Inflationary Cosmology and Alternatives

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Outline

- Lectrure 1
  - Basics of cosmology
  - Cosmological perturbations
    or
    What are we learning about extremely early Universe
    or
    Why are we so excited by WMAP, Planck, galaxy surveys, etc.?

Expanding Universe

- The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean (observational fact!)
All this is encoded in space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$\mathbf{x}$: comoving coordinates, label distant galaxies.

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

$$z(t) = \frac{a_0}{a(t)} - 1 : \quad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength $\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$.

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

$$H_0 \approx 70 \ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9 \ \text{yrs})^{-1}$$

$1 \ \text{Mpc} = 3 \cdot 10^6 \ \text{light yrs} = 3 \cdot 10^{24} \ \text{cm}$

- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Fig.

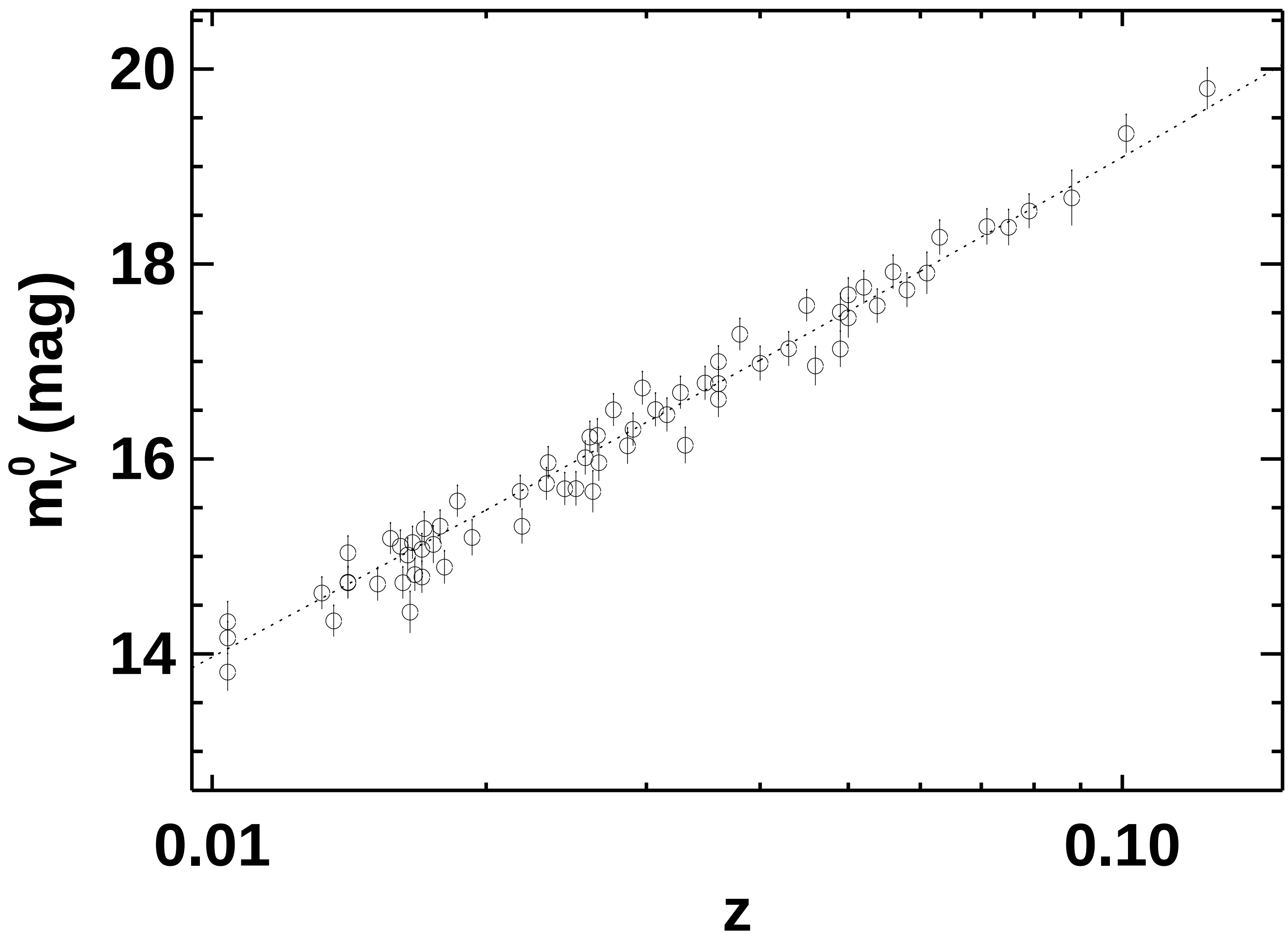
- The Universe is warm: CMB temperature today

$$T_0 = 2.725 \ \text{K}$$

Fig.

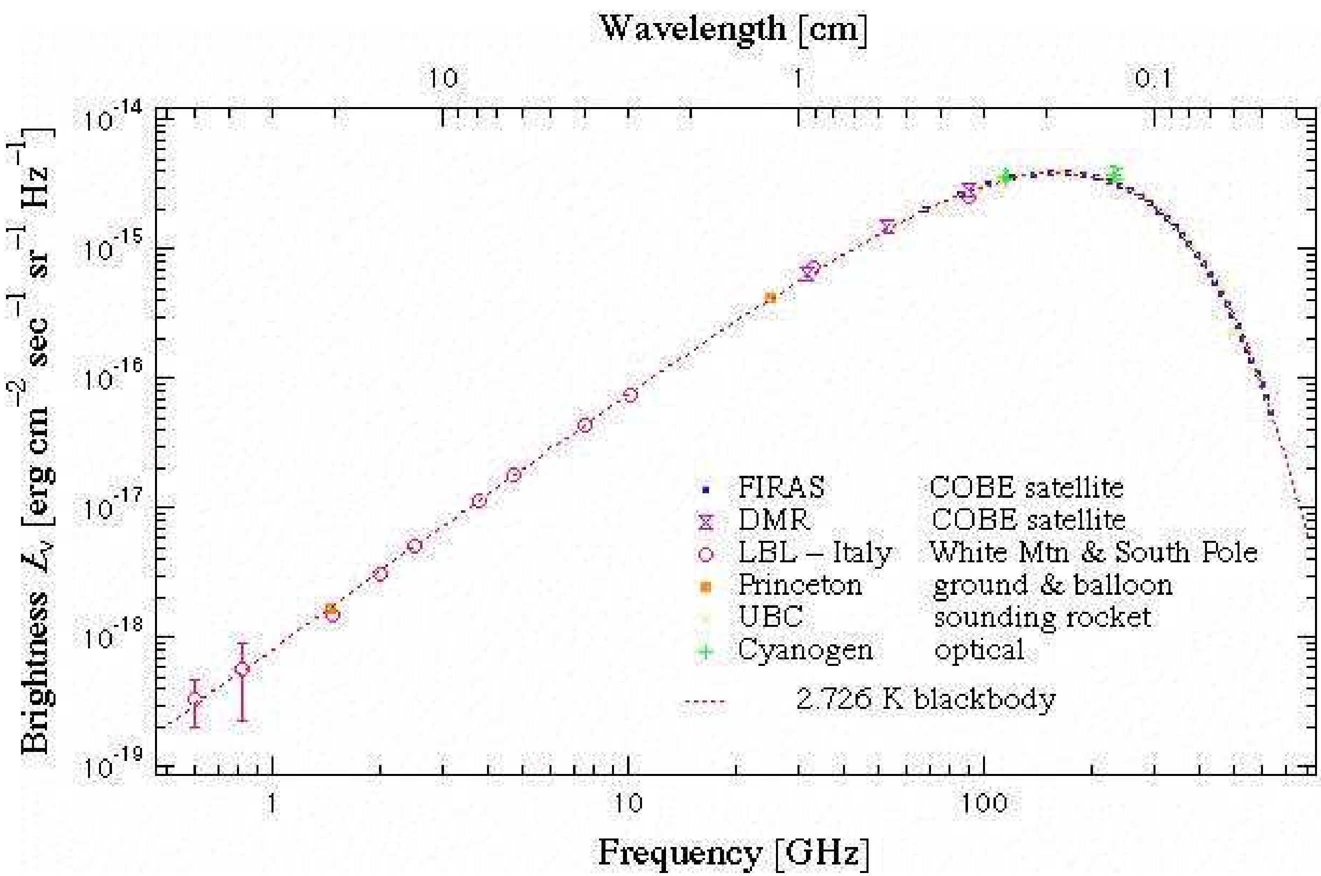
It was denser and warmer at early times.

Hubble diagram for SNe1a



$\text{mag} = 5\log_{10} r + \text{const}$

CMB spectrum



$T = 2.725$ K

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

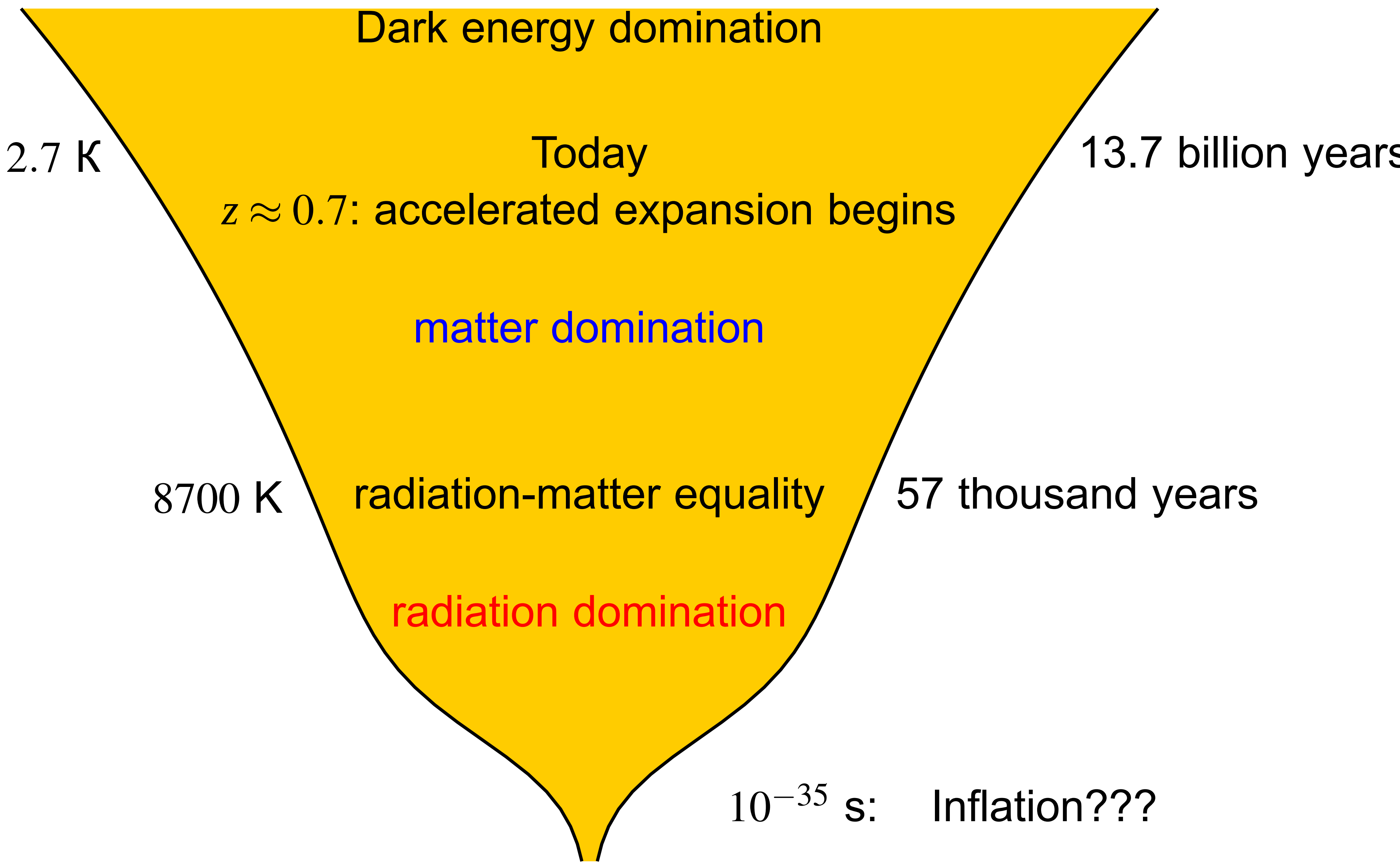
$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

  Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature.

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi} H_0^2 = 5 \cdot 10^{-6}\ \frac{\text{GeV}}{\text{cm}^3}$$

- Number density scales as $a^{-3}(t) \Longrightarrow$ energy (mass) density of non-relativistic matter scales as a^{-3}.
- Temperature scales as $a^{-1} \Longrightarrow$ energy density of relativistic matter scales as $T^4 \propto a^{-4}$. Dominates at early times.


Dark energy domination
2.7 K
Today
13.7 billion years
$z \approx 0.7$: accelerated expansion begins
matter domination
8700 K
radiation-matter equality
57 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}}\rho$$

- Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M^*_{Pl}}$$

with $M^*_{Pl} = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \quad \Longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

- Decelerated expansion: $\ddot{a} < 0$.

Cosmological (particle) horizon

Light travels along $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = 0 \Longrightarrow dx = dt/a(t)$.
If emitted at $t = 0$, travels finite coordinate distance

$$\eta = \int_0^t \frac{dt'}{a(t')} \propto \sqrt{t} \quad \text{at radiation domination}$$

$\eta \propto \sqrt{t} \Longrightarrow$ visible Universe increases in time

Fig.

Physical size of causally connected region at time t (horizon size)

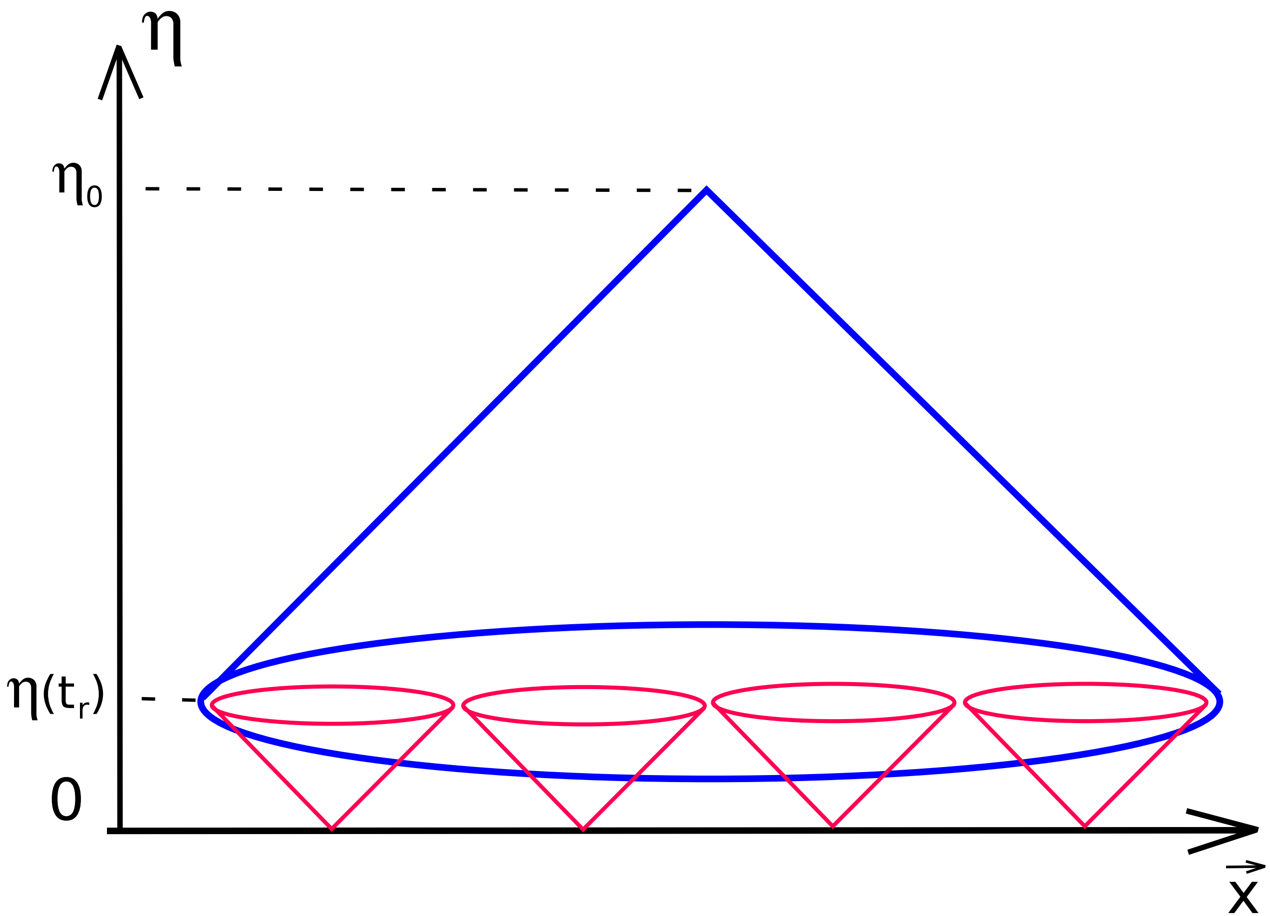
$$l_{H,t} = a(t) \int_0^t \frac{dt'}{a(t')} = 2t \quad \text{at radiation domination}$$

In hot Big Bang theory at both radiation and matter domination

$$l_{H,t} \sim t \sim H^{-1}(t)$$

Today $l_{H,t_0} \approx 15$ Gpc $= 4.5 \cdot 10^{28}$ cm

Causal structure of space-time in hot Big Bang theory



We see many regions that were causally disconnected by time t_r.
Why are they all the same?

With Big Bang nucleosynthesis theory and observations

we are confident of the theory of the early Universe

at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With the LHC, we hope to be able to go

up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: ⊙ density perturbations and associated gravitational potentials (3d scalar), observed;

⊙ gravitational waves (3d tensor), not observed (yet?).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

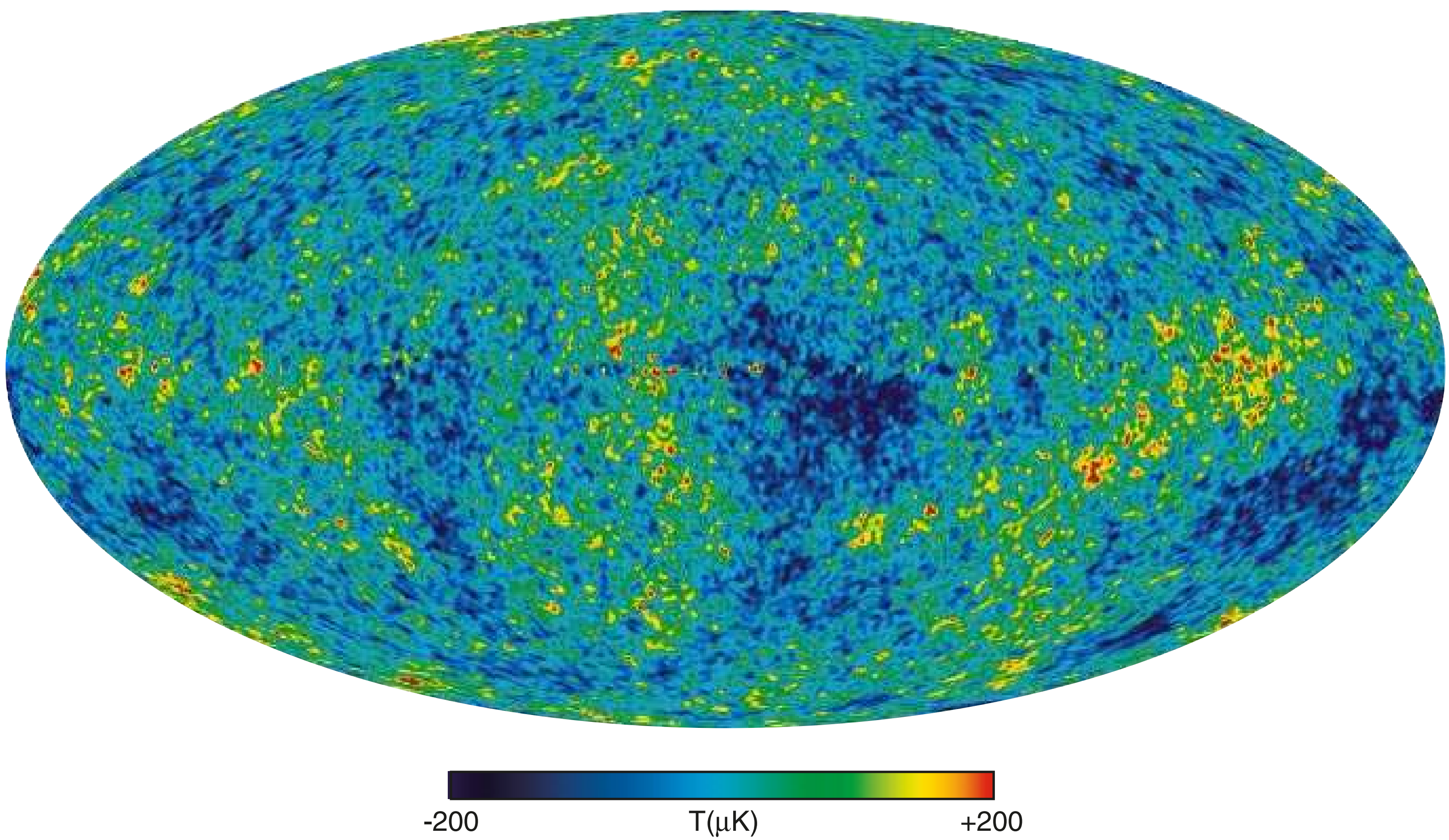
Linear analysis appropriate.

How are they measured?

- Cosmic microwave background: photographic picture of the Universe at age 380 000 yrs, $T = 3000$ K (transition from plasma to neutral gas, mostly hydrogen and helium)
  - Temperature anisotropy fig
  - Polarization fig
- Deep surveys of galaxies and quasars, cover good part of entire visible Universe
- Gravitational lensing, etc.

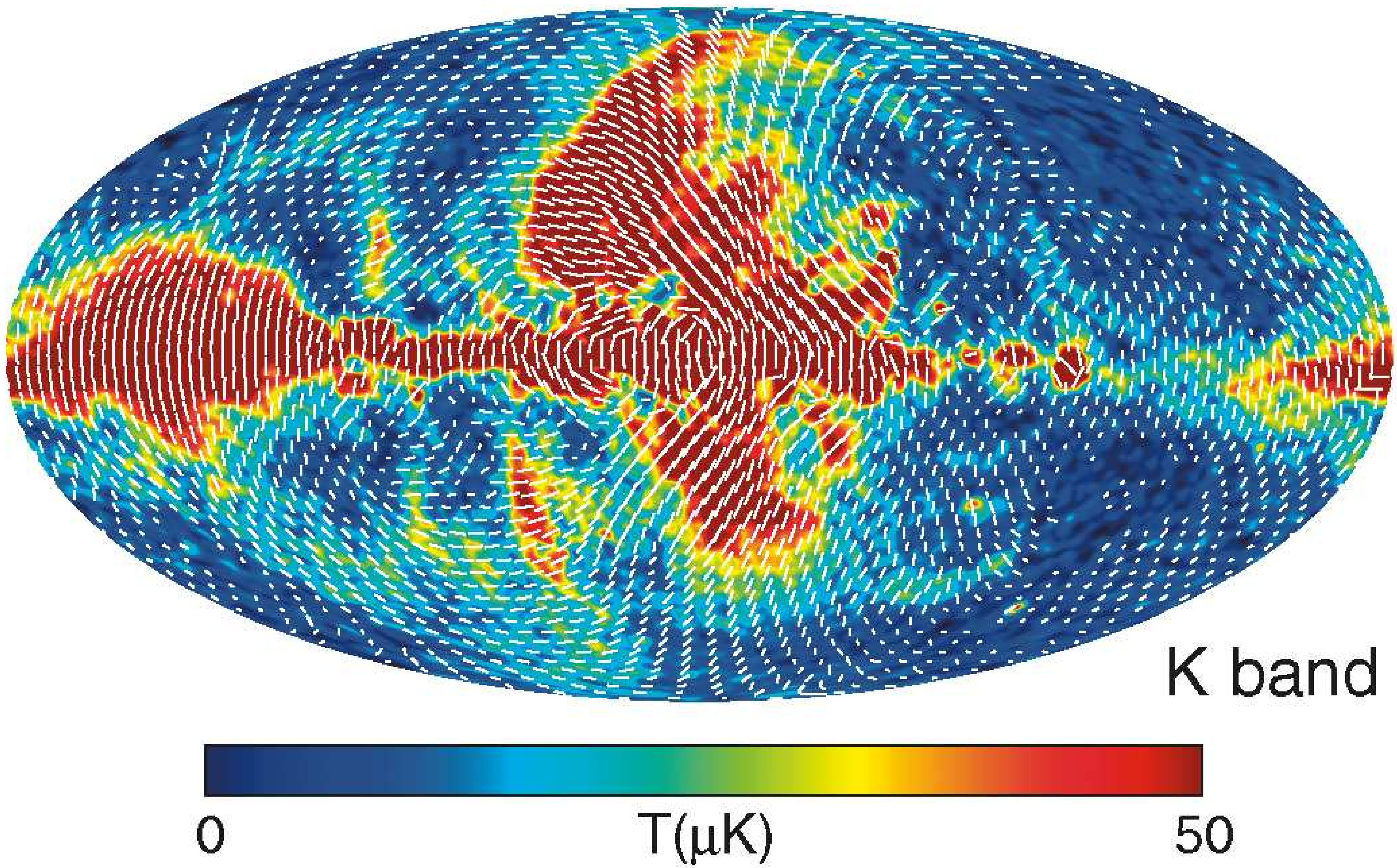
CMB temperature anisotropy

$$T = 2.726^\circ K, \ \ \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$

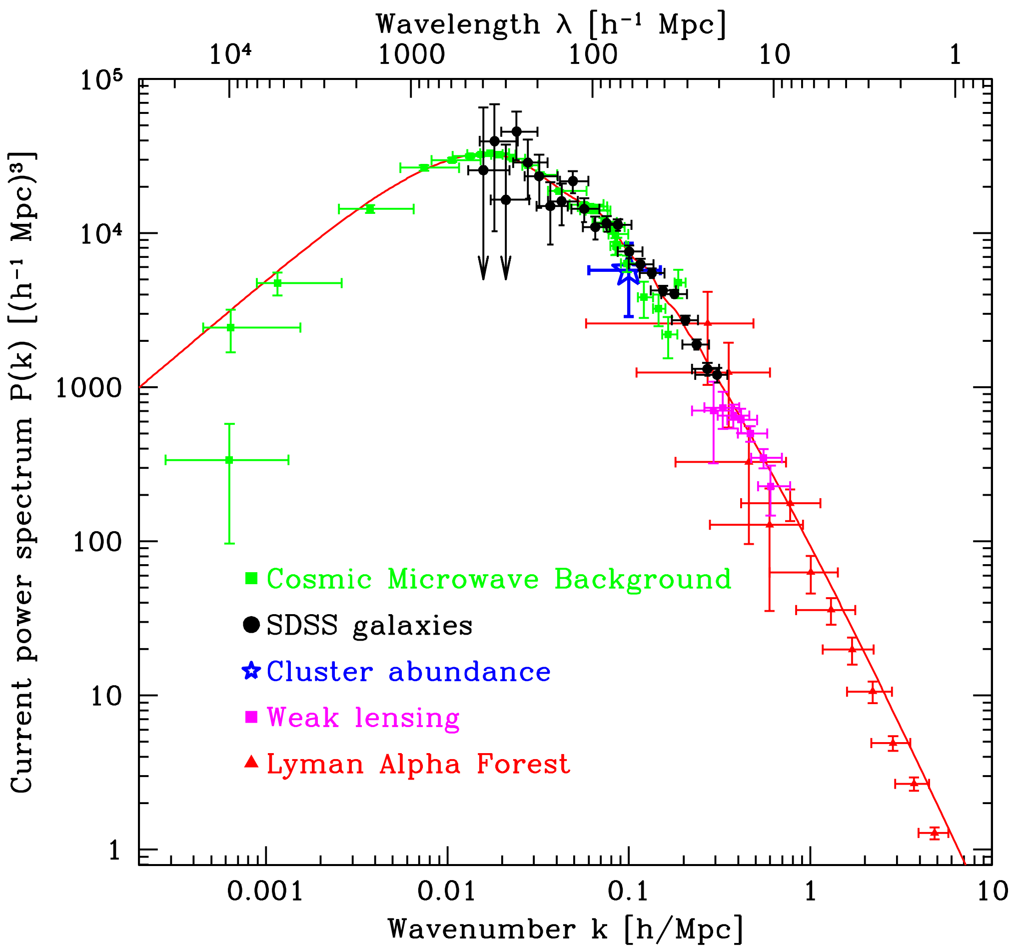


WMAP

CMB polarization map



Overall consistency



NB: density perturbations = random field.
k = wavenumber
$P(k)$ = power spectrum transfered to present epoch using linear theory

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We know that this is not the whole story!

Properties of perturbations in conventional (“hot”) Universe.

Reminder:
$a(t) \propto t^{1/2}$ at radiation domination stage (before $T \simeq 1$ eV,
$t \simeq 60$ thousand years)

Cosmological horizon at time t (assuming that nothing preceeded hot epoch): distance that light travels from Big Bang moment,

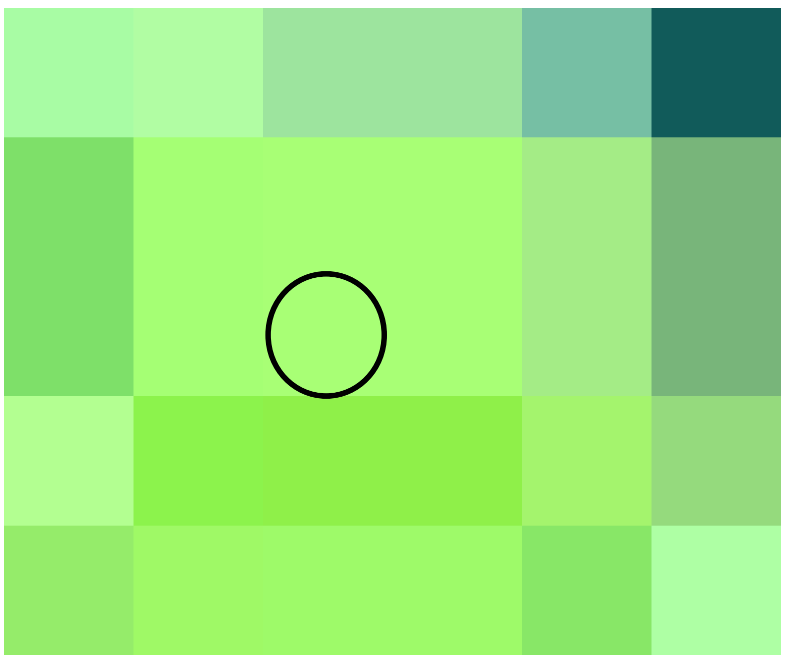
$$l_{H,t} \sim H^{-1}(t) \sim t$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

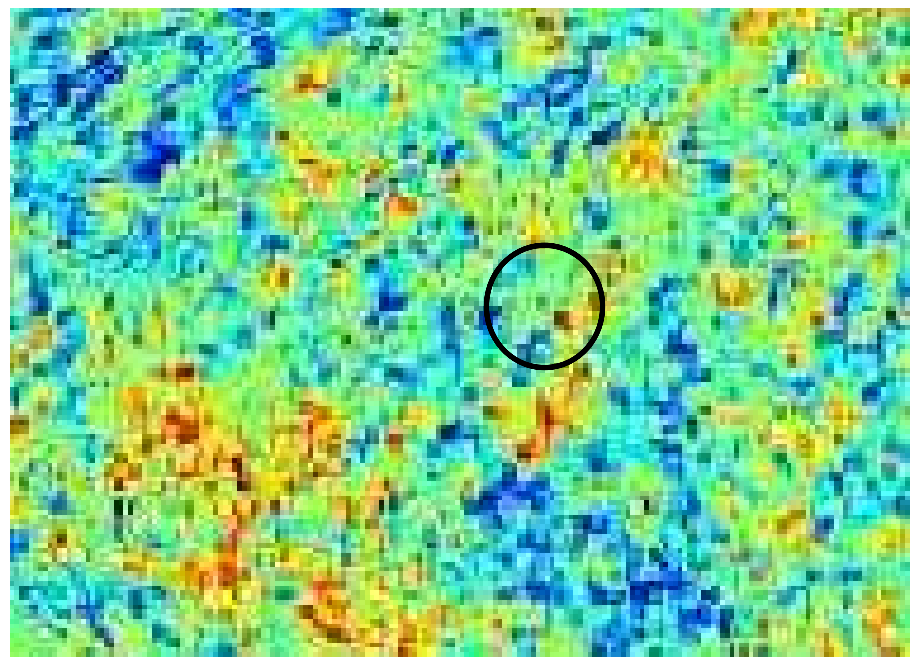
$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_{H,t} \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.



superhorizon mode



subhorizon mode

In other words, physical wavenumber (momentum) gets redshifted,

$$q(t) = \frac{2\pi}{\lambda(t)} = \frac{k}{a(t)}\,, \qquad k = \text{const} = \text{coordinate momentum}$$

Today

$$q > H \equiv \frac{\dot{a}}{a}$$

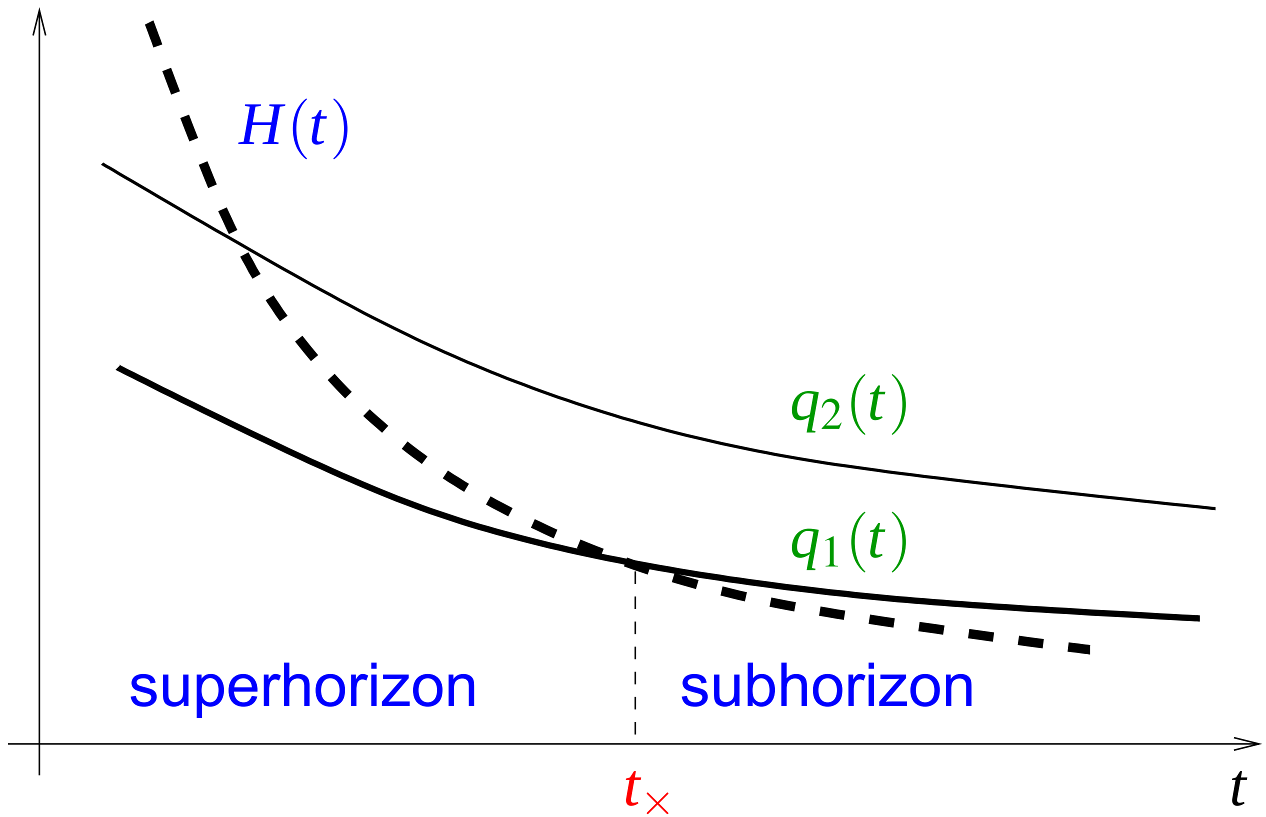
Early on

$$q(t) < H(t)$$

Very different regimes of evolution.

NB: Horizon entry occured after Big Bang Nucleosynthesis epoch for modes of all relevant wavelengths $\Longleftrightarrow$ no guesswork at this point.

Regimes at radiation (and matter) domination



$q_2 > q_1$

Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

  E.g., seeded by topological defects (cosmic strings, etc.)

  The only possibility, if expansion started from hot Big Bang.

  No longer an option!

- Hot epoch was preceeded by some other epoch. Perturbations were generated then.

Perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase. Why?

Prototype example: wave equation in expanding Universe (not exactly the same as equation for sound waves, but captures main properties).

Massless scalar field ϕ in FLRW spacetime: action

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$g_{\mu\nu} = (1, -a^2, -a^2, -a^2)$,
$g^{\mu\nu} = (1, -a^{-2}, -a^{-2}, -a^{-2})$,
$g = \text{det}\,(g_{\mu\nu}) = a^6$.

$$S = \frac{1}{2}\int d^3x dt \; a^3(t)\left(\dot{\phi}^2 - \frac{1}{a^2}\vec{\partial}\phi\cdot\vec{\partial}\phi\right)$$

Field equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\Delta\phi = 0$$

NB. $\dot{a}/a = H$: Hubble parameter.

Fourier decomposition in 3d space

$$\phi(\vec{x},t) = \int d^3k \; \mathrm{e}^{i\vec{k}\vec{x}}\phi_{\vec{k}}(t)$$

NB. $\vec{k}$: coordinate momentum, constant in time.
Physical momentum $q(t) = k/a(t)$ gets redshifted.

Wave equation in momentum space:

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{k^2}{a^2(t)}\phi = 0$$

- Redshift effect: frequency $\omega(t) = k/a(t)$.
- Hubble friction: the second term.

As promised, evoltion is different for $k/a > H$ (subhorizon regime) and $k/a < H$ (superhorizon regime).

Subhorion regime (late times): damped oscillations

$$\phi_{\vec{k}}(t) = \frac{A_{\vec{k}}}{a(t)} \cos\left(\int_0^t \frac{k}{a(t)}\, dt + \psi \right), \qquad \psi = \text{ arbitrary phase}$$

NB. Subhorizon sound waves in baryon-photon plasma:
– Amplitude of $\delta\rho/\rho$ does not decrease
– Sound velocity v_s different from 1 ($v_s \approx 1/\sqrt{3}$).
All the rest is the same

Solution to wave equation in superhorizon regime (early times) at radiation domination, $H = 1/(2t)$:

$$\phi = \text{const} \qquad \text{and} \qquad \phi = \frac{\text{const}}{t^{3/2}}$$

Constant and decaying modes.

NB: decaying mode is sometimes called growing, it grows as $t \to 0$.

Same story for density perturbations.

$\delta\rho/\rho \propto t^{-3/2}$: very inhomogeneous Universe at early times $\Longrightarrow$ inconsistency

Under assumption that modes were superhorizon, the initial condition is unique (up to overall amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \;\Longrightarrow \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\Longrightarrow$ phase of oscillations uniquely defined; $\psi = 0$.

Perturbations come to the time of photon last scattering (= recombination) with different phases, depending on wave vector:

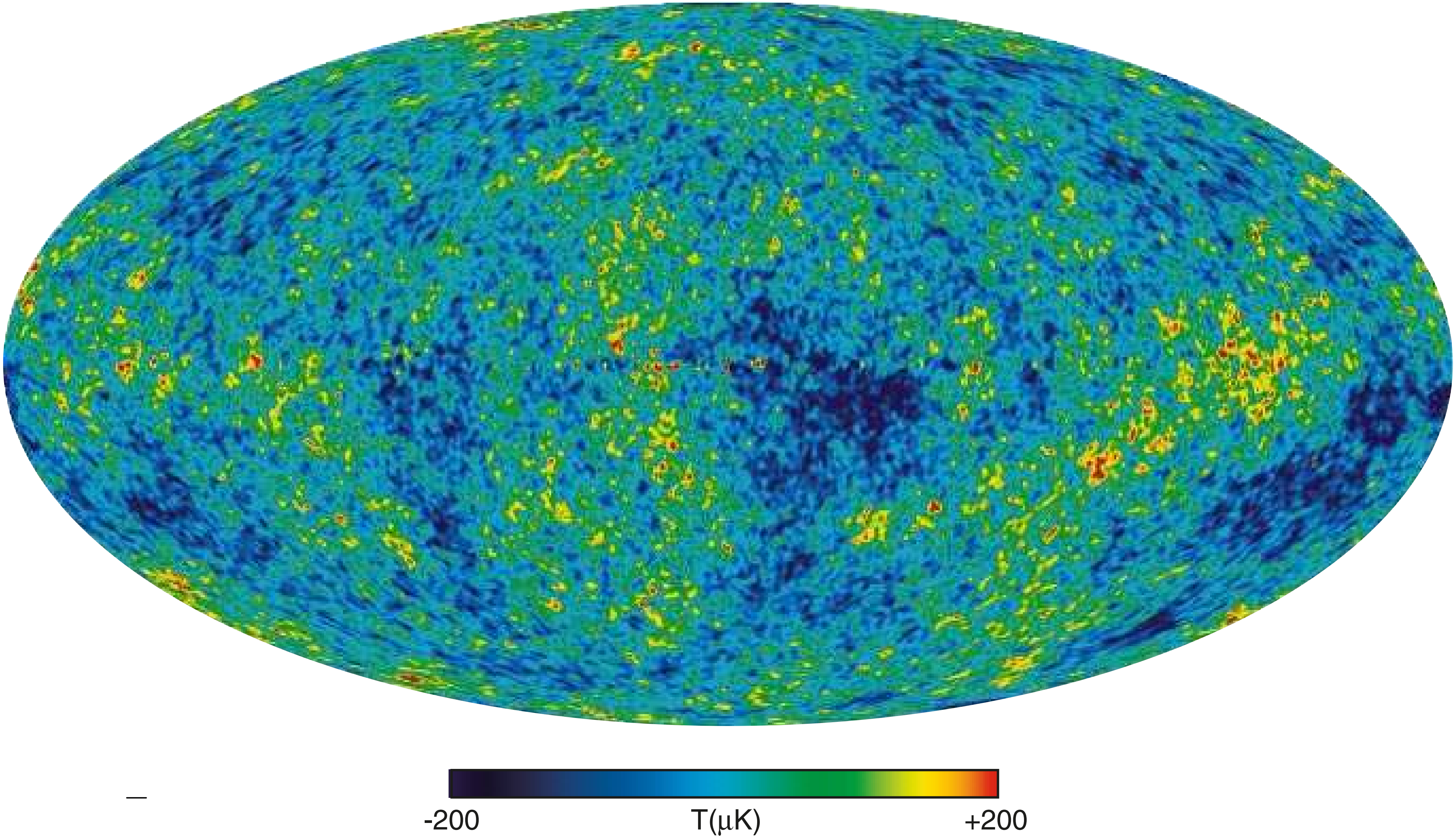
$$\delta(t_r) \equiv \frac{\delta\rho}{\rho}(t_r) \propto \cos\left(k\int_0^{t_r} dt\ \frac{v_s}{a(t)}\right) = \cos(kr_s)$$

r_s: sound horizon at recombination, $a_0 r_s = 150$ Mpc.

Waves with $k = \pi n/r_s$ have large $|\delta\rho|$, while waves with $k = (\pi n + 1/2)/r_s$ have $|\delta\rho| = 0$ in baryon-photon component.

This translates into oscillations in CMB angular spectrum

$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-5}$$



WMAP

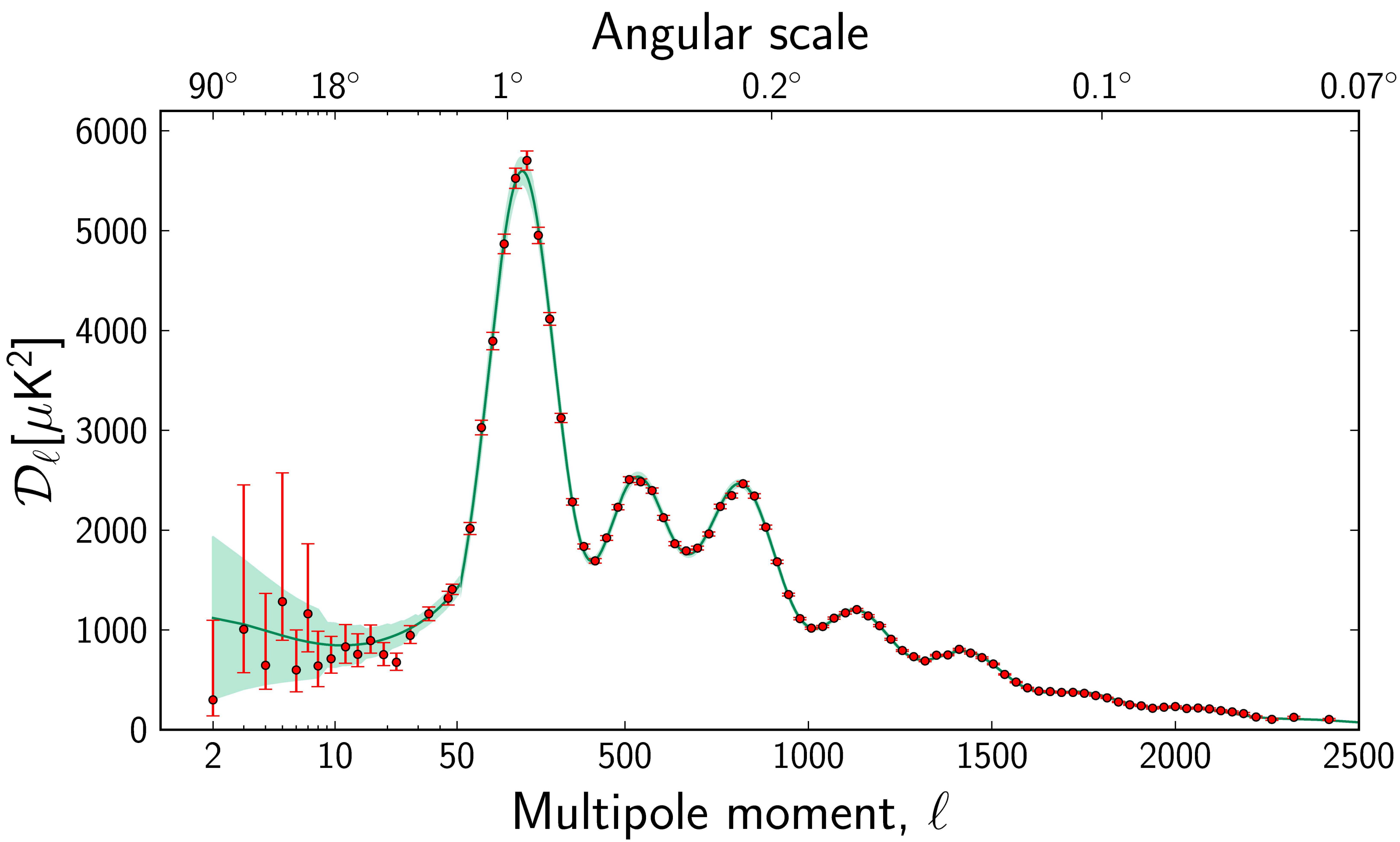
Fourier decomposition of temperatue fluctuations:

$$\frac{\delta T}{T}(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

a_{lm}: independent Gaussian random variables, $\langle a_{lm} a^*_{l'm'} \rangle \propto \delta_{ll'} \delta_{mm'}$
$\langle a^*_{lm} a_{lm} \rangle = C_l$ are measured; usually shown $D_l = \frac{l(l+1)}{2\pi} C_l$
larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths
NB: One Universe, one realization of an ensemble $\Longrightarrow$ cosmic variance $\Delta C_l / C_l \simeq 1/\sqrt{2l}$

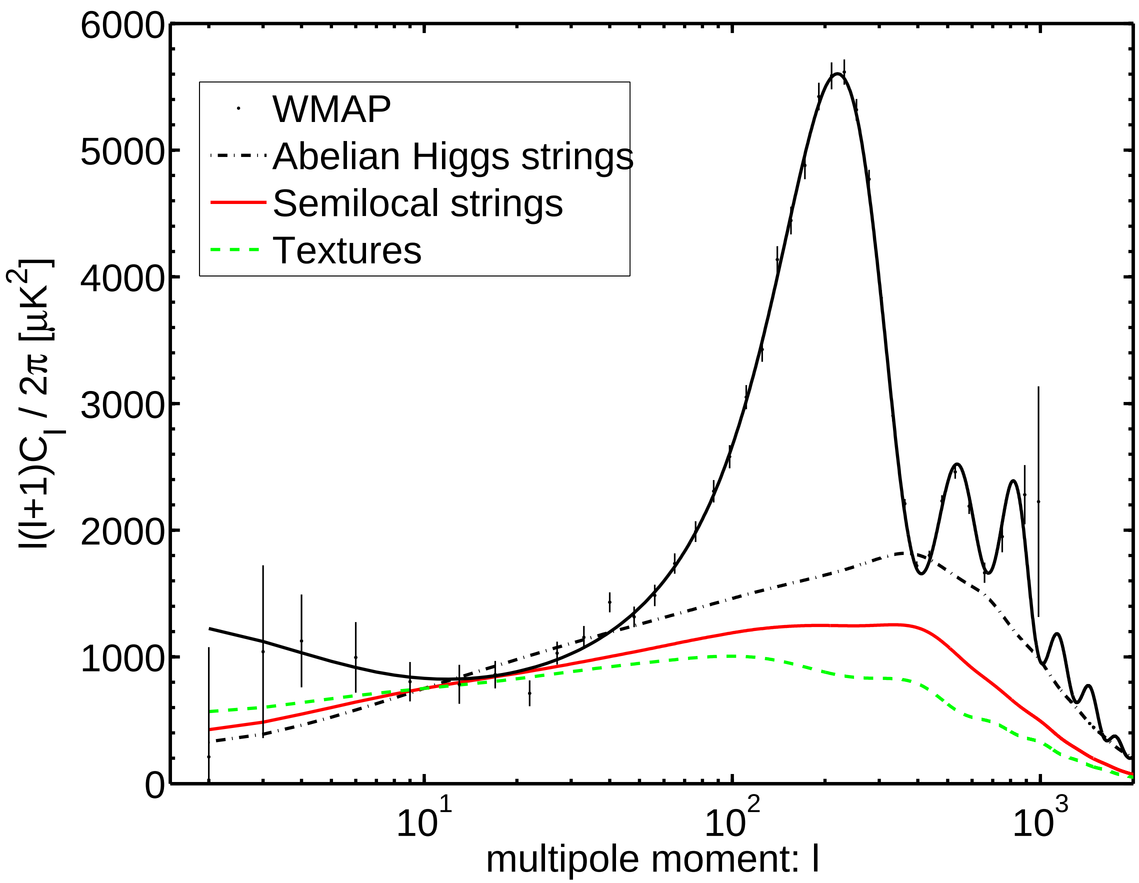
- Physics:
  - Primordial perturbations
  - Development of sound waves in cosmic plasma from early hot stage to recombination
    $\Longrightarrow$ composition of cosmic plasma
  - Propagation of photons after recombination
    $\Longrightarrow$ expansion history of the Universe

CMB angular spectrum



Furthermore, there are perturbations which were superhorizon at the time of photon last scattering (low multipoles, $l \lesssim 50$)

These properties would not be present if perturbations were generated at hot epoch in causal manner: phase ψ would be random function of k, no oscillations in CMB angular spectrum.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long and unusual: perturbations were subhorizon early at that epoch, our visible part of the Universe was in a causally connected region.

- Excellent guess: inflation

Starobinsky’79; Guth’81; Linde’82; Albrecht and Steinhardt’82

- Yet there are alternatives

Other suggestive observational facts about density perturbations (valid within certain error bars!)

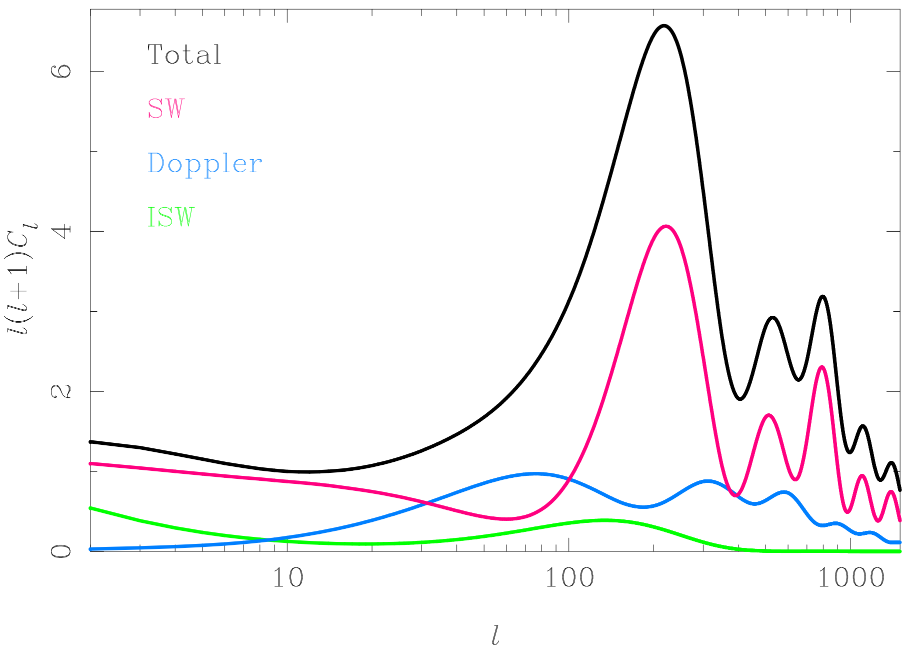
- Perturbations in overall density, not in composition (jargon: “adiabatic”)

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

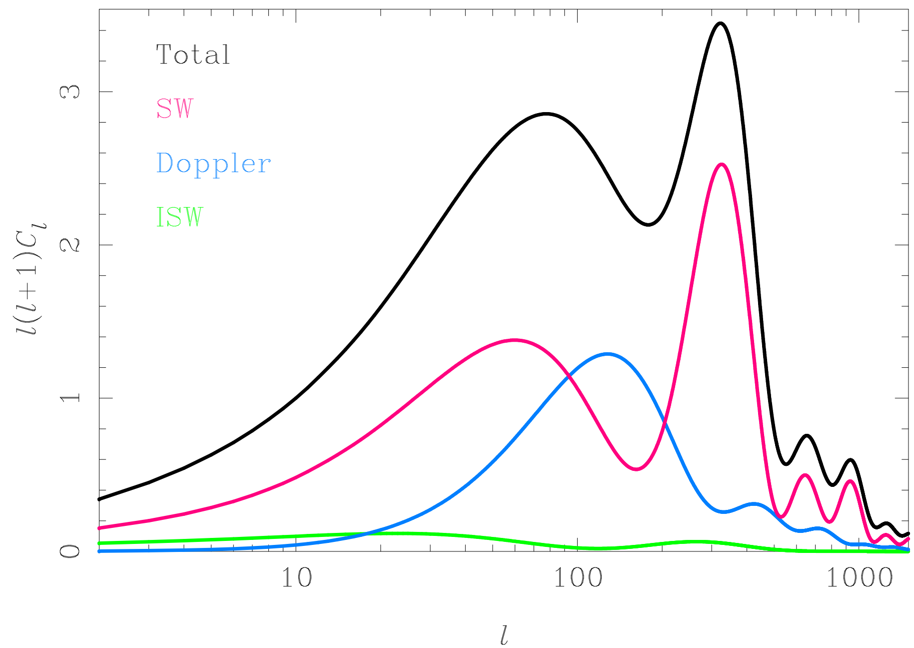
Consistent with generation of baryon asymmetry and dark matter at hot stage.

Perturbation in chemical composition (jargon: “isocurvature” or “entropy”) $\Longrightarrow$ wrong initial condition for acoustic oscillations $\Longrightarrow$ wrong prediction for CMB angular spectrum.

CMB angular spectra



homogeneous chemical composition



inhomogeneous chemical composition

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation

- Primordial perturbations are Gaussian.

Gaussian random field $\delta(\mathbf{k})$: correlators obey Wick's theorem,

$$\begin{aligned} \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle &= 0 \\ \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle &= \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle\cdot\langle\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle \\ &+ \text{permutations of momenta} \end{aligned}$$

- $\langle\delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle$ means averaging over ensemble of Universes. Realization in our Universe is intrinsically unpredictable.
- Hint on the origin: enhanced vacuum fluctuations of free quantum field
  Free quantum field

$$\phi(\mathbf{x},t) = \int d^3k e^{-i\mathbf{k}\mathbf{x}} \left(f_{\mathbf{k}}^{(+)}(t) a_{\mathbf{k}}^{\dagger} + e^{i\mathbf{k}\mathbf{x}} f_{\mathbf{k}}^{(-)}(t) a_{\mathbf{k}} \right)$$

In vacuo $f_{\mathbf{k}}^{(\pm)}(t) = \mathrm{e}^{\pm i\omega_k t}$

Enhanced perturbations: large $f_{\mathbf{k}}^{(\pm)}$. But in any case, Wick's theorem valid

- Inflation does the job very well: fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

  Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

  Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

- Enhancement of vacuum fluctuations is less automatic in alternative scenarios

- Non-Gaussianity: big issue
  - Very small in the simplest inflationary theories
  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with bispectrum (3-point function; vanishes for Gaussian field)

$$\langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\; G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$$

    Shape of $G(k_i^2;\; \vec{k}_1\cdot\vec{k}_2;\; \vec{k}_1\cdot\vec{k}_3)$ different in different models $\Longrightarrow$ potential discriminator.
  - In some models bispectrum vanishes, e.g., due to some symmetries. But trispectrum (connected 4-point function) may be measurable.

Non-Gaussianity has not been detected yet. Strong constraints from Planck

- Primordial power spectrum is nearly, but not exactly, flat

Homogeneity and anisotropy of Gaussian random field:

$$\langle \delta(\vec{k})\delta(\vec{k}')\rangle = \frac{1}{4\pi k^3}\mathscr{P}(k)\delta(\vec{k}+\vec{k}')$$

$\mathscr{P}(k) =$ power spectrum, gives fluctuation in logarithmic interval of momenta,

$$\langle \left(\frac{\delta\rho}{\rho}(\vec{x})\right)^2 \rangle = \int_0^\infty \frac{dk}{k}\,\mathscr{P}(k)$$

Flat spectrum: $\mathscr{P}$ is independent of k

Harrison' 70; Zeldovich' 72

Parametrization

$$\mathscr{P}(k) = A\left(\frac{k}{k_*}\right)^{n_s-1}$$

Flat spectrum $\Longleftrightarrow$ $n_s = 1$. Observations: $n_s = 0.96$.

To summarize:

- Available data on cosmological perturbations (notably, CMB anisotropies) give confidence that the hot stage of the cosmological evolution was preceeded by some other epoch, at which these perturbations were generated.

- Only very basic things are known for the time being.