Alternatives to inflation

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based on papers with
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To solve the horizon problem, one needs

long epoch preceeding hot expansion stage.

Possibilities:

- Contraction — Bounce — Expansion

  Qui et. al.' 2011; Easson, Sawicki, Vikman' 2011; Osipov, V.R.' 2013

- Start up from static state

  Creminelli et.al.'06; '10

Obstacle: conventional matter obeys
null energy condition (NEC)

$$T_{\mu\nu} n^\mu n^\nu > 0$$

for any null vector n^μ.

Penrose theorem:
Anti-trapped surface (Hubble size region) + NEC $\Longrightarrow$
past singularity

Meaning:

Homogeneous isotropic Universe

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \ .$$

Combination of Einstein equations

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

$\rho = T_{00}$ = energy density
$p = T_{11} = T_{22} = T_{33}$ = effective pressure.

Null Energy Condition:
$T_{\mu\nu} n^\mu n^\nu \geq 0, \ n^\mu = (1,1,0,0) \Longrightarrow \rho + p > 0 \Longrightarrow \dot{H} < 0$,
Hubble parameter was greater in the past. At some moment of time $H = \infty$, Big Bang Singularity.

Another face of NEC

Covariant energy conservation:

$$\frac{d\rho}{dt} = -3H(\rho + p)$$

NEC: energy density decreases as the Universe expands.

Consistent with the Friedmann equation

$$H^2 = \frac{8\pi}{3} G\rho \ .$$

Can one violate Null Energy Condition?

Folklore until recently: No!

Pathologies:

- Ghosts:

$$E = -\sqrt{p^2 + m^2}$$

- Gradient instabilities:

$$E^2 = -(p^2 + m^2) \implies \varphi \propto e^{|E|t}$$

- Superluminal propagation of perturbations

Today: Yes, one can

Senatore' 2004;
V.R.' 2006;
Creminelli, Luty, Nicolis, Senatore' 2006

General properties of NEC-violating theories without pathologies:

- Non-standard kinetic terms
- Non-trivial background solution (instability of Minkowski)

Example: scalar field $\pi(x^\mu)$,

$$L = F(Y)\cdot e^{4\pi} + K(Y)\cdot\Box\pi\cdot e^{2\pi}$$

$$Y = e^{-2\pi}\cdot(\partial_\mu\pi)^2$$

Deffayet, Pujolas, Sawicki, Vikman' 2010

Kobayashi, Yamaguchi, Yokoyama' 2010

- Second order field equation (!)
- Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

Homogeneous solution in Minkowski (attractor)

$$\mathrm{e}^{\pi_c} = \frac{1}{\sqrt{Y_*}(t_* - t)}$$

It has $Y \equiv \mathrm{e}^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* =$ const, solution to

$$Z(Y_*) \equiv -F + 2Y_* F' - 2Y_* K + 2Y_*^2 K' = 0$$

$$' \ = d/dY \ .$$

Energy density:

$$\rho = \mathrm{e}^{4\pi_c} Z = 0$$

Effective pressure T_{11}:

$$p = \mathrm{e}^{4\pi_c} (F - 2Y_* K)$$

can be made negative by suitable choice of $F(Y)$ and $K(Y) \Longrightarrow$ $\rho + p < 0$, NEC-violation.

Perturbations about homogeneous background

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

Quadratic Lagrangian for perturbations

$$L^{(2)} = \mathrm{e}^{2\pi_c} Z' (\partial_t \delta\pi)^2 - V(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

Absence of ghosts:

$$Z'(Y_*) > 0$$

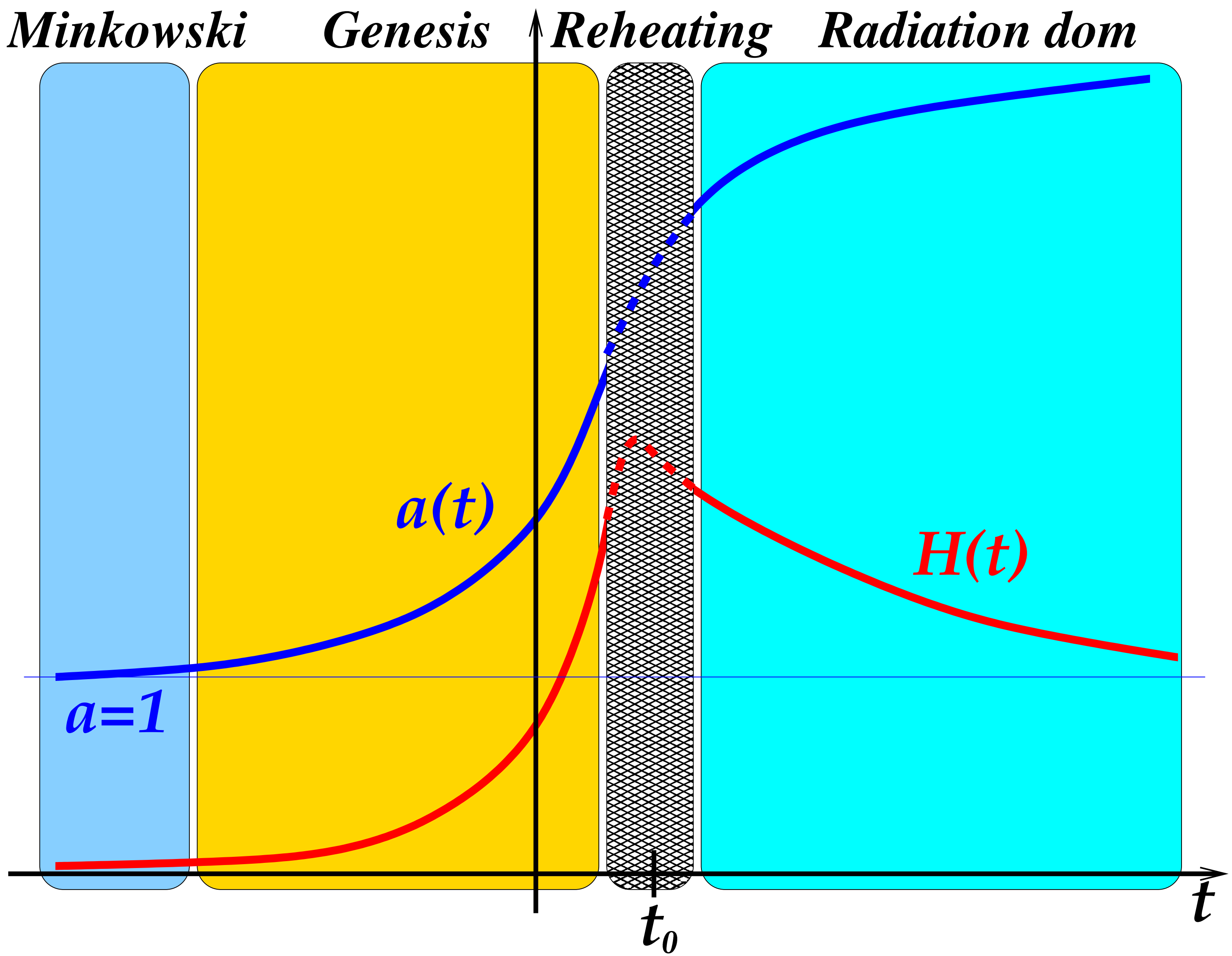
Absence of gradient instabilities and superluminality:

$$V > 0 \, ; \qquad V < \mathrm{e}^{2\pi_c} Z'$$

Holds for suitable choice of $F(Y)$ and $K(Y)$.

If no other fields $\Longrightarrow$ Genesis

Genesis



How can one generate

density perturbations

with nearly flat spectrum?

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time, $SO(4,1)$

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x} \, , \quad t \to t - \frac{1}{2H} \log \lambda$$

Inflation automatically generates nearly flat spectrum.

- Alternative: conformal symmetry $SO(4,2)$

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.

$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97

Concrete models: V.R.' 09;

Creminelli, Nicolis, Trincherini' 10

What if our Universe started off from or passed through

an unstable conformal state

and then evolved to much less symmetric state we see today?

Two ways of getting flat scalar spectrum

Way # 1: conformal rolling V.R. '09

Conformal plus global symmetry instead of de Sitter symmetry

- Main requirement: long evolution before the hot stage. But otherwise insensitive to regime of cosmological evolution. Can work at inflation and its alternatives.

Model:

$$S = S_{G+M} + S_\phi$$

S_{G+M}: gravity plus dominating matter

S_ϕ: conformal complex scalar field ϕ with negative quartic potential. Spectator until late epoch.

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{R}{6} |\phi|^2 - (-h^2 |\phi|^4) \right]$$

Conformal symmetry. Global symmetry $U(1)$. $\phi = 0$: unstable state with unbroken conformal symmetry. [Conformal symmetry explicitly broken at large fields. To be discussed later.]

Homogeneous and isotropic Universe,

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$$

In terms of the field $\chi(\eta,\vec{x}) = a(\eta)\phi(\eta,\vec{x}) = \chi_1 + i\chi_2$, evolution is Minkowskian,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0$$

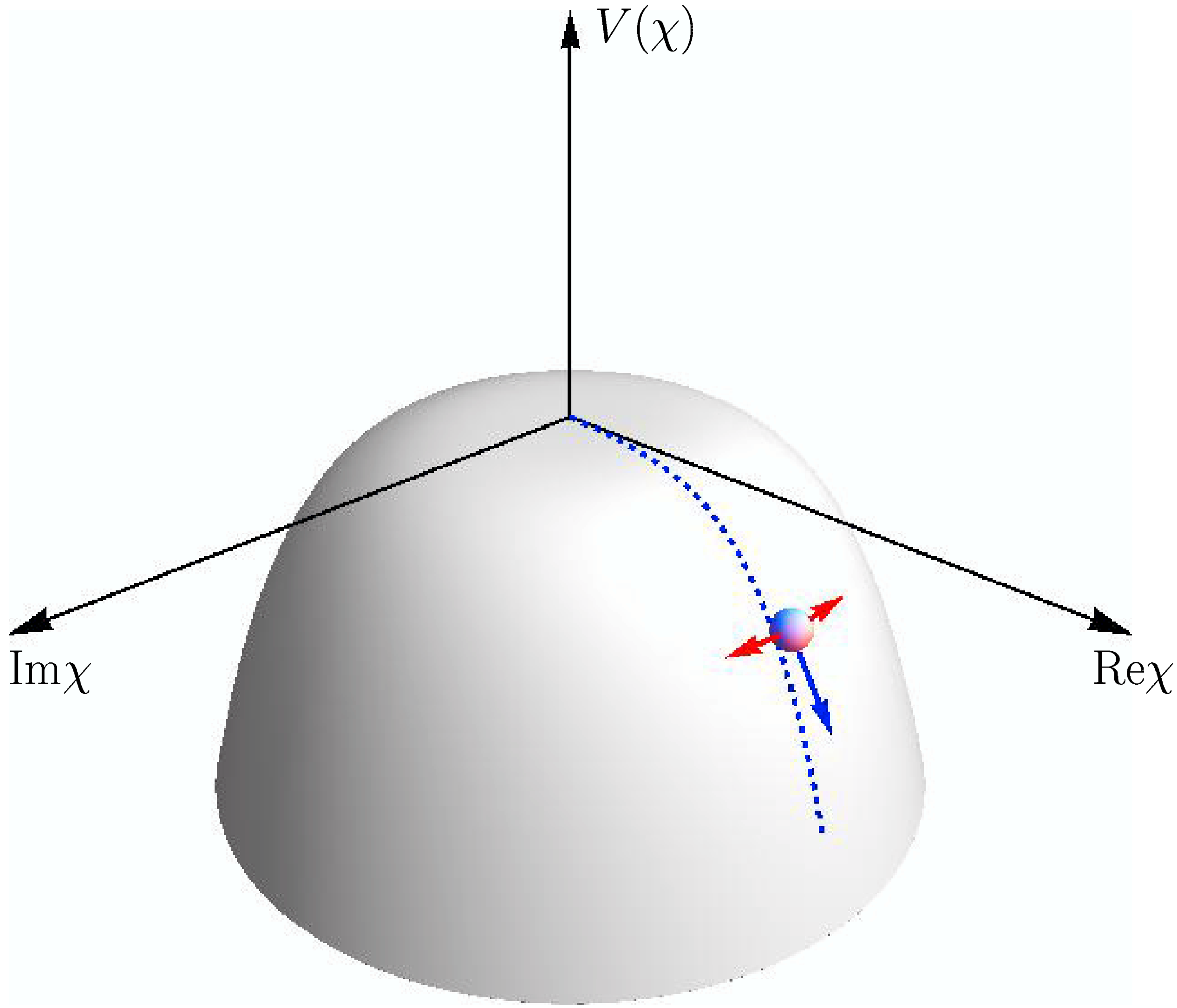
Homogeneous background solution

Attractor (real without loss of generality)

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

$\eta_* =$ constant of integration, end time of roll.

NB: Particular behavior $\chi_c \propto (\eta_* - \eta)^{-1}$
dictated by conformal symmetry.



$V(\chi)$
Imχ
Reχ

Fluctuations of Im χ

automatically have flat power spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \mathrm{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{d\eta^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$]

Regimes of evolution:

- Early times, $k \gg 1/(\eta_* - \eta)$, short wavelength regime, χ_c negligible, free Minkowskian field

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{-ik\eta} A_{\vec{k}} \;+\; \mathrm{h.c.}$$

- Late times, $k \ll 1/(\eta_* - \eta)$, long wavelength regime,
  term with χ_c dominates,

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{1}{k(\eta_* - \eta)} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

  - Phase of the field ϕ freezes out:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{h}{k} \cdot A_{\vec{k}} \quad + \quad \text{h.c.}$$

  - Power specrum of phase is flat:

$$\langle\delta\theta^2\rangle = \frac{h^2}{2(2\pi)^3}\int \frac{d^3k}{k^3} \quad \Longrightarrow \quad \mathscr{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2}$$

  - This is automatic consequence of global $U(1)$
    and conformal symmetry

To see this, consider long wavelength regime:

$\vec{k}$ negligible,
equation for $\delta\chi_2$ is equation for spatially homogeneous perturbation.
χ_c is solution to full field equation, $\mathrm{e}^{i\alpha}\chi_c$ also $\Longrightarrow$
$\delta\chi = i\alpha\chi_c$ is solution to perturbation equation $\Longrightarrow$

$$\delta\chi_2 : \quad \mathrm{e}^{-ik\eta} \Longrightarrow C(k)\chi_c(\eta) = \frac{1}{k(\eta_* - \eta)}$$

NB: $1/k$ on dimensional grounds.

NB: In fact, equation for $\delta\chi_2$ is precisely the same as equation for minimally coupled massless scalar field in inflating Universe

Comments:

- Mechanism requires long cosmological evolution: need

$$(\eta_* - \eta) \gg 1/k$$

early times, short wavelength regime,
well defined vacuum of the field $\delta\chi_2$.

For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved, at least formally.

This is a pre-requisite for most mechanisms that generate density perturbations

- Small explicit breaking of conformal invariance $\Longrightarrow$ tilt of the spectrum

Osipov, V.R. '10

- Depends both on the way conformal invariance is broken and on the evolution of scale factor

Way # 2 of getting flat spectrum

Creminelli, Nicolis, Trincherini '10

Galilean Genesis

Begin with galileon field π, Lagrangian

$$L_\pi = -f^2 \mathrm{e}^{2\pi} \partial_\mu \pi \partial^\mu \pi + \frac{f^3}{\Lambda^3} \partial_\mu \pi \partial^\mu \pi \cdot \Box \pi + \frac{f^3}{2\Lambda^3} (\partial_\mu \pi \partial^\mu \pi)^2$$

Conformally invariant. Under dilatations

$$\mathrm{e}^{\pi(x)} \to \lambda \mathrm{e}^{\pi(\lambda x)}$$

Universe begins from Minkowski space-time. Galileon rolls as

$$\mathrm{e}^{\pi_c} = \frac{1}{H_G(t_* - t)}, \qquad t < t_* ,$$

where $H_G^2 = \frac{2\Lambda^3}{3f}$. Again dictated by conformal invariance.

Initial energy density is zero, then it slowly builds up,

$$H(t) = \frac{1}{3}\frac{f^2}{M_{Pl}^2}\frac{1}{H_G^2(t_* - t)^3}$$

until $(t_* - t_e) \sim H_G^{-1} \cdot f/M_{PL}$. NB: Hubble parameter grows in time. Strong violation of all energy conditions. Yet fully consistent theory, no ghosts, tachyons, other pathologies.

At some point galileon is assumed to transmit its energy to conventional matter, hot epoch begins.

Galileon perturbations are not suitable for generating scalar perturbations.
Introduce another field θ of conformal weight 0,

$$L_\theta = \mathrm{e}^{2\pi}(\partial_\mu \theta)^2 \implies L_\theta = \frac{\text{const}}{(t_* - t)^2} \cdot (\partial_\mu \theta)^2$$

Dynamics of perturbations $\delta\theta$ in background π_c is exactly the same as in conformal rolling model.

Similarity is not an accident

Hinterbichler, Khouri '11

General setting:

- Effectively Minkowski space-time
- Conformally invariant theory
- Field ρ of conformal weight $\Delta \neq 0$
  - $\rho = \text{const} \cdot |\phi|$ in conformal rolling model
  - $\rho = \text{const} \cdot \text{e}^{\pi}$ in Galilean Genesis; $\Delta = 1$ in both models.

Homogeneous classical solution

$$\rho_c(t) = \frac{1}{(t_* - t)^{\Delta}}$$

by conformal invariance.

NB: t is conformal time in conformal rolling scenario

- Another scalar field θ of conformal weight 0.
- Kinetic term dictated by conformal invariance (modulo field rescaling)

$$L_\theta = \rho^{2/\Delta}(\partial_\mu \theta)^2$$

- Assume potential terms negligible $\Longrightarrow$ Lagrangian in rolling background

$$L_\theta = \frac{1}{(t_* - t)^2} \cdot (\partial_\mu \theta)^2$$

Exactly like scalar field minimally coupled to gravity in de Sitter space, with $t =$ conformal time, $a(t) = \text{const}/(t_* - t)$.

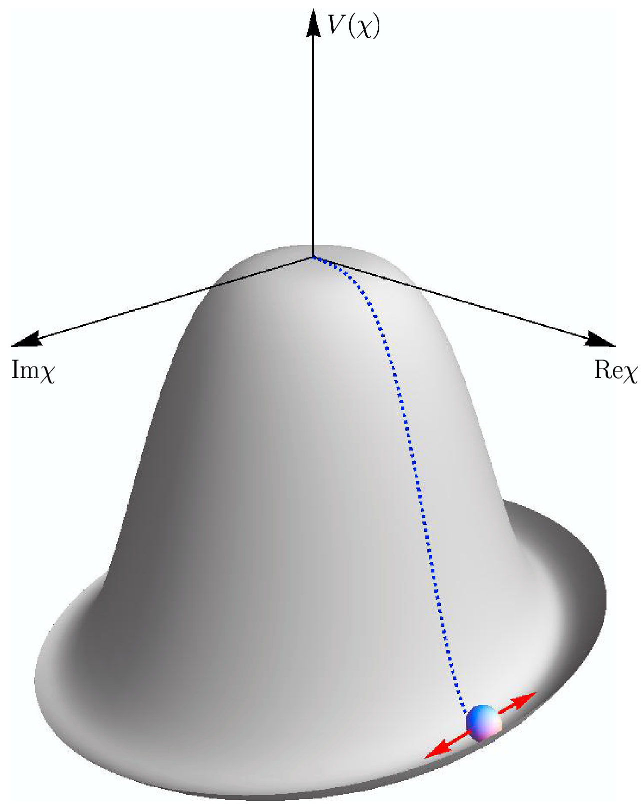
θ develops perturbations with flat power spectrum.

Use conformal rolling model in what follows for definiteness.

phase θ as curvaton

- Assume that conformal evolution ends up at some late time. Scalar potential actually has a minimum at large field.

Modulus of the field ϕ freezes out at the minimum of the scalar potential. Assume that energy density of ϕ is negligible at that time (probably, unimportant).



Density perturbations

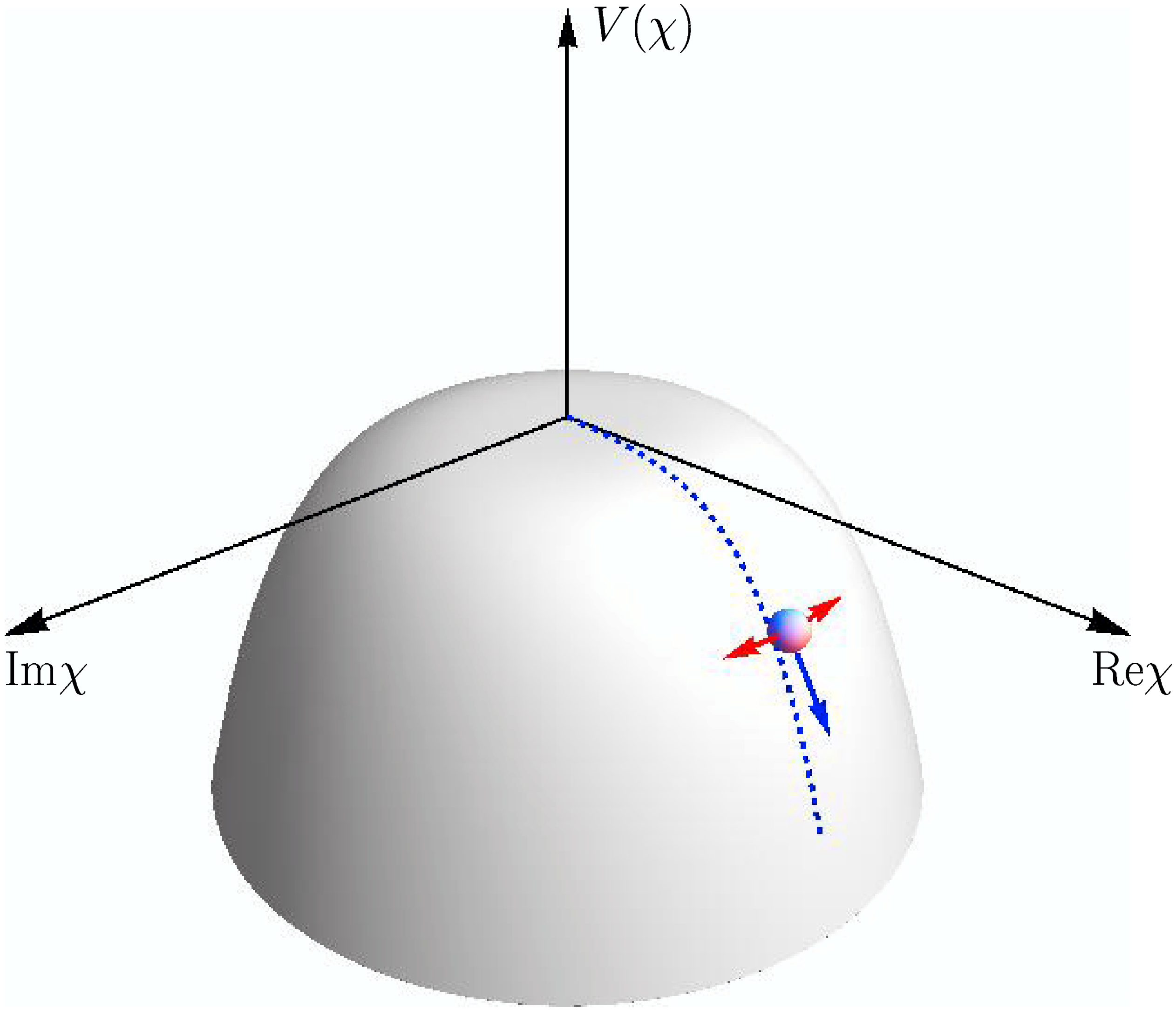
$$\delta = \text{const} \cdot \delta\theta \;\; + \;\; \text{possible non-linear terms}$$

Adiabatic perturbations inherit shape of power spectrum and correlation properties from $\delta\theta$, plus possible additional non-Gaussianity.

const < 1 and may be $\ll 1 \Longrightarrow \delta\theta \gg \delta$ quite possible $\Longrightarrow$ $h < 1$, but not necessarily $h \sim 10^{-4}$.

In any case, no relationship with tensor perturbations

Order h effects: back to conformal evolution



Peculiarity: raidial perturbations.

- Linear analysis of perturbations of $\chi_1 = \mathrm{Re}\chi$ about the homogeneous real solution χ_c:

$$\frac{d^2}{d\eta^2}\delta\chi_1 + k^2\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$].
Again initial condition

$$\delta\chi_1 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}\vec{x} - ik\eta} B_{\vec{k}} \;+\; \text{h.c.}$$

But now the solution is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}} H_{5/2}{}^{(1)}\left[k(\eta_* - \eta)\right] \cdot B_{\vec{k}} \;+\; \text{h.c.}$$

- In long wavelength regime, $k \ll 1/(\eta_* - \eta)$,

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{k^2\sqrt{k}(\eta_* - \eta)^2} B_{\vec{k}} \quad + \quad \text{h.c.}$$

  - Red spectrum:

$$\langle \delta\chi_1^2 \rangle \propto \int \frac{d^3k}{k^5}$$

  - Large $\delta\chi_1$ at small $(\eta_* - \eta)$

    [Recall $\chi_c = 1/[h(\eta_* - \eta)]$]

- Again by symmetry: now translations of conformal time:

$\chi_c \propto 1/(\eta_* - \eta) \Longrightarrow$ spatially homogeneous solution to perturbation equation $\delta\chi = \partial_\eta \chi_c$.

Modulo field redefinition and notations, properties of galileon perturbations are exactly the same as properties of radial perturbations in conformal rolling scenario.

Libanov, Mironov, V.R. ’11

Furthermore, these properties are unambiguously determined by conformal invariance

Libanov, Mironov, V.R. ’11

Hinterbichler, Khouri’ 11

In fact, invariance with respect to dilatations is sufficient.

Hence, we are dealing with the whole class of models

- Interpretation: time shift $\eta_* \longrightarrow \eta_* + \delta\eta_*(\vec{x})$

$$\text{Re}\chi = \chi_c(\eta) + \delta\chi_1(\eta, \vec{x})$$
$$= \frac{1}{\eta_* - \eta} + \frac{F(\vec{x})}{(\eta_* - \eta)^2} = \frac{1}{\eta_* + \delta\eta_*(\vec{x}) - \eta}$$

- Background for perturbations $\delta\chi_2 = \text{Im}\chi$ (in other words, for phase θ) is no longer spatially homogeneous.

- Red spectrum of $\delta\eta_*(\vec{x})$: $\sqrt{\mathscr{P}_{\delta\eta_*}} = \frac{3h}{2\pi k}$

- η_* itself is irrelevant: overall time shift. Relevant are derivatives

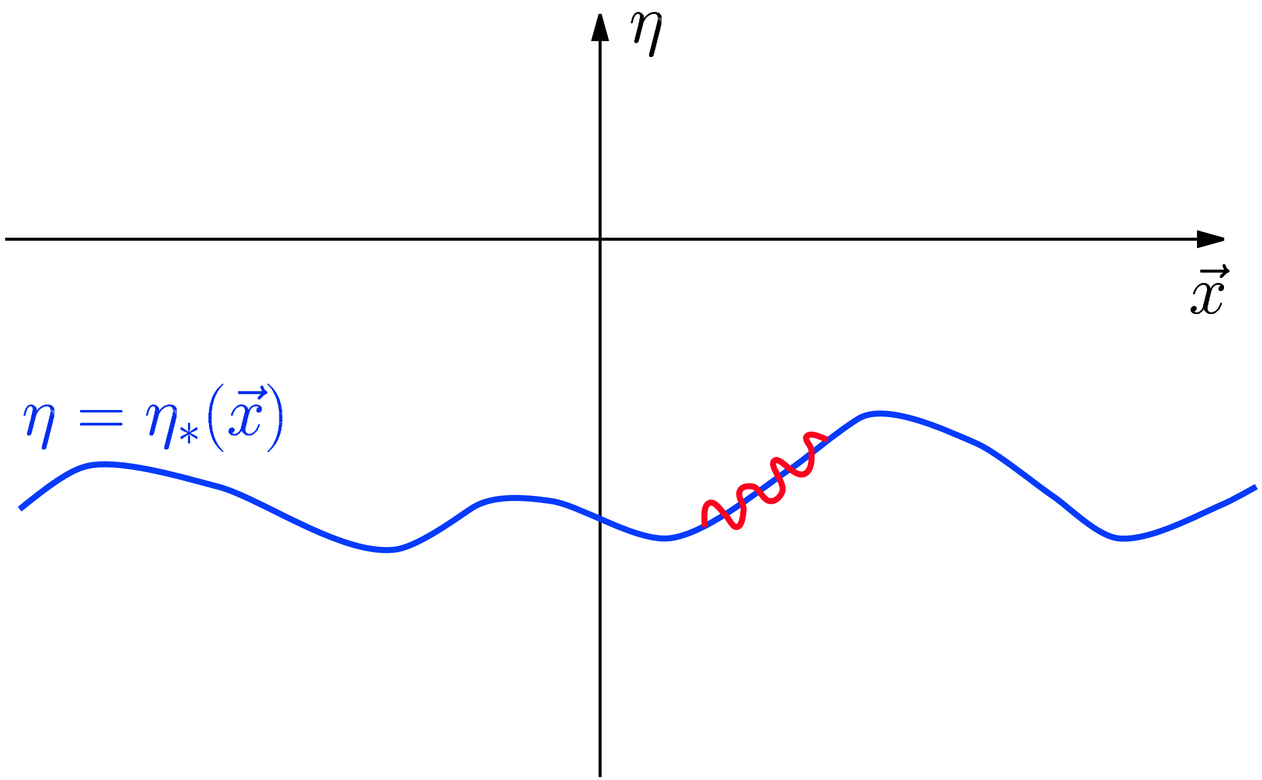
$$\vec{\partial}\eta_*(\vec{x}) \equiv -\vec{v}\,, \qquad \text{etc.}$$

- $\vec{v}$ has flat power spectrum

$$\sqrt{\mathscr{P}_{\vec{v}}} = \frac{3h}{2\pi}$$

- Potentially dangerous effects of infrared radial modes: have to study perturbations of $\mathrm{Im}\chi$ in *spatially inhomogeneous background*, slowly varying in space,

$$\chi_c = \frac{1}{h(\eta_*(\vec{x}) - \eta)}$$



- Back to equation for perturbations of $\delta\chi_2 = \mathrm{Im}\chi$

$$\frac{d^2}{d\eta^2}\delta\chi_2 - \frac{\partial^2}{\partial\vec{x}^2}\delta\chi_2 - \frac{2}{(\eta_*(\vec{x}) - \eta)}\delta\chi_2 = 0$$

  - Initial condition as $\eta \to -\infty$:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}\vec{x} - ik\eta} A_{\vec{k}} \quad + \quad \mathrm{h.c.}$$

  - $\eta_*(\vec{x})$: long ranged field, derivative expansion appropriate
    - Zeroth order in $\partial_i\eta_*$: local shift of conformal time.
    - First order in $\partial_i\eta_*$:

      $\eta_*(\vec{x}) - \eta = \eta_*(0) - (\eta - \vec{\partial}\eta_* \cdot \vec{x})$

      $\Longrightarrow$ local Lorentz boost with $\vec{v} = -\vec{\partial}\eta_*$;
      background is locally homogeneous and isotropic in a reference frame other than cosmic frame.

Solution to the first order in derivative expansion: time shift and Lorentz boost of the original solution $f(k,\eta) \propto H_{3/2}[k(\eta_* - \eta)]$:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}(\vec{x}+\vec{v}\eta) - i(k+\vec{k}\vec{v})\eta_*(\vec{x})} f(q(\vec{x}), \eta - \eta_*(\vec{x})) \cdot A_{\vec{k}} \;\; + \;\; \mathrm{h.c.}$$

$\vec{q} = \vec{k} + k\vec{v} =$ boosted momentum; $v_i(\vec{x}) = -\partial_i \eta_*(\vec{x})$.

Phase field freezes out at

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{h}{(2\pi)^{3/2}\sqrt{2k}(k+\vec{k}\vec{v})} \mathrm{e}^{i\vec{k}\vec{x} - ik\eta_*(\vec{x})} \cdot A_{\vec{k}} \;\; + \;\; \mathrm{h.c.}$$

Potentially observable effects depend on what happens to phase perturbations after the end of conformal rolling stage.

NB: Once radial field $|\phi|$ has settled down to minimum of its potential, phase θ is massless scalar field minimally coupled to gravity

Two sub-scenarios

- Sub-scenario # 1. Phase perturbations superhorizon in conventional sense after end of conformal rolling stage

Libanov, V.R. ’10

- Sub-scenario # 2 (more natural in conformal rolling model, less natural in Galilean Genesis): Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

Libanov, Ramazanov, V.R. ’11

- sub-scenario # 1. Phase perturbations superhorizon in conventional sense after end of conformal rolling stage

$\delta\theta$ remains frozen until the time it gets reprocessed into adiabatic perturbations $\Longrightarrow$

$$\mathscr{P}_\zeta \propto \mathscr{P}_{\delta\theta}$$

Effects of infrared radial modes: derivative expansion.

No effect to the order $\vec{\partial}\eta_*$ (!!)

Lorentz-invariance does the job.

- Derivative expansion to the second order: perturbative solution.

Long wavelength regime:

$$\delta\theta = \delta\theta^{(0)} \cdot \left(1 - \frac{\pi}{2k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_*\right)$$

Scalar power spectrum

$$\mathscr{P}(\vec{k}) = \mathscr{P}_0(k)\left(1 - \frac{\pi}{k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_*\right)$$

Statistical anisotropy due to constant in space tensor $\partial_i\partial_j\eta_*|_{\text{long wavelengths}} \Longrightarrow$ CMB correlators $\langle a_{l,m} a^*_{l\pm2,m}\rangle$, etc.

- Quadrupole of general form
- Momentum dependence $1/k$
- Difficult case because of cosmic variance

- Non-Gaussianity to order h^2

Libanov, Mironov, V.R. '10, 11

Over and beyond non-Gaussianity which can be generated when perturbations in θ are converted into density perturbations.

Invariance $\theta \to -\theta \Longrightarrow$ bispectrum (3-point function) vanishes.

Trispectrum fully calculated. Most striking property: singularity in "folded" limit:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = \text{const} \cdot \delta \left(\sum_{i=1}^{n} \vec{k}_i \right) \cdot \frac{1}{k_{12} k_1^4 k_3^4} \left[1 - 3 \left(\frac{\vec{k}_{12} \cdot \vec{k}_1}{k_{12} k_1} \right)^2 \right] \left[\quad \vec{k}_1 \leftrightarrow \vec{k}_3 \right]$$

$$\vec{k_{12}} = \vec{k}_1 + \vec{k}_2 \to 0$$

This is in sharp contrast to single field inflation.

Origin: infrared enhancement of radial perturbations $\delta\chi_1$

- Sub-scenario # 2 (more natural in conformal rolling model, less natural in Galilean Genesis): Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

Libanov, Ramazanov, V.R. ’11

$\delta\theta$ evolves non-trivially before it becomes super-horizon and freezes out again.

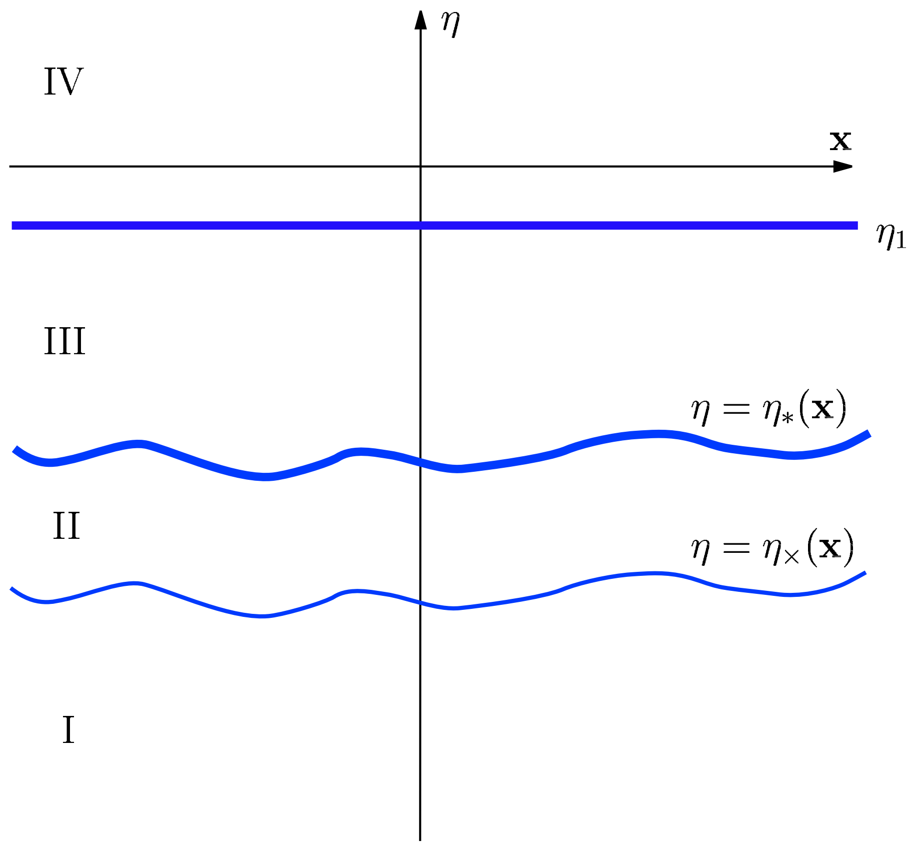
Motivation. Fairly generic feature of alternatives to inflation: long stage of almost Minkowskian evolution

Example: Ekpyrotic models (in broad sense): contracting stage with

$$a(t) = |t|^p , \qquad p \ll 1$$

(to avoid Belinsky–Lifshits–Khalatnikov phenomenon)

Almost Minkowski.



- Nearly Minkowskian evolution at intermediate stage III

Otherwise power spectrum becomes tilted!

- Two-fold effect of radial perturbations
  - $\eta \approx \eta_*$: initial field $\delta\theta(\vec{x})$ non-trivial
  - Cauchy hypersurface $\eta = \eta_*(\vec{x})$ non-trivial
- For given $\vec{k}$, phase perturbation after second freeze-out is a linear combination of waves coming from direction of $\vec{k}$ and from opposite direction and traveling distance $r = \eta_1 - \eta_* \Longrightarrow$ Imprint on $\delta\theta(\vec{k})$ of random field $\delta\eta_*(\pm\vec{n}_k r)$, which depends on $\vec{n}_k$ only.

Statistical anisotropy with all even multipoles.

$$\mathscr{P}_\zeta(\vec{k}) = \mathscr{P}_\zeta^{(0)}(k)\left[1 + \mathscr{Q}\cdot w_{ij}\left(\frac{k_i k_j}{k^2} - \frac{1}{3}\delta_{ij}\right) + \text{higher multipoles}\right]$$

with $w_{ij}w_{ij} = 1$ and $\langle\mathscr{Q}^2\rangle = \frac{675}{32\pi^2}h^2$.

NB: multipoles $\mathscr{Q}$, etc., are independent of $k \Longrightarrow$ no suppression of effect on CMB at large l, unlike in sub-scenario # 1.

non-Gaussianity

- Non-trivial part of tri-spectrum: dependence on directions of momenta

$$\langle \zeta(\vec{k})\zeta(\vec{k}')\zeta(\vec{q})\zeta(\vec{q}')\rangle = \frac{\mathscr{P}_\zeta^{(0)}(k)}{4\pi k^3}\frac{\mathscr{P}_\zeta^{(0)}(q)}{4\pi q^3}\delta(\vec{k}+\vec{k}')\delta(\vec{q}+\vec{q}')\cdot[1+F_{NG}(\vec{n}_k,\vec{n}_q)]$$

$$+ \ \text{permutations}$$

with

$$F_{NG} = \frac{3h^2}{\pi^2}\log\frac{\text{const}}{|\vec{n}_k - \vec{n}_q|}$$

NB: recall that power spectrum of $\vec{\partial}\eta_*$ is flat $\Longrightarrow$ log behavior of F_{NG}.

To summarize:

- Flat (or nearly flat) spectrum of scalar perturbations may be a consequence of conformal symmetry (+ possibly global symmetry), rather than de Sitter symmetry

  - Models of this sort: (i) conformally coupled complex scalar field with negative quartic potential
    (ii) Galilean Genesis

  - Properties of perturbations dictated by conformal invariance

  - Predictions are model-independent, at least to the leading non-linear level (modulo effects due to conversion of field perturbations into density perturbations)

- Peculiar property which has potentially observable consequences: fluctuations along rolling direction
  - Interpretation in terms of local time shift
- Interplay between phase perturbations, responsible for density perturbations in the end, and local time shift $\delta\eta_*(\vec{x}) \Longrightarrow$ non-trivial correlation properties of density perturbations

Sub-scenario # 1:

- Statistical anisotropy of quadrupole form

$$\mathscr{P}(\vec{k}) = A_s(k)\left(1 + \frac{hH_0}{k} w_{ij} n_{ki} n_{kj}\right)$$

- Trispectrum singular in folded limit

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle \propto \frac{1}{|\vec{k}_1 + \vec{k}_2|}$$

Sub-scenario # 2:

- Statistical anisotropy of a general form
- Non-Gaussianity of a peculiar kind

All this is in sharp contrast to inflationary mechanism.

No primordial gravity waves expected, unlike in the simplest inflationary models

What if the world started out

conformal indeed?