DIAS-13

Etudes in mathematical cosmology: integrable and nonintegrable cosmological and static reductions in extended theories of gravity.

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General properties and some spherical/cylindrical reductions, expansion near horizons, vecton – scalaron duality, topological portrait of Static (BH) - Cosmology solutions, an approach to search for (partially) integrable dilaton gravity coupled to scalars.

arXiv:1302.6372; 1302.6969 (E.A.Davydov + ATF); 1112.3023

The **affine generalized Einstein gravity** and its reductions to **dilaton gravity coupled to vecton and scalars** were presented in the papers

arXiv: 1011.2445, 1008.2333, 1003.0782, 0812.2616

Integrability of **general DG coupled to scalars** and **unified treatment** of static and cosmological solutions were considered in: MPLA **11**(1996)1691-1704, arXiv: hep-th/0307269*, 0505060, 0612258*, 0811.4501*, 0902.4445*, refs. therein.

(* V.de Alfaro + ATF)

Content of the talk

Why **Ein.** is **Int. ?**

Who ordered scalar fields? Where they come from? Brief discussion of models based on Superstrings, SUGRA. Scalar particles coupled to (super)gravity.

Briefly on **affine models** based on ideas of Weyl, Einstein, Eddington
Vecton – Scalaron equivalence in Dilaton Gravity for D = 2,1

Dilaton Gravity (**DG**) coupled to **scalars** (**DGS**).
Unified treatment of **static** (**BH**) and **cosmological** solutions.
Dynamical systems of static & cosmological states in DGS

Integrability *vs*. Nonintegrability: new integrable models and **`Master Integral Equation' or MIE,** in DVectG and DScalG

Partially integrable DSG (insufficient # of integrals) and approximate methods for BHs and cosmologies (cosmostat)

Topological portraits (idea and the simplest example)

$$\sqrt{-g^{(10)}}\left(e^{-2\phi^{(10)}}R^{(10)}+4e^{-2\phi^{(10)}}(D\phi^{(10)})^2-\frac{1}{12}H^{(10)2}\right)$$

10-D SUGRA is dim. reduced to dimension 6

$$ds^2=g^{(6)}_{\mu\nu}dx^\mu dx^\nu+e^{\psi}dx^m dx^m \qquad H^{(10)}=dB$$

NS - 3-form

$$g^{(6)}_{\mu\nu}=\begin{pmatrix} g^{(5)}_{\alpha\beta}+e^{\psi_1}A_\alpha A_\beta & e^{\psi_1}A_\alpha \\ e^{\psi_1}A_\beta & e^{\psi_1}\end{pmatrix}$$

Kaluza reduction to 5-brane $S^1\times T^4$

and then spherical reduction to DGS

$$H'=H-A\wedge F_2 \qquad F_{2\alpha\beta}=H_{\alpha\beta x^5}$$

In D=2: $H'_{ijk}=H_0\epsilon_{ijk}$

$$\sqrt{-g}e^{-2\phi}\Big(R+6e^{2\psi_2}+4(D\phi)^2-(D\psi)^2-\frac{1}{4}(D\psi_1)^2-3(D\psi_2)^2$$

D=2

$$-\frac{1}{2}e^{6\psi_2}H_0^2-\frac{1}{4}e^{-\psi_1}F_2^2-\frac{1}{4}e^{\psi_1}F^2\Big)$$

e.g.,Y.Kiem e.a. hep-th/9806182

$$ds^2=g_{\alpha\beta}dx^\alpha dx^\beta+e^{-2\psi_2}d\Omega_{(3)}$$

Spherical reduction on 5-brane

Generalized cylindrical reduction of 4-dimensional Einstein gravity
(Kaluza - type reduction)

$$ds_4^2 = (g_{ij} + \varphi\,\sigma_{mn}\,\varphi_i^m \varphi_j^n)\,dx^i dx^j + 2\varphi_{im}\,dx^i dy^m + \varphi\,\sigma_{mn}\,dy^m dy^n$$

x-coordinates (t, r) $\qquad y^m = (\varphi, z)$ (coordinates on torus)

$$\sigma_{22} = e^{\eta}\cosh\xi, \quad \sigma_{33} = e^{-\eta}\cosh\xi, \quad \sigma_{23} = \sigma_{32} = \sinh\xi$$

$$\sqrt{-g}\left[\varphi R(g) + \frac{1}{2\varphi}(\nabla\varphi)^2 - \frac{\varphi}{2}[(\nabla\xi)^2 + (\cosh\xi)^2\,(\nabla\eta)^2] + V_{\text{eff}}(\varphi,\xi,\eta)\right]$$

$$V_{\text{eff}}(\varphi,\xi,\eta) = -\frac{\cosh\xi}{2\varphi^2}\left[Q_1^2 e^{-\eta} - 2Q_1Q_2\tanh\xi + Q_2^2 e^{\eta}\right]$$

Interesting **consistent truncation**: $\qquad Q_1 \neq 0,\ Q_2 = 0,\ \xi \equiv 0$:

$$\mathcal{L}_{\text{eff}}^{(2)} = \sqrt{-g}\left[\varphi R(g) + \frac{1}{2\varphi}(\nabla\varphi)^2 - \frac{Q_1^2}{2\varphi^2}\,e^{-\eta} - \frac{\varphi}{2}\,(\nabla\eta)^2\right]$$

GEOMETRY OF SYMMETRIC CONNECTIONS

General symmetric connection

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + a^i_{jk}$$

$$\Gamma^i_{jk}[g] = \frac{1}{2} g^{il} (g_{lj,k} + g_{lk,j} - g_{jk,l})$$

$$r^i_{jkl} = -\gamma^i_{jk,l} + \gamma^i_{mk}\gamma^m_{jl} + \gamma^i_{jl,k} - \gamma^i_{ml}\gamma^m_{jk}$$

NONSYMMETRIC RICCI CURVATURE

$$r_{jk} = -\gamma^i_{jk,i} + \gamma^i_{mk}\gamma^m_{ji} + \gamma^i_{ji,k} - \gamma^i_{mi}\gamma^m_{jk}$$

Symmetric part of the Ricci curvature

$$s_{ij} \equiv \frac{1}{2}(r_{ij} + r_{j\,i})$$

Anti-symmetric part of the Ricci curvature

$$a_{ij} \equiv \frac{1}{2}(r_{ij} - r_{j\,i}) = \frac{1}{2}(\gamma^m_{jm,i} - \gamma^m_{im,j})$$

$$a_{ij,\,k} + a_{jk,i} + a_{ki,j} \equiv 0$$

VECTON: $\quad a_i \equiv a^m_{im}$

$$a_i \equiv \gamma^m_{mi} - \Gamma^m_{mi} \equiv \gamma_i - \partial_i \ln \sqrt{|g|}$$

$$a_{ij} \equiv -\frac{1}{2}(a_{i,j} - a_{j,i}) \equiv -\frac{1}{2}(\gamma_{i,j} - \gamma_{j,i})$$

The simplest examples of generalized symmetric connection

$\alpha\beta$ - CONNECTION

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha(\delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j) - (\alpha - 2\beta)g_{jk}\,\hat{a}^i$$

Weyl: $\beta = 0$

geo-Riemannian: $\alpha = 2\beta$.

Einstein $\alpha = -\beta = \frac{1}{6}$

Main principles (suggested by Einstein's approach)

1. **Geometry:** **dimensionless** *`action'* constructed of a ***scalar** density*; its variations give the geometry and main equations ***without complete specification of the analytic form of the Lagrangian .***

2. **Dynamics:** a concrete Lagrangian constructed of the ***geometric variables*** - homogeneous function of order D (e.g. , the square root of the determinant of the curvature) produces a physical **effective Lagrangian**.

3. **Duality** between the geometrical and physical variables and Lagrangians.
   NB: This looks more artificial than the first two principles and, possibly, works for rather special models (actually giving *exotic fields*, *tachyons* etc.)

A simple nontrivial choice of a **geometric Lagrangian** density, which generalizes the Eddington – Einstein Lagrangian ,

$$\mathcal{L} \equiv \sqrt{-\det(r_{ij})} \equiv \sqrt{-r}$$

is the following (depending on one dimensionless parameter):

$$\mathcal{L}_{geom} = \sqrt{-\det(s_{ij} + \bar{l} a_{ij})} \equiv \sqrt{-\Delta_s}$$

It is more convenient to take below: $\mathcal{L} \equiv 2\tilde{\gamma}\sqrt{-\Delta_s}$

Now we define (following Einstein) the metric and field densities by a Legendre-like transformation

$$\frac{\partial \mathbf{L}}{\partial s_{ij}} \equiv \mathbf{g}^{ij}\,, \qquad \frac{\partial \mathbf{L}}{\partial a_{ij}} \equiv \mathbf{f}^{ij}$$

dual to

Here we better introduce a dimensional parameter

$$s_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{g}^{ij}}\,, \qquad a_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{f}^{ij}}$$

$$\nabla_i^{\gamma}\, \mathbf{f}^{ki} \;=\; \partial_i \mathbf{f}^{ki} \;\equiv\; \mathbf{a}^k. \qquad \nabla_i^{\gamma}\, \mathbf{g}^{ik} \;=\; -\frac{D+1}{D-1}\hat{\mathbf{a}}^k$$

The main equation $$\nabla_i^{\gamma}\, \mathbf{g}^{jk} \;=\; -\frac{1}{D-1}(\delta_i^j \hat{\mathbf{a}}^k + \delta_i^k \hat{\mathbf{a}}^j)$$

for any dimension D

Neglect hats!

Defining the Riemann metric

tensor g_{ij} by the equations

$$g^{ij}\sqrt{-g} = \mathbf{g}^{ij}\,, \quad g_{ij}\,g^{jk} = \delta_i^k$$

$$\nabla_i\, g_{jk} = 0, \quad \nabla_i\, g^{jk} = 0 \qquad \hat{a}^k \equiv \hat{\mathbf{a}}^k / \sqrt{-g}$$

we can **derive** the expression for the connection coefficients

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha_D\,[\,\delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j - (D-1)\,g_{jk}\,\hat{a}^i]$$

$$\alpha_D \equiv [(D-1)(D-2)]^{-1}\,, \qquad \beta_D \equiv -[2(D-1)]^{-1}$$

We thus have derived the connection using a rather general dynamics!

Not using any particular form of the geometric Lagrangian!

It is easy to derive the important relation

$$|\Delta_g| \equiv |\det(\mathbf{g}^{ij} + l\,\mathbf{f}^{ij})| = \tilde{\gamma}^D\,|\det(s_{ij} + l^{-1}\,a_{ij})|^{(D-2)/2}\,.$$

and using the relations between geometrical and `physical' variables
to find the complete effective Lagrangian replacing the EH one

$$\mathcal{L}_{phys} = \sqrt{-g}\left[-2\Lambda\,[\det(\delta_i^j + l f_i^j)]^\nu + R(g) - m^2\,g^{ij}a_i a_j\right]$$

Λ having the dimension L^{-2}

l is a dimensionless

$$[s_{ij}] = [a_{ij}] = \mathrm{L}^{-2}$$

$$[\tilde{\gamma}] = \mathrm{L}^{D-2}$$

Dimensional reductions of

$$\mathcal{L}_{\rm ph} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^\nu + R(g) + c_a\, g^{ij} a_i a_j\right]$$

Spherical reduction of the theory

$$ds^2_D = ds^2_2 + ds^2_{D-2} = g_{ij}\, dx^i\, dx^j \; + \; \varphi^{2\nu}\, d\Omega^2_{D-2}(k)$$

$$\mathcal{L}_D^{(2)} = \sqrt{-g}\left[\varphi R(g) + k_\nu\, \varphi^{1-2\nu} + \frac{1-\nu}{\varphi}(\nabla\varphi)^2 + X(\varphi, \mathbf{f}^2) - m^2 \varphi\, \mathbf{a}^2\right]$$

$$X(\varphi, \mathbf{f}^2) \equiv -2\Lambda\varphi\left[1 + \frac{1}{2}\lambda^2 \mathbf{f}^2\right]^\nu \qquad \mathbf{f}^2 \equiv f_{ij} f^{ij} \qquad \nu \equiv (D-2)^{-1}$$

D=3: Maxwell+Einstein+cosm. constant

Weyl rescaling

$$g_{ij} = \hat{g}_{ij}\, w^{-1}(\varphi), \quad w(\varphi) = \varphi^{1-\nu} \qquad \mathbf{f}^2 = w^2\, \hat{\mathbf{f}}^2, \quad \mathbf{a}^2 = w\, \hat{\mathbf{a}}^2$$

$$\mathcal{L}^{(2)}_{DW} = \sqrt{-g}\left[\varphi R(g) + k_\nu\, \varphi^{-\nu} - 2\Lambda\varphi^{\nu}\left[1 + \frac{1}{2}\lambda^2\varphi^{2(1-\nu)}\mathbf{f}^2\right]^{\nu} - m^2\varphi\,\mathbf{a}^2\right]$$

3-dimensional theory

$$\mathcal{L}^{(2)}_{3} = \sqrt{-g}\left[\varphi R(g) - 2\Lambda\varphi - \lambda^2\Lambda\varphi\,\mathbf{f}^2 - m^2\varphi\,\mathbf{a}^2\right]$$

Vecton – Scalaron DUALITY

in LC coordinates

$$ds^2 = -4\,h(u,v)\,du\,dv\,, \quad \sqrt{-g} = 2h \qquad f^n_{uv} \equiv a^n_{u,v} - a^n_{v,u}$$

L /2*h* = $\varphi R + V(\varphi,\psi) + X(\varphi,\psi;\mathbf{f}^2_n)$ $\qquad -2\mathbf{f}^2_n = (f^n_{uv}/h)^2$

L' /2*h* = $\varphi R + V(\varphi,\psi) + X_{\text{eff}}(\varphi,\psi;q_n)$ $\qquad q_n(u,v) \equiv h^{-1}X_n\,f^n_{uv}$

Effective action on `mass shell'; *f* – from eq, & $\quad$ & $\quad X_n \equiv \dfrac{\partial X}{\partial\,\mathbf{f}^2_n}$

$$X_{\text{eff}}(\varphi, \psi; q_n) = X(\varphi, \psi; \bar{\mathbf{f}}_n^2) + \sum q_n(u,v)\sqrt{-2\,\bar{\mathbf{f}}_n^2}$$

where: $2\,\bar{\mathbf{f}}_n^2 = -(q_n/\bar{X}_n)^2$ $\bar{X}_n \equiv \frac{\partial}{\partial\,\bar{\mathbf{f}}_n^2} X(\varphi, \psi; \bar{\mathbf{f}}_n^2)$

$$\partial_u(h^{-1} X_n f_{uv}^n) = -Z_n a_u^n = \partial_u\; q_n(u,v)$$

This defines a_u^n in terms of $q_n(u,v)$ and (φ, ψ)

$$X_{\text{eff}} = -2\Lambda\sqrt{\varphi}\left[1 + q^2/\lambda^2\Lambda^2\varphi^2\right]^{\frac{1}{2}}$$ for $D = 4$

$V = 2k\varphi^{-\frac{1}{2}}$, $\bar{Z} = -1/m^2\varphi$ **N.B**: normally, ***Z*** ~ to dilaton φ

$$X_{\text{eff}}(\varphi;\; q(u,v)) = -q^2/\lambda^2\Lambda\varphi - 2\Lambda\varphi$$ for $D = 3$

The result: we can study **DSG** instead of **DVG**

General **dilaton gravity** coupled to massless vectors and eff. massive scalars

$$\mathcal{L}^{(2)} = \sqrt{-g}\,[\,U(\varphi)R(g) + V(\varphi) + W(\varphi)(\nabla\varphi)^2 + \\ + X(\varphi, \psi, F^2_{(1)}, ..., F^2_{(A)}) + Y(\varphi, \psi) + \sum_n Z_n(\varphi, \psi)(\nabla\psi_n)^2\,]\,.$$

Dilaton gravity **dual** to vecton gravity with **massless Abelian vector fields,** Weyl frame

$$\mathcal{L}^{(2)}_{\rm eff} = \sqrt{-g}\left[\varphi R + V(\varphi, \psi) + X_{\rm eff}(\varphi, \psi; q) + \sum Z(\varphi, \psi)(\nabla\psi)^2\right].$$

DSG dual to **massive** vecton gravity (Weyl frame) has kinetic q-term

$$\mathcal{L}_{\rm dsg} = \sqrt{-g}\left[\varphi R + U(\varphi, \psi, q) + \bar{Z}(\varphi)(\nabla q)^2 + \sum Z(\varphi, \psi)(\nabla\psi)^2\right]$$

A general theory of **HORIZONS** in DSG

L' /2h = $\varphi R + U(\varphi, \psi, q) + \bar{Z}(\varphi)(\nabla q)^2$ (omitting normal scalars)

Consider **STATIC** solutions that normally **have horizons** when there are **no scalars**

All the equations can be derived from the Hamiltonian (constraint)

H = $\dot{\varphi}\,\dot{h}/h + hU + \bar{Z}\,\dot{q}^2 + Z\,\dot{\psi}^2$ (**= 0** in the end)

Without the scalars the EXACT solutions is: $h = C_0^2\,[N_0 - N(\varphi)]$

where $N(\varphi) \equiv \int U(\varphi) d\varphi$ $C_0 \tau = \int d\varphi\,[N_0 - N(\varphi)]^{-1}$

There is always a horizon, i.e. $h \to 0 \text{ for } \varphi \to \varphi_0$

Horizons are classified into:

:

regular **simple**, regular **degenerate**, **singular**

We find a gen. sol. near horizon as locally convergent power series in: $\tilde{\varphi} \equiv \varphi - \varphi_0$

$$h = \sum h_n \tilde{\varphi}^n, \qquad \chi = \sum \chi_n \tilde{\varphi}^n, \qquad q = \sum q_n \tilde{\varphi}^n, \qquad \chi(\varphi) \equiv \dot{\varphi}$$

$$h_0 = \chi_0 = 0 \qquad q_0 \neq 0 \qquad \tilde{\varphi} \equiv \varphi - \varphi_0$$

The equations for these functions are **not integrable** and we do not know exact solutions of the recurrence relations

Practically the **same equations** are applicable to studies of the **cosmological models** with vecton.

However, we can show that the global picture cannot be found without knowledge of horizons connecting static and cosmological solutions.

It is important to use local language. BUT! The physics can not be completely understood without global picture.

Main differential equations

$$\psi' = E(\xi)\,, \quad H' = -E^2 H(\xi)\,;$$

$$\chi' = -Z(\varphi)\,U(\varphi,\psi)\,H\,, \quad \eta' = -\frac{1}{2}Z(\varphi)\,U_\psi(\varphi,\psi)\,H\,.$$

$$\psi' \equiv \frac{d\psi}{\partial\xi}\,, \quad U_\psi \equiv \frac{\partial U}{\partial\psi}\,, \quad \xi \equiv \int d\varphi\, Z^{-1}(\varphi)\,, \quad Z(\varphi) = 1/\xi'(\varphi)\,.$$

$$E(\xi) \equiv \frac{\eta(\xi)}{\chi(\xi)}\,, \quad H(\xi) \equiv \frac{h(\xi)}{\chi(\xi)}\,. \qquad \frac{d\eta}{d\chi} = \frac{U_\psi}{2U}\,, \quad \frac{d\ln H}{d\psi} = -E\,.$$

$$\psi(\xi) = \psi_0 + \int_{\xi_0}^{\xi} E(\bar{\xi}) \equiv \mathcal{I}\{E;\xi\}\,,$$

Solutions in terms of **one** function ***E***

Basic solutions

$$H(\xi) = H_0 \exp\int_{\xi_0}^{\xi} E^2(\bar{\xi}) \equiv H_0\, \exp\mathcal{I}\{E^2;\xi\}\,,$$

$$\chi(\xi) = \chi_0 - \mathcal{I}_1\{E;\xi\}\,, \quad \eta = \eta_0 - \mathcal{I}_2\{E;\xi\}\,,$$

$$\mathcal{I}_1\{E;\xi\} = -H_0 \int_{\xi_0}^{\xi} d\bar{\xi}\, Z[\varphi(\bar{\xi})]\; U[\varphi(\bar{\xi}, \mathcal{I}\{E;\bar{\xi}\}]\; e^{\mathcal{I}\{E^2;\bar{\xi}\}}\,.$$

$$\mathcal{I}_2\{E;\xi\} = -\frac{1}{2} H_0 \int_{\xi_0}^{\xi} d\bar{\xi}\, Z[\varphi(\bar{\xi})]\; U_\psi[\varphi(\bar{\xi}), \mathcal{I}\{E;\bar{\xi}\}]\; e^{\mathcal{I}\{E^2;\bar{\xi}\}}$$

THE MASTER INTEGRAL EQUATION

$$E(\xi) = \frac{\eta_0 - \mathcal{I}_2\{E;\xi\}}{\chi_0 - \mathcal{I}_1\{E;\xi\}}$$

$$U = u(\varphi)v(\psi) \Rightarrow \frac{U_\psi}{2U} = v_\psi(\psi)/2v(\psi) = g_1 \quad \text{if} \quad v(\psi) = e^{2g_1\psi}$$

Then $\mathcal{I}_2 = \mathcal{I}_1$ and we find the second-order diff. eq. for $E(\xi)$

It is integrable if, in addition, $Z(\varphi)\,u(\varphi) = g\,e^{g_2\,\xi(\varphi)}$.

Without it we have a 'partial integrability as $\eta = g_1\chi + C_0$.

This is a generalization of the model in ATF `96

If $C_0 = 0$, one can analytically derive the solutions depending on three parameters.

With this condition, we solve the essentially nonlinear system using **effective iterations of the MIE** from ***E = g_1***

Extended dynamical system

$$\rho + UH + \eta^2/\chi = \chi' + UH = \eta' + U_\psi H/2 = \rho' + U_\xi H = 0$$

$$\psi\,\eta' + \psi\,U_\psi H/2 = \eta\,\psi' - \eta^2/\chi = \xi\,\rho' + \xi\,U_\xi H = \xi'\rho - \rho = 0$$

generates integrals

$$F \equiv \ln h$$

$$\chi F' - \rho = 0$$

$$[c_1\chi + c_2\,\eta + c_3\,\rho + c_4(\psi\,\eta + \xi\,\rho)]' =$$

$$-H[(c_1 + c_4)U + c_2\,U_\psi/2 + c_3\,U_\xi + c_4(\psi\,U_\psi/2 + \xi\,U_\xi)]$$

$$c_1\chi + c_2\,\eta + c_3\,\rho + c_4(\psi\,\eta + \xi\,\rho) = I_1$$ = **integral**

IF $$(c_1 + c_4)U + c_2\,U_\psi/2 + c_3\,U_\xi + c_4(\psi\,U_\psi/2 + \xi\,U_\xi) = 0$$

$$U = U_0\exp(2g\psi + g_1\xi) \;\Rightarrow\; \chi' = (g^2 + g_1)\,\chi + (\rho_0 + 2\,\eta_0\,g) + \eta_0^2/\chi$$

Gauge independent dim. reduction: 2 to 1

$$ds_2^2 = e^{2\alpha(t,r)}dr^2 - e^{2\gamma(t,r)}dt^2$$

$$\epsilon \mathcal{L}_{\rm V}^{(1)} = e^{\epsilon(\alpha-\gamma)}\varphi\left[\dot\psi^2 - 2\dot\alpha_\epsilon\,\frac{\dot\varphi}{\varphi} - (1-\nu)\left(\frac{\dot\varphi}{\varphi}\right)^2\right] -$$

\epsilon = -1 for static solutions

$$-- \cdot e^{\epsilon(\gamma-\alpha)}\mu_v\,\varphi\,a_\epsilon^2 + \epsilon\,e^{\alpha+\gamma}\left[V + X(\mathbf{f}^2)\right]$$

$$\mathcal{L}_a \equiv h\,X(\mathbf{f}^2) - \tilde\mu\,\varphi\,a^2\,; \qquad \tilde\mu = e^{\epsilon(\gamma-\alpha)}\mu_v\,, \quad h = \epsilon\,e^{\alpha+\gamma}\,.$$

$$-\lambda^2\left(\frac{\dot a}{h}\right)^2 = -y^2\,, \quad p_a \equiv \frac{\partial\mathcal{L}_a}{\partial\dot a} = \lambda\,\frac{\partial X}{\partial y}\,, \Rightarrow -2q\,, \quad a \Rightarrow p/2$$

$$\mathcal{H}_a(p_a\,,a) \equiv p_a\,\dot a - \mathcal{L}_a = -h\left[X - y\frac{\partial X}{\partial y}\right] + \tilde\mu\,\varphi\,a^2$$

$$l_\epsilon^{-1}\left[\varphi\,\dot{\psi}^2 - 2\dot{\alpha}_\epsilon\,\dot{\varphi} - (1-\nu)\,\frac{\dot{\varphi}^2}{\varphi} + \frac{\dot{q}^2}{m^2\varphi}\right]$$

Scalaron Lagrangian and effective potential

$$+\, l_\epsilon\,\epsilon\,e^{2\alpha_\epsilon}\,U(\varphi,\psi,q)$$

$$X_{\text{eff}} = X - y\frac{\partial X}{\partial y} = -2\Lambda\varphi\,\frac{x}{y}\left[1-(1-2\nu)y^2\right]$$

$$y = x\left[1-(1-\nu)\,x^2 + \ldots\right]$$

$$1-y^2 = |x|^{-\sigma} - (\sigma/2)\,|x|^{-2\sigma} + \ \ldots \qquad \sigma \equiv 1/(1-\nu)$$

$$y = x\,(1-y^2)^{1-\nu} \qquad x \equiv q/(-2\nu\lambda\Lambda\,\varphi)$$

Equation for y(x)

$$X_{\text{eff}} = -2\Lambda\varphi\,[1 + q^2/4\nu\lambda^2\Lambda^2\varphi^2 \;+ O(x^4)]$$

Universal asymptotic behavior

$$X_{\text{eff}} = -2\sqrt{q^2/\lambda^2} + O(|x|^{-\sigma})$$

$$X_{\text{eff}}/(-2\Lambda\varphi) \equiv v_\nu(x)$$

Reduced potential

$$\mathcal{L}_q^{(1)} = \bar{l}_\epsilon^{-1}\,[\,\dot{\bar{q}}^2 - \dot{\xi}\,\dot{\alpha}_\epsilon\,] - \bar{l}_\epsilon\,2\Lambda h\,\varphi^{\nu-1}\,v_\nu\,(mx)$$

$$v_\nu(x) = 1 + \nu x^2 + O(x^4)$$

Monotonic concave function

$$v_\nu(x) = 2\nu x\left[1 + \frac{1-\nu}{2\nu}x^{-\sigma} + \right.$$ .....

$$\mathcal{L}_q^{(1)} = \bar{l}_\epsilon^{-1}\,[\,\dot{\bar{q}}^2 - \xi\,\dot{\alpha}_\epsilon\,] - \bar{l}_\epsilon\,2\Lambda h\,\varphi^{\nu-1}\,v_\nu\,(mx)$$

Hamiltonian: $$p_q^2/4 - p_\alpha p_\xi \,+\, 2\Lambda h\,\varphi^{\nu-1}\,v_\nu\,(mx) \,=\, 0$$

$$\dot{\bar{q}} = \bar{l}_\epsilon\,p_q/\,2\,, \quad \dot{\xi} = -\,\bar{l}_\epsilon\,p_\alpha\,, \quad \dot{\alpha} = -\,\bar{l}_\epsilon\,p_\xi$$

D = 2: $$2\Lambda h\,v_1\,(mx) \equiv hV\,(m^2q^2/\xi)$$

$$\dot{p}_q = \,-2hV'\,m^2\bar{q}/\xi\,, \quad \dot{p}_\xi = hV'\,m^2\bar{q}^2/\xi^2$$

$$\dot{p}_\alpha = -2hV(m^2\bar{q}^2/\xi)$$

$$p_q^2 - 4p_\alpha\,p_\xi + 4hV = 0$$

$$\bar{q}\,p_q + 2\,\xi\,p_\xi - p_\alpha = \,C_0$$

Hamiltonian

Integral

Simple linear PDE for V leads to integrals

As distinct from the standard Einstein theory, the generalized one is **not integrable** even in dimension one (static states and cosmologies). Therefore, in addition to the above solutions we need a global information on the system, which we may attempt to present as

topological portrait.

We try to demonstrate that the portrait must include **both static and cosmological** solutions, and that the most important info is in the structure of horizons. Actually, it is not less important for cosmologies than for static states. We prefer to use the **local language** and do not use the term Black Hole which should be reserved for real physical objects

For the moment, the idea can be explained only on integrable systems and only on the plane. For nonintegrable systems we need **3D portraites**

Rather a general integrable models, ATF `96

Massless scalar field INTEGRAL

$$V = W(\bar{g}_4 w^2 - \bar{g}_1), \quad Z^{-1} = W(\bar{g}_3 + \bar{g}_2 \log w)$$

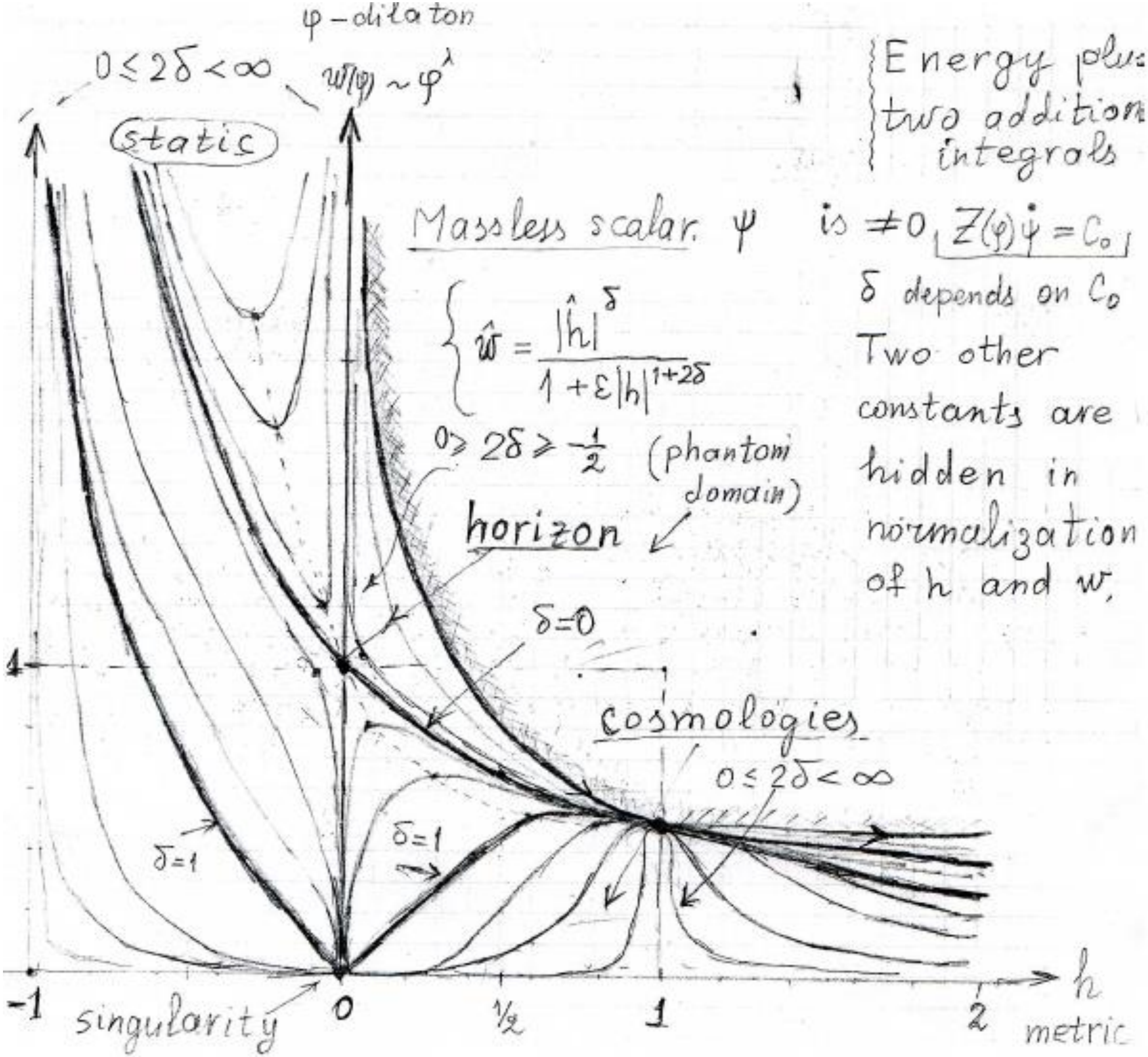
$$(F/W)^2 + 4\bar{g}_1 h + 2\bar{g}_2 C_0^2 \log h = \bar{C}_1$$

$$W = (1-\nu)/\phi,\ w = \phi^{1-\nu},\ Z = -\gamma\phi$$

$$w = \frac{|h|^\delta}{|1 + \varepsilon|h|^{1+2\delta}|}\,.$$

$$\bar{g}_1 = \bar{g}_2 = 0$$

$$w(\phi),\ w'/w \equiv W/U'$$



φ - dilaton
0 ≤ 2δ < ∞
w(φ) ~ φ^λ
static
Energy plus two additional integrals
Massless scalar ψ is ≠ 0, Z(φ)ψ̇ = C₀
δ depends on C₀
ŵ = |ĥ|^δ / (1 + ε|h|^(1+2δ))
Two other constants are hidden in normalization of h and w.
0 > 2δ ≥ -1/2 (phantom domain)
horizon
δ=0
cosmologies
0 ≤ 2δ < ∞
δ=1
δ=1
1
-1
0
½
1
2
h
metric
singularity

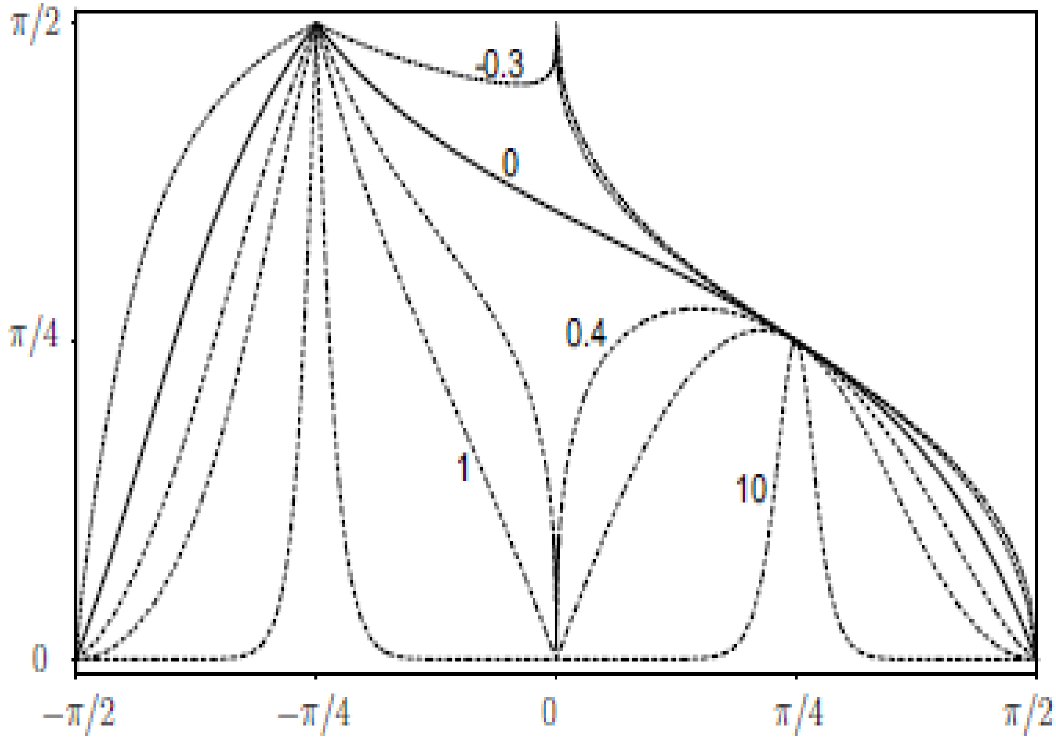


Figure 1: Topological portrait $\arctan w(x)$ of the dilaton-scalar configuration. Values of δ are given on plot. Solid line — separatrix with zero δ, dashed line — separatrix with arbitrary δ.

Picture by E.Davydov

We briefly described the simplest class of **affine theories of gravity in multidimensional space-times with symmetric connections** and their reductions to two-dimensional **dilaton - vecton gravity** field theories (DVG).

The distinctive feature of these theories is existence of an **absolutely neutral massive** (tachyonic) vector field (**vecton**) with essentially nonlinear coupling to dilaton gravity.

We consistently replaced the vecton field by a new effectively massive scalar (**scalaron**) having an unusual coupling to DVG.

Using vecton - scalaron duality, we applied methods and results of DSG to more complex DVG.

In DVG derived from affine theories, we studied *one-dimensional dynamical systems simultaneously describing cosmological and static states.*

Our approach is fully applicable to static, cosmological, and simple wave solutions in multidimensional theories (esp., by using the **scalaron - vecton duality**) and to **general** one-dimensional DGS models.

The global structure of the solutions of integrable DGS-1 models can be usefully visualized by drawing their **`topological portraits'** resembling the phase portraits of dynamical systems and simply visualizing

static – cosmological duality

Though the **MIE** provides us with an ***apparently new*** approach to solving most difficult global problems (e.g., a transition to **chaotic behavior**?) the considered examples are, of course, simply `warming up' etudes.

Crucial thing is to learn of how to find (partial) **portraits** in **not integrable** case (**3D portraits!)**

A very important field for further studies is to generalize our **S-C duality** to a possible **S-C-Waves triality**. It was uncovered in the integrable *N*-Liouville models (VdA and ATF) *(effectively one-dimensional waves of matter)* but a generalization to non-integrable case looks difficult although really important for cosmological applications.

Effects of *nonlinear Lagrangians*
are being studied (like in `B-I cosmology')

(we make it easier!)

Vecton dark matter can be produced
in ***strong*** *gravitational fields* only.

Quantum gravity is necessary!? (`partial' QG ?)

Inflation and ***dark matter***
are crucial things to study and test
the theory in cosmological models

THE

END

The simplest Superstring – SUGRA cosmology

$$\sqrt{|g|}e^{-\varphi}\left[R+(\nabla\varphi)^2-\frac{1}{2}(\nabla\beta)^2-\frac{1}{2}e^{2\varphi}(\nabla\sigma)^2\right]$$

Cosmology with moving 3-brane (from D=11)

$$H^{\mu\nu\lambda}=\epsilon^{\mu\nu\lambda\kappa}\,e^{\varphi}\,\nabla_\kappa\sigma$$

$$\sqrt{-g}\left[\frac{1}{2}R+\frac{1}{4}(\nabla\varphi)^2+\frac{3}{4}(\nabla\beta)^2+\frac{q_5}{2}e^{(\beta-\varphi)}(\nabla z)^2\right]$$

$$ds^2 = -e^{2\nu}d\tau^2+e^{2\alpha}d\mathbf{x}^2$$

z –position of the 5-brane

$$\alpha = \frac{1}{3}\ln\left|\frac{t-t_0}{T}\right|+\alpha_0$$

$$\beta = p_{\beta,i}\ln\left|\frac{t-t_0}{T}\right|+(p_{\beta,f}-p_{\beta,i})\ln\left(\left|\frac{t-t_0}{T}\right|^{-\delta}+1\right)^{-\frac{1}{\delta}}+\beta_0$$

$$\varphi = p_{\varphi,i}\ln\left|\frac{t-t_0}{T}\right|+(p_{\varphi,f}-p_{\varphi,i})\ln\left(\left|\frac{t-t_0}{T}\right|^{-\delta}+1\right)^{-\frac{1}{\delta}}+\varphi_0$$

$$z = d\left(1+\left|\frac{T}{t-t_0}\right|^{-\delta}\right)^{-1}+z_0\ .$$

See E.Copeland hep-th/0202028