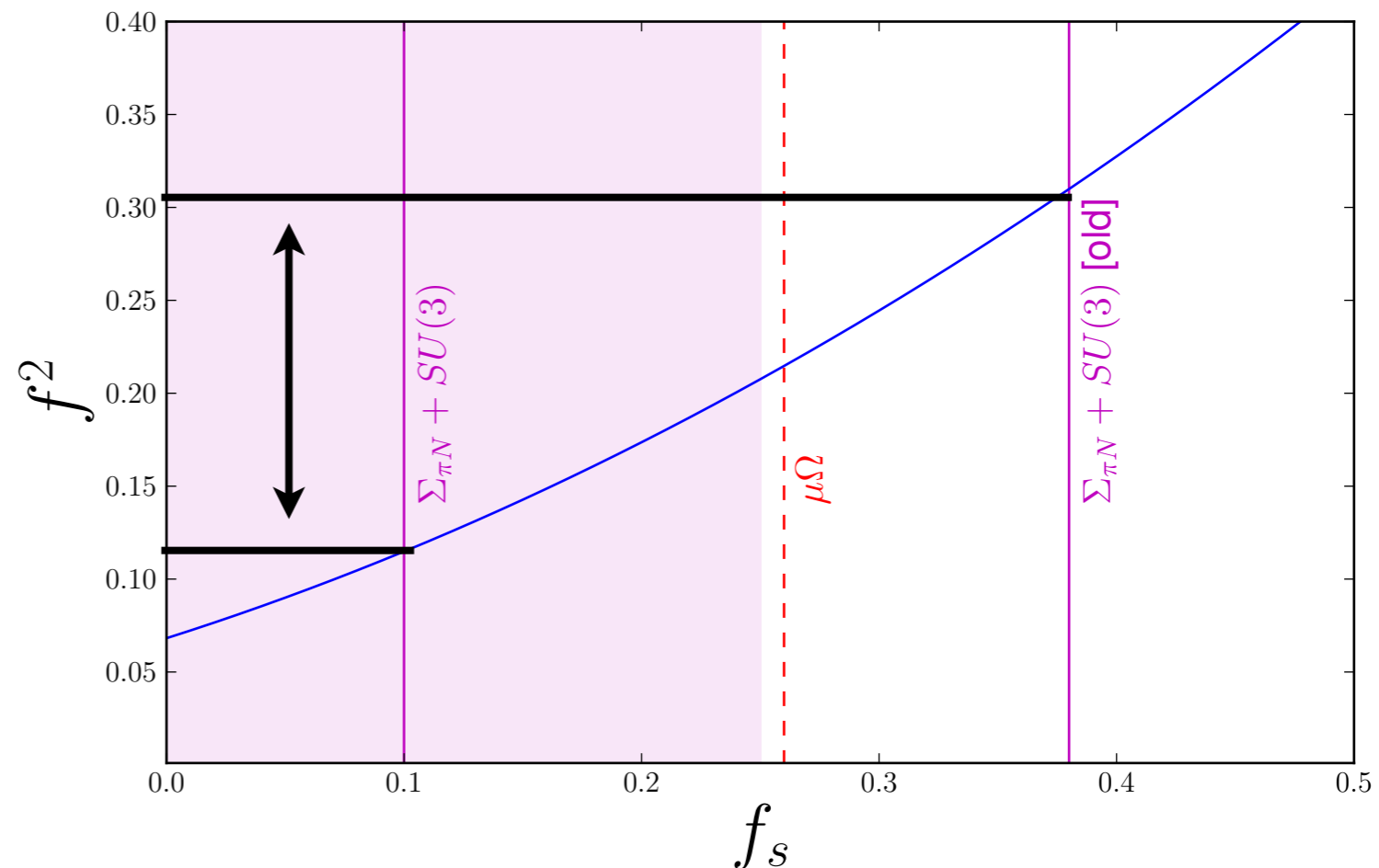


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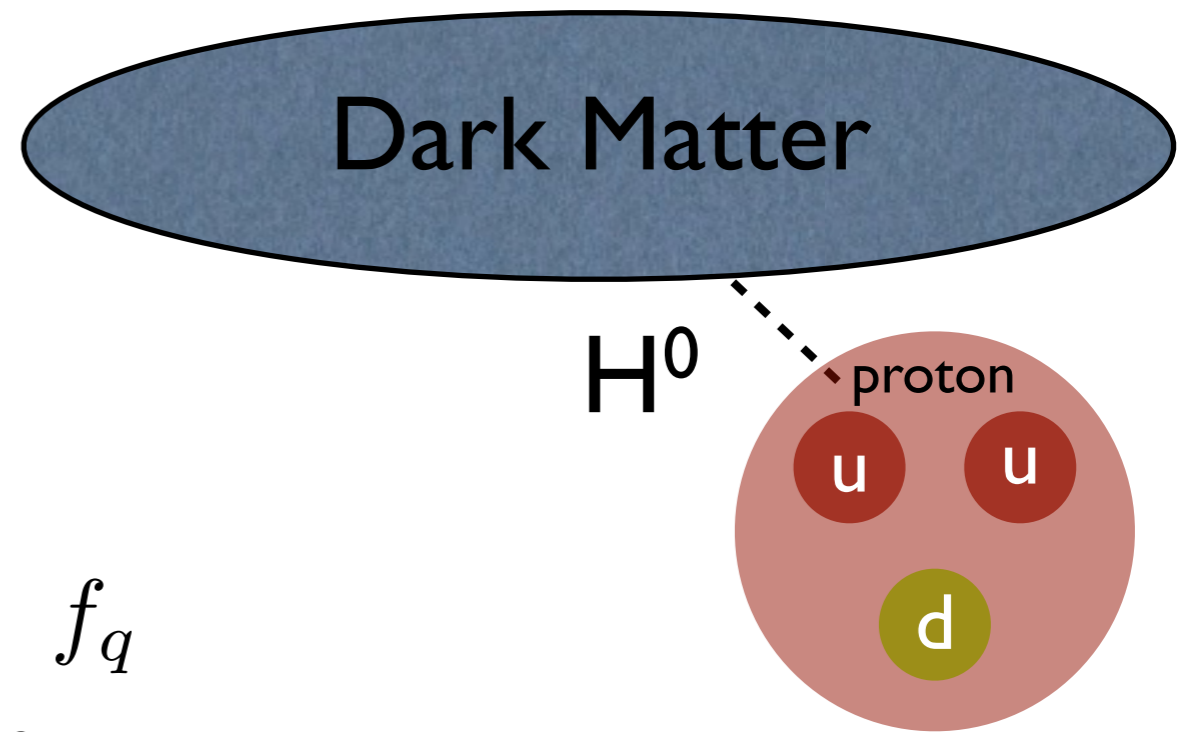
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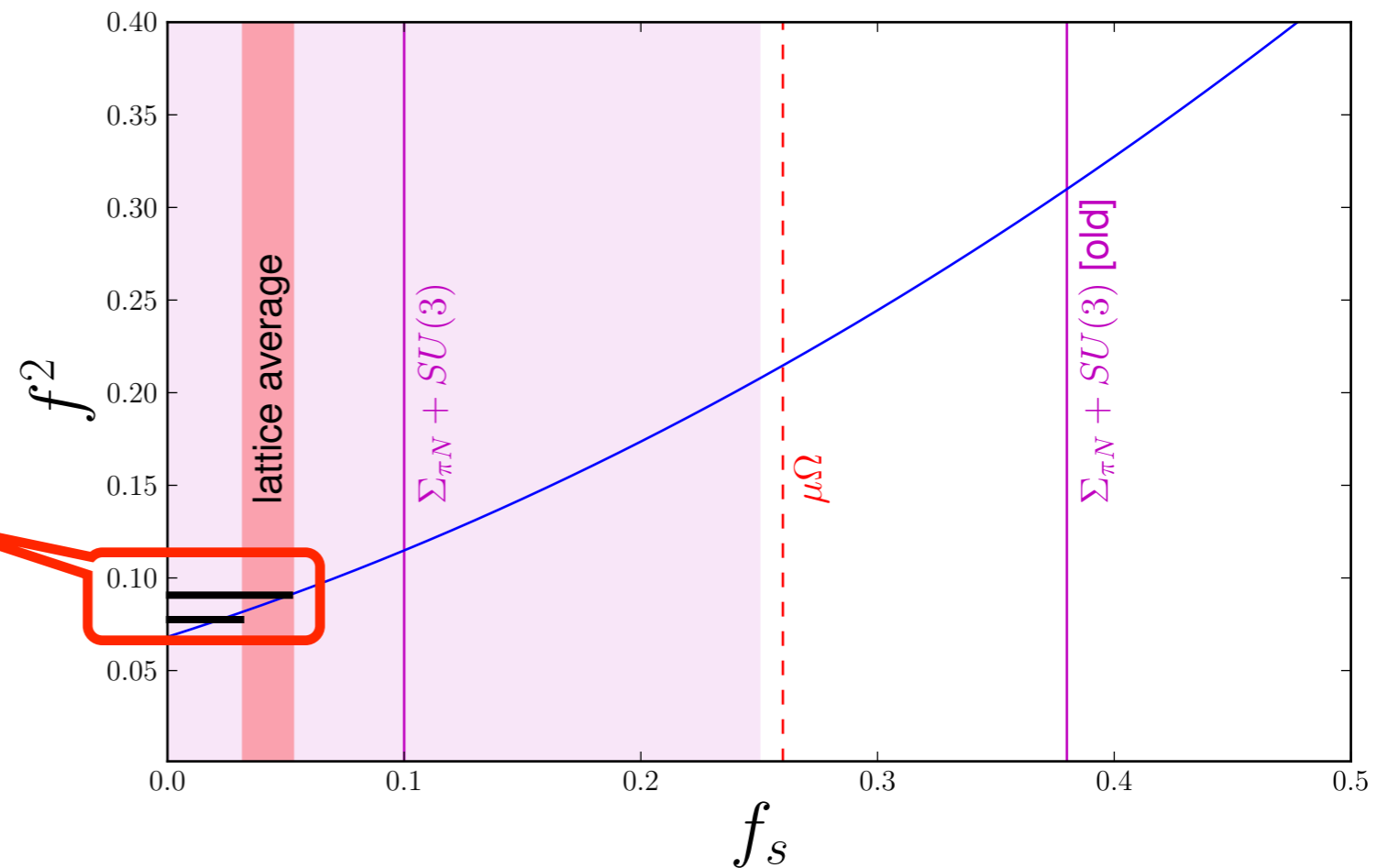


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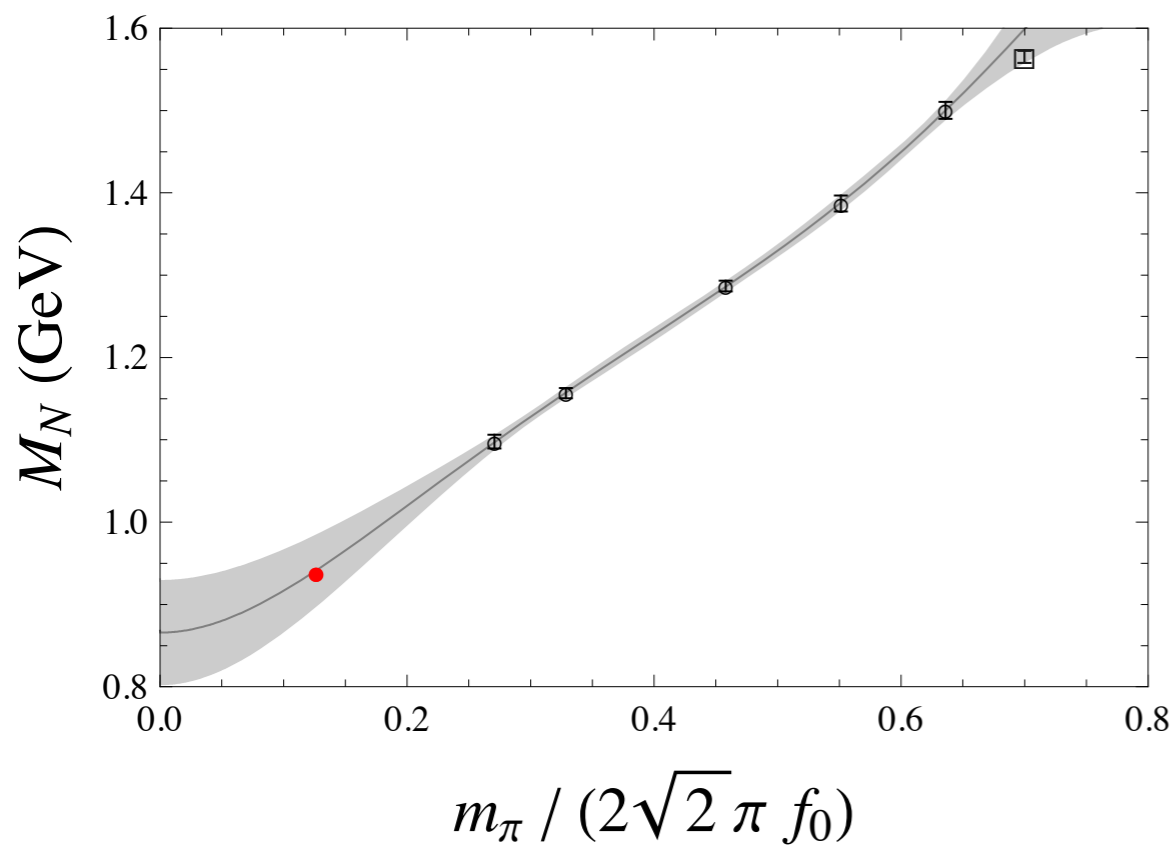
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dramatic reduction in uncertainty of cross section

now $f_{u,d}$ gives larger uncertainty - but harder

NNLO - m_π^4 , with $g_A=1.2(1)$, $g_{\Delta N}=1.5(3)$



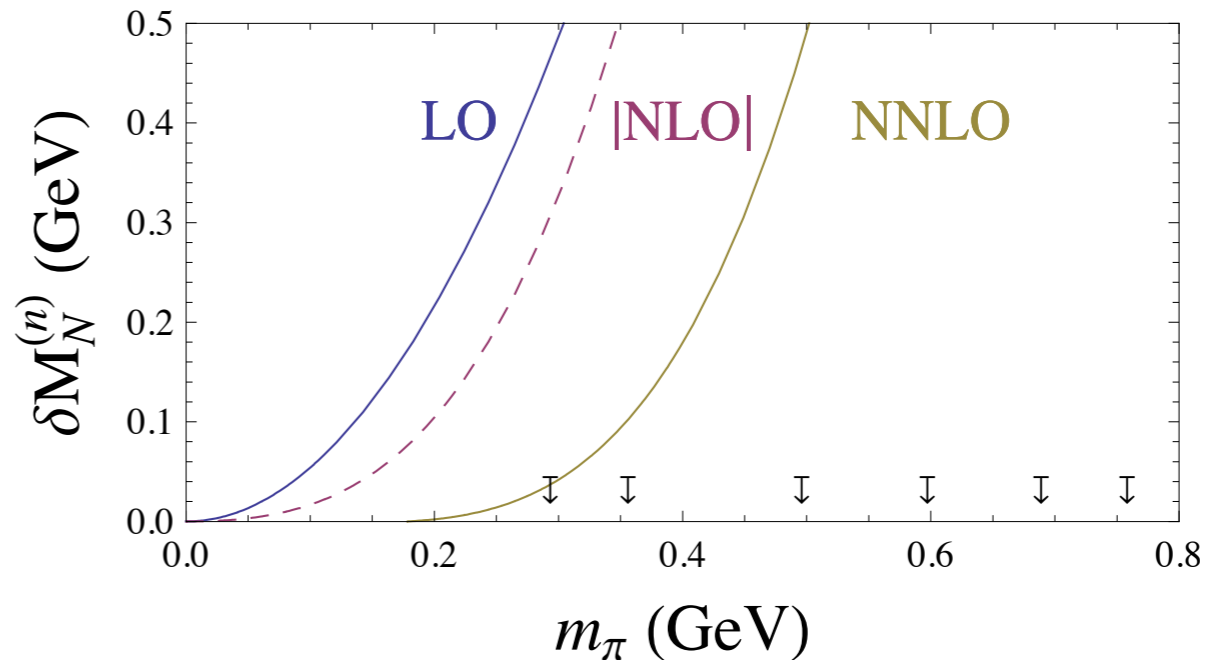
NNLO Heavy Baryon Fit

$$M_N = 954 \pm 42 \pm 20 \text{ MeV}$$

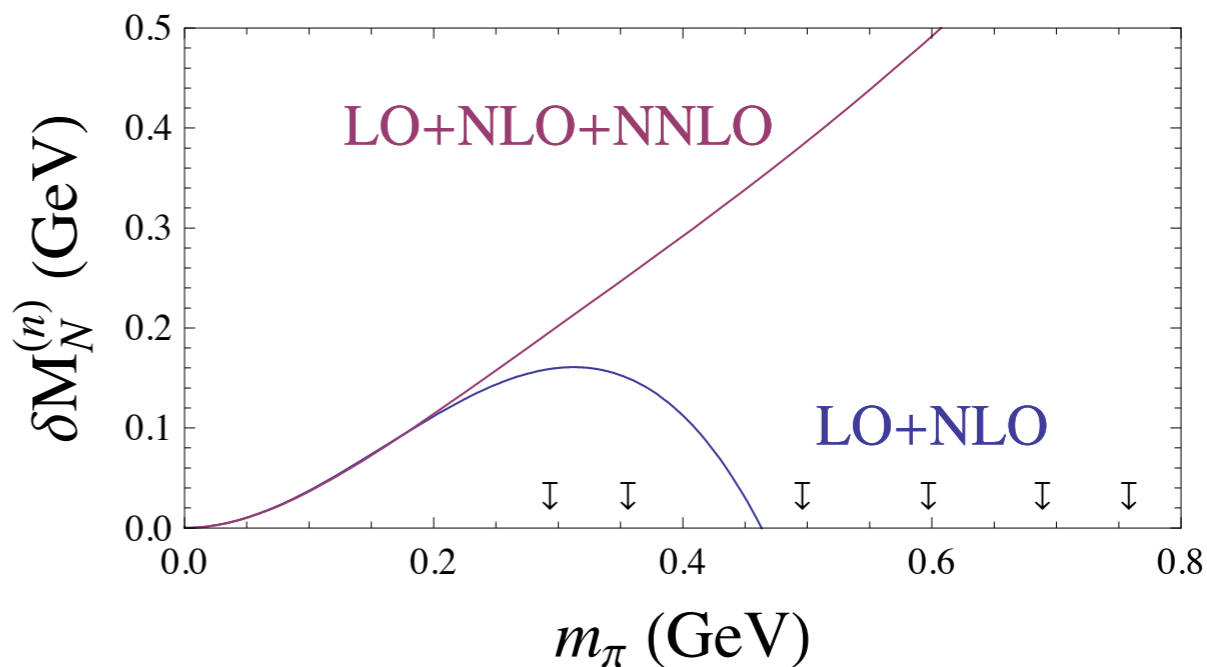
statistical

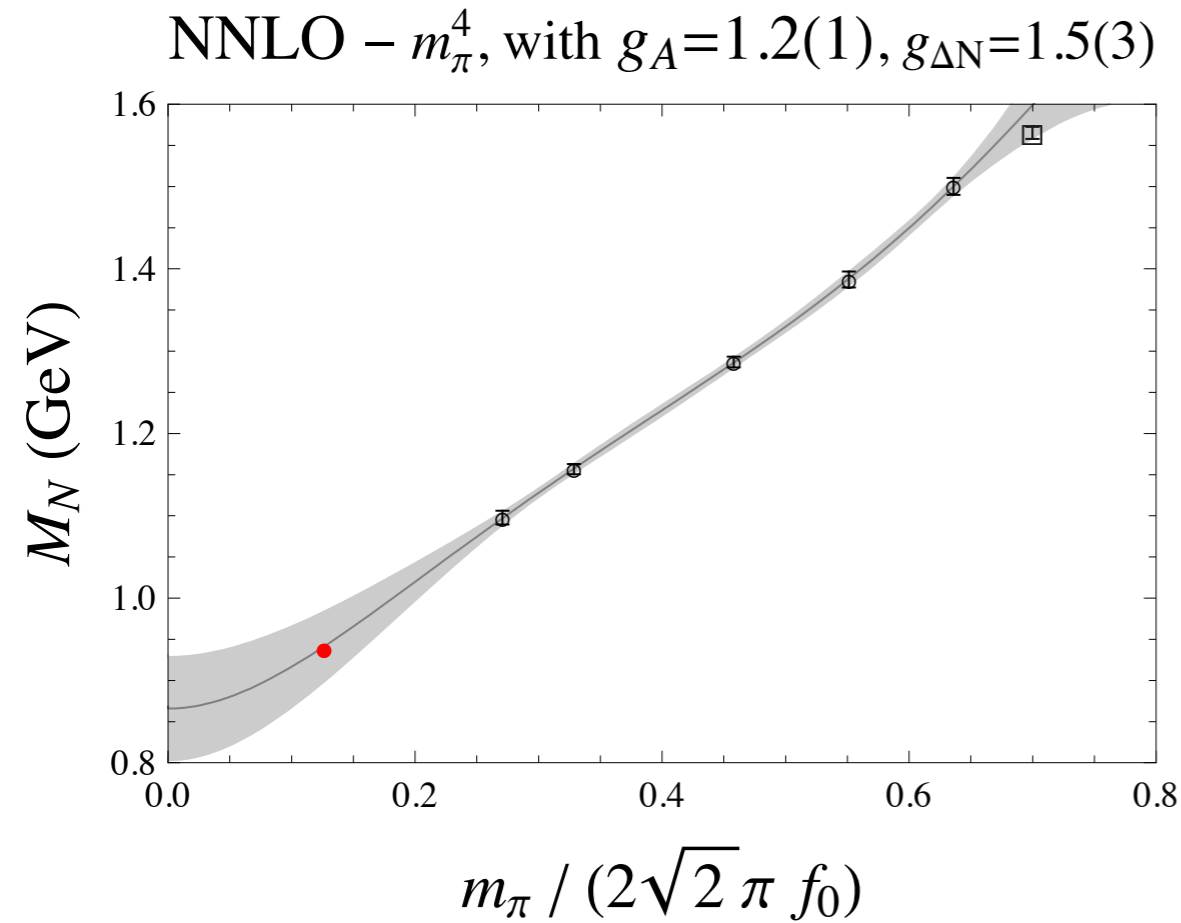
varying inputs

$g_A=1.2(1)$, $g_{\Delta N}=1.5(3)$



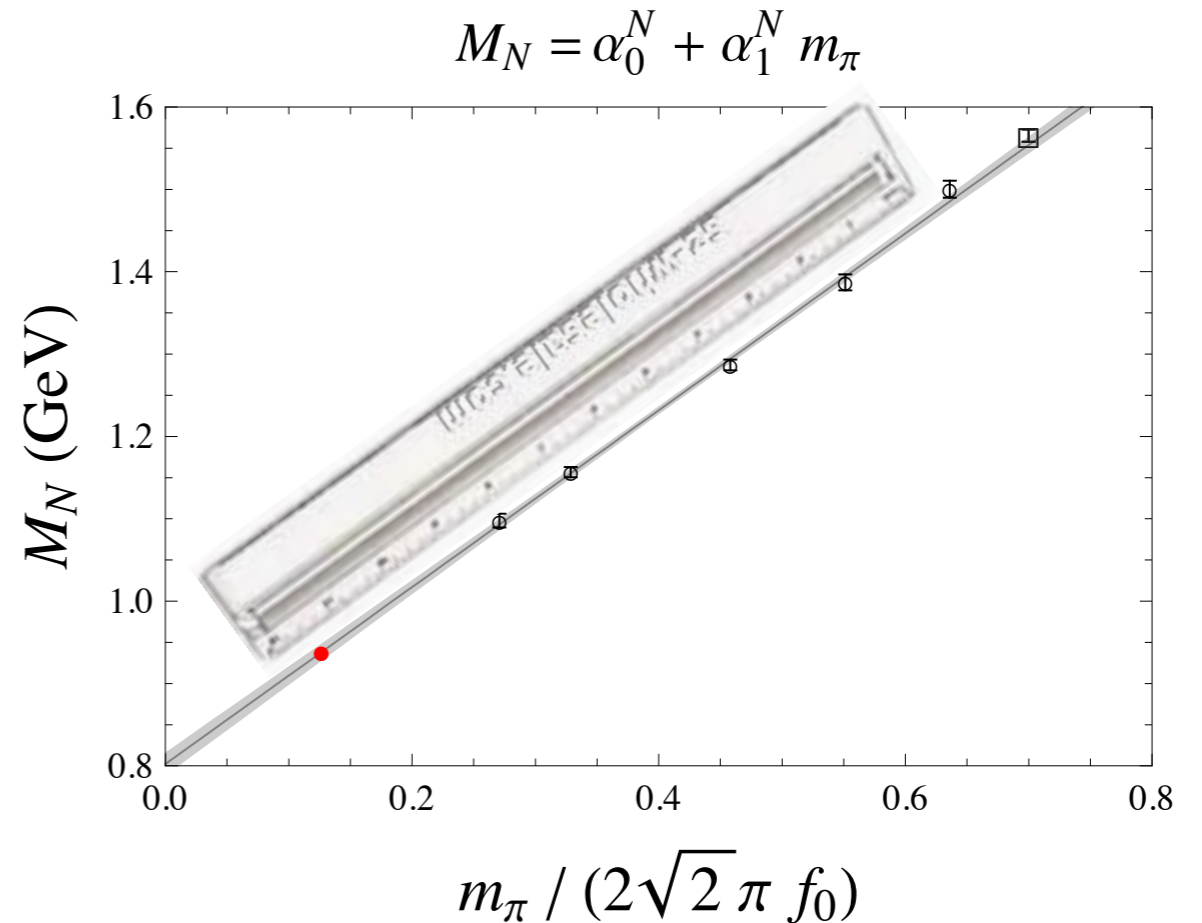
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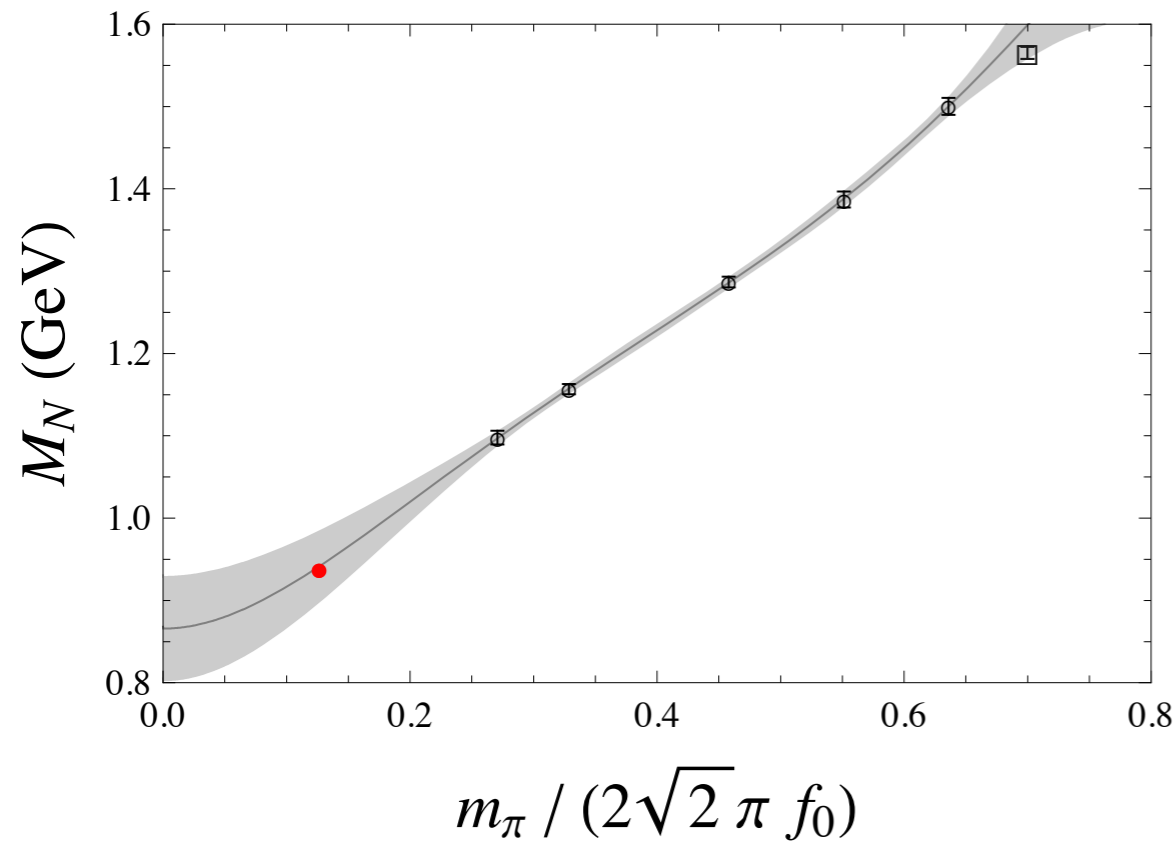
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Ruler Approximation

$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$

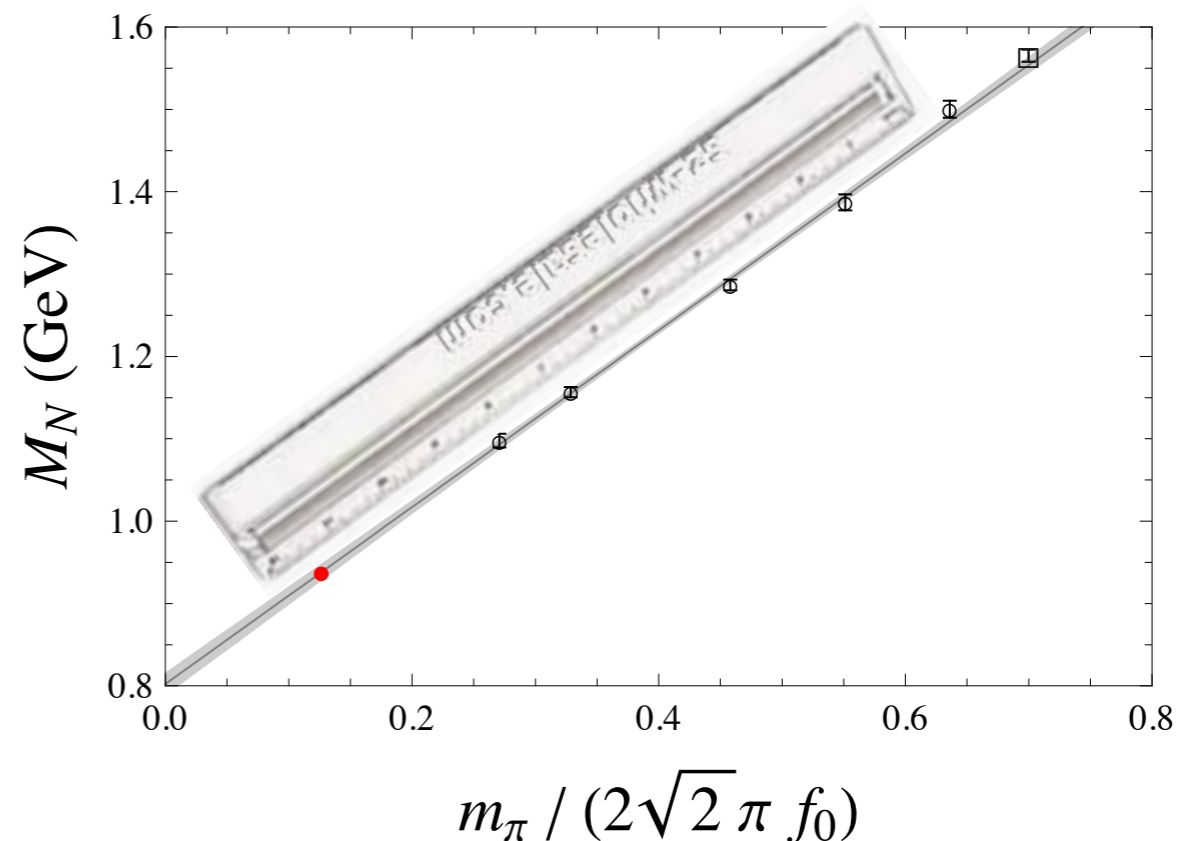
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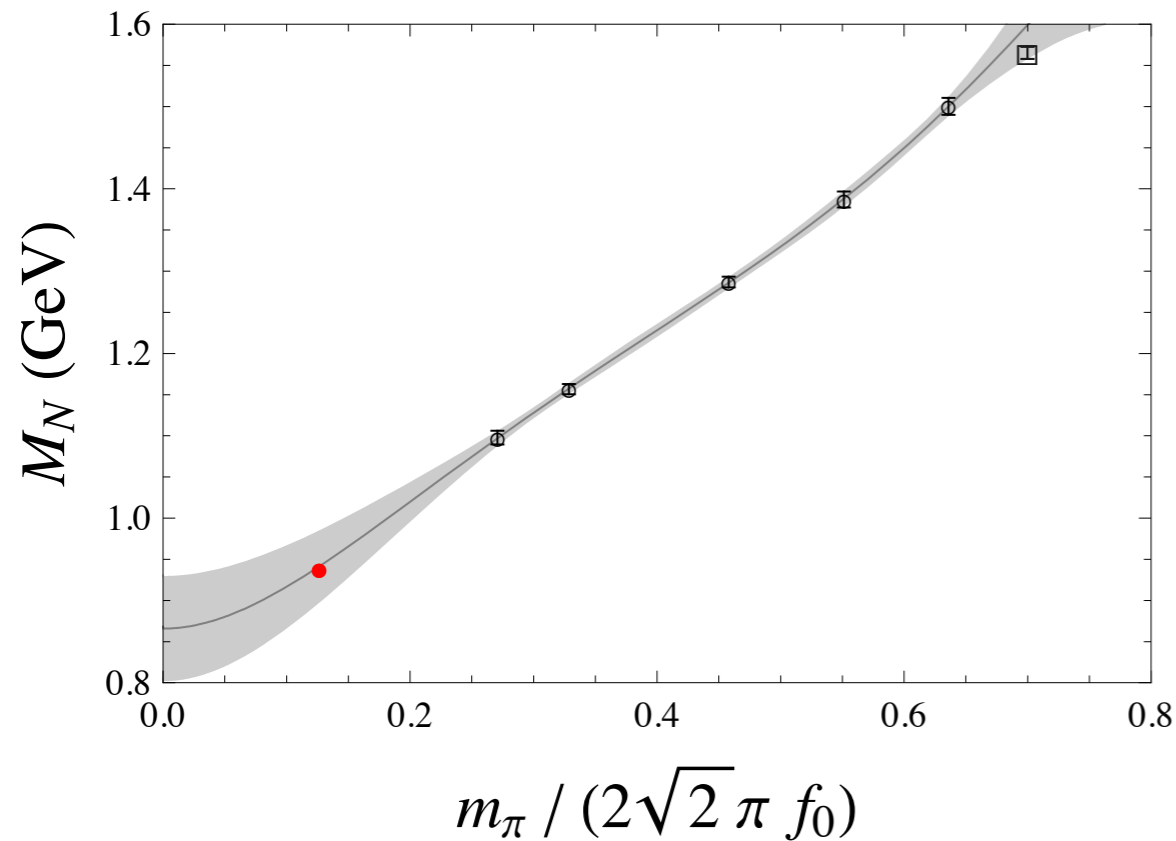
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Ruler Approximation

$$\begin{aligned} M_N &= \alpha_0^N + \alpha_1^N m_\pi \\ &= 938 \pm 9 \text{ MeV} \end{aligned}$$

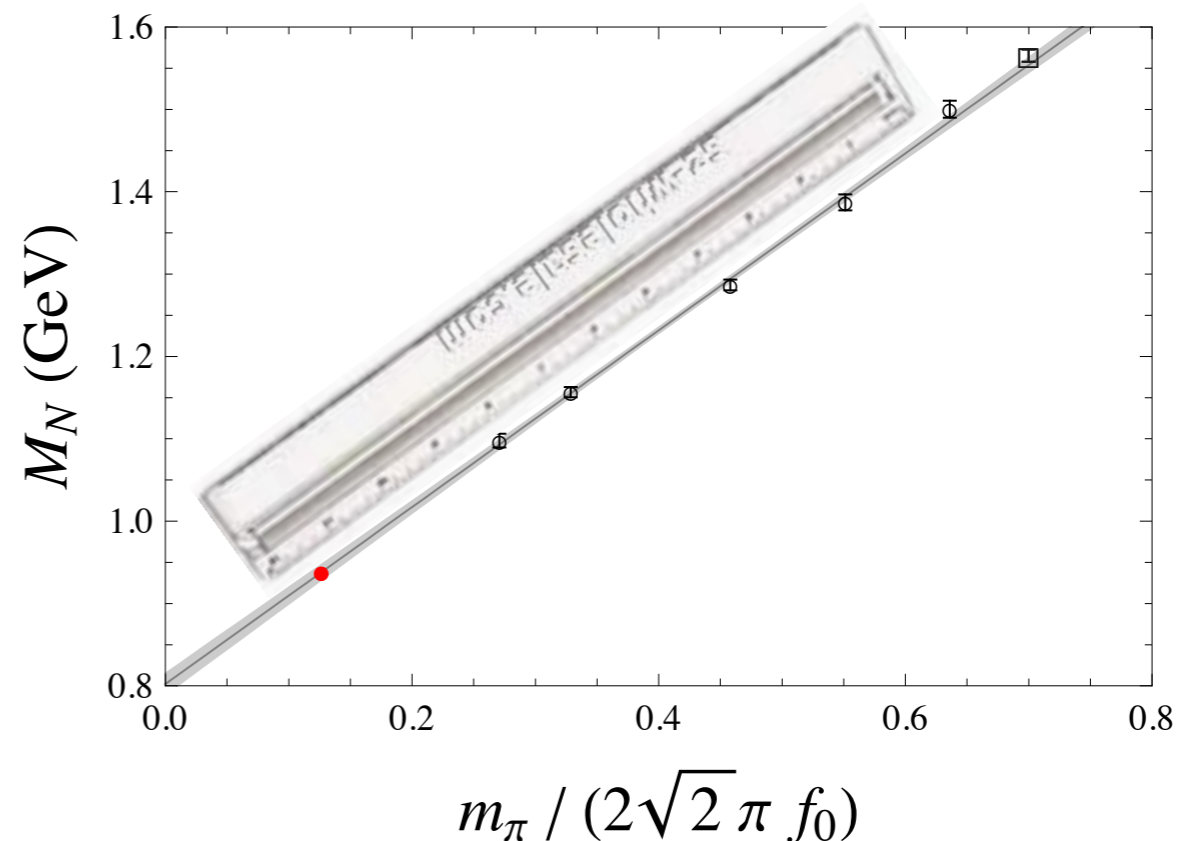
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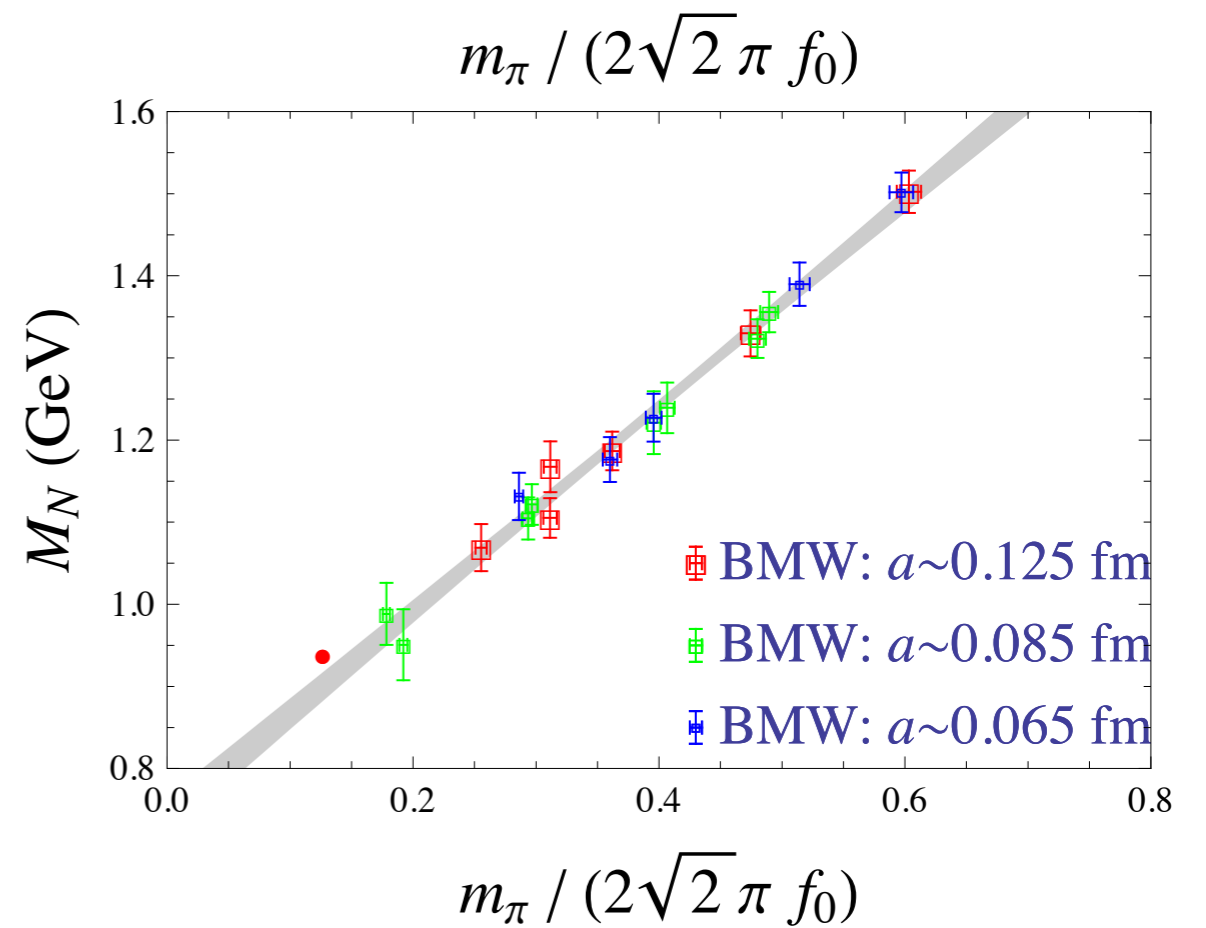
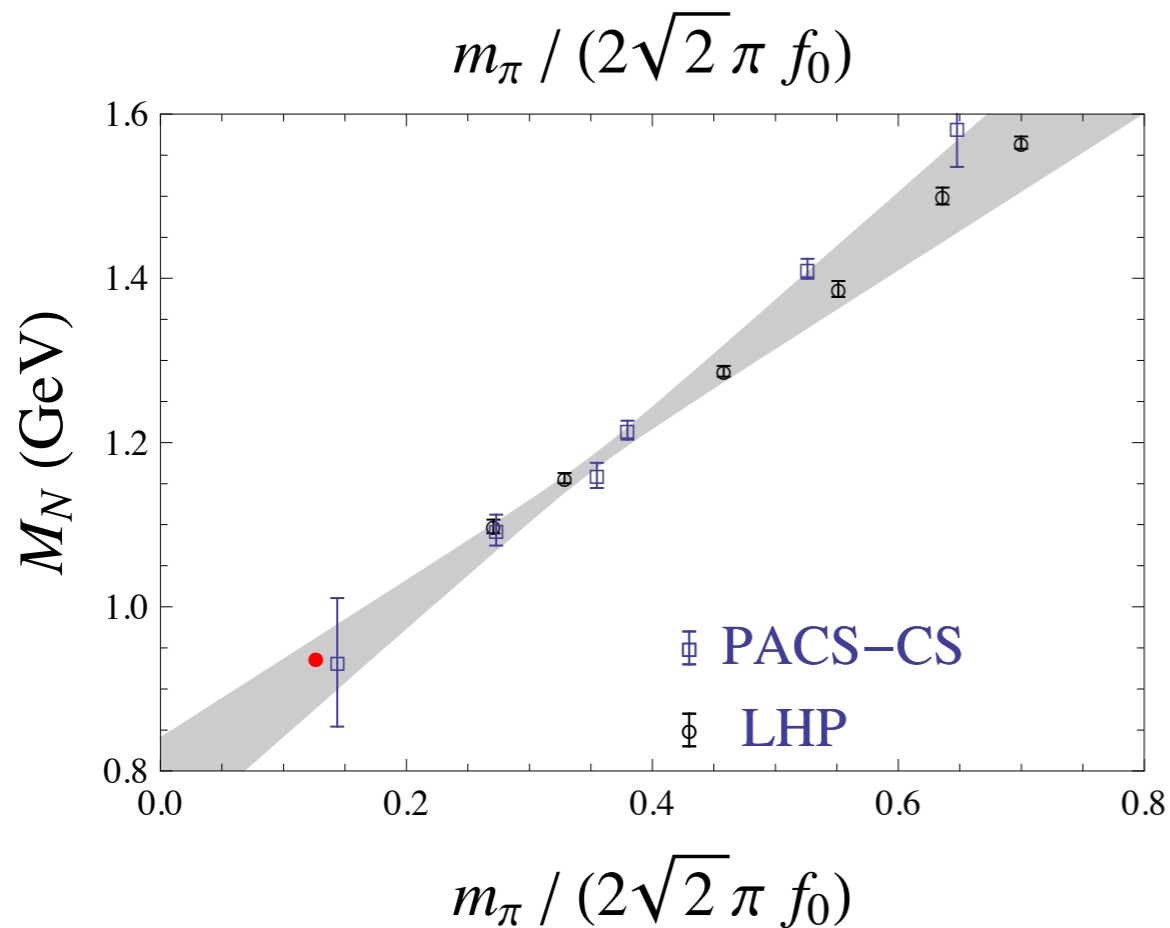
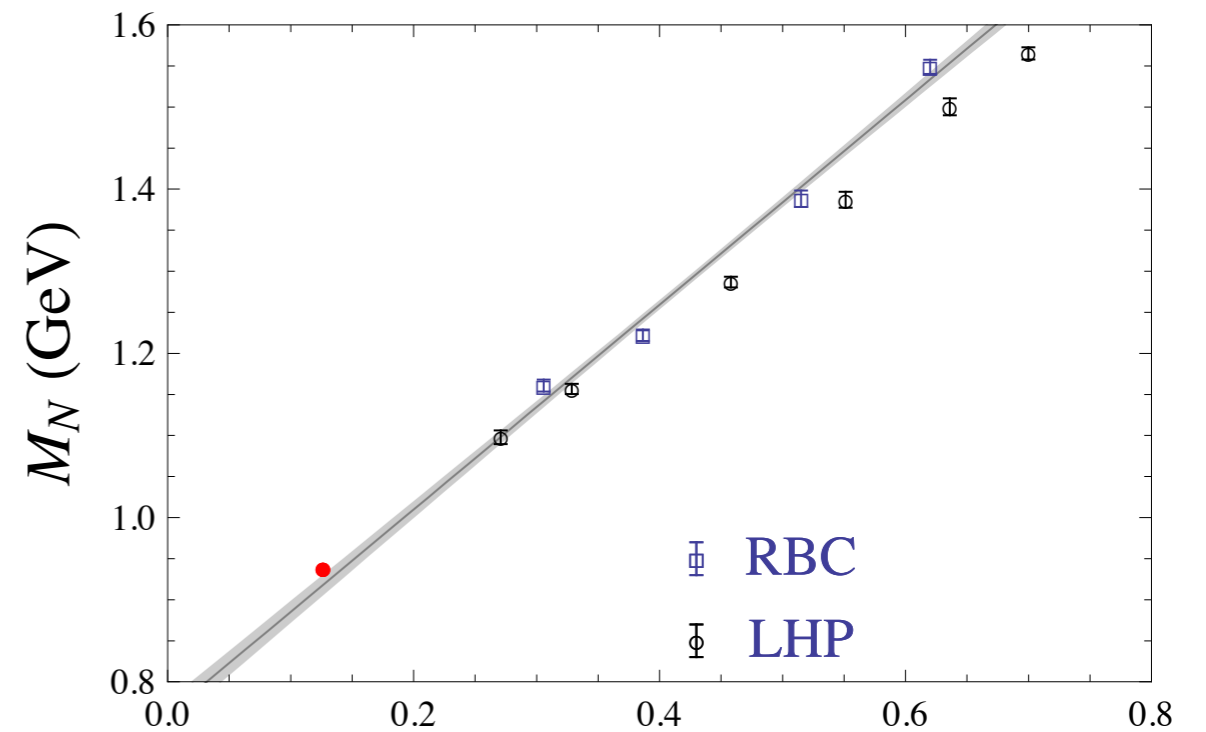
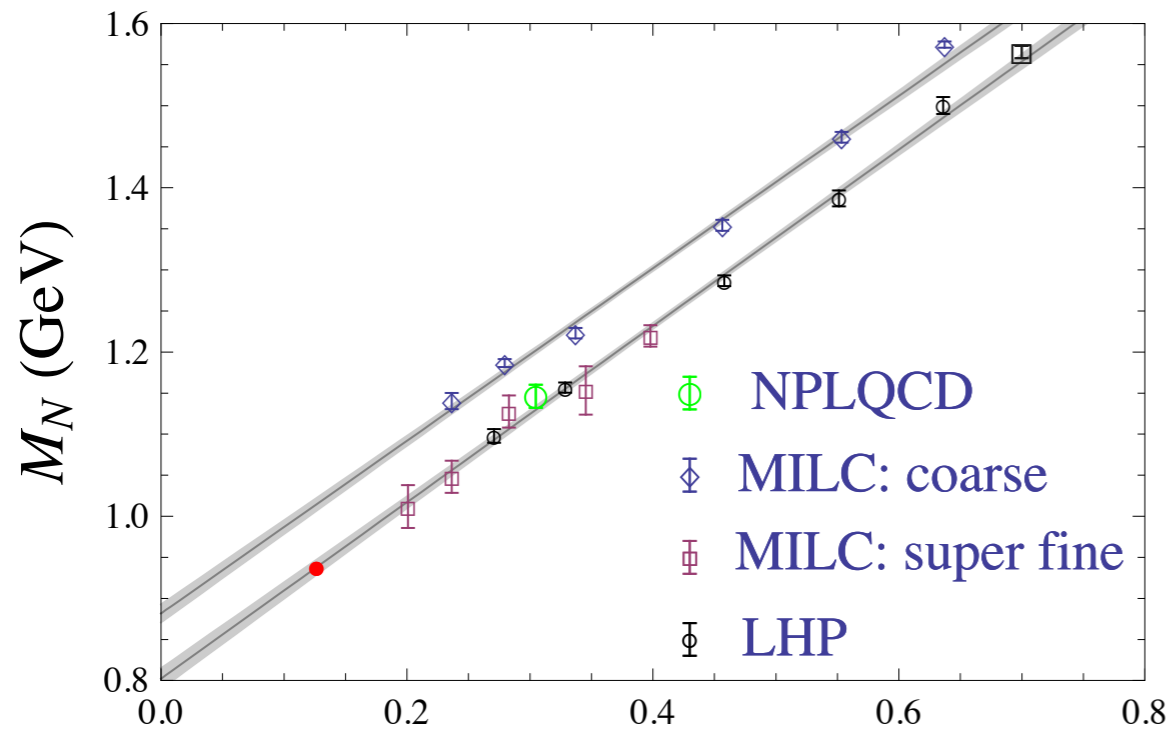
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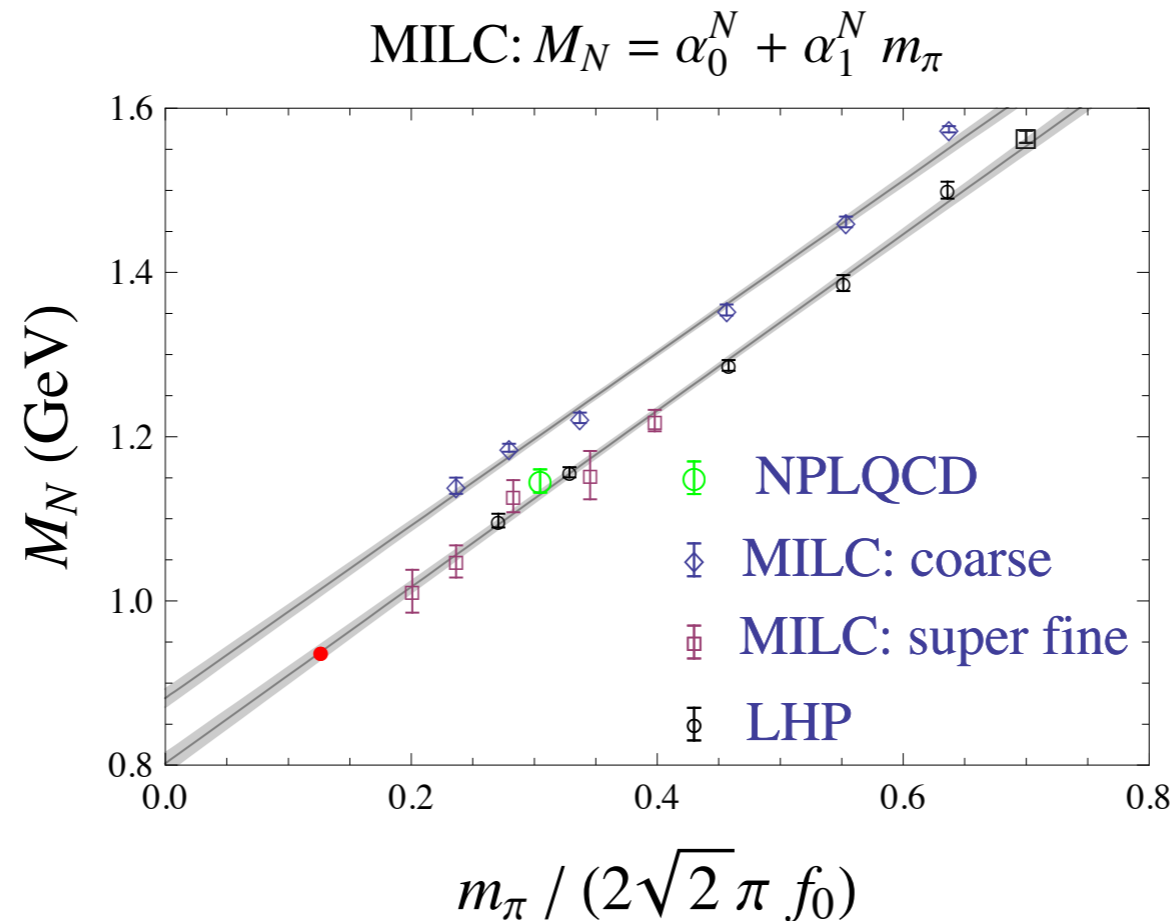
I am not advocating this as a good model for QCD!

MILC: $M_N = \alpha_0^N + \alpha_1^N m_\pi$

Latt 2008, arXiv:0810.0663

$M_N = \alpha_0^N + \alpha_1^N m_\pi$

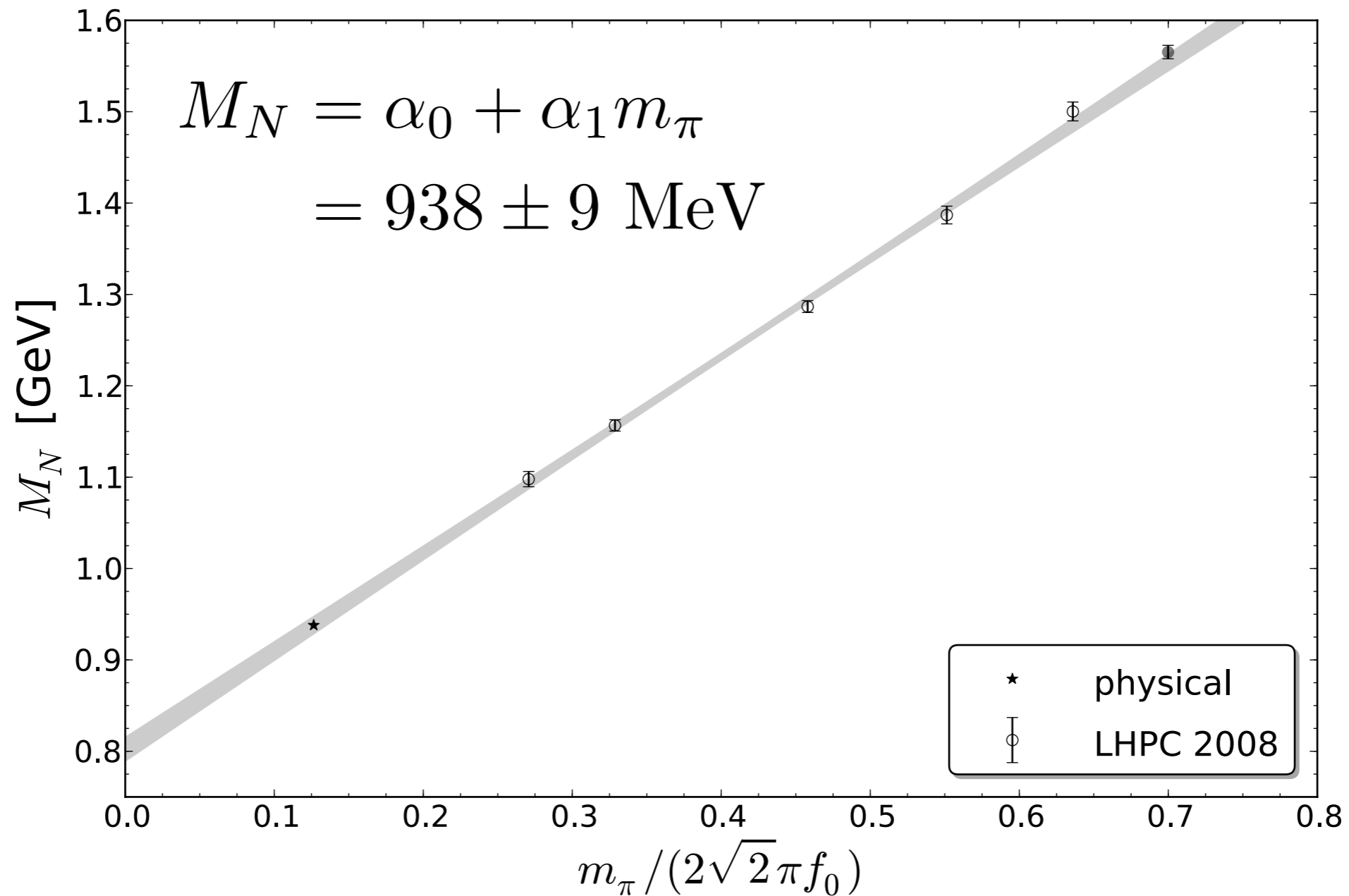




What does this teach us?

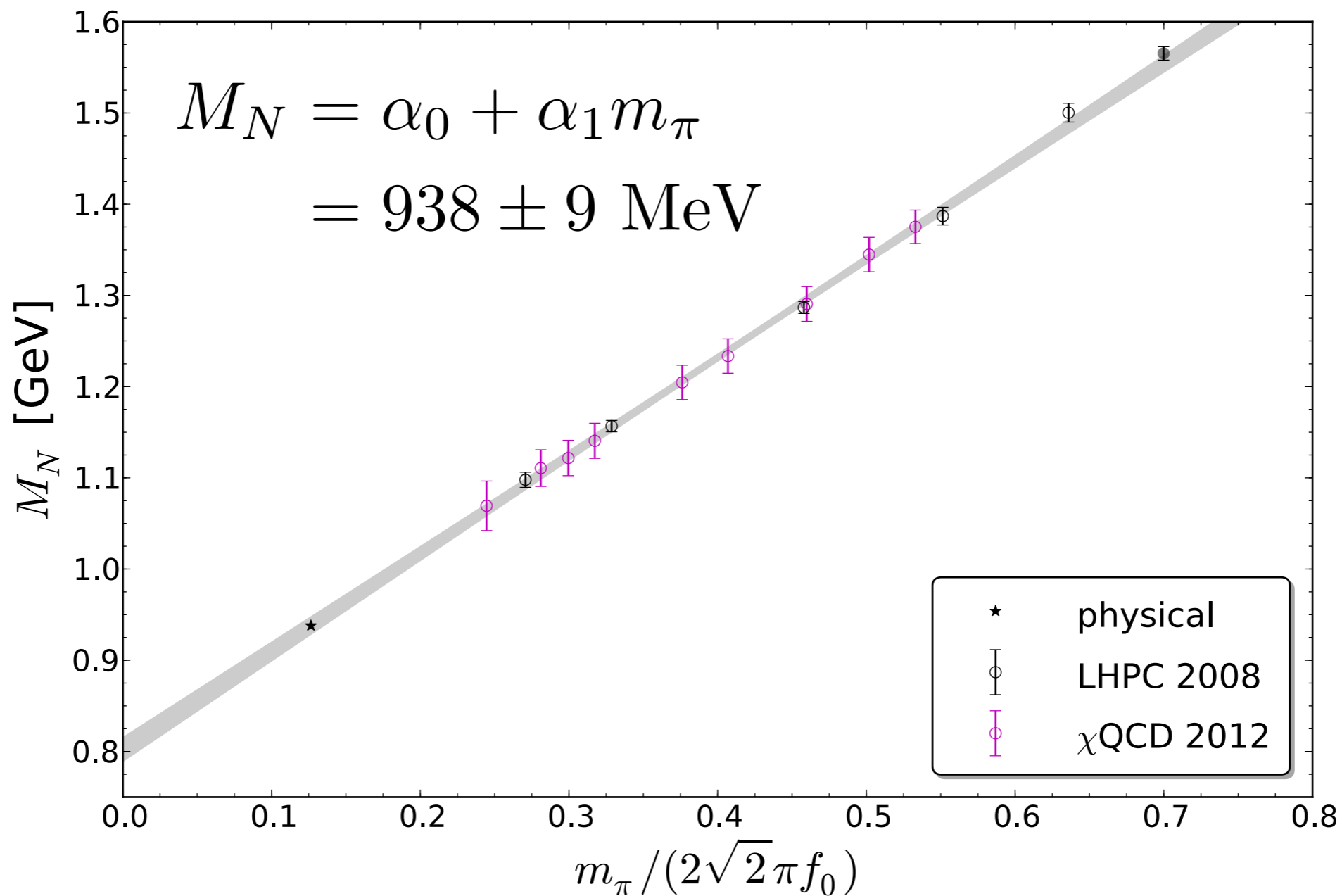
For these pion masses, there is a strong cancelation between LO, NLO and NNLO χ^{PT} contributions perhaps should have been expected given poor convergence (but just not a straight line!!!)

What is the status now (2012)?



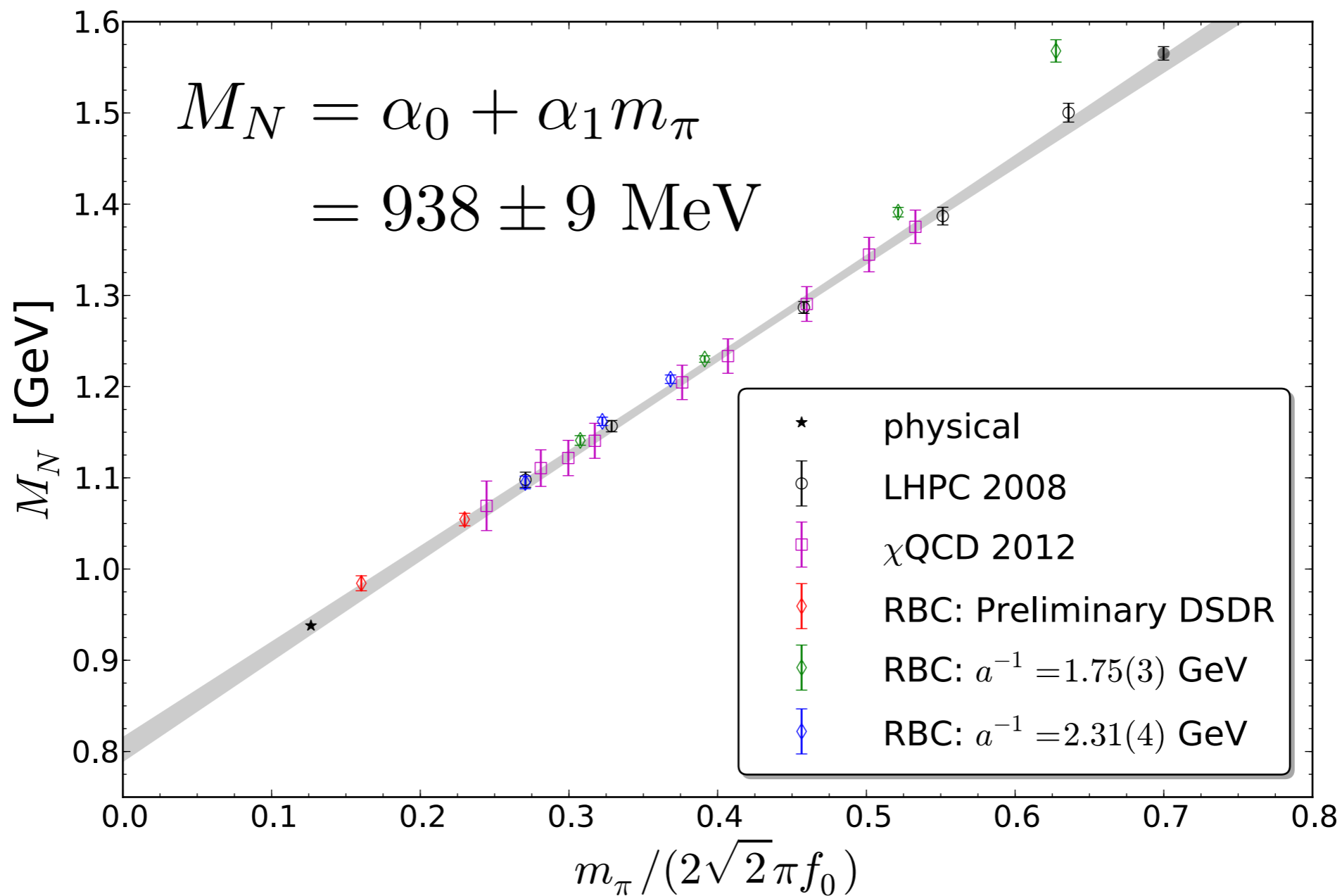
Physical point **NOT** included in fit

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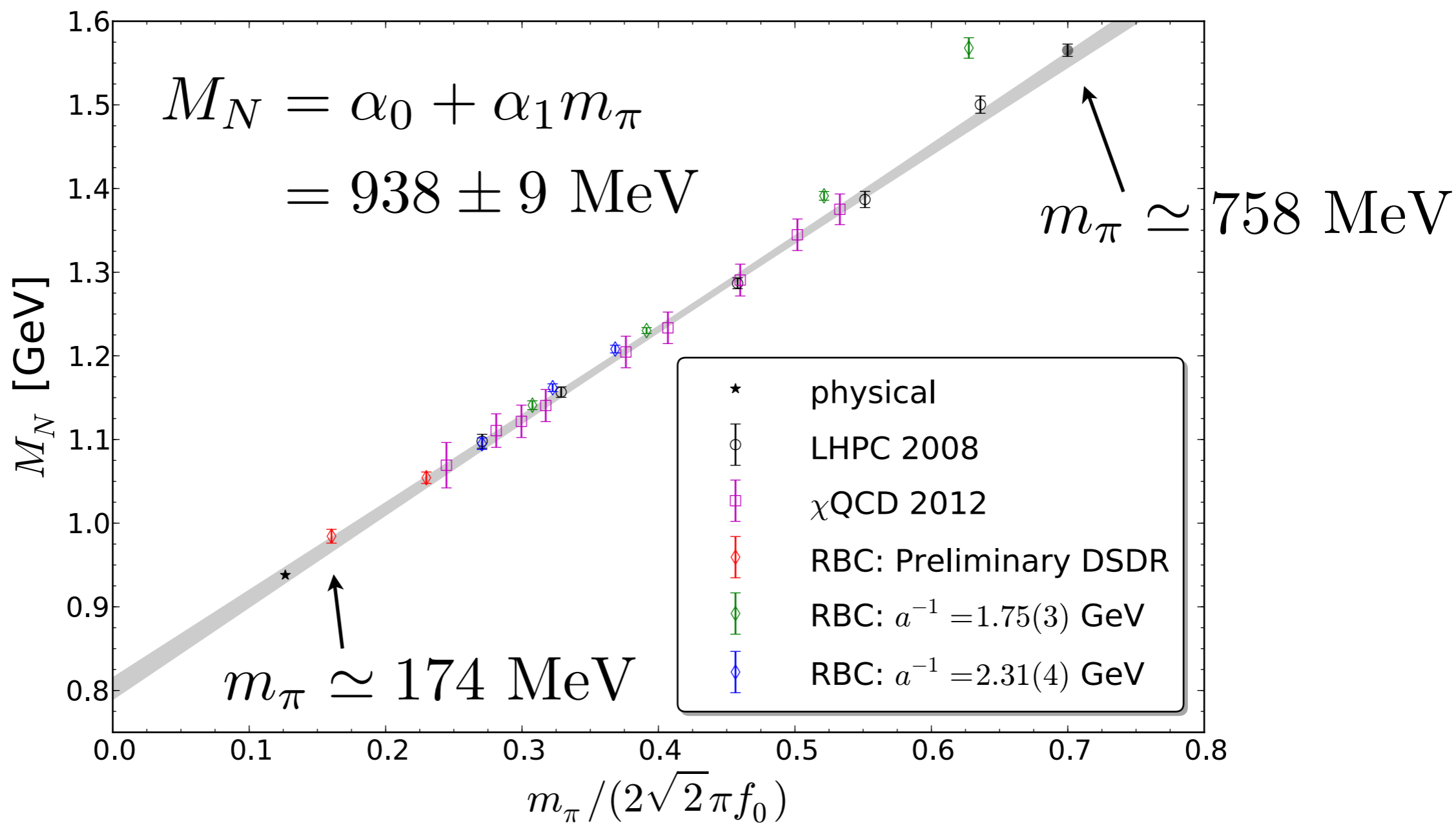
χ QCD Collaboration uses **Overlap Valence** fermions on **Domain-Wall** (RBC-UKQCD) sea fermions

What is the status now (2012)?



RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions

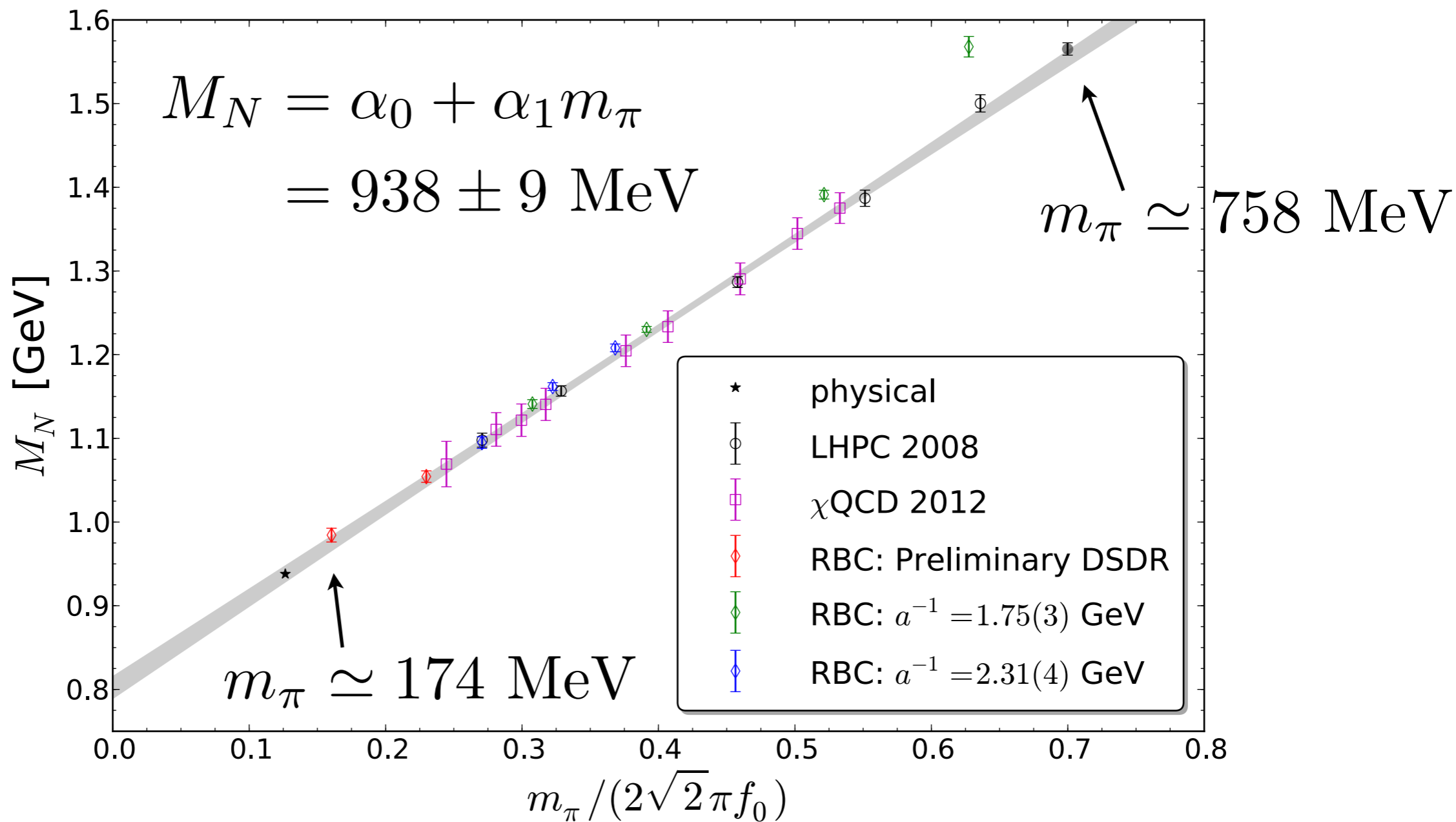
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Taking this seriously yields

$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

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Thank You