Introduction
For simplicity we consider

1) constant
2) Abelian external fields
Why do we need external fields?

1) Suppose we calculate (analytically or on the lattice) the mass of the particle, $M$, in the external electric field, $E$, then

$$M \rightarrow M + \frac{1}{2} 4\pi E^2 \alpha_E + \ldots$$

thus we know the electric polarizability, \(\bigcirc\).

$$M \rightarrow M + \frac{1}{2} E^2 (4\pi \alpha_E - \frac{\mu^2}{4M^3} + \ldots)$$ for spin ½ hadrons

2) Strong electromagnetic fields can interfere with QCD interactions

In heavy ion collisions extremely strong magnetic fields can be generated
Constant Electric and Magnetic fields on continuum and lattice torus
From Minkowski to Euclidean space-time

from (+ - - -) to (+ + + + +)

Metric tensor

\[ t \equiv x_0 = -ix_4 \]

Time

\[
\begin{align*}
A_{k}^{\text{Minkowsky}} &= -A_{k}^{\text{Euclidean}} \quad (k = 1, 2, 3), \\
A_{0}^{\text{Minkowsky}} &= iA_{4}^{\text{Euclidean}}
\end{align*}
\]

Vector potential
From Minkowski to Euclidean space-time

\[ F_{a,Minkowsky}^{k,l} = F_{a,\text{Euclidean}}^{k,l}, \quad (k, l = 1, 2, 3), \]
\[ F_{0l}^{a,Minkowsky} = -i F_{a,\text{Euclidean}}^{4l} \]

\[ \vec{B}^{\text{Minkowsky}} = \vec{B}^{\text{Euclidean}}, \]
\[ \vec{E}^{\text{Minkowsky}} = i \vec{E}^{\text{Euclidean}} \]
How to introduce constant Abelian external field?

For any covariant derivative corresponding to the charged particle we have:

$$\nabla_\mu \rightarrow \nabla_\mu - iqA^\text{ext}_\mu (x)$$

For constant electric field, $E$, along $\mathbb{O}=3$ direction

$$A^\text{ext}_\mu (x) = (0, 0, -Ex_4, 0)$$

For constant electric field, $B$, along $\mathbb{O}=3$ direction

$$A^\text{ext}_\mu (x) = \frac{1}{2} \left( Bx_2, -Bx_1, 0, 0 \right)$$

In agreement with usual formulae in Murkowski space

$$\vec{E} = \frac{\partial \vec{A}}{\partial t}; \quad B_3 = \partial_2 A_1 - \partial_1 A_2$$
How to introduce constant Abelian external field on the lattice? **Example:** electric field

**Continuum**

\[ \nabla_\mu \to \nabla_\mu - iqA_{\mu}^{ext}(x); \quad A_{\mu}^{ext}(x) = (0, 0, -Ex_4, 0) \]

**Lattice**

\[ U_{x_{\mu}} \to U_{x_{\mu}} U_{x_{\mu}}^{ext}; \quad U_{x_{\mu}}^{ext} = \exp[iqA_{\mu}^{ext}(x)] \]

\[ U_{x_{3}}^{ext} = \exp[-iqEx_4]; \]

\[ U_{x_{\nu}}^{ext} = 1(\nu = 1, 2, 4) \]
Usually lattice theory is formulated on 4D torus

1. On the torus constant external fields are quantized

2. On the lattice torus (lattice with periodical boundary conditions) there exist additional twists
Constant gauge fields on the torus are quantized


Move test scalar charged particle along closed contour and return to the starting point A, consider constant electric field

\[ A_{\mu}^{\text{ext}}(x) = (0, 0, -Ex_4, 0) \]

\[ i q \int_{x}^{y} A_{\mu} dx_{\mu} \]

\[ \Phi(x) \Rightarrow e^{i q \int_{x}^{y} A_{\mu} dx_{\mu}} \Phi(y) \]
Constant gauge fields on the torus are quantized

\[ A^\text{ext}_\mu (x) = (0, 0, -Ex_4, 0) \]

\[ \Phi(x) \Rightarrow e^{i q \int A_\mu dx_\mu} \Phi(y) \]

\[ \Phi(A) \Rightarrow e^{i q N_t \cdot 0} \Phi(y) \]
Constant gauge fields on the torus are quantized

\[ A^e_{\mu}(x) = (0, 0, -Ex_4, 0) \]

\[ \Phi(x) \Rightarrow e^{iq\int A_{\mu} dx_{\mu}} \Phi(y) \]

\[ \Phi(A) \Rightarrow e^{-iqELN_t} \Phi(y) \]
Constant gauge fields on the torus are quantized

\[ A^\text{ext}_\mu(x) = (0, 0, -Ex_4, 0) \]

\[ \Phi(x) \Rightarrow e^{iq\int A_\mu dx_\mu} \Phi(y) \]

\[ \Phi(A) \Rightarrow e^{-iqELN_t} \Phi(y) \]
Constant gauge fields on the torus are quantized

\[ A^\text{ext}_\mu (x) = (0, 0, -Ex_4, 0) \]

\[ \Phi(x) \Rightarrow e^{iqE_{LN}t} \Phi(y) \]

\[ \Phi(A) \Rightarrow e^{-iqE_{LN}t} \Phi(A) \]
Constant gauge fields on the torus are quantized

We return to the starting point and get the phase factor

$$\Phi(A) \Rightarrow e^{-iqELN_{t}} \Phi(A)$$

But after each movement we return to the initial point, thus the phase factor should be UNITY
Constant gauge fields on the torus are quantized

We return to the starting point and get the phase factor

\[ \Phi(A) \Rightarrow e^{-iqELN_t} \Phi(A) \]

Thus

\[ qELN_t = 2\pi n \]
Constant gauge fields on the torus are quantized.

We return to the starting point and get the phase factor

\[ \Phi(A) \Rightarrow e^{-i qELN_t} \Phi(A) \]

Thus

\[ qELN_t = 2\pi n \]

Electric field is quantized:

\[ E = \frac{2\pi n}{qLN_t} \]
Constant gauge fields on the torus are quantized.

For lattice $L^3 \cdot N_t$

Electric field is quantized:
$$E = \frac{2\pi n}{qLN_t}$$

Magnetic field is quantized:
$$B = \frac{2\pi n}{qL^2}$$
For lattice with periodic boundary conditions (lattice torus) we have

(a) ‘t Hooft quantization of constant gauge fields

(b) additional twist on the boundary for links which contribute to the covariant derivative for matter fields

Additional twist on the boundary for constant electric field

\[ A_{\mu}^{\text{ext}}(x) = (0, 0, -Ex_4, 0) \]

\[ U_{\text{plaq}}^{\text{ext}} = \exp[\text{i}qE] \]

in \( x_3-x_4 \) plane
Additional twist on the boundary for constant electric field

\[ A_{\mu}^{ext}(x) = \left(0, 0, -Ex_4, 0\right) \]

\[ U_{plaq}^{ext} = \exp[iqE] \]

Periodic boundary conditions:

points A and B are the same
Additional twist on the boundary for constant electric field

\[ A_\mu^{ext}(x) = (0, 0, -Ex_4, 0) \]

\[ U_{plaq}^{ext} = \exp[iqE] \]

Periodic boundary conditions:

points A and B are the same

Points C and D are also the same
Additional twist on the boundary for constant electric field

\[ A^\text{ext}_\mu (x) = (0, 0, -Ex_4, 0) \]

\[ U^\text{ext}_{\text{plaq}} = \exp[iqE] \]

Periodic boundary conditions:

points A and B are the same

Points C and D are also the same

\[ U^\text{ext}_{x3} = \exp[-iqEx_4] \]
Additional twist on the boundary for constant electric field

\[ A^{\text{ext}}_\mu(x) = (0, 0, -E x_4, 0) \]

\[ U^{\text{ext}}_{\text{plaq}} = \exp[iqE] \]

All red links we multiply by

\[ U^{\text{ext}}_{x3} = \exp[-iqE x_4]; \]

then all pink plaquettes (excluding upper row) carry electric field:

\[ U^{\text{ext}}_{\text{plaq}} = \exp[iqE(x_4 + 1) - iqE x_4] = \exp[iqE] \quad \text{OK!} \]
$U_{x_3}^{ext} = \exp[-i q E x_4]$;

$U^{ext}_{plaq} = \\
\exp[iqE(x_4 + 1) - iqE x_4] \\
= \exp[iqE]$
Additional twist on the boundary for constant electric field

\[ A_{\mu}^{ext}(x) = (0,0,-Ex_4,0) \]

then all pink plaquettes (excluding upper row) carry electric field:

\[ U_{\text{plaq}}^{ext} = \exp[iqE(x_4 + 1) - iqEx_4] = \exp[iqE] \quad \text{OK!} \]

All blue plaquettes carry electric field:

\[ U_{\text{plaq}}^{ext} = \exp[iqE(1 - N_4)] \]
Additional twist on the boundary for constant electric field

\[ A^\text{ext}_\mu(x) = (0, 0, -Ex_4, 0) \]

All blue plaquettes carry electric field:

\[ U^\text{ext}_{\text{plaq}} = ??? \]

\[ = \exp[iqE(1 - N_4)] \]

Multiply all green links on \( U^\text{ext}_{x_4} = \exp[iqEN_4x_3] \)

Then all blue plaquettes carry electric field:

\[ U^\text{ext}_{\text{plaq}} = \exp[iqE] \]

OK!!!
Additional twist on the boundary for constant electric field

\[ A^\text{ext}_\mu(x) = (0,0,-E x_4,0) \]

Multiply all **green** links on

\[ U_{x_4}^\text{ext bound} = \exp[iqEN_4x_3] \]

Then all **blue** plaquettes carry electric field:

\[ U_{\text{plaq}}^\text{ext} = \exp[iqE] \]

The **red** plaquette carry electric field:

\[ U_{\text{plaq}}^\text{ext} = \exp[iqE(1-N_4L)] \]

If the quantization condition is satisfied

\[ E = \frac{2\pi n}{qLN_t} \]

then

the **red** plaquette carry electric field:

\[ U_{\text{plaq}}^\text{ext} = \exp[iqE] \text{ OK!!!} \]
Summary

1. Electric field in Minkowski space-time corresponds to \textit{imaginary} electric field in Euclidean space-time.

2. Magnetic field in Minkowski space-time corresponds to \textit{real} magnetic field in Euclidean space-time.

3. Constant electric and magnetic fields on the torus are quantized.

4. There exists additional twist on the boundary for constant electric and magnetic fields on the lattice with periodical boundary conditions.
External Fields as Additional Parameters of the Theory
Magnetic Moments and Electric Polarizabilities from lattice QCD


The main idea is the calculation of the mass shift due to the background fields, e.g. for spin $\frac{1}{2}$ baryons in the external electric field

$$M \rightarrow M + \frac{1}{2} E^2 (4\pi \alpha_E - \frac{\mu^2}{4M^3} + \ldots),$$

for spin $\frac{1}{2}$ baryons in the external magnetic field


$$M \rightarrow M \pm \mu B,$$

It's very interesting, but I will not speak about all that
Very Strong Magnetic Fields in Heavy Ion Collisions
Very strong magnetic fields in heavy ion collisions
Very strong magnetic fields in heavy ion collisions
Very strong magnetic fields in heavy ion collisions

First collisions of Pb+Pb (Lead+lead) seen by the ALICE experiment at LHC

1) QG plasma is thermalized

2) Viscosity of QG plasma is very small

3) Multiplicity is very high
Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran ’07–’08]

Reaction plane ($\Psi_R$)

$X$ (defines $\Psi_R$)

Heavy ion

Quarks and gluons

Heavy ion
Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran ’07–’08]

Magnetic fields in non-central collisions

The medium is filled by electrically charged particles.

Large orbital momentum, perpendicular to the reaction plane.

Large magnetic field along the direction of the orbital momentum.

Charge is large, velocity is high.

Thus we have two very big currents.
Magnetic fields in non-central collisions

The medium is filled by electrically charged particles.

Large orbital momentum, perpendicular to the reaction plane.

Large magnetic field along the direction of the orbital momentum.

Two very big currents produce a very strong magnetic field.

“Instanton”

Reaction plane \( (\Psi_R) \)

\( \Psi_R \) (defines \( \Psi_R \))

\( x \) (defines \( \Psi_R \))

Mclerran, Kharzeev, Fukushima
## Comparison of magnetic fields

<table>
<thead>
<tr>
<th>Description</th>
<th>Magnetic Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Earth's magnetic field</td>
<td>0.6 Gauss</td>
</tr>
<tr>
<td>A common, hand-held magnet</td>
<td>100 Gauss</td>
</tr>
<tr>
<td>The strongest steady magnetic fields achieved so far in the laboratory</td>
<td>$4.5 \times 10^5$ Gauss</td>
</tr>
<tr>
<td>The strongest man-made fields ever achieved, if only briefly</td>
<td>$10^7$ Gauss</td>
</tr>
<tr>
<td>Typical surface, polar magnetic fields of radio pulsars</td>
<td>$10^{13}$ Gauss</td>
</tr>
<tr>
<td>Surface field of Magnetars</td>
<td>$10^{15}$ Gauss</td>
</tr>
</tbody>
</table>

[http://solomon.as.utexas.edu/~duncan/magnetar.html](http://solomon.as.utexas.edu/~duncan/magnetar.html)

At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon

\[ eB(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss} \]
In heavy ion collisions, magnetic forces are of the order of strong interaction forces.

\[ eB \approx \Lambda^2_{QCD} \]
Magnetic forces are of the order of strong interaction forces.

\[ eB \approx \Lambda_{QCD}^2 \]

We expect the influence of magnetic field on strong interaction physics.
Chiral Magnetic Effect (CME)
1. Massless quarks in external magnetic field.

Red: momentum  Blue: spin
1. Massless quarks in external magnetic field.

Red: momentum
Blue: spin
2. Quarks in the instanton field.

**Red:** momentum  
**Blue:** spin

**Effect of topology:**

\[ u_L \rightarrow u_R \]  
\[ d_L \rightarrow d_R \]
Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

3. Electric current along magnetic field

Red: momentum
Blue: spin

Effect of topology:

\[ u_L \rightarrow u_R \]
\[ d_L \rightarrow d_R \]

u-quark: q=+2/3

d-quark: q= - 1/3
Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

3. Electric current is along magnetic field
In the *instanton* field

Red: momentum
Blue: spin

Effect of topology:
\[ u_L \rightarrow u_R \]
\[ d_L \rightarrow d_R \]

u-quark: \( q = +\frac{2}{3} \)
d-quark: \( q = -\frac{1}{3} \)
3D time slices of topological charge density, lattice calculations

D. Leinweber

Topological charge density after vacuum cooling

P.V. Buividovich, T.K. Kalaydzhyan, M.I. Polikarpov

Fractal topological charge density without vacuum cooling
Summary

1. In heavy ion noncentral collisions the very strong magnetic fields can be generated

2. The interference between strong and electromagnetic interactions can produce new physical effects (CME)
Very Strong Magnetic Fields in Lattice Calculations
Magnetic forces are of the order of strong interaction forces

\[ eB \approx \Lambda_{\text{QCD}}^2 \]

We expect the influence of magnetic field on strong interaction physics.

The effects are nonperturbative, and we use Lattice Calculations.
ITEP lattice group publications on gluodynamics with strong magnetic fields


We calculate \( \langle \bar{\psi} \Gamma \psi \rangle; \quad \Gamma = 1, \gamma_{\mu}, \sigma_{\mu\nu} \)
in the external magnetic field and in the presence of the vacuum gluon fields. We consider SU(2) gauge fields and quenched approximation.
Quenched vacuum, overlap Dirac operator, external magnetic field

\[ eB = \frac{2\pi k}{L^2} ; \quad eB \geq (250 \text{ Mev})^2 \]
Density of electric charge in the vacuum vs. magnetic field
Density of the electric charge vs. magnetic field, 3D time slices

\[ B = 0 \]

\[ B = (500 \text{ MeV})^2 \]

\[ B = (780 \text{ MeV})^2 \]

\[ B = (1.1 \text{ GeV})^2 \]
Chiral Magnetic Effect on the lattice, numerical results \( T=0 \)

Regularized electric current:

\[
\langle j_3^2 \rangle_{IR} = \langle j_3^2 (H,T) \rangle - \langle j_3^2 (0,0) \rangle, \quad j_3 = \overline{\psi} \gamma_3 \psi
\]
Chiral Magnetic Effect on the lattice, numerical comparison of results near $T_C$ and near zero

$T = 0$
$F_{12} \neq 0$

$\langle j_1^2 \rangle \ll \langle j_2^2 \rangle$

$\langle j_3^2 \rangle \ll \langle j_0^2 \rangle$

Regularized electric current:

$\langle j_i^2 \rangle_{IR} = \langle j_i^2(H,T) \rangle - \langle j_i^2(0,0) \rangle$, \quad $j_i = \bar{\psi} \gamma_i \psi$
We really observe two opposite flows of negative and positive particles.
Magnetic Field Induced Conductivity of the Vacuum

Qualitative definition of conductivity,

\[ \langle j_\mu(x) j_\nu(y) \rangle = C + A \cdot \exp\{ -m |x - y| \} r^\alpha \]

\[ j_\mu(x) = \overline{q}(x) \gamma_\mu q(x) \]

\[ \sigma \propto C \]
Magnetic Field Induced Conductivity of the Vacuum

\[ \sigma_{ij} = \frac{\rho_{ij}(0)}{4T} \]

- Conductivity (Kubo formula)

\[ G_{ij}(\tau) = \int_{-\infty}^{+\infty} \frac{dw}{2\pi} K(w, \tau) \rho_{ij}(w), \]

\[ K(w, \tau) = \frac{w}{2T} \frac{\cosh \left( w \left( \tau - \frac{1}{2T} \right) \right)}{\sinh \left( \frac{w}{2T} \right)}, \]

\[ G_{ij}(\tau) = \int d^3 \vec{x} \langle j_i \left( \vec{0}, 0 \right) j_j (\vec{x}, \tau) \rangle \]
Magnetic Field Induced Conductivity of the Vacuum

\[ \sigma_{ij} = \frac{\rho_{ij}(0)}{4T} \]

- Conductivity (Kubo formula)

For weak constant electric field

\[ < j_i > = \sigma_{ik} E_k \]
Magnetic Field Induced Conductivity of the Vacuum
Calculations in SU(2) gluodynamics

\[
\langle \bar{q} (x) \gamma_i q (x) \bar{q} (y) \gamma_j q (y) \rangle \\
= \int D A_\mu \, e^{-S_{YM}[A_\mu]} \, \text{Tr} \left( \frac{1}{D + m} \gamma_i \frac{1}{D + m} \gamma_j \right)
\]

We use overlap operator + Shifted Unitary Minimal Residue Method (Borici and Allococi (2006)) to obtain fermion propagator

\[
G_{ij} (\tau) = \int d^3 \vec{x} \langle j_i \left( \vec{0}, 0 \right) j_j (\vec{x}, \tau) \rangle
\]
Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T/T_c=0.45$

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

$\vec{B}$ is parallel to 0Z axis
Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T/T_c=0.45$

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

At $T=0$, $B=0$ vacuum is insulator

Critical value of magnetic field?
Calculations in SU(2) gluodynamics, conductivity along magnetic field at $T<T_c$, $T>T_c$

$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$

$\tilde{H}$ is parallel to 0Z axis
Calculations in SU(2) gluodynamics, conductivity at \( T/T_c = 0.45 \), variation of the quark mass

\[ \sigma_{zz} (m_q, qB) \sim m_q^{-\alpha} (|qB|)^\beta \]

\( \alpha \approx 1 \)
\( \beta \approx 1 \)
Calculations in SU(2) gluodynamics, conductivity at $T/T_c=0.45$, variation of the quark mass and magnetic field

$$\sigma_{ij} \propto \frac{B_i B_j}{Bm_q} \propto \frac{B_i B_j}{Bm^2_\pi}$$

Why?
1.3 Dilepton emission rate


\[ \frac{R}{V} = -4e^4 \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_1}{2E_1} \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_1}{2E_1} L^{\mu\nu}(p_1, p_2) \frac{\rho_{\mu\nu}(q)}{q^4}, \]  

(7)

where \( p_1 \) and \( p_2 \) are the momenta of the leptons, \( q = p_1 + p_2 \), \( m \) is their mass and \( L^{\mu\nu} = \left( (p_1 \cdot p_2 + m^2) \eta^{\mu\nu} - p_{1\mu} p_{2\nu} - p_{2\mu} p_{1\nu} \right) \) is the dilepton tensor. If the electric conductivity is nonzero in the direction of the magnetic field, for sufficiently small \( p_1, p_2 \) one has \( \rho_{ij}(q) \approx \rho_{ij}(0) \sim \sigma_{ij} \sim B_i B_j / |B| \), and hence

\[ \frac{R}{V} \sim \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_1}{2E_1} \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_1}{2E_1} (p_1 \cdot B) (p_2 \cdot B) \frac{(p_1 \cdot B)(p_2 \cdot B)}{|B|}. \]  

(8)
• There should be more soft dileptons in the direction \textit{perpendicular} to magnetic field

\[
\frac{d\sigma}{dp_1 dp_2} \propto \frac{|B|}{m_\pi} \sin^2 \theta
\]

\(\theta\) is the angle between the spatial momentum of the leptons and the magnetic field, in the center of mass of dilepton pair
Can nothing be a superconductor and a superfluid?
M.N. Chernodub arXiv:1104.4404

Superconductivity of the vacuum can exist at T=0

Model explanation: Superconductivity is due to the condensation of the \( \square \) mesons
Preliminary lattice results

Condensate of the charged $\rho$-mesons vs value of the magnetic field

$$\eta, \text{ GeV}^3$$

$$eB, \text{ GeV}^2$$

Condensate of the charged $\rho$-mesons vs value of the magnetic field
CME Summary

a) Visualization

b) Large fluctuations of the electric current in the direction of the magnetic field

c) Conductivity of the vacuum in the direction of the magnetic field at T>0

d) Dilepton emission rate

e) Superconductivity of the vacuum at T=0
Other effects induced by magnetic field

1. Chiral symmetry breaking

2. Magnetization of the vacuum

3. Electric dipole moment of quark along the direction of the magnetic field
1. Chiral condensate in QCD

\[ \Sigma = - \langle \bar{\psi} \psi \rangle \]

\[ m_{\pi}^2 f_{\pi}^2 = m_q \langle \bar{\psi} \psi \rangle \]
We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!
Localization of Dirac Eigenmodes

Typical densities of the nearzero eigenmodes vs. the strength of the external magnetic field

\[ B=0 \]

\[ B=(780 \text{MeV})^2 \]
Localization of Dirac Eigenmodes

Typical densities of the nearzero eigenmodes vs. the strength of the external magnetic field

\[ B = 0 \]

\[ B = (780 \text{ MeV})^2 \]

\[ B = (780 \text{ MeV})^2 \]
Chiral condensate, simulations with dynamical quarks


• Michael Abramczyk, Tom Blum, and Gregory Petropoulos, R. Zhou, arXiv:0911.1348
2. Magnetization of the vacuum as a function of the magnetic field

Spins of virtual quarks turn parallel to the magnetic field

\[
\langle \bar{\psi} \sigma_{\alpha\beta} \psi \rangle = \chi \langle \bar{\psi} \psi \rangle F_{\alpha\beta}
\]

\[
\sigma_{\alpha\beta} = \frac{1}{2i} [\gamma_\alpha, \gamma_\beta]
\]

\[
\langle \bar{\psi} \psi \rangle \chi = -46(3) \text{ MeV} \leftrightarrow \text{our result}
\]

\[
\langle \bar{\psi} \psi \rangle \chi \approx -50 \text{ MeV} \leftrightarrow \text{QCD sum rules}
\]

(I.I. Balitsky, 1985, P. Ball, 2003.)
3. Generation of the anomalous quark electric dipole moment along the axis of magnetic field

Large correlation between square of the electric dipole moment and chirality

\[ \sigma_{0i} = i \bar{\psi} [\gamma_0, \gamma_i] \psi \quad \text{and chirality} \quad \rho_5 = \bar{\psi} \gamma_5 \psi \]
4. Deconfinement phase transition in the presence of magnetic field

Massimo D’Elia, Swagato Mukherjee and Francesco Sanfilippo (2011)
Summary

• Chiral condensate

• Magnetization of the vacuum

• Quark local electric dipole Moment

• B-T phase diagram
Systematic errors

• SU(2) gluodynamics instead of QCD
• Moderate lattice volumes
• Not large number of gauge field configurations
• In some cases we calculate the overlap propagator using summation over eigenfunctions:

\[
\langle \bar{\Psi} \Sigma_{\alpha\beta} \Psi \rangle = 2m \left\{ \sum_{\lambda_k > 0} \frac{\psi^\dagger_k(x) \Sigma_{\alpha\beta} \psi_k(x)}{\lambda_k^2 + m^2} \right\}
\]
Proposals for calculations

\[ \langle \bar{\psi} \sigma_{\alpha \beta} \psi \rangle \quad \langle \bar{\psi} \psi \rangle \quad \langle i \bar{\psi} [\gamma_0, \gamma_i] \psi \cdot \bar{\psi} \gamma_5 \psi \rangle \quad \langle j_3^2 \rangle_{IR} \]

\[ \langle j_\mu(x) j_\nu(y) \rangle \]

- CME, vacuum conductivity
- Chiral condensate
- Magnetization of the vacuum
- Quark local electric dipole moment
- Dilepton pair angular distribution
- Shift of the phase transition
Proposals for calculations

- D. Kharzeev

\[ \mu \leftrightarrow \mu_5 \quad L_\mu = i \mu \bar{\psi} \gamma_0 \psi + \mu_5 \bar{\psi} \gamma_5 \psi \]

\[ \sigma \leftrightarrow \sigma_5 \quad \sigma \rightarrow <j_\mu(x)j_\nu(y)> \quad \sigma_5 \rightarrow <j_{5\mu}(x)j_{5\nu}(y)> \]

**SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks**

Chiral Magnetic Effect (CME) + Chiral Separation Effect (CSE) = Chiral Magnetic Wave

Dmitri E. Kharzeev, Ho-Ung Yee, arXiv:1012.6026
Proposals for calculations

• D. Kharzeev

\[ \mu = 0, \mu_5 \neq 0 \quad L_\mu = \mu_5 \bar{\psi} \gamma_5 \gamma_0 \psi \]

\[ \sigma_{5ij}(\mu_5, B, T) \]

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD (imaginary unity is absent)
Proposals for calculations


\[ < j_\mu (-q) j^5_\nu (q) > = - \frac{q^2}{4\pi^2} P_\mu^{\alpha\beta} [P_\nu^{\beta\gamma} \omega_T (q^2) + P_\nu^{\beta=} \omega_L (q^2)] \tilde{F}_{\alpha\beta} \]

\[ \omega_L (q^2) = \frac{2N_c}{q^2} \iff \text{no quantum corrections} \]

\[ \omega_T (q^2) = \frac{N_c}{q^2} \iff \text{there are nonperturbative corrections} \]

**SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD**
New proposals for calculations


\[ \omega_L(q^2) = \frac{2N_c}{q^2} \quad \iff \quad \text{no quantum corrections} \]

\[ \omega_T(q^2) = \frac{N_c}{q^2} \quad \iff \quad \text{there are nonperturbative corrections} \]

\[ \omega_T(q^2) = \frac{N_c}{q^2} - \frac{N_c}{f_{\pi}^2} [< j_\mu^5 j_\mu^5 > - < j_\mu j_\mu >] \]

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Proposals for calculations


\[ < j_{\mu} (-q) j^5_{\nu} (q) > = - \frac{q^2}{4\pi^2} P_{\mu}^{\alpha \perp} \left[ P_v^{\beta \perp} \left( \frac{N_c}{q^2} - \frac{N_c}{f_\pi^2} \right) [< j^5_{\mu} j^5_{\nu} > - < j_{\mu} j_{\mu} >] \right) \]

\[ + P_{\nu}^{\beta= \frac{2N_c}{q^2}} F_{\alpha \beta} \]

\[ \begin{aligned} P_{\mu}^{\alpha \perp} &= \eta_{\mu}^\alpha - \frac{q_{\mu} q^\alpha}{q^2}, \quad P_{\mu}^{\alpha=} = \frac{q_{\mu} q^\alpha}{q^2} \end{aligned} \]

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD
Proposals for calculations

- D.T. Son, N. Yamamoto: \[ \langle \bar{\psi} \gamma_\mu \psi(x) \bar{\psi} \gamma_\mu \gamma_5 \psi(y) \rangle \]

- D. Kharzeev: \[ \langle \bar{\psi} \psi(x) \bar{\psi} \gamma_5 \psi(y) \rangle \]

**SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD**
Proposals for calculations

• L. McLerran “Nonsymmetric condensate”

Calculate \[ \langle \overline{\psi}\psi(x)\overline{\psi}\psi(y) \rangle \] parallel and perpendicular to the field. Chiral condensate may depend on the direction!

\[ SU(2), SU(3) \text{ gluodynamics, } SU(2) \text{ with dynamical quarks, } QCD \]
Lattice simulations with magnetic fields, status

1. Chiral Magnetic Effect
   1.1 CME on the lattice
   1.2 Vacuum conductivity induced by magnetic field
   1.3 Quark mass dependence of CME (+ talk of P. Buividovich)
   1.4 Dilepton emission rate (+ talk of P. Buividovich)

2. Other effects induced by magnetic field
   2.1 Chiral symmetry breaking
   2.2 Magnetization of the vacuum
   2.3 Electric dipole moment of quark along the direction of the magnetic field
Lattice simulations with magnetic fields, future

1. Calculations of “OLD” quantities in $SU(3)$, $SU(2)$ with dynamical quarks, QCD. Decreasing systematic errors in $SU(2)$ calculations

2. Calculation of “NEW” physical quantities