Axial Anomaly and chiral (magnetic and vortical) effects

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Outline

Lecture 1 (on the blackboard)
Physical interpretation
Anomaly as Landau levels flow
Triangle diagram
Lecture 2
Anomaly and hadronic spectrum: t’Hooft principle
Abelian vs non-Abelian Anomalies
Chiral Magnetic Effect
Chiral Vortical Effect
Observation of chiral effects
Symmetries and conserved operators

- (Global) Symmetry -> conserved current \( \partial^\mu J_\mu = 0 \)

- Exact:
  - U(1) symmetry – charge conservation - electromagnetic (vector) current
  - Translational symmetry – energy momentum tensor \( \partial^\mu T_{\mu\nu} = 0 \)
Massless fermions (quarks) – approximate symmetries

- Chiral symmetry (mass flips the helicity)
  \[ \partial^\mu J^5_\mu = 0 \]

- Dilatational invariance (mass introduce dimensional scale – c.f. energy-momentum tensor of electromagnetic radiation)
  \[ T_{\mu\mu} = 0 \]
Quantum theory

- Currents -> operators
- Not all the classical symmetries can be preserved -> anomalies
- Enter in pairs (triples?...)
- Vector current conservation <-> chiral invariance
- Translational invariance <-> dilatational invariance
Calculation of anomalies

- Many various ways
- All lead to the same operator equation

\[ \partial_\mu j_5^{(0)} = 2i \sum_q m_q q^\gamma_5 q - \left( \frac{N_f \alpha_s}{4\pi} \right) G^a_{\mu\nu} G^{\mu\nu, a} \]

- UV vs IR languages - understood in physical picture (Gribov, Feynman, Nielsen and Ninomiya) of Landau levels flow (E||H)
Anomaly and virtual photons

- Often assumed that only manifested in real photon amplitudes
- Not true – appears at any $Q^2$
- Natural way – dispersive approach to anomaly (Dolgov, Zakharov’70) - anomaly sum rules
- One real and one virtual photon – Horejsi, OT’95

$\int_{4m^2}^{\infty} A_3(t; q^2, m^2) dt = \frac{1}{2\pi}$

$$F_j(p^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{A_j(t)}{t - p^2} dt, \quad j = 3, 4$$

$$T_{\alpha\mu\nu}(k, q) = F_1 \varepsilon_{\alpha\mu\nu\rho} k^\rho + F_2 \varepsilon_{\alpha\mu\nu\rho} q^\rho + F_3 q_\nu \varepsilon_{\alpha\mu\rho\sigma} k^\rho q^\sigma + F_4 q_\nu \varepsilon_{\alpha\mu\rho\sigma} k^\rho q^\sigma + F_5 k_\mu \varepsilon_{\alpha\nu\rho\sigma} k^\rho q^\sigma + F_6 q_\mu \varepsilon_{\alpha\nu\rho\sigma} k^\rho q^\sigma$$
Dispersive derivation

- Axial WI
  \[ F_2 - F_1 = 2mG + \frac{1}{2\pi^2} \]

- GI
  \[ F_2 - F_1 = (q^2 - p^2)F_3 - q^2F_4 \]

- No anomaly for imaginary parts

- Anomaly as a finite subtraction

\[
(q^2 - t)A_3(t) - q^2A_4(t) = 2mB(t)
\]

\[
F_j(p^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{A_j(t)}{t - p^2} dt, \quad j = 3, 4
\]

\[
F_2 - F_1 - 2mG = \frac{1}{\pi} \int_{4m^2}^{\infty} A_3(t) dt
\]

\[
\int_{4m^2}^{\infty} A_3(t; q^2, m^2) dt = \frac{1}{2\pi}
\]
Properties of anomaly sum rules

- Valid for any $Q^2$ (and quark mass)
- No perturbative QCD corrections (Adler-Bardeen theorem)
- No non-perturbative QCD corrections (t’Hooft consistency principle)
- Exact – powerful tool
Mesons contributions
(Klopot, Oganesian, OT)

- Pion – saturates sum rule for real photons
  \[ \text{Im} F_3 = \sqrt{2} f_\pi F_{\pi \gamma^* \gamma}(Q^2) \delta(s - m_\pi^2) \]
  \[ F_{\pi \gamma^* \gamma}(0) = \frac{1}{2\sqrt{2\pi^2 f_\pi}} \]

- For virtual photons – pion contribution is rapidly decreasing
  \[ F_{\pi \gamma^* \gamma}^{\text{asymp}}(Q^2) = \frac{\sqrt{2} f_\pi}{Q^2} + O(1/Q^4) \]

- This is also true also for axial and higher spin mesons (longitudianl components are dominant)

- Heavy PS decouple in a chiral limit
Anomaly as a collective effect

- One can never get constant summing finite number of decreasing function
- Anomaly at finite $Q^2$ is a collective effect of meson spectrum
- General situation – occurs for any scale parameter (playing the role of regulator for massless pole)
- For quantitative analysis – quark-hadron duality
Mesons contributions within quark hadron duality – transition FF (talks of P. Kroll, S. Mikhailov, A. Pimikov)

- Pion:
  \[ F_{\pi\gamma\gamma^*}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} \]

- Cf Brodsky&Lepage, Radyushkin – comes now from anomaly!

- Axial meson contribution to ASR

\[ \int_0^\infty A_3(s; Q^2)ds = \frac{1}{2\pi} = I_\pi + I_{a_1} + I_{cont}. \quad I_{a_1} = \frac{1}{2\pi} Q^2 \left( \frac{s_1 - s_0}{(s_1 + Q^2)(s_0 + Q^2)} \right) \]
Content of Anomaly Sum Rule ("triple point")

Figure 1: Relative contributions of \( \pi \) (blue line) and \( a_1 \) (orange line) mesons, intervals of duality are \( s_0 = 0.7 \text{ GeV}^2 \) and \( s_1 - s_0 = 1.8 \text{ GeV}^2 \) respectively, and continuum (black line), continuum threshold is \( s_1 = 2.5 \text{ GeV}^2 \)
ASR and BaBar data

- In the BaBar(2009) region – main contribution comes from the continuum
- Small relative correction to continuum – due to exactness of ASR must be compensated by large relative contributions to lower states!
- Amplification of corrections

\[
\frac{\delta I_{\text{cont}}}{I_{\text{cont}}} / \frac{\delta I_\pi}{I_\pi} = \frac{s_0}{Q^2} \sim \frac{1}{30}
\]

\[Q^2 = 20 \text{ GeV}^2, \ s_0 = 0.7 \text{ GeV}^2\]

- Smaller for eta because of larger duality interval (supported by BaBar)
Corrections to Continuum

- Perturbative – zero at 2 loops level (massive-Pasechnik&OT – however cf Melnikov; massless-Jegerlehner&Tarasov)
- Non-perturbative (e.g. instantons)
- The general properties of ASR require decrease at asymptotically large $Q^2$ (and $Q^2=0$)
- Corresponds to logarithmic contribution (cf Radyushkin, Polyakov, Dorokhov).

\[
I_{\text{cont}} = \frac{1}{2\pi s_0} \frac{Q^2}{s_0 + Q^2} - c_{s_0} \frac{\ln(Q^2/s_0) + b}{Q^2},
\]

\[
I_{\pi} = \frac{1}{2\pi s_0} \frac{s_0}{s_0 + Q^2} + c_{s_0} \frac{\ln(Q^2/s_0) + b}{Q^2}.
\]
Modelling of corrections

- Continuum vs pion

- Fit
  \[ b = -2.74, \quad c = 0.045. \]

- Continuum contribution similar for Radyushkin’s approach
Interplay of pion with lower resonances

- Small (NP) corrections to continuum – interplay of pion with higher states
- A1 – decouples for real photons
- Relation between transition FF’s of pion and A1 (testable!)
Generalization for $\eta(\prime)$

- Octet channel sum rule (gluon anomaly free)
Conclusions/Discussion-I

- New manifestation of Axial Anomaly - Anomaly Sum Rule – exact NPQCD tool – do not require QCD factorization
- Anomaly for virtual photons – collective effect (with fast excitation of collective mode)

- Exactness of ASR – very unusual situation when small pion contribution can be studied on the top of large continuum – amplification of corrections to continuum
- BaBar data – small negative correction to continuum
- If continuum is precisely described by Born term – interplay with A1 (TO BE STUDIED THEORETICALLY AND Experimentally)
- Similar collective effect is expected for finite temperature and/or chemical potential
Anomaly in Heavy Ion Collisions - Chiral Magnetic Effect (D. Kharzeev)

From QCD back to electrodynamics: Maxwell-Chern-Simons theory

\[ \mathcal{L}_{MCS} = -\frac{1}{4} F^{\mu \nu} F_{\mu \nu} - A_\mu J^\mu + \frac{c}{4} P_\mu J^\mu_{CS}. \]

\[ J^\mu_{CS} = \epsilon^{\mu \nu \rho \sigma} A_\nu F_{\rho \sigma}, \quad P_\mu = \partial_\mu \theta = (M, \vec{P}). \]

\[ \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c \left( M \vec{B} - \vec{P} \times \vec{E} \right), \]

\[ \vec{\nabla} \cdot \vec{E} = \rho + c \vec{P} \cdot \vec{B}, \]

\[ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \]

\[ \vec{\nabla} \cdot \vec{B} = 0, \]
Comparison of magnetic fields

The Earth's magnetic field: 0.6 Gauss

A common, hand-held magnet: 100 Gauss

The strongest steady magnetic fields achieved so far in the laboratory: $4.5 \times 10^5$ Gauss

The strongest man-made fields ever achieved, if only briefly: $10^7$ Gauss

Typical surface, polar magnetic fields of radio pulsars: $10^{13}$ Gauss

Surface field of Magnetars: $10^{15}$ Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html

At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon:

\[ eB(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss} \]
Induced current for (heavy - with respect to magnetic field strength) strange quarks

- Effective Lagrangian

\[ L = c (F\tilde{F})(G\tilde{G}) / m^4 + d (FF)(GG) / m^4 \]

- Current and charge density from \( c \) (~7/45) term

\[ j^\mu = 2c\tilde{F}^{\mu\nu}\partial_\nu(G\tilde{G}) / m^4 \]

- \( \rho \sim \tilde{H}\tilde{\nabla}\theta \) (multiscale medium!)

\[ \theta \sim (G\tilde{G}) / m^4 \rightarrow \int d^4xG\tilde{G} \]

- Light quarks --> matching with D. Kharzeev et al’ --> correlation of density of electric charge with a gradient of topological one (Lattice ?)
Properties of perturbative charge separation

- Current carriers are obvious - strange quarks -> matching -> light quarks?
- No relation to topology (also pure QED effect exists)
- Effect for strange quarks is of the same order as for the light ones if topological charge is localized on the distances $\sim 1/m_s$, strongly ($4^{\text{th}}$ power!) depends on the numerical factor: Ratio of strange/light – sensitive probe of correlation length
- Universality of strange and charm quarks separation - charm separation suppressed as $(m_s/m_c)^4 \sim 0.0001$
- Charm production is also suppressed – relative effects may be comparable at moderate energies (NICA?) – but low statistics
Anomaly in medium – new external lines in VVA graph

- Gauge field $\rightarrow$ velocity
- CME $\rightarrow$ CV(ortical)$\xi$
- Kharzeev, Zhitnitsky (07) – EM current
- Straightforward generalization: any (e.g. baryonic) current – neutron asymmeries@NICA - Rogachevsky, Sorin, OT - Phys.Rev.C82:054910,2010.
Baryon charge with neutrons – (Generalized) Chiral Vortical Effect

- **Coupling:** \[ e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha \]

- **Current:**
  - Uniform chemical potentials:
  - Rapidly (and similarly) changing chemical potentials:

\[
J_\gamma = \frac{N_c}{4\pi^2 N_f} \varepsilon^{\gamma\beta\alpha\rho} \partial_\alpha V_\rho \partial_\beta (\theta \sum_j e_j \mu_j)
\]

\[
J'_i = \frac{\sum_j g_{i(j)} \mu_j}{\sum_j e_j \mu_j} J'_e
\]

\[
J^0_i = \frac{|\vec{\nabla} \sum_j g_{i(j)} \mu_j|}{|\vec{\nabla} \sum_j e_j \mu_j|} J^0_e
\]
Comparing CME and CVE

- Orbital Angular Momentum and magnetic moment are proportional – Larmor theorem
- CME for 3 flavours – no baryon charge separation \(2/3-1/3-1/3=0!\) (Kharzeev, Son) - but strange mass!
- Same scale as magnetic field
Observation of chiral effects

- Sign of topological field fluctuations unknown – need quadratic (in induced current) effects
- CME – like-sign and opposite-sign correlations – S. Voloshin
- No antineutrons, but like-sign baryonic charge correlations possible
- Look for neutron pairs correlations!
- MPD@NICA (lecture of A. Sorin) may be well suited for neutrons!
Charge asymmetry w.r.t. reaction plane:
how to detect it?

\[
\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle = \\
= \langle \cos \Delta \phi_\alpha \cos \Delta \phi_\beta \rangle - \langle \sin \Delta \phi_\alpha \sin \Delta \phi_\beta \rangle \\
= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B^{\text{in}}] - [\langle a_\alpha a_\beta \rangle + B^{\text{out}}].
\]

S. Voloshin, hep-ph/0406311

A sensitive measure of the asymmetry:

\[
a^k a^m = \sum_{ij} \sin(\varphi_i^k - \Psi_R) \sin(\varphi_j^m - \Psi_R)
\]

Expect \(a^+ a^+ = a^- a^- > 0; \ a^+ a^- < 0\).
Figure 2. (Taken from [17]) STAR results compared to simulations for 200 GeV Au+Au. Blue symbols mark opposite-charge correlations, and red are same-charge. The shaded bands show the systematic error due to uncertainty in $v_2$ measurements. In simulations the true reaction plane from the generated event was used. Thick solid lighter colored lines represent non reaction-plane dependent contribution as estimated by HIJING. Corresponding estimates from UrQMD are about factor of two smaller.
Estimates of statistical accuracy at NICA MPD (months of running)

- UrQMD model: \( Au + Au \) at \( \sqrt{s_{NN}} = 9 \text{ GeV} \)
- 2-particles -> 3-particles correlations
  - no necessity to fix the event plane
- 2 neutrons from mid-rapidity \(|\eta| < 1\)
- +1 from ZDC \(|\eta| > 3\)
Background effects

- Can correlations be simulated by UrQMD generator?
Other sources of quadratic effects

- Quadratic effect of induced currents – not necessary involve (C)P-violation
- May emerge also as C&P even quantity
- Complementary probes of two-current correlators desirable
- Natural probe – dilepton angular distributions
Observational effects of current correlators in medium

- McLerran Toimela’85
  \[ W^{\mu\nu} = \int d^4x e^{-i q \cdot x} \langle J^\mu(x) J^\nu(0) \rangle \]

- Dileptons production rate
  \[
  \frac{d(R/V)}{d^4q d^3p d^3p'} = - \frac{1}{E_p E_{p'}} \frac{1}{(2\pi)^6} \times \delta^{(4)}(p + p' - q) L^{\mu\nu}(p, p') \\
  \times (1/q^4) W_{\mu\nu}(q) .
  \]

- Structures – similar to DIS F1, F2 (p -> v)
Tensor polarization of in-medium vector mesons (Bratkovskaya, Toneev, OT’95)

- Hadronic in-medium tensor – analogs of spin-averaged structure functions: p -> ν
- Only polar angle dependence
- Tests for production mechanisms - recently performed by HADES in Ar+KCl at 1.75 A GeV!
General hadronic tensor and dilepton angular distribution

- Angular distribution

\[ d\sigma \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi + \rho \sin 2\theta \sin \phi + \sigma \sin^2 \theta \sin 2\phi \]

- Positivity of the matrix (= hadronic tensor in dilepton rest frame)

\[
\begin{pmatrix}
\frac{1-\lambda}{2} & \mu & \rho \\
\mu & \frac{1+\lambda+\nu}{2} & \sigma \\
\rho & \sigma & \frac{1+\lambda-\nu}{2}
\end{pmatrix}
\]

\[ |\lambda| \leq 1, \ |\nu| \leq 1 + \lambda, \ \mu^2 \leq \frac{(1 - \lambda)(1 + \lambda - \nu)}{4}, \ \rho^2 \leq \frac{(1 - \lambda)(1 + \lambda + \nu)}{4}, \ \sigma^2 \leq \frac{(1 - \lambda)^2 - \nu^2}{4} \]

- + cubic – det M > 0

- 1st line – Lam&Tung by SF method
Magnetic field conductivity and asymmetries

- zz-component of conductivity (\sim hadronic) tensor dominates
- \lambda = -1
- Longitudinal polarization with respect to magnetic field axis
- Effects of dilepton motion – work in progress
Other signals of rotation

- Hyperons (in particular, $\Lambda$) polarization (self-analyzing in weak decay)
- Searched at RHIC (S. Voloshin et al.) – oriented plane (slow neutrons) - no signal observed
- No tensor polarizations as well
Why rotation is not seen?

- Possible origin – distributed orbital angular momentum and local spin-orbit coupling
- Only small amount of collective OAM is coupled to polarization
- The same should affect lepton polarization
- Global (pions) momenta correlations (handedness)
New sources of $\Lambda$ polarization coupling to rotation

- Bilinear effect of vorticity – generates quark axial current (Son, Surowka)
- Strange quarks - should lead to $\Lambda$ polarization
- Proportional to square of chemical potential – small at RHIC – may be probed at FAIR & NICA

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2 \mu n}{3 (\varepsilon + P)}\right) \varepsilon^{\mu \nu \lambda \rho} V_\nu \partial_\lambda V_\rho$$
Conclusions/Discussion - II

- Anomalous coupling to fluid vorticity – new source of neutron asymmetries
- Related to the new notion of relativistic chaotic flows
- Two-current effects – dilepton tensor polarization
- New source of hyperon polarization in heavy ions collisions