BPS branes in 10 and 11 dimensional supergravity

arXiv:1301.2139, arXiv:1107.4089

A. A. Golubtsova[1],[a,b] in collaboration with V.D. Ivashchuk[a,c]

(a) PFUR IGC, (b) The Paris Observatory, (c) VNIIMS

DIAS Summer School,
Dubna 2013

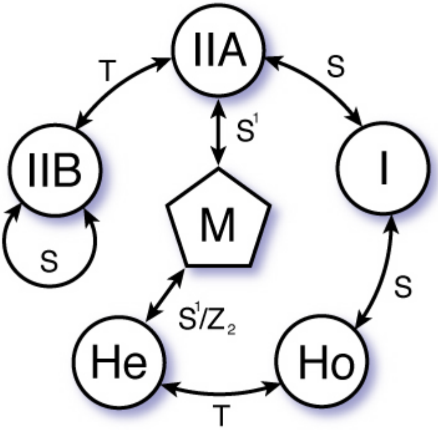
[1] siedhe@gmail.com

Outline

Table: The correspondence between the string theories, M-theory and the supergravity theories

Quantum theory	Effective theory
M-theory	$D = 11$, $\mathcal{N} = 1$ supergravity
Type IIA string theory	Non-chiral $D = 10$, $\mathcal{N} = 2$ IIA supergravity
Type IIB string theory	Chiral $D = 10$, $\mathcal{N} = 2$ IIB supergravity
Type I string theory	$D = 10$, $\mathcal{N} = 1$ supergravity + Yang Mills with $SO(32)$ gauge group
Heterotic $SO(32)$ string theory	$D = 10$, $\mathcal{N} = 1$ supergravity + Yang Mills with $SO(32)$ gauge group
Heterotic $E_8 \times E_8$ string theory	$D = 10$, $\mathcal{N} = 1$ supergravity + Yang Mills with $E_8 \times E_8$ gauge group

Duality and reduction



$10D$ and $11D$ Supergravity

11 dimensional $\mathcal{N} = 1$ SUGRA

$$S_{11D} = \int d^{11}z\sqrt{|g|}\left\{R[g] - \frac{1}{2(4!)}\hat{F}^2\right\} - \frac{1}{6}\int \hat{A}\wedge\hat{F}\wedge\hat{F},$$

where

$$\hat{F} = d\hat{A} = \frac{1}{4!}\hat{F}_{NPQR}dz^N \wedge dz^P \wedge dz^Q \wedge dz^R$$

is the 4-form field strength of the 3-form potential A.

$10D$ and $11D$ Supergravity

11 dimensional $\mathcal{N}=1$ SUGRA

$$S_{11D} = \int d^{11}z\sqrt{|g|}\left\{R[g] - \frac{1}{2(4!)}\hat{F}^2\right\} - \frac{1}{6}\int \hat{A}\wedge\hat{F}\wedge\hat{F},$$

where

$$\hat{F} = d\hat{A} = \frac{1}{4!}\hat{F}_{NPQR}dz^N\wedge dz^P\wedge dz^Q\wedge dz^R$$

is the 4-form field strength of the 3-form potential A.

10 dimensional $\mathcal{N}=2$ SUGRA

$$S_{IIA,SF} = \int d^{10}x\sqrt{|g|}\Big\{e^{-2\varphi}\left[R[g] + 4\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2(3!)}|F_{(3)}|^2\right] - \frac{1}{2(2!)}|F_{(2)}|^2 - \frac{1}{2(4!)}|\tilde{F}_{(4)}|^2\Big\} - \frac{1}{2}\int A_2\wedge F_{(4)}\wedge F_{(4)},$$

where φ is the dilaton, $F_{(3)} = dA_2$ is the field strength of the NS-NS two form, $F_{(2)} = dA_1$ is the field strength of the R-R 1-form, $F_{(4)} = dA_3$, $\tilde{F}_{(4)} = dA_3 + F_{(3)}\wedge A_1$ are the Ramond-Ramond field strengths.

P-brane Solution

Ansatz for the metric

$$\underbrace{M_0}_{\text{the transverse space}} \times \underbrace{M_1}_{\text{the worldvolume}}$$

P-brane Solution

Ansatz for the metric

$$\underbrace{M_0}_{\text{the transverse space}} \times \underbrace{M_1}_{\text{the worldvolume}}$$

$$SO(D-p-1) \times (\text{Poincaré}_{(p+1)})$$

P-brane Solution

Ansatz for the metric

$$\underbrace{M_0}_{\text{the transverse space}} \times \underbrace{M_1}_{\text{the worldvolume}}$$

$$SO(D-p-1) \times (\text{Poincaré}_{(p+1)})$$

$$g = e^{2\phi(x)} g^0 + e^{2\gamma(x)} g^1 .$$

The harmonic function

A p-brane solution depends on a harmonic function H

$$\nabla^2 H = 0,$$

where (∇^2) is the Laplacian on the transverse space.

The SUSY algebra

$$\frac{1}{p!}(C\Gamma_{\mu_1\ldots\mu_p})_{\alpha\beta}Z^{\mu_1\cdots\mu_p},$$

$$Z^{\mu_1\cdots\mu_p} = Q_{(p)}\int dX^{\mu_1}\wedge dX^{\mu_2}\wedge\ldots\wedge dX^{\mu_p}$$

is the topological charge, C is the charge conjugation matrix, X^μ are spacetime coordinates and Γ is an antisymmetric combination of Gamma matrices.

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = Z_{\alpha\beta}$$

The BPS bound

$$T = Q_{(p)}$$

The SUSY algebra

$$\frac{1}{p!}(C\Gamma_{\mu_1\ldots\mu_p})_{\alpha\beta}Z^{\mu_1\cdots\mu_p},$$

$$Z^{\mu_1\cdots\mu_p} = Q_{(p)}\int dX^{\mu_1}\wedge dX^{\mu_2}\wedge\ldots\wedge dX^{\mu_p}$$

is the topological charge, C is the charge conjugation matrix, X^μ are spacetime coordinates and Γ is an antisymmetric combination of Gamma matrices.

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = Z_{\alpha\beta}$$

The BPS bound

$$T = Q_{(p)}$$

$11D$ SUGRA: $M2$-brane, $M5$-brane, KK monopole, $M9$-brane

The SUSY algebra

$$\frac{1}{p!}(C\Gamma_{\mu_1\ldots\mu_p})_{\alpha\beta}Z^{\mu_1\ldots\mu_p},$$

$$Z^{\mu_1\ldots\mu_p} = Q_{(p)}\int dX^{\mu_1}\wedge dX^{\mu_2}\wedge\ldots\wedge dX^{\mu_p}$$

is the topological charge, C is the charge conjugation matrix, X^μ are spacetime coordinates and Γ is an antisymmetric combination of Gamma matrices.

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = Z_{\alpha\beta}$$

The BPS bound

$$T = Q_{(p)}$$

$11D$ SUGRA: $M2$-brane, $M5$-brane, KK monopole, $M9$-brane
$10D$ IIA SUGRA: $D0$-brane, $D2$-brane, $D4$-brane, $D6$-brane, $D8$-brane, F-string, NS5, NS9, KK monopole

The SUSY algebra

$$\frac{1}{p!}(C\Gamma_{\mu_1\dots\mu_p})_{\alpha\beta}Z^{\mu_1\dots\mu_p},$$

$$Z^{\mu_1\dots\mu_p} = Q_{(p)}\int dX^{\mu_1}\wedge dX^{\mu_2}\wedge\dots\wedge dX^{\mu_p}$$

is the topological charge, C is the charge conjugation matrix, X^μ are spacetime coordinates and Γ is an antisymmetric combination of Gamma matrices.

$$\{\hat{Q}_\alpha, \hat{Q}_\beta\} = Z_{\alpha\beta}$$

The BPS bound

$$T = Q_{(p)}$$

$11D$ SUGRA: $M2$-brane, $M5$-brane, KK monopole, $M9$-brane
$10D$ IIA SUGRA: $D0$-brane, $D2$-brane, $D4$-brane, $D6$-brane, $D8$-brane, F-string, NS5, NS9, KK monopole
$10D$ IIB SUGRA: $D1$-brane, $D3$-brane, $D5$-brane, $D7$-brane, $D9$-brane, F-string, NS5, NS9, KK monopole

Outline

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16,\ \nu = 32.$

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16,\ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16,\ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY
- M-brane configurations on flat factor spaces

$$M_0$$

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16,\ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY
- M-brane configurations on flat factor spaces

$$M_0 \times M_1$$

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16, \ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY
- M-brane configurations on flat factor spaces

$$M_0 \times M_1 \times \dots$$

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16,\ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY
- M-brane configurations on flat factor spaces

$$M_0 \times M_1 \times \ldots \times M_n, \quad M_i = \mathbb{R}^{k_i}$$

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16, \ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY
- M-brane configurations on flat factor spaces

$$M_0 \times M_1 \times \ldots \times M_n, \quad M_i = \mathbb{R}^{k_i}$$
$$\mathcal{N} = 2^{-k}, \quad k = 1, 2, 3, 4, 5$$

E. Bergshoeff, M. de Roo, et al., *Multiple Intersections of D-branes and M-branes*, *Class. Quantum Grav.*, 14, 2757(1997).

The amount of preserved supersymmetries

$$\mathcal{N} = n/\nu,$$

where ν — a number of maximal supersymmetries of the system

$11D$ SUGRA

- $n = 0, 1, 2, 4, 5, 6, 8, 16, \ \nu = 32.$
- Basic M2- and M5- branes solutions preserve 1/2 SUSY
- M-brane configurations on flat factor spaces

$$M_0 \times M_1 \times \ldots \times M_n, \quad M_i = \mathbb{R}^{k_i}$$
$$\mathcal{N} = 2^{-k}, \quad k = 1, 2, 3, 4, 5$$

E. Bergshoeff, M. de Roo, et al., *Multiple Intersections of D-branes and M-branes, Class. Quantum Grav.*, 14, 2757(1997).

- $\Rightarrow$ P-branes on Ricci-flat manifolds, $\mathcal{N} = ?$

Generalization to curved manifolds

- The generalization of worldvolume manifold to Ricci-flat manifold admitting Killing spinors

M. J. Duff, H. Lü, C. N. Pope and E. Sezgin, *Phys. Lett. B* **371**, (1996) 206

D. R. Brecher and M. J. Perry, *Nucl.Phys. B* **566**, 51-172 (2000).

Generalization to curved manifolds

- The generalization of worldvolume manifold to Ricci-flat manifold admitting Killing spinors

M. J. Duff, H. Lü, C. N. Pope and E. Sezgin, *Phys. Lett. B* **371**, (1996) 206

D. R. Brecher and M. J. Perry, *Nucl.Phys. B* **566**, 51-172 (2000).

- The solutions with indecomposable Ricci-flat brane worldwolumes

A. Kaya, Nucl. Phys. B **583** , 411 (2000).

J. M. Figueroa-O'Farrill, *More Ricci-flat branes*, *Phys.Lett. B* **471** , 128-132 (1999).

Orthogonally intersecting branes

Orthogonally intersecting branes

$(p + q_1) -$ brane :
$(p + q_2) -$ brane :
$(p + q_3) -$ brane :

Orthogonally intersecting branes

$$\begin{array}{ll} (p+q_1) - \text{brane} : & \overbrace{X \cdots X}^{p+1} \\ (p+q_2) - \text{brane} : & X \cdots X \\ (p+q_3) - \text{brane} : & X \cdots X \end{array}$$

Orthogonally intersecting branes

$$
\begin{array}{lcccc}
 & \overbrace{\quad\quad}^{p+1} & \overbrace{\quad\quad}^{q_1} & \overbrace{\quad\quad}^{q_2} & \overbrace{\quad\quad}^{q_3} \\
(p+q_1)-\text{brane}: & X\cdots X & X\cdots X & -\cdots- & -\cdots- \\
(p+q_2)-\text{brane}: & X\cdots X & -\cdots- & X\cdots X & -\cdots- \\
(p+q_3)-\text{brane}: & X\cdots X & -\cdots- & -\cdots- & X\cdots X
\end{array}
$$

Orthogonally intersecting branes

$$
\begin{array}{lccccc}
 & \overbrace{X \cdots X}^{p+1} & \overbrace{X \cdots X}^{q_1} & \overbrace{- \cdots -}^{q_2} & \overbrace{- \cdots -}^{q_3} & \overbrace{- \cdots -}^{\tilde{D}} \\
(p+q_1) - \text{brane}: & & & & & \\
(p+q_2) - \text{brane}: & X \cdots X & - \cdots - & X \cdots X & - \cdots - & - \cdots - \\
(p+q_3) - \text{brane}: & X \cdots X & - \cdots - & - \cdots - & X \cdots X & - \cdots -
\end{array}
$$

Orthogonally intersecting branes

	$\overbrace{}^{p+1}$	$\overbrace{}^{q_1}$	$\overbrace{}^{q_2}$	$\overbrace{}^{q_3}$	$\overbrace{}^{\tilde{D}}$
$(p+q_1)$ − brane :	$X \cdots X$	$X \cdots X$	$- \cdots -$	$- \cdots -$	$- \cdots -$
$(p+q_2)$ − brane :	$X \cdots X$	$- \cdots -$	$X \cdots X$	$- \cdots -$	$- \cdots -$
$(p+q_3)$ − brane :	$X \cdots X$	$- \cdots -$	$- \cdots -$	$X \cdots X$	$- \cdots -$

The three branes have

- the $(p+1)$-dimensional common worldvolume space
- the relative transverse $(q_1+q_2+q_3)$-dimensional space
- the totally transverse $\tilde{D}$-dimensional space.

Configurations on Ricci-flat factor spaces

The product of manifolds

$$M = M_0 \times M_1 \times \ldots \times M_n$$
$$g = e^{2\gamma(x)} g^0 + \sum_{i=1}^{n} e^{2\phi^i(x)} g^i.$$

The diagonalizing D-beins

$$g_{MN} = \eta_{AB} e^A_M e^B_N,$$
$$\text{with} \quad \eta_{AB} = \eta^{AB} = \eta_A \delta_{AB}, \quad e^A = e^A_M dx^M$$

The field strength for composite intersecting branes

$$F = \sum_{s=1}^{m} c_s \mathcal{F}_s,$$

where $\mathcal{F}_s$ is an elementary 4-form corresponding to s-th p-brane, $c_s = \pm 1$ is the sign factor of s-th p -brane, which defines the orientation of the worldvolume.

SUSY equations

Killing Spinor Equations

$$\delta\psi = (D_M + B_M)\varepsilon = 0, \quad \text{where} \quad D_M = \partial_M + \frac{1}{4} w_{ABM} \hat{\Gamma}^A \hat{\Gamma}^B,$$

ε is a SUSY transformation parameter, w_{ABM} is a spin connection.

$$\hat{\Gamma}^A \hat{\Gamma}^B + \hat{\Gamma}^B \hat{\Gamma}^A = 2\eta^{AB} \mathbf{1}_{32},$$

$$B_M = \frac{1}{288} \left(\Gamma_M \Gamma^N \Gamma^P \Gamma^Q \Gamma^R - 12 \delta_M^N \Gamma^P \Gamma^Q \Gamma^R \right) F_{NPQR}$$

and Γ_M are world gamma matrices satisfying Clifford algebra relations

$$\Gamma_M = e_M^A \Gamma_A, \quad \Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2 g_{MN} \mathbf{1}_{32}$$

SUSY equations

Killing Spinor Equations

$$\delta\psi = (D_M + B_M)\varepsilon = 0, \quad \text{where} \quad D_M = \partial_M + \frac{1}{4} w_{ABM} \hat{\Gamma}^A \hat{\Gamma}^B,$$

ε is a SUSY transformation parameter, w_{ABM} is a spin connection.

$$\hat{\Gamma}^A \hat{\Gamma}^B + \hat{\Gamma}^B \hat{\Gamma}^A = 2\eta^{AB} \mathbf{1}_{32},$$

$$B_M = \frac{1}{288} \left(\Gamma_M \Gamma^N \Gamma^P \Gamma^Q \Gamma^R - 12 \delta_M^N \Gamma^P \Gamma^Q \Gamma^R \right) F_{NPQR}$$

and Γ_M are world gamma matrices satisfying Clifford algebra relations

$$\Gamma_M = e_M^A \Gamma_A, \quad \Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2 g_{MN} \mathbf{1}_{32}$$

The number of unbroken SUSY

$$\mathcal{N} = N/32,$$

where N is the dimension of the linear space of solutions to differential equation $(D_M + B_M)\varepsilon = 0$.

The chirality conditions

The solutions admit spinors in the form

$$\varepsilon = \left(\prod_{s\in S_e} H_s^{-1/6}\right)\left(\prod_{s\in S_m} H_s^{-1/12}\right)\eta$$

with the parallel spinor η

$$\bar{D}^l_{m_l}\eta = 0, \quad \bar{D}^l_{m_l} = \partial_{m_l} + w^{(l)}_{a_l b_l m_l}, \quad l = 0, \ldots, n, \tag{1}$$

satisfying brane chirality conditions

$$\hat{\Gamma}_{[s]}\eta = c_s\eta.$$

The chirality operators

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{A_1}\hat{\Gamma}^{A_2}\hat{\Gamma}^{A_3}, \quad \text{for} \quad s \in S_e$$

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{B_1}\hat{\Gamma}^{B_2}\hat{\Gamma}^{B_3}\hat{\Gamma}^{B_4}\hat{\Gamma}^{B_5}, \quad \text{for} \quad s \in S_m$$

$$\left(\hat{\Gamma}_{[s]}\right)^2 = 1$$

Outline

$M2 \cap M2 \cap M2$

The product of manifolds

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4$$

$d_0 = 4,\ d_1 = d_2 = d_3 = 2$ and $d_4 = 1$

$$g = \left\{ \begin{array}{ccccccccccccc} - & - & - & - & \times & \times & - & - & - & - & \times & & : H_1 \\ - & - & - & - & - & - & \times & \times & - & - & \times & & : H_2 \\ \underbrace{- \quad - \quad - \quad -}_{M_0} & & & & \underbrace{- \quad -}_{M_1} & & \underbrace{- \quad -}_{M_2} & & \underbrace{\times \quad \times}_{M_3} & & \underbrace{\times}_{M_4} & & : H_3. \end{array} \right.$$

The solution

$$g = H_1^{1/3} H_2^{1/3} H_3^{1/3} \{ \hat{g}^0 + H_1^{-1} \hat{g}^1 + H_2^{-1} \hat{g}^2 + H_3^{-1} \hat{g}^3 + H_1^{-1} H_2^{-1} H_3^{-1} \hat{g}^4 \},$$

$$F = c_1 dH_1^{-1} \wedge \hat{\tau}_1 \wedge \hat{\tau}_4 + c_2 dH_2^{-1} \wedge \hat{\tau}_2 \wedge \hat{\tau}_4 + c_3 dH_3^{-1} \wedge \hat{\tau}_3 \wedge \hat{\tau}_4,$$

where $c_1^2 = c_2^2 = c_3^2 = 1$. The metrics g^i, $i = 0, 1, 2, 3$, have Euclidean signatures and $g^4 = -dt \otimes dt$.

$M2 \cap M2 \cap M2$

The set of gamma matrices

$$(\hat{\Gamma}^A) = \begin{array}{ccccccccc} (\hat{\Gamma}^{a_0}_{(0)} & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & 1, \\ \hat{\Gamma}_{(0)} & \otimes & \hat{\Gamma}^{a_1}_{(1)} & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & 1, \\ i\hat{\Gamma}_{(0)} & \otimes & \hat{\Gamma}_{(1)} & \otimes & \hat{\Gamma}^{a_2}_{(2)} & \otimes & \mathbf{1}_2 & \otimes & 1, \\ \hat{\Gamma}_{(0)} & \otimes & \hat{\Gamma}_{(1)} & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}^{a_3}_{(3)} & \otimes & 1, \\ \hat{\Gamma}_{(0)} & \otimes & \hat{\Gamma}_{(1)} & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & 1), \end{array}$$

where

$$\hat{\Gamma}_{(0)} = \hat{\Gamma}^{1_0}_{(0)} \ldots \hat{\Gamma}^{4_0}_{(0)}, \quad \hat{\Gamma}_{(i)} = \hat{\Gamma}^{1_i}_{(i)} \hat{\Gamma}^{2_i}_{(i)},$$

obey

$$(\hat{\Gamma}_{(0)})^2 = \mathbf{1}_4, \qquad (\hat{\Gamma}_{(i)})^2 = -\mathbf{1}_2, \quad i = 1, 2, 3.$$

The monomial spinor reads

$$\eta = \eta_0(x) \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4(y_4),$$

where $\eta_0 = \eta_0(x)$ is 4-component spinor on M_0, $\eta_i = \eta_i(y_i)$ is 2-component spinor on M_i, $i = 1, 2, 3$, and $\eta_4 = \eta_4(y_4)$ is 1-component spinor on M_4.

$M2 \bigcap M2 \bigcap M2$

$$\bar{D}^{(0)}_{m_0}\eta = (D^{(0)}_{m_0}\eta_0)\otimes\eta_1\otimes\eta_2\otimes\eta_3\otimes\eta_4, \quad \bar{D}^{(1)}_{m_1}\eta = \eta_0\otimes(D^{(1)}_{m_1}\eta_1)\otimes\eta_2\otimes\eta_3\otimes\eta_4,$$
$$\bar{D}^{(2)}_{m_2}\eta = \eta_0\otimes\eta_1\otimes(D^{(2)}_{m_2}\eta_2)\otimes\eta_3\otimes\eta_4, \quad \bar{D}^{(3)}_{m_3}\eta = \eta_0\otimes\eta_1\otimes\eta_2\otimes(D^{(3)}_{m_3}\eta_3)\otimes\eta_4,$$
$$\bar{D}^{(4)}_{m_4}\eta = \eta_0\otimes\eta_1\otimes\eta_2\otimes\eta_3\otimes(D^{(4)}_{m_4}\eta_4),$$

where $D^{(i)}_{m_i}$ correspond to M_i, $i = 0,1,2,3$. Here $D^{(4)}_{m_4} = \partial_{m_4}$.

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_1}\hat{\Gamma}^{2_1}\hat{\Gamma}^{1_4} = -\hat{\Gamma}_{(0)}\otimes\mathbf{1}_2\otimes\hat{\Gamma}_{(2)}\otimes\hat{\Gamma}_{(3)}\otimes 1, \quad \text{for} \quad s = I_1,$$
$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_2}\hat{\Gamma}^{2_2}\hat{\Gamma}^{1_4} = -\hat{\Gamma}_{(0)}\otimes\hat{\Gamma}_{(1)}\otimes\mathbf{1}_2\otimes\hat{\Gamma}_{(3)}\otimes 1, \quad \text{for} \quad s = I_2,$$
$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3}\hat{\Gamma}^{1_4} = -\hat{\Gamma}_{(0)}\otimes\hat{\Gamma}_{(1)}\otimes\hat{\Gamma}_{(2)}\otimes\mathbf{1}_2\otimes 1, \quad \text{for} \quad s = I_3.$$

The chirality restrictions are satisfied if

$$\hat{\Gamma}_{(0)}\eta_0 = c_{(0)}\eta_0, \qquad c^2_{(0)} = 1,$$
$$\hat{\Gamma}_{(j)}\eta_j = c_{(j)}\eta_j, \qquad c^2_{(j)} = -1,$$

$j = 1,2,3$ with

$$c_{(0)} = c_1 c_2 c_3, \qquad c_{(j)} = \pm i c_j.$$

$M2 \bigcap M2 \bigcap M2$

The following solution to SUSY equations corresponding to the field configuration from

$$\varepsilon = H_1^{-1/6} H_2^{-1/6} H_3^{-1/6} \eta_0(x) \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4.$$

Here η_i, $i = 0, 1, 2, 3$, are chiral parallel spinors defined on M_i, respectively ($D^{(i)}_{m_i} \eta_i = 0$), η_4 is constant.

The number of linear independent solutions to $(D_M + B_m)\varepsilon = 0$

$$N = 32\mathcal{N} = n_0(c_1c_2c_3) \sum_{c=\pm 1} n_1(icc_1)n_2(icc_2)n_3(icc_3),$$

where $n_j(c_{(j)})$ is the number of chiral parallel spinors on M_j, $j = 0, 1, 2, 3$.

M	M_0	M_1	M_2	M_3	$\mathcal{N} = 1/16 n_0(c_1c_2c_3)$
	$\mathbb{R}^4$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	1/8
	$K3 = CY_2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$1/8$ for $c_1c_2c_3 = 1$ / 0 for $c_1c_2c_3 = -1$
	$\mathbb{C}^2_*/Z_2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$1/8$ for $c_1c_2c_3 = 1$ / 0 for $c_1c_2c_3 = -1$

$M5 \cap M5 \cap M5$

The product manifold

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4,$$

where $d_0 = 3$, $d_1 = d_2 = d_3 = d_4 = 2$.

$$g = \begin{cases} - \; - \; - \; - \; - \; \times \; \times \; \times \; \times \; \times \; \times & : H_1 \\ - \; - \; - \; \times \; \times \; - \; - \; \times \; \times \; \times \; \times & : H_2 \\ \underbrace{- \; - \; -}_{M_0} \; \underbrace{\times \; \times}_{M_1} \; \underbrace{\times \; \times}_{M_2} \; \underbrace{- \; -}_{M_3} \; \underbrace{\times \; \times}_{M_4} & : H_3. \end{cases}$$

The solutions for the metric and field strengths

$$g = H_1^{2/3} H_2^{2/3} H_3^{2/3} \Big\{ g^0 + H_2^{-1} H_3^{-1} g^1 + H_1^{-1} H_3^{-1} g^2 + H_1^{-1} H_2^{-1} g^3 + H_1^{-1} H_2^{-1} H_3^{-1} g^4 \Big\},$$

$$F = c_1 (*_0 dH_1) \wedge \tau_1 + c_2 (*_0 dH_2) \wedge \tau_2 + c_3 (*_0 dH_3) \wedge \tau_3,$$

where $c_1^2 = c_2^2 = c_3^2 = 1$.

$M5 \cap M5 \cap M5$

The set of gamma matrices

$$(\hat{\Gamma}^A) = \begin{pmatrix} 1 & \otimes & \hat{\Gamma}_{(1)} & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)}, \\ 1 & \otimes & i\hat{\Gamma}^{a_1}_{(1)} & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)}, \\ 1 & \otimes & \mathbf{1}_2 & \otimes & \hat{\Gamma}^{a_2}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)}, \\ 1 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & i\hat{\Gamma}^{a_3}_{(3)} & \otimes & \hat{\Gamma}_{(4)}, \\ 1 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & \hat{\Gamma}^{a_4}_{(4)} \end{pmatrix}$$

Here the operators $\hat{\Gamma}_{(i)}$, $i = 1, 2, 3, 4$, are given by

$$\hat{\Gamma}_{(1)} = \hat{\Gamma}^{1_1}_{(1)}\hat{\Gamma}^{2_1}_{(1)}, \quad \hat{\Gamma}_{(2)} = \hat{\Gamma}^{1_2}_{(2)}\hat{\Gamma}^{2_2}_{(2)}, \quad \hat{\Gamma}_{(3)} = \hat{\Gamma}^{1_3}_{(3)}\hat{\Gamma}^{2_3}_{(3)}, \quad \hat{\Gamma}_{(4)} = \hat{\Gamma}^{1_4}_{(4)}\hat{\Gamma}^{2_4}_{(4)}\hat{\Gamma}^{3_4}_{(4)}\hat{\Gamma}^{4_4}_{(4)}$$

obey $\quad (\hat{\Gamma}_{(i)})^2 = -\mathbf{1}_2, \quad (\hat{\Gamma}_{(4)})^2 = -\mathbf{1}_4, \quad$ with $\quad i = 1, 2, 3.$

$$\eta = \eta_0(x) \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4(y_4),$$

where $\eta_0(x)$ is a 1-component spinor on M_0, $\eta_i = \eta_i(y_i)$ is a 2-component spinor on M_i, $i = 1, 2, 3$, $\eta_4 = \eta_4(y_4)$ is a 4-component spinor on M_4.

$M5 \cap M5 \cap M5$

The covariant derivatives can be written down as

$$\bar{D}_{m_1} = \partial_{m_1} + \frac{1}{4}\omega^{(1)}_{a_1 b_1 m_1}\left(1 \otimes \hat{\Gamma}^{a_1}_{(1)}\hat{\Gamma}^{b_1}_{(1)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_4\right),$$

$$\bar{D}_{m_2} = \partial_{m_2} + \frac{1}{4}\omega^{(2)}_{a_2 b_2 m_2}\left(1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}^{a_2}_{(2)}\hat{\Gamma}^{b_2}_{(2)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_4\right),$$

$$\bar{D}_{m_3} = \partial_{m_3} + \frac{1}{4}\omega^{(3)}_{a_3 b_3 m_3}\left(1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}^{a_3}_{(3)}\hat{\Gamma}^{b_3}_{(3)} \otimes \mathbf{1}_4\right),$$

$$\bar{D}_{m_4} = \partial_{m_4} + \frac{1}{4}\omega^{(4)}_{a_4 b_4 m_4}\left(1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}^{a_4}_{(4)}\hat{\Gamma}^{b_4}_{(4)}\right),$$

where $\omega^{(i)a_i}{}_{b_i c_i}$ are components of the spin connection corresponding to the manifold M_i, $D^{(i)}_{m_i}$ is a covariant derivatives corresponding to M_i, $i = 1, 2, 3, 4$, $\bar{D}_{m_0} = \partial_{m_0}$ and $D^{(0)}_{m_0} = \partial_{m_0}$.

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{1_2}\hat{\Gamma}^{2_2}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3} = 1 \otimes \hat{\Gamma}_{(1)} \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(4)} \quad \text{for} \quad s = I_1,$$

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{2_1}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3} = 1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(2)} \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(4)} \quad \text{for} \quad s = I_2,$$

$$\hat{\Gamma}_{[s]} = \hat{\Gamma}^{1_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{2_1}\hat{\Gamma}^{1_2}\hat{\Gamma}^{2_2} = 1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \quad \text{for} \quad s = I_3.$$

The supersymmetry constraints

$$\hat{\Gamma}_{(j)}\eta_j = c_{(j)}\eta_j, \qquad c_{(j)}^2 = -1, \quad j = 1,2,3,4,$$

and

$$c_{(1)}c_{(4)} = c_1, \quad c_{(2)}c_{(4)} = c_2, \quad c_{(3)}c_{(4)} = c_3.$$

$$\varepsilon = \prod_{s=1}^{3} H_s^{-\frac{1}{12}}\eta_0 \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_2(y_3) \otimes \eta_4(y_4),$$

where η_i, $i = 1,2,3,4$ are parallel spinors defined on M_i, respectively.

$$c_{(1)} = -ic_1, \quad c_{(2)} = -ic_2, \quad c_{(3)} = -ic_3, \quad c_{(4)} = i,$$
$$c_{(1)} = ic_1, \quad c_{(2)} = ic_2, \quad c_{(3)} = ic_3, \quad c_{(4)} = -i.$$

The number of preserved supersymmetries

$$N = 32\mathcal{N} = n_1(-ic_1)n_2(-ic_2)n_3(-ic_3)n_4(i) + n_1(ic_1)n_2(ic_2)n_3(ic_3)n_4(-i),$$

where $n_j(c_j)$ is the number of chiral parallel spinors on M_j, $j = 1,2,3,4$.

Exapmles

Let $M_0 = \mathbb{R}$ and $M_1 = M_2 = M_3 = \mathbb{R}^2$. Then all $n_j(c) = 1$, $j = 1, 2, 3$, with $c = \pm i$, and hence

$$N = 32\mathcal{N} = n_4(i) + n_4(-i). \tag{2}$$

M	M_0	M_1	M_2	M_3	M_4	$\mathcal{N}$
	$\mathbb{R}$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^{1,3}$	1/8
	$\mathbb{R}$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$(\mathbb{R}^{1,1}_*/Z_2) \times \mathbb{R}^2$	1/16
	$\mathbb{R}$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	a $4d$ pp-wave manifold	1/16

$M2 \cap M2 \cap M5$

The product manifold

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4 \times M_5 \times M_6,$$

$d_0 = 3$, $d_1 = d_2 = d_4 = d_5 = d_6 = 1$ and $d_3 = 3$.

$$g = \left\{ \begin{array}{ccccccccccccc} - & - & - & \times & - & - & - & - & - & \times & \times & & : H_1 \\ - & - & - & - & \times & - & - & - & \times & - & \times & & : H_2 \\ \underbrace{- \quad - \quad -}_{M_0} & & & \underbrace{-}_{M_1} & \underbrace{-}_{M_2} & \underbrace{\times \quad \times \quad \times}_{M_3} & & & \underbrace{\times}_{M_4} & \underbrace{\times}_{M_5} & \underbrace{\times}_{M_6} & & : H_3. \end{array} \right.$$

The solution reads

$$g = H_1^{1/3} H_2^{1/3} H_3^{2/3} \Big\{ g^0 + H_1^{-1} g^1 + H_2^{-1} g^2 + H_3^{-1} g^3 + H_2^{-1} H_3^{-1} g^4 + H_1^{-1} H_3^{-1} g^5 + H_1^{-1} H_2^{-1} H_3^{-1} g^6 \Big\},$$

$$F = c_1 dH_1^{-1} \wedge \tau_1 \wedge \tau_5 \wedge \tau_6 + c_2 dH_2^{-1} \wedge \tau_2 \wedge \tau_4 \wedge \tau_6 + c_3 (*_0 dH_3) \wedge \tau_1 \wedge \tau_2,$$

$M2 \cap M2 \cap M5$

The set of Γ-matrices

$$\begin{aligned}
(\hat{\Gamma}^A) = (&\hat{\Gamma}^{a_0}_{(0)} \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2, \\
&\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2, \\
&\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \mathbf{1}_2, \\
&\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \hat{\Gamma}^{a_3}_{(3)} \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_1 \otimes \mathbf{1}_2, \\
&\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_3, \\
&\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_1, \\
&\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes i \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2).
\end{aligned}$$

$$\hat{\Gamma}_{(0)} = \hat{\Gamma}^{1_0}_{(0)}\hat{\Gamma}^{2_0}_{(0)}\hat{\Gamma}^{3_0}_{(0)}, \quad \hat{\Gamma}_{(3)} = \hat{\Gamma}^{1_3}_{(3)}\hat{\Gamma}^{2_3}_{(3)}\hat{\Gamma}^{3_3}_{(3)}$$

$$(\hat{\Gamma}^{a_i}_{(i)}) = (\sigma_1, \sigma_2, \sigma_3)\,, \quad \hat{\Gamma}_{(i)} = i\mathbf{1}_2, \quad i = 0, 6$$

$M2 \cap M2 \cap M5$

$$\eta = \eta_0(x) \otimes \eta_1(y_1) \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4(y_4) \otimes \eta_5(y_5) \otimes \eta_6(y_6) \otimes \chi,$$

where $\eta_i = \eta_i(y_i)$ is a 1-component spinor on M_i, $i = 1, 2, 4, 5, 6$, $\eta_0 = \eta_0(x)$ is a 2-component spinor on M_0, $\eta_3 = \eta_3(y_3)$ is a 2-component spinor on M_3 and χ belongs to $V = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$. The covariant derivatives $\bar{D}_{m_i}$ act on η as follows

$$\bar{D}_{m_0}\eta = \left(D^{(0)}_{m_0}\eta_0\right) \otimes \eta_1 \otimes \eta_2 \otimes \eta_3 \otimes \eta_4 \otimes \eta_5 \otimes \eta_6 \otimes \chi,$$

$$\cdots$$

$$\bar{D}_{m_3}\eta = \eta_0 \otimes \eta_1 \otimes \eta_2 \otimes \left(D^{(3)}_{m_3}\eta_3\right) \otimes \eta_4 \otimes \eta_5 \otimes \eta_6 \otimes \chi,$$

$$\cdots$$

$$\bar{D}_{m_6}\eta = \eta_0 \otimes \eta_1 \otimes \eta_2 \otimes \eta_3 \otimes \eta_4 \otimes \eta_5 \otimes \left(D^{(6)}_{m_6}\eta_6\right) \otimes \chi,$$

where

$$\bar{D}_{m_0} = \partial_{m_0} + \frac{1}{4}\omega^{(0)}_{a_0 b_0 m_0}(\hat{\Gamma}^{a_0}_{(0)}\hat{\Gamma}^{b_0}_{(0)} \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2),$$

$$\bar{D}_{m_3} = \partial_{m_3} + \frac{1}{4}\omega^{(3)}_{a_3 b_3 m_3}(\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \hat{\Gamma}^{a_3}_{(3)}\hat{\Gamma}^{b_3}_{(3)} \otimes 1 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2).$$

$M2 \cap M2 \cap M5$

The operators corresponding to the $M2$-branes and the $M5$-brane

$$\hat\Gamma_{[s]} = \hat\Gamma^{1_1}\hat\Gamma^{1_5}\hat\Gamma^{1_6} = -\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes B_1,$$

for $s = I_1$,

$$\hat\Gamma_{[s]} = \hat\Gamma^{1_2}\hat\Gamma^{1_4}\hat\Gamma^{1_6} = -\mathbf{1}_2 \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes B_2,$$

for $s = I_2$,

$$\hat\Gamma_{[s]} = \hat\Gamma^{1_0}\hat\Gamma^{2_0}\hat\Gamma^{3_0}\hat\Gamma^{1_1}\hat\Gamma^{1_2} = i\hat\Gamma_{(0)} \otimes 1 \otimes 1 \otimes \mathbf{1}_2 \otimes 1 \otimes 1 \otimes 1 \otimes B_3,$$

for $s = I_3$.

B_s are self-adjoint commuting idempotent operators acting on $V = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$

$$B_1 = \sigma_1 \otimes \mathbf{1}_2 \otimes \sigma_3, \quad B_2 = \sigma_2 \otimes \sigma_3 \otimes \sigma_1, \quad B_3 = \mathbf{1}_2 \otimes \sigma_3 \otimes \mathbf{1}_2.$$

$$\varepsilon = H_1^{-1/6} H_2^{-1/6} H_3^{-1/12} \eta_0(x) \otimes \eta_1 \otimes \eta_2 \otimes \eta_3(y_3) \otimes \eta_4 \otimes \eta_5 \otimes \eta_6 \otimes \psi_{\varepsilon_1,\varepsilon_2,\varepsilon_3},$$

where $\eta_0(x)$ and $\eta_3(y_3)$ are parallel spinors defined on M_0 and M_3, respectively, η_i is a constant 1-dimensional spinor on M_i, $i = 1, 2, 4, 5, 6$.

$$-\varepsilon_s = c_s. \tag{3}$$

The number of preserved SUSY

$$\mathcal{N} = n_0 n_3/32$$

where n_j is the number of parallel spinors on the 3-dimensional manifolds M_j, $j = 0, 3$.

M	M_0	M_1	M_2	M_3	M_4	M_5	M_6	$\mathcal{N}$
	$\mathbb{R}^3$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}^3$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	1/8

$M2 \cap M5 \cap M5$

The product of manifolds

$$M_0 \times M_1 \times M_2 \times M_3 \times M_4 \times M_5,$$

$d_0 = d_2 = d_3 = d_4 = d_5 = 2$ и $d_1 = 1$.

$$g = \left\{ \begin{array}{cccccccccccccc} - & - & \times & - & - & - & - & - & - & \times & \times & & : H_1 \\ - & - & - & \times & \times & - & - & \times & \times & \times & \times & & : H_2 \\ \underbrace{- \quad -}_{M_0} & & \underbrace{-}_{M_1} & \underbrace{- \quad -}_{M_2} & & \underbrace{\times \quad \times}_{M_3} & & \underbrace{\times \quad \times}_{M_4} & & \underbrace{\times \quad \times}_{M_5} & & & : H_3. \end{array} \right.$$

The solution

$$g = H_1^{1/3} H_2^{2/3} H_3^{2/3} \Big\{ \hat{g}^0 + H_1^{-1} \hat{g}^1 + H_2^{-1} \hat{g}^2 + H_3^{-1} \hat{g}^3 + H_2^{-1} H_3^{-1} \hat{g}^4 + H_1^{-1} H_2^{-1} H_3^{-1} \hat{g}^5 \Big\},$$

$$F = c_1 dH_1^{-1} \wedge \hat{\tau}_1 \wedge \hat{\tau}_5 + c_2 (*_0 dH_2) \wedge \hat{\tau}_1 \wedge \hat{\tau}_3 + c_3 (*_0 dH_3) \wedge \hat{\tau}_1 \wedge \hat{\tau}_2,$$

$c_1^2 = c_2^2 = c_3^2 = 1$. g^i $(i = 0, 1, 2, 3, 4)$ have Euclidean signatures g^5 has the signature $(-, +)$.

$M2 \cap M5 \cap M5$

Γ-matrices

$$
(\hat{\Gamma}^A) = \begin{array}{ccccccccccc}
(\hat{\Gamma}^{a_0}_{(0)} & \otimes & 1 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2, \\
\hat{\Gamma}_{(0)} & \otimes & 1 & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)} & \otimes & \hat{\Gamma}_{(5)} \\
i\hat{\Gamma}_{(0)} & \otimes & 1 & \otimes & \hat{\Gamma}^{a_2}_{(2)} & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 \\
i\hat{\Gamma}_{(0)} & \otimes & 1 & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}^{a_3}_{(3)} & \otimes & \mathbf{1}_2 & \otimes & \mathbf{1}_2 \\
\hat{\Gamma}_{(0)} & \otimes & 1 & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}^{a_4}_{(4)} & \otimes & \mathbf{1}_2 \\
\hat{\Gamma}_{(0)} & \otimes & 1 & \otimes & \hat{\Gamma}_{(2)} & \otimes & \hat{\Gamma}_{(3)} & \otimes & \hat{\Gamma}_{(4)} & \otimes & \hat{\Gamma}^{a_5}_{(5)}),
\end{array}
$$

$$
\hat{\Gamma}_{(5)} = \hat{\Gamma}^{1_5}_{(5)}\hat{\Gamma}^{2_5}_{(5)}, \quad \hat{\Gamma}_{(i)} = \hat{\Gamma}^{1_i}_{(i)}\hat{\Gamma}^{2_i}_{(i)},
$$

satisy

$$
(\hat{\Gamma}_{(i)})^2 = -\mathbf{1}_2, \quad (\hat{\Gamma}_{(5)})^2 = \mathbf{1}_2, \quad i = 0, 2, 3, 4.
$$

$$
\eta = \eta_0(x) \otimes \eta_1 \otimes \eta_2(y_2) \otimes \eta_3(y_3) \otimes \eta_4(y_4) \otimes \eta_5(y_5)
$$

where $\eta_i = \eta_i(y_i)$ is a 2-component spinor on M_i, $i = 0, 2, 3, 4, 5$, η_1 is a 1-component spinor on M_1.

$M2 \cap M5 \cap M5$

The operator $\bar{D}^{(i)}_{m_i}$ acts on η as

$$\bar{D}^{(i)}_{m_i}\eta = \ldots \otimes \eta_{i-1} \otimes \left(D^{(i)}_{m_i}\eta_i\right) \otimes \eta_{i+1} \otimes \ldots$$

The chirality operators

$$\begin{aligned}
\hat{\Gamma}_{[s]} &= \hat{\Gamma}^{1_1}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3} = \hat{\Gamma}_{(0)} \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \otimes \mathbf{1}_2, \quad \text{for} \quad s = I_1,\\
\hat{\Gamma}_{[s]} &= \hat{\Gamma}^{1_0}\hat{\Gamma}^{2_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{1_3}\hat{\Gamma}^{2_3} = \mathbf{1}_2 \otimes 1 \otimes \hat{\Gamma}_{(2)} \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(4)} \otimes \hat{\Gamma}_{(5)}, \quad \text{for} \quad s = I_2,\\
\hat{\Gamma}_{[s]} &= \hat{\Gamma}^{1_0}\hat{\Gamma}^{2_0}\hat{\Gamma}^{1_1}\hat{\Gamma}^{1_2}\hat{\Gamma}^{2_2} = \mathbf{1}_2 \otimes 1 \otimes \mathbf{1}_2 \otimes \hat{\Gamma}_{(3)} \otimes \hat{\Gamma}_{(4)} \otimes \hat{\Gamma}_{(5)}, \quad \text{for} \quad s = I_3.
\end{aligned}$$

The restrictions are satisfied if

$$\begin{aligned}
\hat{\Gamma}_{(3)}\eta_3 = c_{(3)}\eta_3, &\qquad c_{(3)}^2 = 1,\\
\hat{\Gamma}_{(j)}\eta_j = c_{(j)}\eta_j, &\qquad c_{(j)}^2 = -1, \quad j = 0, 2, 4, 5\\
c_{(0)}c_{(2)}c_{(3)}c_{(4)} = c_1, \quad c_{(2)}c_{(4)}c_{(5)} &= c_2, \quad c_{(3)}c_{(4)}c_{(5)} = c_3.
\end{aligned}$$

$M2 \cap M5 \cap M5$

The solution to SUSY equations

$$\varepsilon = H_1^{-1/6} H_2^{-1/12} H_3^{-1/12} \eta_0(x) \otimes \eta_1 \otimes \eta_2(y_2) \otimes \eta_2(y_3) \otimes \eta_4(y_4) \otimes \eta_5(y_5),$$

where η_i, $i = 0, 2, 3, 4, 5$ are chiral parallel spinors defined on M_i, η_1 is constant.

The number of linear independent solutions to $(D_M + B_m)\varepsilon = 0$

$$N = 32\mathcal{N} = \sum_{\substack{\varepsilon_2=\pm1,\\ \varepsilon_4=\pm1}} n_0(i\varepsilon_4 c_1 c_2 c_3) n_2(i\varepsilon_2) n_3(i\varepsilon_2 c_2 c_3) n_4(i\varepsilon_4) n_5(-\varepsilon_2\varepsilon_4 c_2),$$

where $n_j(c_j)$ is the number of chiral parallel spinors on M_j, $j = 0, 2, 3, 4, 5$, $\varepsilon_2 = \pm 1$, $\varepsilon_4 = \pm 1$.

M	M_0	M_1	M_2	M_3	M_4	M_5	$\mathcal{N}$
	$\mathbb{R}^2$	$\mathbb{R}$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^{(1,1)}$	1/8
	$\mathbb{R}^2$	$\mathbb{R}$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}^2$	$\mathbb{R}_*^{(1,1)}/Z_2$	1/16

Outline

Conclusions

- We have obtained relations for computing the amount of preserved SUSY
  - $M2$-/$M5$-branes defined on product spaces including Ricci-flat manifolds and flat spaces with non-trivial topology.
  - All possible orthogonal intersections of two M-branes: $M2 \cap M2$, $M2 \cap M5$, $M5 \cap M5$.
  - All possible triple intersections: $M2 \cap M2 \cap M2$, $M5 \cap M5 \cap M5$(three configurations), $M2 \cap M5 \cap M5$ (two configurations), $M2 \cap M2 \cap M5$.

Conclusions

- We have obtained relations for computing the amount of preserved SUSY
  - $M2$-/$M5$-branes defined on product spaces including Ricci-flat manifolds and flat spaces with non-trivial topology.
  - All possible orthogonal intersections of two M-branes: $M2 \cap M2$, $M2 \cap M5$, $M5 \cap M5$.
  - All possible triple intersections: $M2 \cap M2 \cap M2$, $M5 \cap M5 \cap M5$(three configurations), $M2 \cap M5 \cap M5$ (two configurations), $M2 \cap M2 \cap M5$.
- The amount of preserved supersymmetries for the model defined on the product of factor spaces including Ricci-flat manifolds or manifolds with non trivial topology is less than for the case of the product of flat spaces.

Conclusions

- We have obtained relations for computing the amount of preserved SUSY
  - $M2$-/$M5$-branes defined on product spaces including Ricci-flat manifolds and flat spaces with non-trivial topology.
  - All possible orthogonal intersections of two M-branes: $M2 \cap M2$, $M2 \cap M5$, $M5 \cap M5$.
  - All possible triple intersections: $M2 \cap M2 \cap M2$, $M5 \cap M5 \cap M5$(three configurations), $M2 \cap M5 \cap M5$ (two configurations), $M2 \cap M2 \cap M5$.
- The amount of preserved supersymmetries for the model defined on the product of factor spaces including Ricci-flat manifolds or manifolds with non trivial topology is less than for the case of the product of flat spaces.
- For the intersections $M2 \cap M2 \cap M2$, $M5 \cap M5 \cap M5$ we have presented examples where $\mathcal{N}$ depend on upon brane sign factors $c_s = \pm 1$.

V. D. Ivashchuk, *More M-branes on product of Ricci-flat manifolds, IJGMMP* **9** 8 (2012); [arXiv:1107.4089v3 [hep-th]].

A.A. Golubtsova and V.D. Ivashchuk, *Triple M-brane configurations and preserved supersymmetries, Nucl. Phys.* **B872**, 289-312(2013).

THANK YOU FOR YOUR ATTENTION!