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Topological effects in Double Field Theory.

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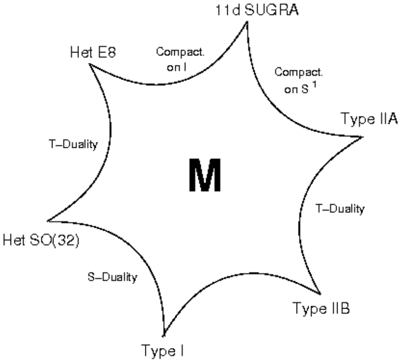
Overview

INTRODUCTION

Dualitites in string theory

- net of theories
- string cosmology
- moduli stabilization
- string vacuum



T-duality in Type II theories on a torus $\mathbb{T}^d$

$$S = \int d^2\sigma \left(\sqrt{-h}h^{\alpha\beta}G_{mn} - \epsilon^{\alpha\beta}B_{mn}\right)\partial_\alpha X^m \partial_\beta X^n \tag{1}$$

Equations of motion $\Longleftrightarrow$ Bianchi identities

$\partial_\alpha \tilde{\mathcal{G}}^\alpha_m(X^m) = 0$ $\qquad$ $\epsilon^{\alpha\beta}\partial_\alpha\partial_\beta X^m = 0$

Momentum $\Longleftrightarrow$ Winding mode

radius of the torus R $\Longleftrightarrow$ α'/R (for $\mathbb{T}^1$, $d=1$)

$$M^2 = \frac{n^2}{R^2} + \frac{m^2R^2}{\alpha'^2} + 2(N + \tilde{N} - 2).$$

Dual coordinates $\tilde{X}_\mu$, whose Bianchi identities are EOM for X^μ.

$X = X_+ + X_-$, $\tilde{X} = X_+ - X_-$ – independent

[A. Tseytlin, P. West, M. Duff, J. Lu]

The Buscher rules

$$S = \int d^2\sigma (G_{mn} + B_{mn}) \partial_+ X^m \partial_- X^n = \int d^2\sigma E_{mn} \partial_+ X^m \partial_- X^n \quad (2)$$

The Buscher rules for $\mathbb{S}^1$, $X^m = (\theta, X^{\hat{m}})$:

$$\begin{aligned} G'_{\theta\theta} &= \frac{1}{G_{\theta\theta}}, & E'_{\theta\hat{a}} &= \frac{1}{G_{\theta\theta}} E_{\theta\hat{a}}, \\ E'_{\hat{a}\theta} &= -\frac{1}{G_{\theta\theta}} E_{\hat{a}\theta}, & E'_{\hat{a}\hat{b}} &= E_{\hat{a}\hat{b}} - E_{\hat{a}\theta} \frac{1}{G_{\theta\theta}} E_{\theta\hat{b}}. \end{aligned} \quad (3)$$

The Buscher rules for $\mathbb{T}^d$ (non-linear):

$$E' = \frac{aE + b}{cE + d}, \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in O(d,d) \quad (4)$$

Non-geometric backgrounds

T-duality mixes

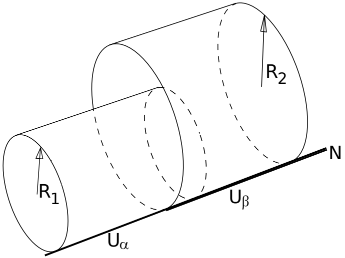
- the metric G_{mn} and the gauge field B_{mn}.
- diffeomorphisms and gauge transformations.

New class of consistent string-theory backgrounds — T-folds.

Transition functions

- Manifold: diffeos;
- T-fold: T-duality;
- U-fold: U-duality;
- Mirror-fold: mirror symmetry;

schematically:



Non-geometry (a toy-model of $\mathbb{T}^3$)

Backgrounds with fluxes f and Q.

- Torsion $f_{abc} \neq 0$ (Kaluza-Klein monopole/$5^1_2 - brane$)

$$ds^2 = (dx + Nzdy)^2 + dy^2 + dz^2, \quad B = 0,$$

$$z \to z + 1, \quad x \to x + Ny, \quad N \in \mathbb{Z}$$

- Non-geometric flux $Q_{abc} \neq 0$ ($5^2_2 - brane$)

$$ds^2 = \frac{1}{1+N^2z^2}(dx^2 + dy^2 + dz^2), \quad B = \frac{Nz}{1+N^2z^2} dx \wedge dy.$$

$z \to z + 1$, glued by T-duality

$$H_{xyz} \xrightarrow{T_x} f^x{}_{yz} \xrightarrow{T_y} Q^{xy}{}_z \xrightarrow{T_z} R^{xyz}. \quad (5)$$

[B. Wecht (lectures) hep-th/0708.3984]

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Non-geometry

- non-commutativity

$$[X^m, X^n] \sim \oint_C Q^{mn}{}_a \, dX^a$$

- non-associativity

$$[X^m, X^n, X^a] \sim R^{mna}$$

[O. Hohm, D. Lust hep-th/1204.1979]

T-fold backgrounds:

- string compactifications in the presence of fluxes
- gauged supergravities (Θ_M^α contains fluxes)
- moduli stabilization

EXTENDED GEOMETRY

Poincaré symmetry (illustration of the idea)

$ISO(3,1)$: symmetry of Maxwell equations

$$\begin{aligned} div\vec{E} &= \rho, \qquad & div\vec{H} &= 0 \\ rot\vec{H} &= \dot{\vec{E}} + \vec{j}, \qquad & rot\vec{E} &= -\dot{\vec{H}}. \end{aligned} \tag{6}$$

- ☑ Extra coordinate: time;
- ☑ New fundamental fields $A^\mu \in \mathcal{R}_v$ of $SO(3,1)$;
- ☑ $\vec{E}$ & $\vec{H}$ appear on the same footing in $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$;
- ! Equations become simple

$$\partial_\mu F^{\mu\nu} = j^\nu; \quad \epsilon^{\alpha\beta\mu\nu}\partial_\beta F_{\mu\nu} = 0. \tag{7}$$

The fundamental field A_μ has a geometric interpretation!

Doubled (extended) geometry

Duff's procedure $\Longrightarrow$ the dual coordinate $\tilde{X}_m$.

$$\mathbb{X}^M = \begin{bmatrix} X^m \\ \tilde{X}_n \end{bmatrix}, \mathcal{H}_{MN} = \begin{bmatrix} G_{ab} - B_{ar}B^r{}_b & -B_a{}^n \\ B^m{}_b & G^{mn} \end{bmatrix}, \eta_{MN} = \begin{bmatrix} 0 & \delta^m_n \\ \delta^a_b & 0 \end{bmatrix}.$$

- G_{mn} & B_{mn} are on equal footing;
- T-duality acts linearly by an $O(d,d)$ rotation;

$$\mathbb{X}'^M = \mathcal{O}^M{}_N \mathbb{X}^N, \quad \mathcal{H}'_{MN} = \mathcal{O}^K{}_M \mathcal{H}_{KL} \mathcal{O}^L{}_N$$

- Diffeos+gauge transformations = Generalised Diffeos;

$$\mathcal{L}_\xi V^M = L_\xi V^M + \eta^{MN}\eta_{KL}\partial_N \xi^K V^L$$

[C. Hull, O. Hohm, B. Zwiebach, D. Berman, M. Perry, M. Grana et al.]

Local properties

Transformation of a (generalised) vector

$$\delta_\Sigma V^M = \left(\hat{\mathcal{L}}_\Sigma V\right)^M = (L_\Sigma V)^M + \eta^{MN}\eta_{AB}\partial_N \Sigma^A V^B. \qquad (8)$$

The Jacobi identities fail to satisfy

$$[\delta_{\Sigma_1}, \delta_{\Sigma_2}, \delta_{\Sigma_3}]V^M \neq 0 \qquad (9)$$

C-bracket (in general the algebra is not closed)

$$\begin{aligned} [V_1, V_2]_C &= \frac{1}{2}\left(\hat{\mathcal{L}}_{V_1} V_2 - \hat{\mathcal{L}}_{V_2} V_1\right) \\ [\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}] &= \hat{\mathcal{L}}_{[\xi_1,\xi_2]_C} + F_0. \end{aligned} \qquad (10)$$

The section condition:

$$\eta^{AB}\partial_A \bullet \partial_B \bullet = 0 \implies F_0 = 0 \qquad (11)$$

The effective action

The effective action for DFT (T-duality covariant)

$$S = \int dX d\tilde{X} e^{-2d} \left(\Phi[\mathcal{H}, \partial\mathcal{H}, d] + \eta^{MN} \eta_{KL} \partial_M E^K{}_{\bar{A}} \partial_N E^L{}_{\bar{B}} \mathcal{H}^{\bar{A}\bar{B}} \right).$$

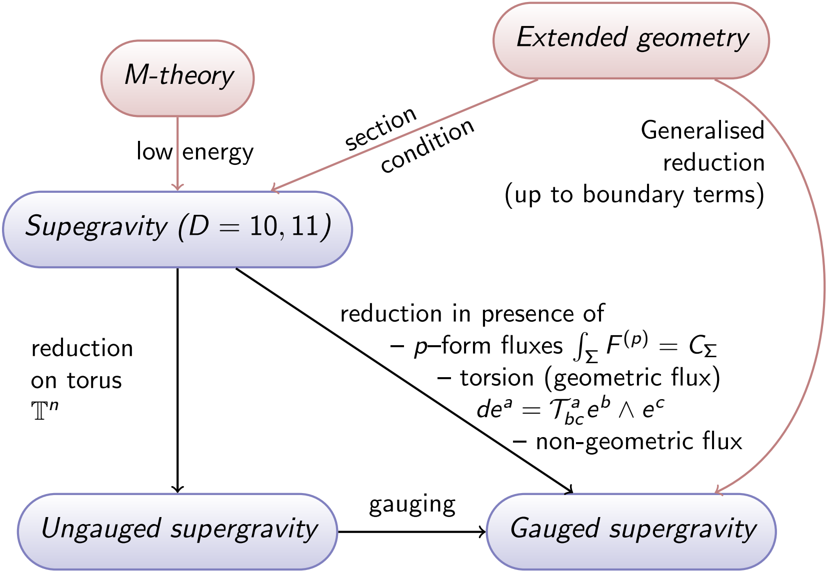
Two natural choices for the vielbein $\mathcal{H}_{MN} = E_M{}^{\bar{A}} E_N{}^{\bar{B}} \mathcal{H}_{\bar{A}\bar{B}}$

$$\hat{E}^M_{\bar{A}} = \begin{bmatrix} e^m_{\bar{a}} & 0 \\ e^k_{\bar{a}} B_{nk} & e^{\bar{b}}_n \end{bmatrix}, \qquad \tilde{E}^M_{\bar{A}} = \begin{bmatrix} e^m_{\bar{a}} & e^{\bar{b}}_k \beta^{mn} \\ 0 & e^{\bar{b}}_n \end{bmatrix}. \qquad (12)$$

The 2-vector $\beta = \beta^{mn} \partial_m \wedge \partial_n$ is a sign of non-geometry.

$$[X^m, X^n] \sim \beta^{mn}$$

Gauged supergravities



Generalized flux

Ordinary torsion ($g_{mn} = e^{\bar{a}}_m e^{\bar{b}}_n \eta_{\bar{a}\bar{b}}$):

$$[e_{\bar{a}}, e_{\bar{b}}] = f^{\bar{a}}{}_{\bar{b}\bar{c}} e_{\bar{a}}. \tag{13}$$

Generalized "torsion" ($\mathcal{H}_{MN} = E^{\bar{A}}{}_M E^{\bar{B}}{}_N \mathcal{H}_{\bar{A}\bar{B}}$)

$$[E_{\bar{A}}, E_{\bar{B}}]_C = F_{\bar{A}\bar{B}}{}^{\bar{C}} E_{\bar{C}}, \tag{14}$$

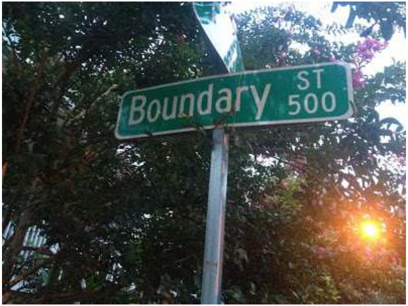
$F_{\bar{A}\bar{B}}{}^{\bar{C}}$ encodes all fluxes (schematically):

$$F_{MN}{}^K = \begin{bmatrix} f_{mn}{}^k & H_{mnk} \\ Q_m{}^{nk} & R^{mnk} \end{bmatrix} \tag{15}$$

The embedding tensor $\Longleftrightarrow$ generalized flux.

$$X_{MN}{}^K = F_{MN}{}^K + Z_{MN}{}^K \tag{16}$$

BOUNDARY TERMS



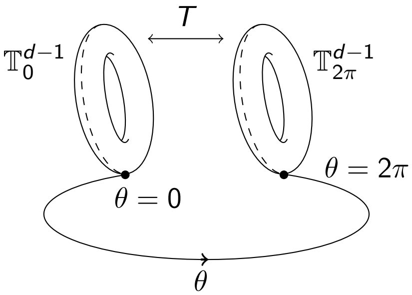
The action

Boundary terms $\Longrightarrow$ Gibbons-Hawking terms.

The full action differs from S_G by a full derivative

$$S_{Full} = S_G + 2\oint_{\partial} \partial_A \left(e^{-2d} \mathcal{H}^{AB} N_B \right)$$

Non-trivial monodromy when going around one of the cycles of the torus.

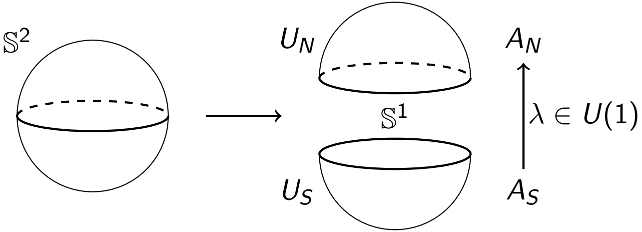


Non-trivial monodromies

Electrodynamics: the classical monopole

$$\int_{\mathbb{S}^2} F = \int_{\mathbb{S}^2} dA = \int_{U_N} dA_N + \int_{U_S} dA_S = \int_{\mathbb{S}^1} d\lambda. \tag{17}$$

Gauge transformation $A_N = A_S + d\lambda$.



Non-trivial topology $\Longrightarrow$ quantization of the monopole charge.

The result

$$\begin{array}{ccc} \theta & \to & \theta + 2\pi \\ \text{Tilde gauge (with } B\text{)} & & \text{Hat gauge (with } \beta\text{)} \end{array}$$

$$S_B = 2v\left[\int_{\mathbb{T}^{d-1}_\theta \times \tilde{\mathbb{T}}^d} n^a Q^{mn}{}_a B_{mn} e^{-2\lambda} + \int_{\mathbb{T}^d \times \tilde{\mathbb{T}}^{d-1}_{\tilde{\theta}}} \tilde{n}_b f^b{}_{mn} \beta^{mn} e^{-2\lambda}\right] +$$
$$+ 2v\left[\int_{\mathbb{T}^{d-1}_\theta \times \tilde{\mathbb{T}}^d} n^m B_{mn} \tilde{\partial}^n e^{-2\lambda} - \int_{\mathbb{T}^d \times \tilde{\mathbb{T}}^{d-1}_{\tilde{\theta}}} \tilde{n}_m \beta^{mn} \partial_n e^{-2\lambda}\right].$$

- Fluxes f and Q couple magnetically to the B-field and the 2-vector;
- The expression is T-duality invariant;
- Looks like the Gibbons-Hawking term for a charged black hole

$$S_{GH} \sim (M - Q\Phi)$$

.

Exotic states: the 5^2_2 brane primer

	1	2	3	4	5	6	7	8	9
NS5			$\times$	$\times$	$\times$	$\times$	$\times$	$\cdot$	$\cdot$
KKM			$\times$	$\times$	$\times$	$\times$	$\times$	$\odot$	$\cdot$
5^2_2			$\times$	$\times$	$\times$	$\times$	$\times$	$\odot$	$\odot$

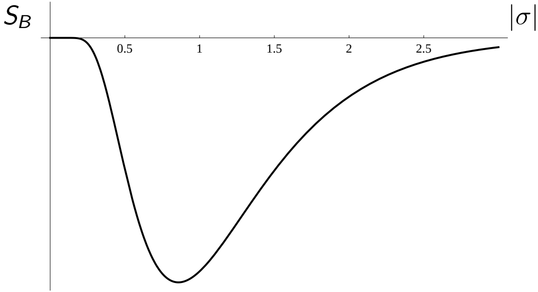
The non-geometric flux $Q^{89}{}_{\theta} = \sigma = \text{const}$

$$
\begin{aligned}
ds^2 &= H(dr^2 + r^2 d\theta^2) + \frac{H}{H^2 + \sigma^2\theta^2} ds^2_{89} + ds^2_{034567}, \\
B^{(2)} &= \frac{H\sigma\theta}{H^2 + \sigma^2\theta^2} dx^8 \wedge dx^9, \quad e^{-2\phi} = \frac{H}{H^2 + \sigma^2\theta^2},
\end{aligned}
\tag{18}
$$

The torus $\mathbb{T}^2_{8,9}$ is glued by a T-duality transformation when going around the brane (the cycle $\theta \to \theta + 2\pi$).

Exotic states: the 5^2_2 brane primer

$$S_B = 2\int \sqrt{-g}\, d^n x Q^{89}{}_\theta B_{89}\Big|_{\theta=2\pi} \tag{19}$$



The contribution of the topological term for configurations with flux σ close to $|\sigma| = 1$ is dominant in the partition function.

Conclusion

- ☑ The boundary contribution to the DFT action was presented in terms of the fluxes f and Q;
- ☑ The exotic state corresponding to the 5^2_2-brane gives non-zero contribution to the partition function;
- ☑ For a single 5^2_2-brane contributions with the flux $\sigma = 1$ dominate;

Future problems

- ☐ Consider backgrounds with non-zero R-, f- and H-flux.
- ☐ Formulate a Gibbons-Hawking-like thermodynamics for these objects using the calculated boundary action.
- ☐ Consider generalised reduction with the boundary terms included.

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THANK YOU!