Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research

Dubna International Advanced School of Theoretical Physics

Helmholtz International Summer School

Lattice QCD, Hadron Structure and Hadronic Matter

Dubna, Russia, September 5 - 17, 2011

Lattice QCD in external fields

M.I. Polikarpov, ITEP, Moscow

Introduction

For simplicity

we consider

1) constant

2) Abelian

external fields

Why do we need external fields?

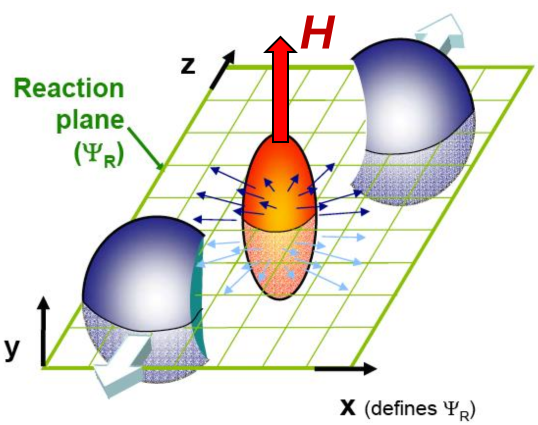
1) Suppose we calculate (analytically or on the lattice) the mass of the particle, *M*, in the external *electric* field, *E*, then

$$M \to M + \frac{1}{2} 4\pi E^2 \alpha_E + \ldots$$

thus we know the electric polarizability, ♋ ☜

$$M \to M + \frac{1}{2} E^2 (4\pi\alpha_E - \frac{\mu^2}{4M^3} + \ldots)$$ for spin ½ hadrons

2) Strong electromagnetic fields can interfere with QCD interactions



In heavy ion collisions extremely strong magnetic fields can be generated

Constant Electric and Magnetic fields on continuum and lattice torus

From Murkowski to Euclidean space-time

from (+ - - -) to (+ + + +)

Metric tensor

$$t \equiv x_0 = -ix_4$$

Time

$$A_k^{Minkowsky} = -A_k^{Euclidean} \quad (k = 1, 2, 3),$$

$$A_0^{Minkowsky} = iA_4^{Euclidean}$$

Vector potential

From Murkowski to Euclidean space-time

$$F_{kl}^{a,Minkowsky} = F_{kl}^{a,Euclidean} \quad (k,l=1,2,3),$$

$$F_{0l}^{a,Minkowsky} = -iF_{4l}^{a,Euclidean}$$

$$\vec{B}^{Minkowsky} = \vec{B}^{Euclidean},$$

$$\vec{E}^{Minkowsky} = i\vec{E}^{Euclidean}$$

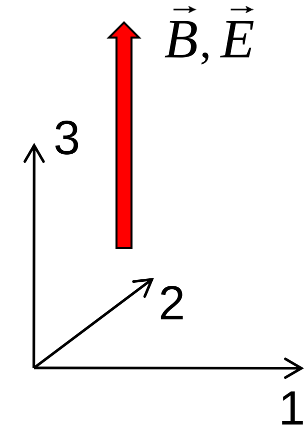
How to introduce constant Abelian external field?

For any covariant derivative corresponding to the charged particle we have:

$$\nabla_\mu \to \nabla_\mu - iqA_\mu^{ext}(x)$$

For constant electric field, ***E***, along **O*=3*** direction

$$A_\mu^{ext}(x) = \left(0, 0, -Ex_4, 0\right)$$



For constant electric field, ***B***, along **O*=3*** direction

$$A_\mu^{ext}(x) = \frac{1}{2}\left(Bx_2, -Bx_1, 0, 0\right)$$

In agreement with usual formulae in Murkowski space $\vec{E} = \frac{\partial \vec{A}}{\partial t}; \quad B_3 = \partial_2 A_1 - \partial_1 A_2$

How to introduce constant Abelian external field on the lattice? Example: electric field

Continuum

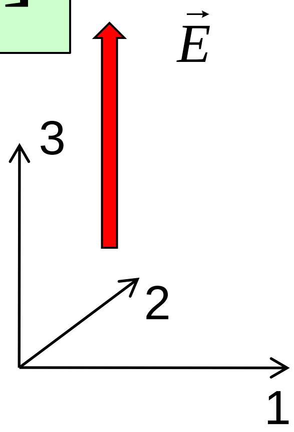
$$\nabla_\mu \to \nabla_\mu - iqA_\mu^{ext}(x); \quad A_\mu^{ext}(x) = \left(0, 0, -Ex_4, 0\right)$$

Lattice

$$U_{x\mu} \to U_{x\mu} U_{x\mu}^{ext}; \quad U_{x\mu}^{ext} = \exp[iqA_\mu^{ext}(x)]$$

$$U_{x3}^{ext} = \exp[-iqEx_4];$$

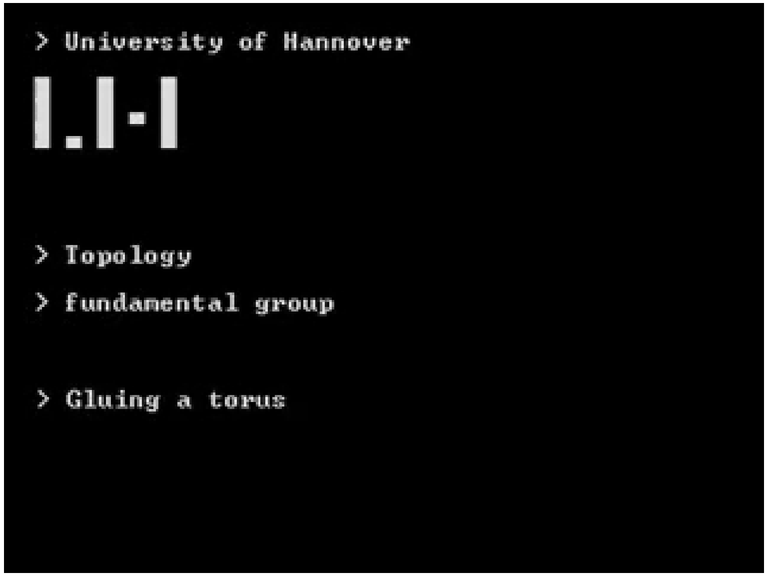
$$U_{x\nu}^{ext} = 1 (\nu = 1, 2, 4)$$



Usually lattice theory is formulated on 4D torus

1. On the torus constant external fields are quantized

2. On the lattice torus (lattice with periodical boundary conditions) there exist additional twists

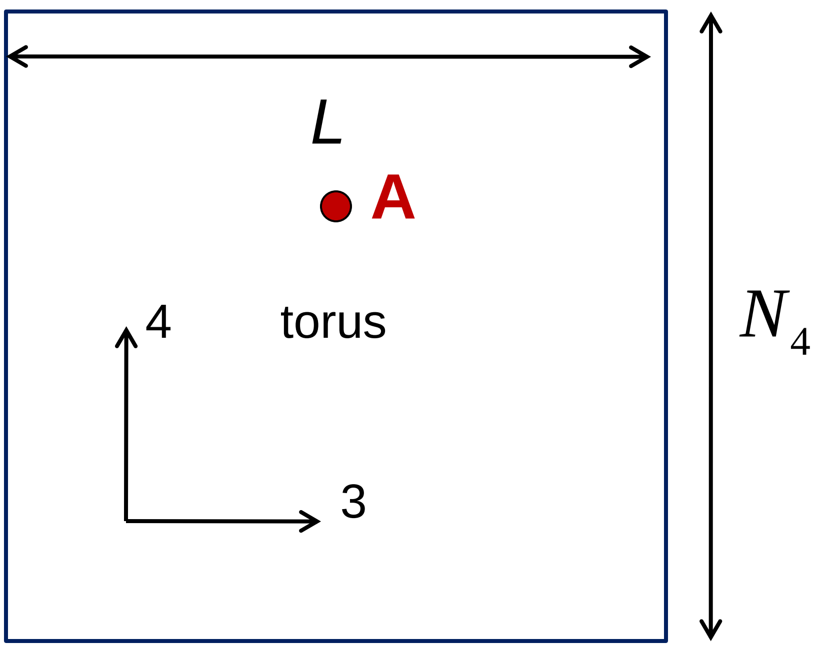


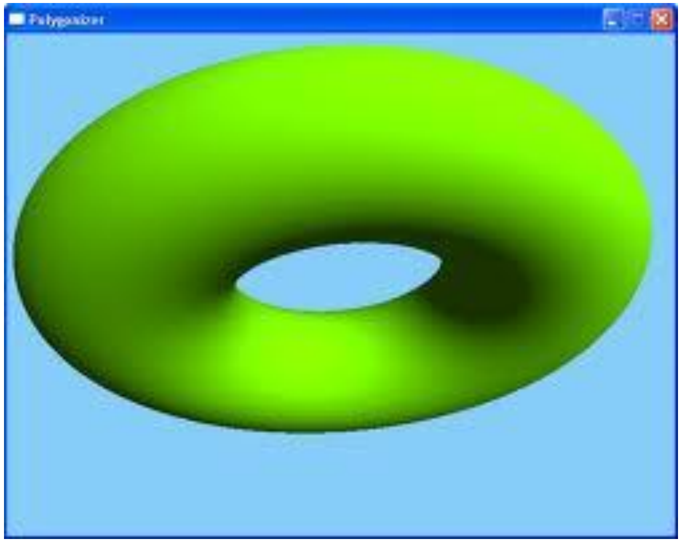
Constant gauge fields on the torus are quantized

G. 't Hooft, Nucl. Phys. B153, 141 (1979).
G. 't Hooft, Commun. Math. Phys. 81, 267 (1981).
P. van Baal, Commun. Math. Phys. 85, 529 (1982).

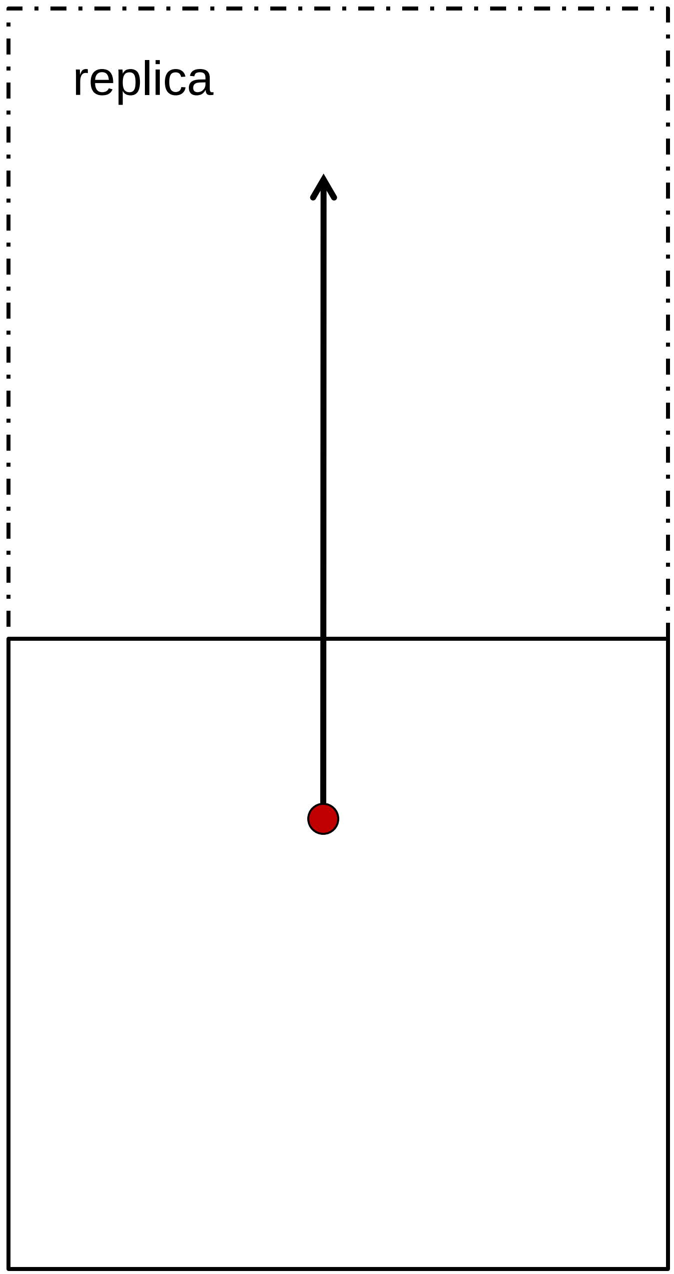
Move test scalar charged particle along closed contour and return to the starting point A, consider constant electric field $A_\mu^{ext}(x) = \left(0, 0, -Ex_4, 0\right)$

$$\Phi(x) \Rightarrow e^{iq\int_x^y A_\mu dx_\mu}\,\Phi(y)$$



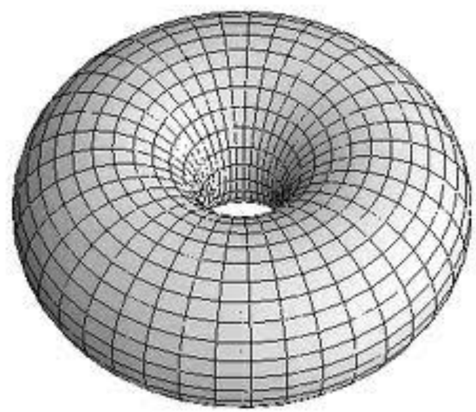


Constant gauge fields on the torus are quantized

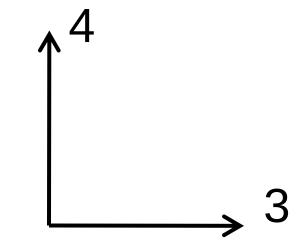


$$A_\mu^{ext}(x) = \left(0, 0, -Ex_4, 0\right)$$

$$\Phi(x) \Rightarrow e^{iq\int_x^y A_\mu dx_\mu} \Phi(y)$$



$$\Phi(A) \Rightarrow e^{iqN_t \bullet 0}\Phi(y)$$



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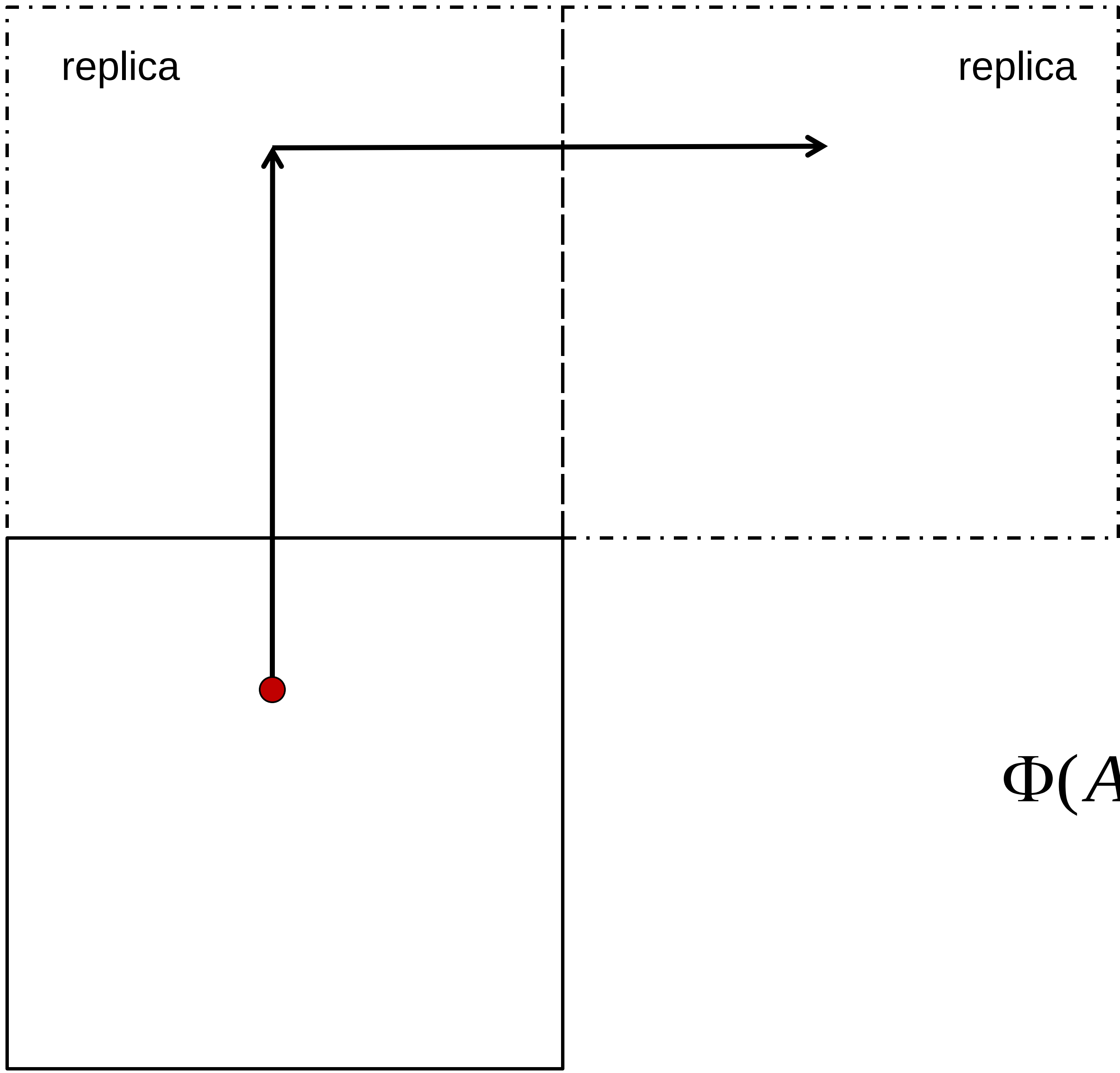

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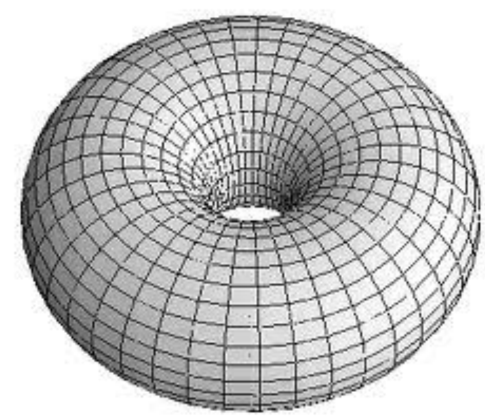
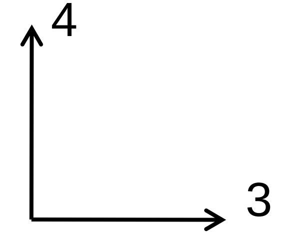
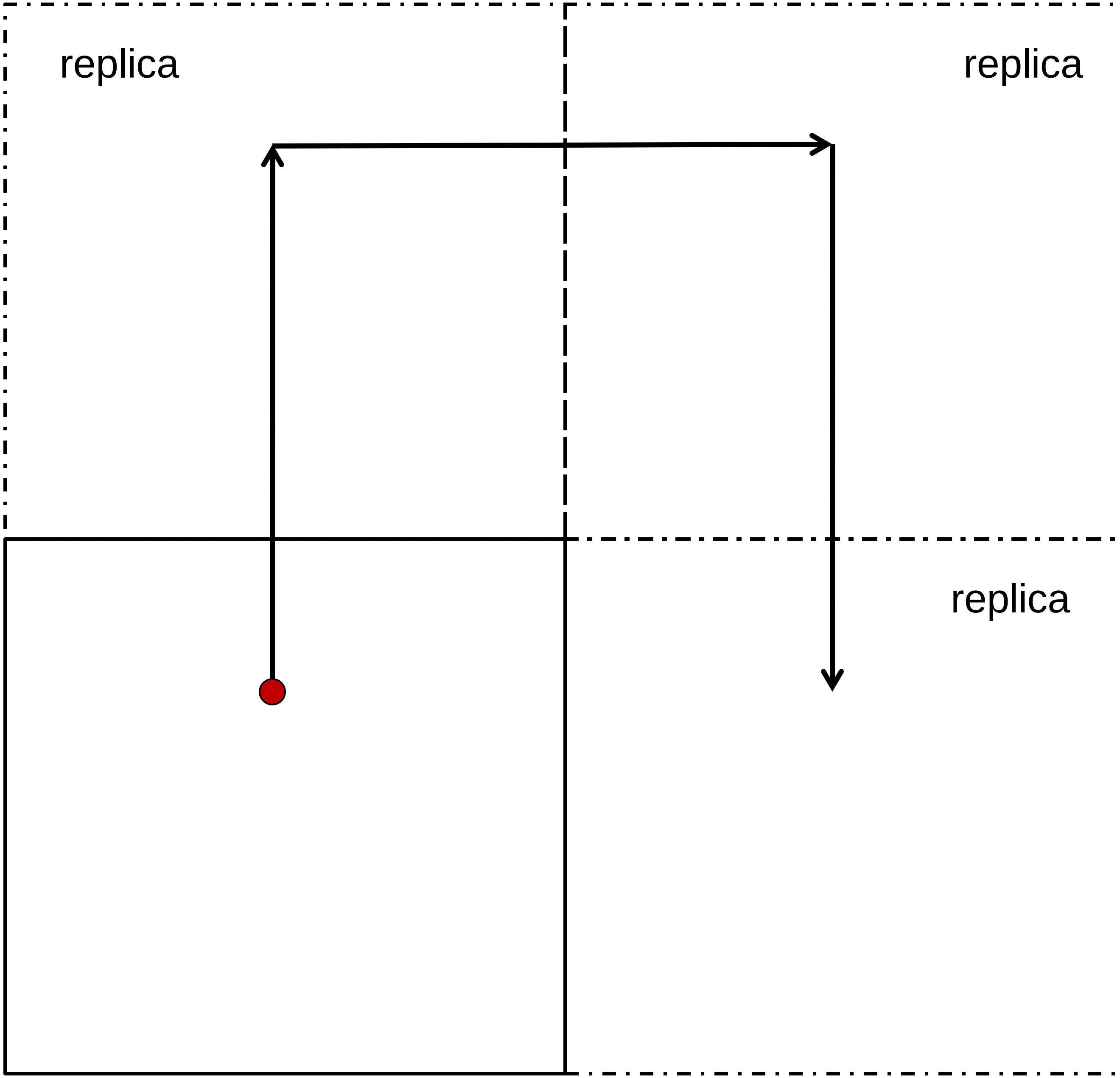


$$\Phi(A) \Rightarrow e^{-iqELN_t} \Phi(y)$$



Constant gauge fields on the torus are quantized



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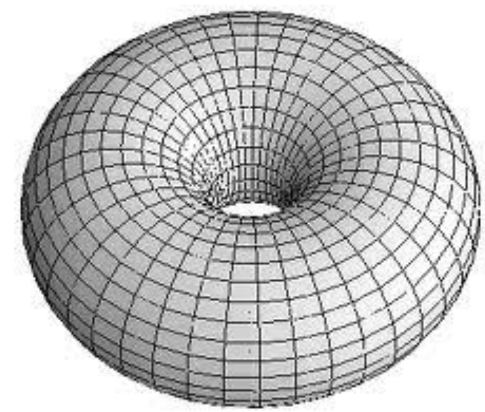
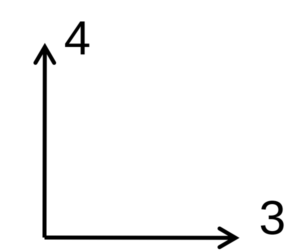
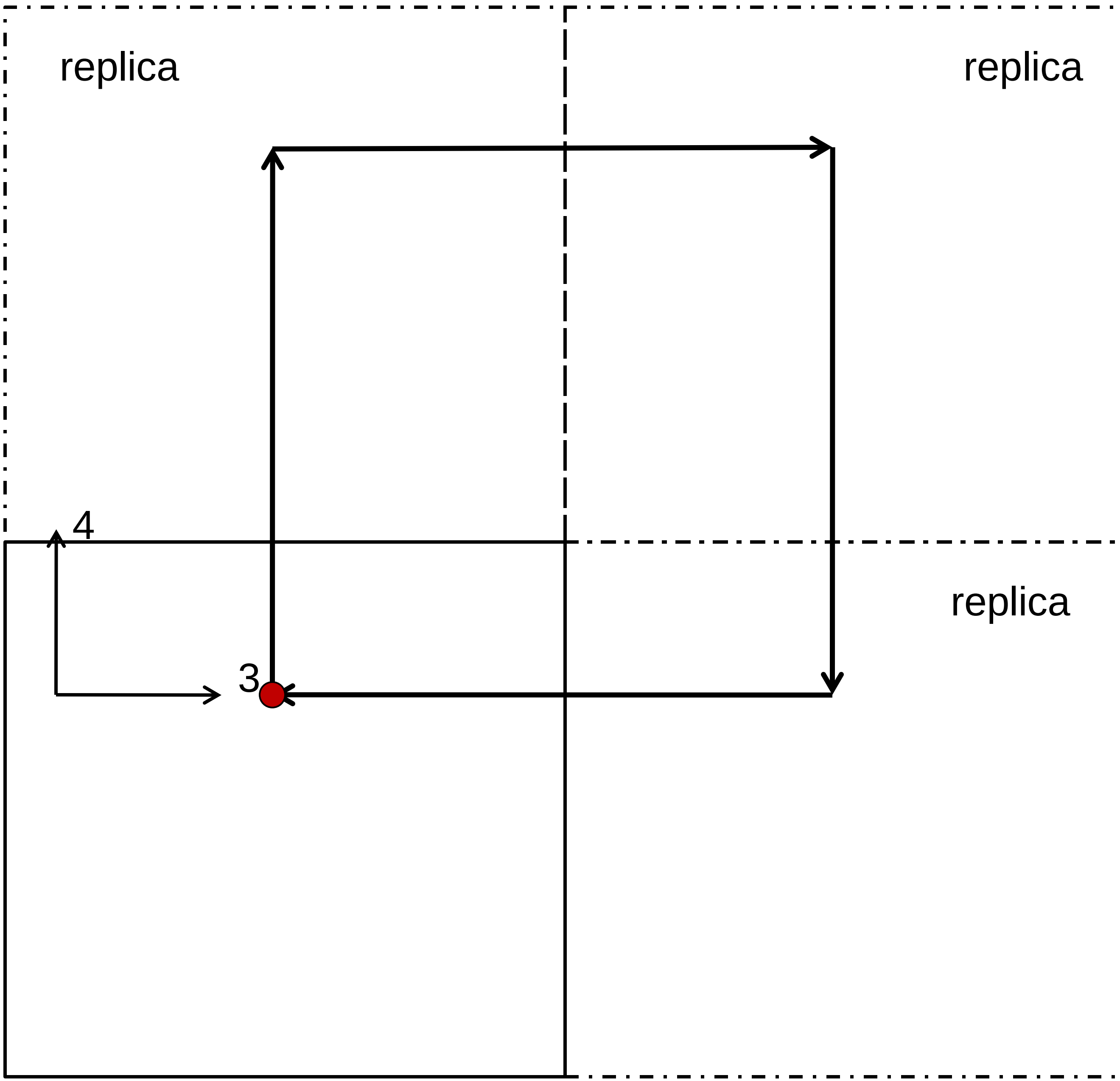


$$\Phi(A) \Rightarrow e^{-iqELN_t} \Phi(y)$$



Constant gauge fields on the torus are quantized



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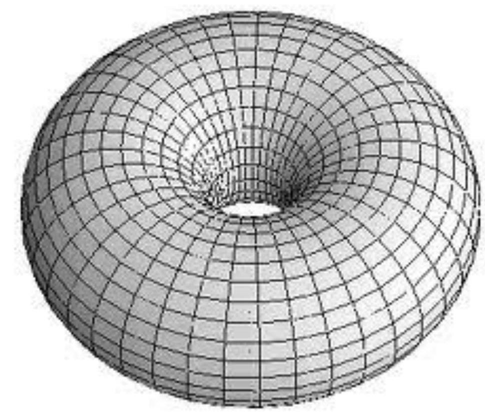

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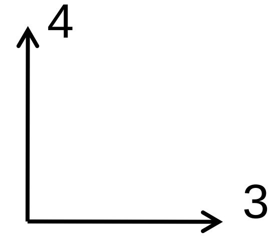


Constant gauge fields on the torus are quantized

We return to the starting point and get the phase factor

$$\Phi(A) \Rightarrow e^{-iqELN_t}\Phi(A)$$

But after each movement we return to the initial point, thus the phase factor shod be UNITY

Constant gauge fields on the torus are quantized

We return to the starting point and get the phase factor

$$\Phi(A) \Rightarrow e^{-iqELN_t}\Phi(A)$$

Thus

$$qELN_t = 2\pi n$$

Constant gauge fields on the torus are quantized

We return to the starting point and get the phase factor

$$\Phi(A) \Rightarrow e^{-iqELN_t}\Phi(A)$$

Thus

$$qELN_t = 2\pi n$$

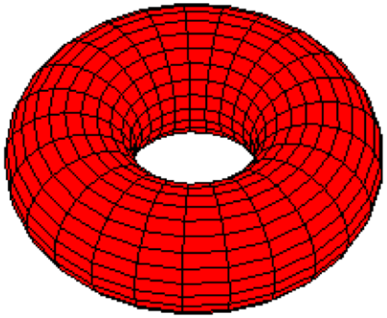
Electric field is quantized:

$$E = \frac{2\pi n}{qLN_t}$$

Constant gauge fields on the torus are quantized

For lattice $$L^3 \bullet N_t$$

Electric field is quantized:



$$E = \frac{2\pi n}{qLN_t}$$

Magnetic field is quantized:

$$B = \frac{2\pi n}{qL^2}$$

For lattice with periodic boundary conditions (lattice torus) we have

(a) ‘t Hooft quantization of constant gauge fields

(b) additional twist on the boundary for links which contribute to the covariant derivative for matter fields

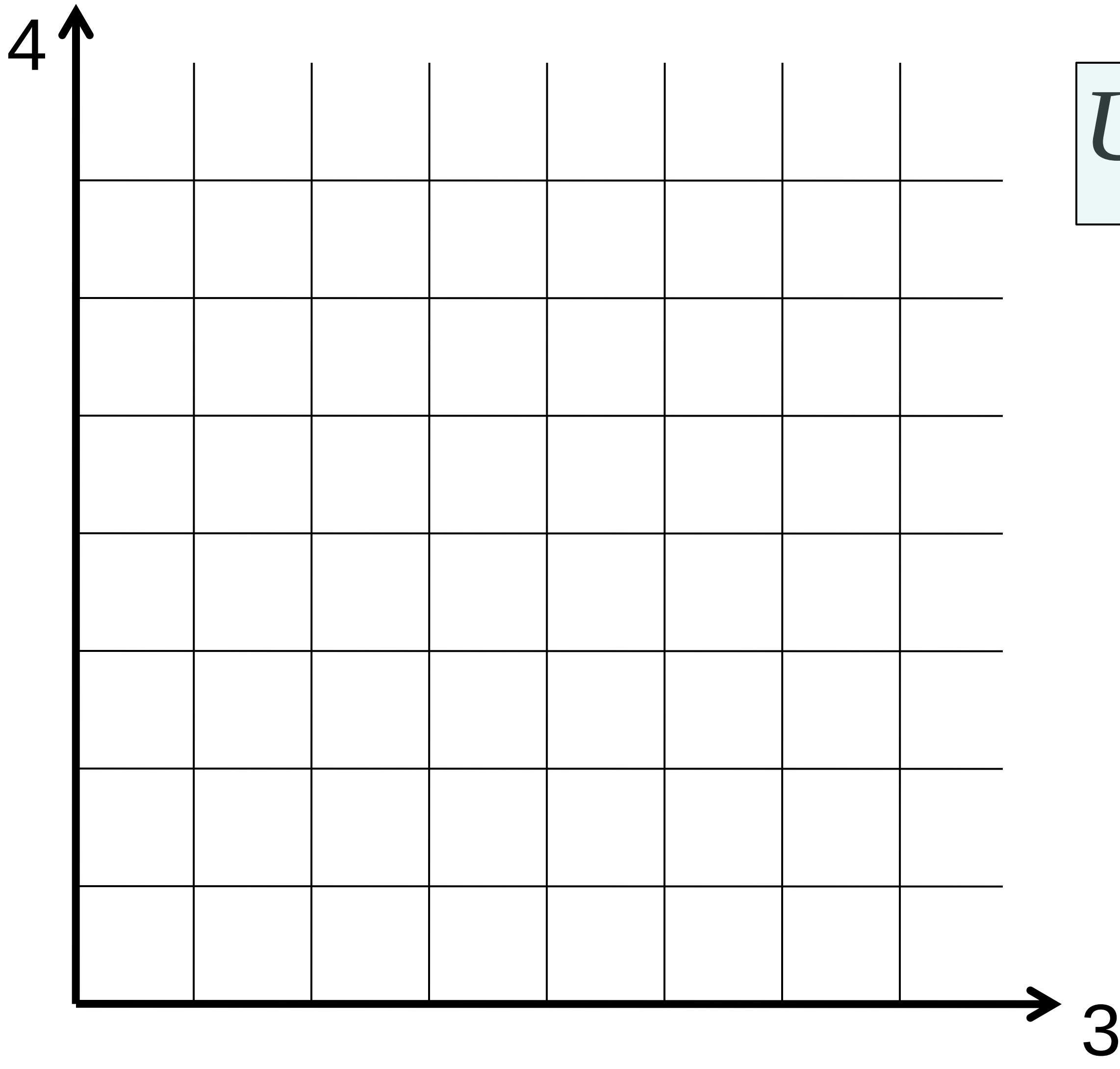
J. Smit and J. C. Vink, Nucl. Phys. B286, 485 (1987).
H. R. Rubinstein, S. Solomon, and T. Wittlich, Nucl. Phys. B457, 577 (1995).
M. H. Al-Hashimi and U. J. Wiese, Annals Phys. 324, 343 (2009), 0807.0630.

Additional twist on the boundary for constant electric field

$$A_\mu^{ext}(x) = \left(0, 0, -Ex_4, 0\right)$$

$$U_{plaq}^{ext} = \exp[iqE]$$

in x_3-x_4 plane



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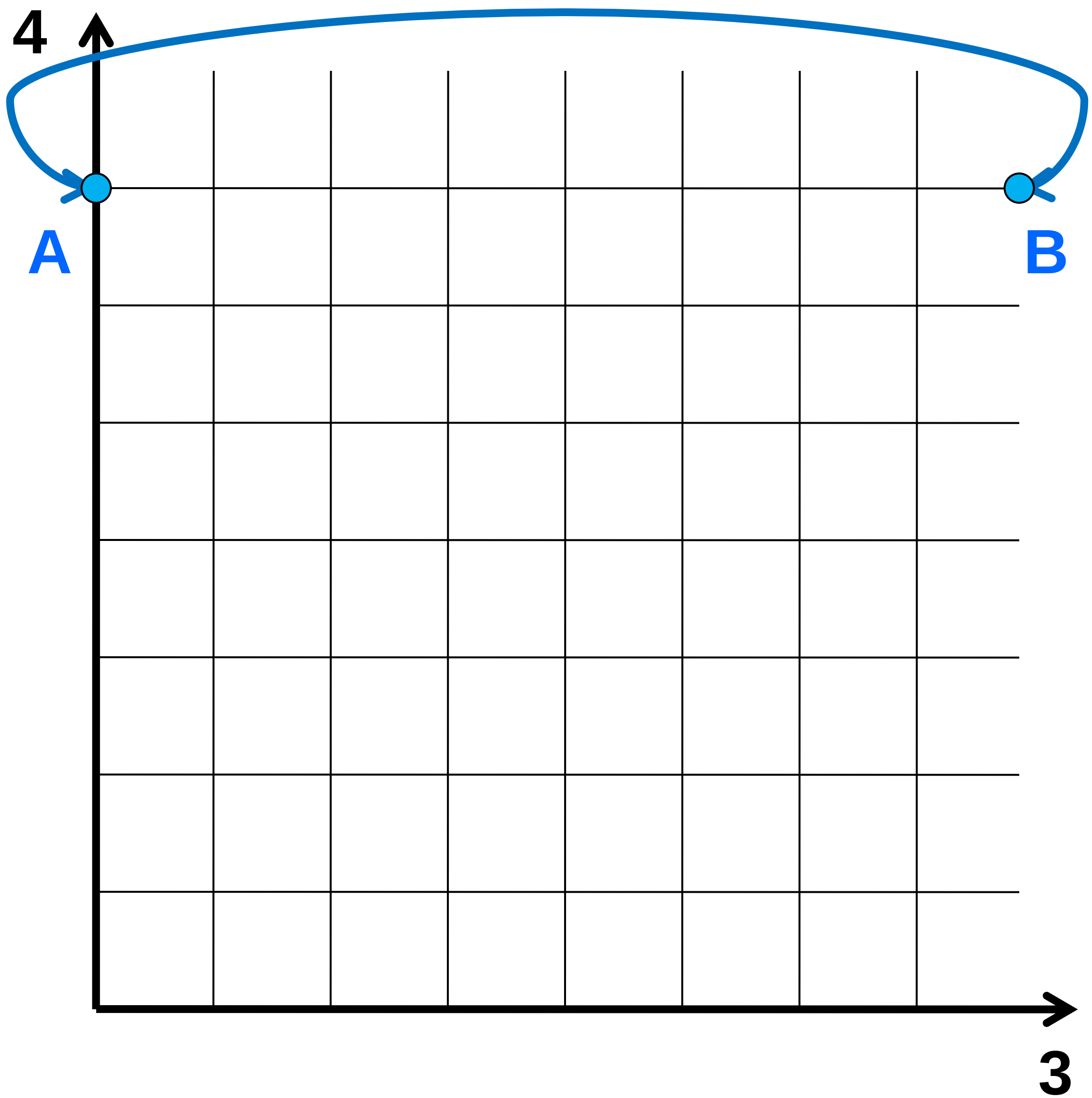
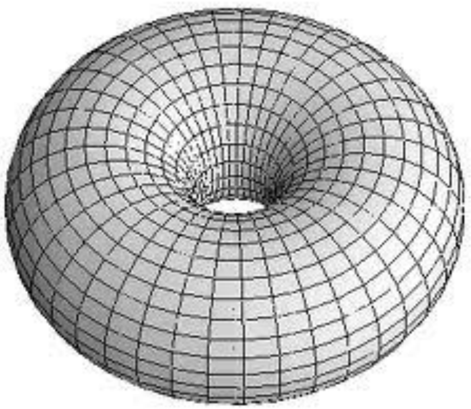


Periodic boundary conditions:

points A and B are the same



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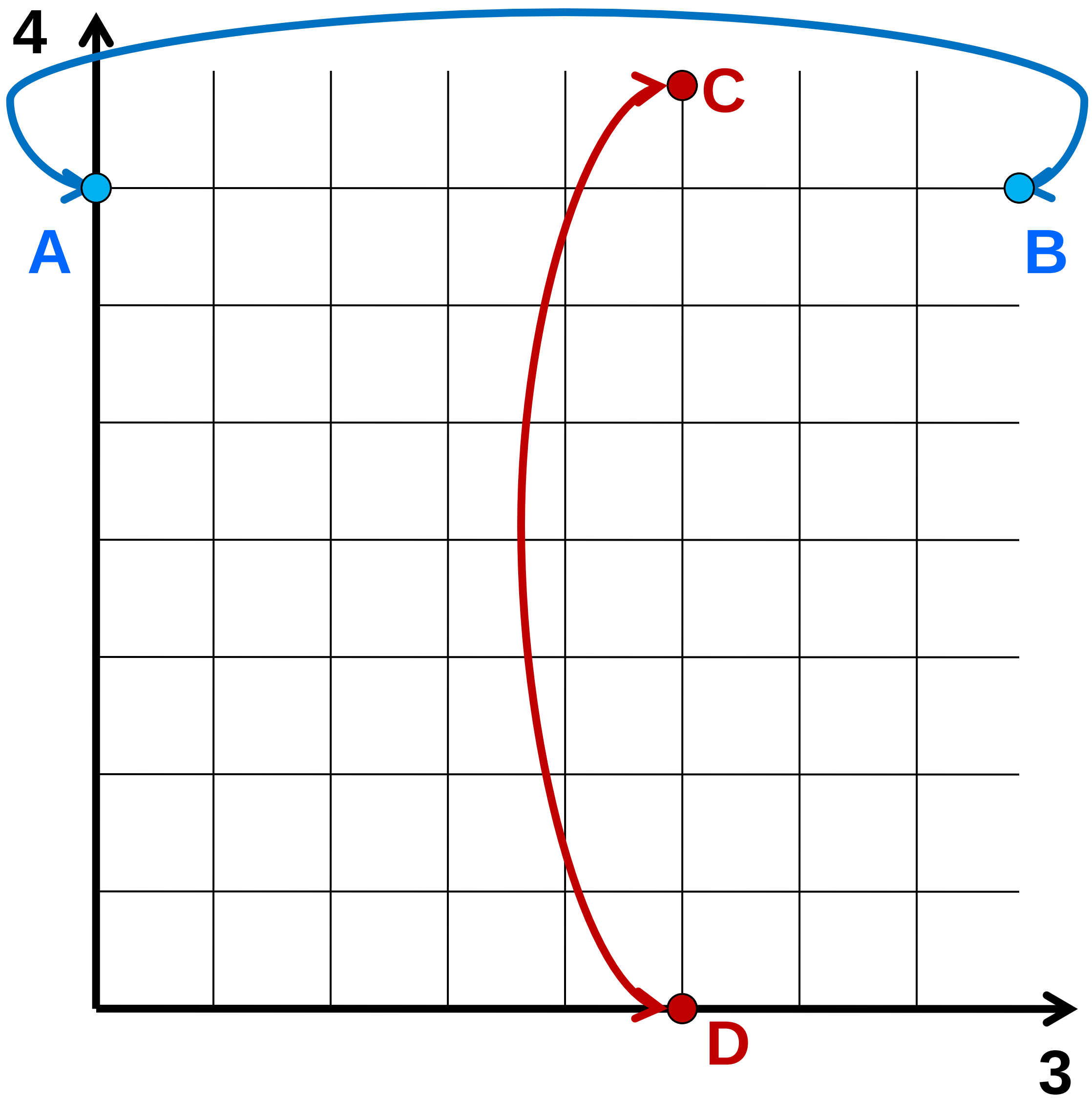
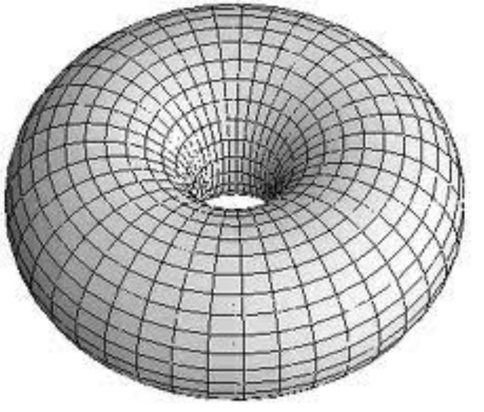


Periodic boundary conditions:

points A and B are the same

Points C and D are also the same



Additional twist on the boundary for constant electric field

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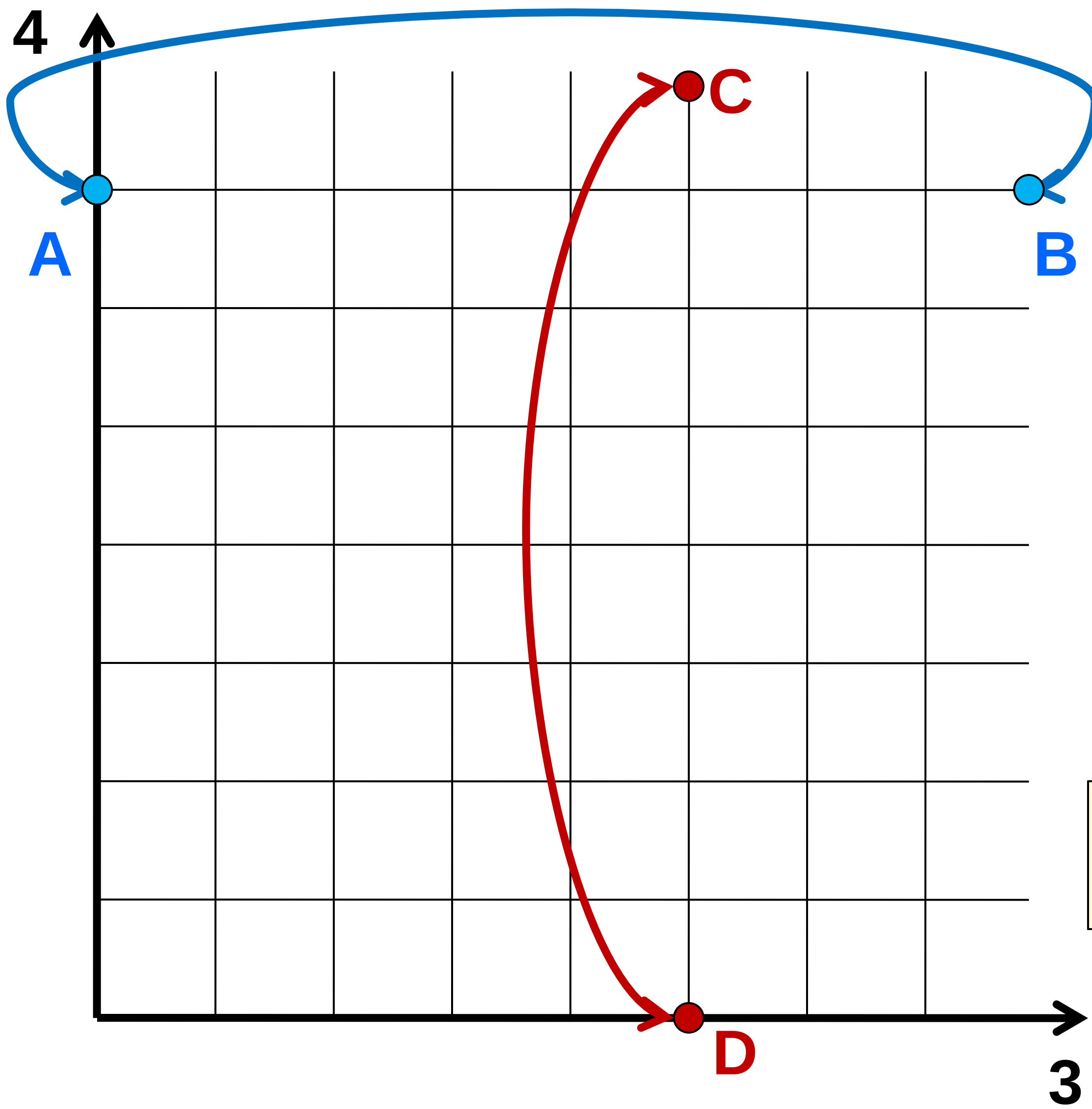


Periodic boundary conditions:

points A and B are the same

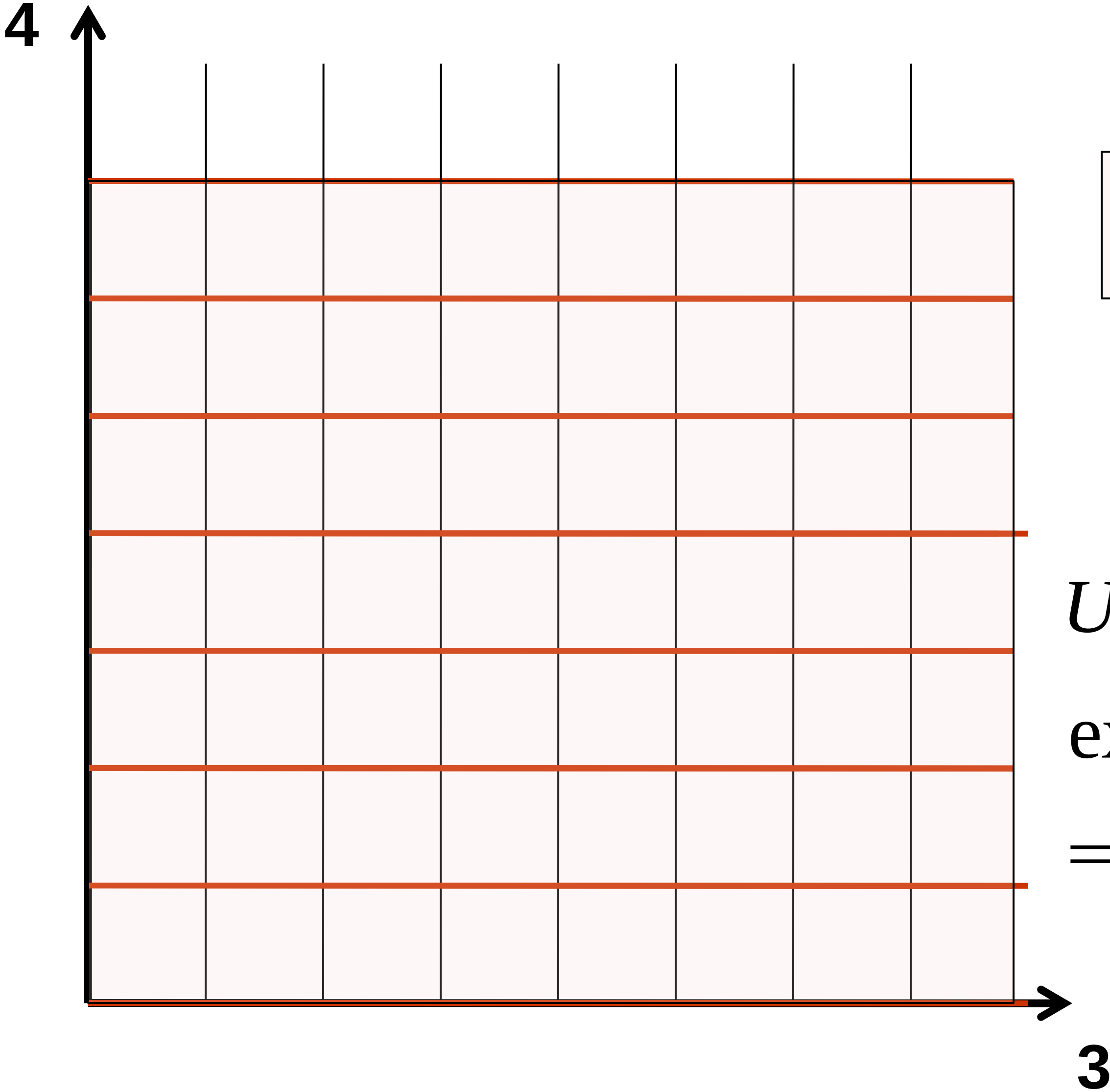
Points C and D are also the same

$$U_{x3}^{ext} = \exp[-iqEx_4]$$

Additional twist on the boundary for constant electric field

$$A_\mu^{ext}(x) = (0, 0, -Ex_4, 0)$$

$$U_{plaq}^{ext} = \exp[iqE]$$



All red links we multiply by

$$U_{x3}^{ext} = \exp[-iqEx_4];$$

then all pink plaquettes (excluding upper row) carry electric field:

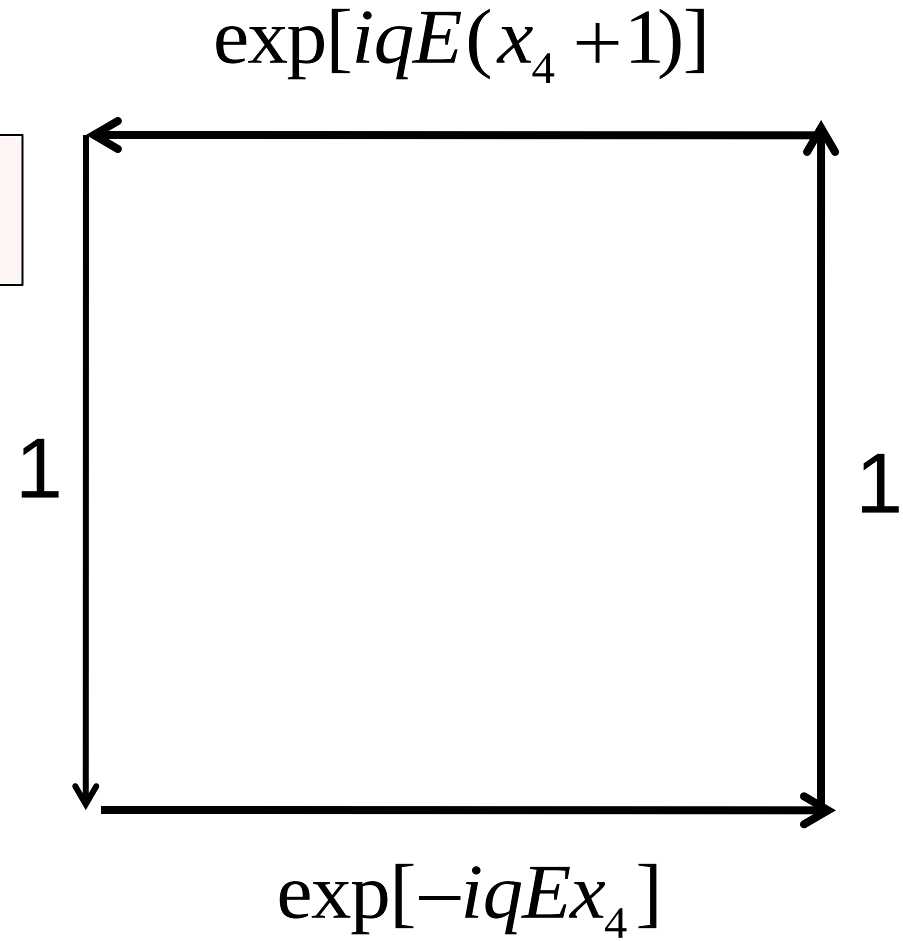
$$U_{plaq}^{ext} =$$

$$\exp[iqE(x_4+1) - iqEx_4]$$

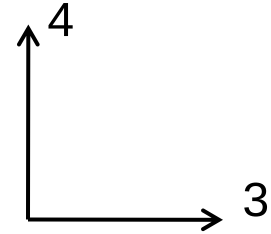
$$= \exp[iqE]$$

OK!

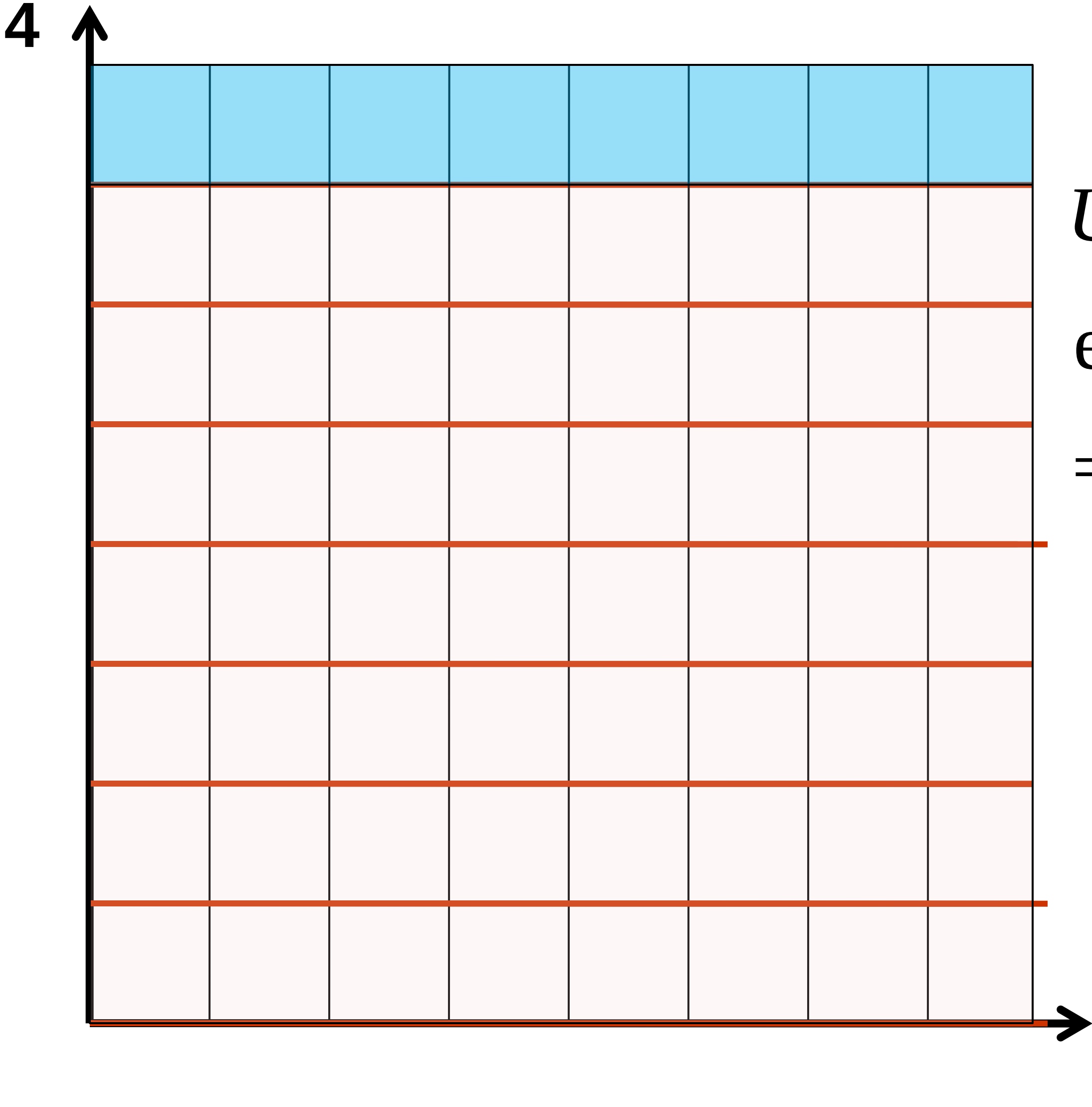
$$U^{ext}_{x3} = \exp[-iqEx_4];$$



$$U^{ext}_{plaq} =$$
$$\exp[iqE(x_4+1) - iqEx_4]$$
$$= \exp[iqE]$$



Additional twist on the boundary for constant electric field $A^{ext}_{\mu}(x) = \left(0, 0, -Ex_4, 0\right)$



then all pink plaquettes (excluding upper row) carry electric field:

$$U^{ext}_{plaq} =$$

$$\exp[iqE(x_4+1) - iqEx_4]$$

$$= \exp[iqE]$$ **OK!**

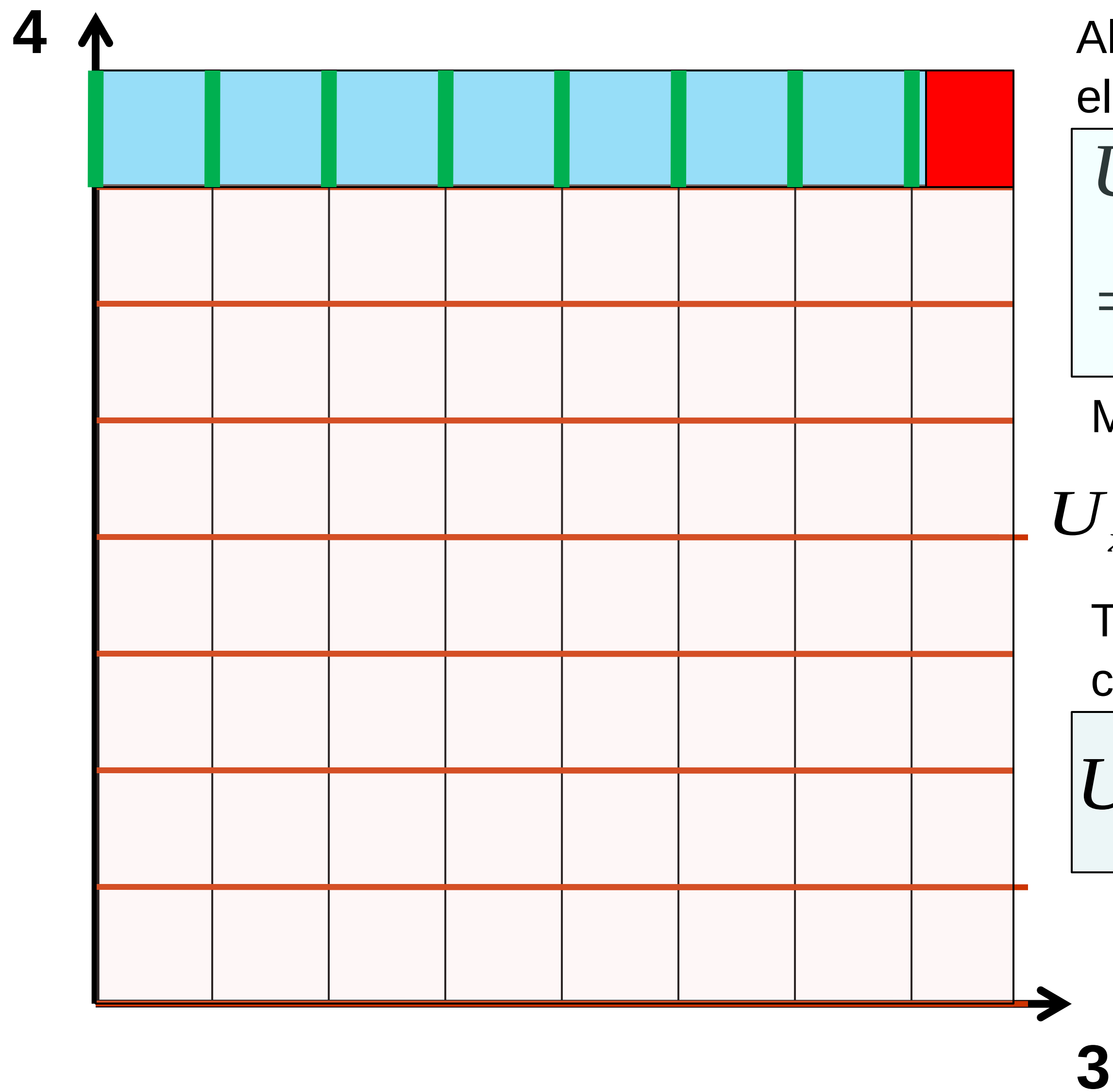
All blue plaquettes carry electric field:

$$U^{ext}_{plaq} =$$

$$= \exp[iqE(1 - N_4)]$$

???

Additional twist on the boundary for constant electric field $A_\mu^{ext}(x) = (0, 0, -E x_4, 0)$



All blue plaquettes carry electric field:

$$U_{plaq}^{ext} = \;???$$
$$= \exp[iqE(1 - N_4)]$$

Multiply all green links on

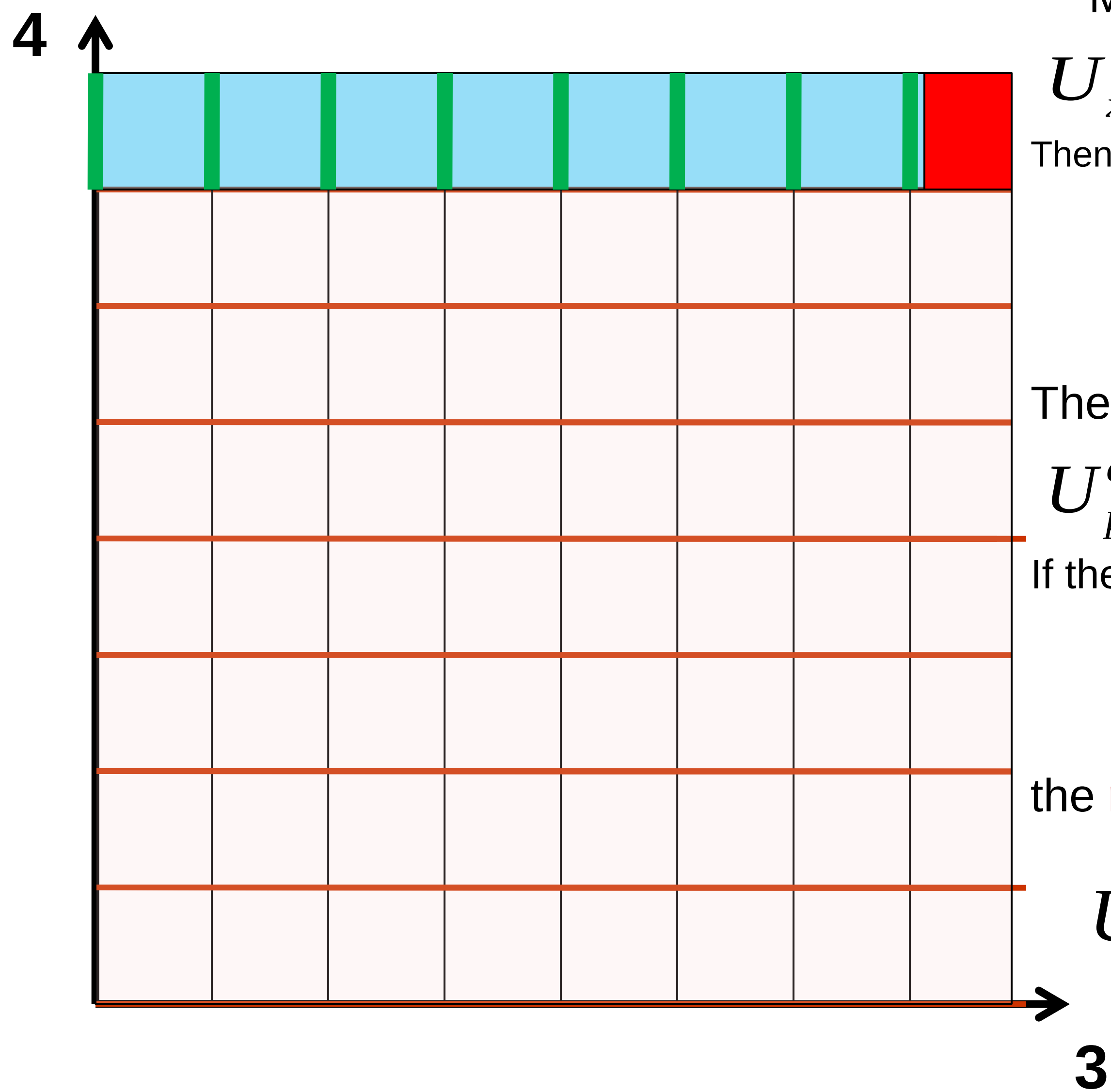
$$U_{x4}^{ext\,bound} = \exp[iqEN_4 x_3]$$

Then all blue plaquettes carry electric field:

$$U_{plaq}^{ext} = \exp[iqE]$$

OK!!!

Additional twist on the boundary for constant electric field $A_\mu^{ext}(x) = \left(0, 0, -Ex_4, 0\right)$



Multiply all **green** links on

$$U_{x4}^{ext\,bound} = \exp[iqEN_4 x_3]$$

Then all blue plaquettes carry electric field:

$$U_{plaq}^{ext} = \exp[iqE]$$

The **red** plaquette carry electric field:

$$U_{plaq}^{ext} = \exp[iqE(1 - N_4 L)]$$

If the quantization condition is satisfied

$$E = 2\pi n / qLN_t$$

then

the **red** plaquette carry electric field:

$$U_{plaq}^{ext} = \exp[iqE]$$ **OK!!!**

Summary

1. Electric field in Minkowski space-time corresponds to *imaginary* electric field in Euclidean space-time.

2. Magnetic field in Minkowski space-time corresponds to *real* magnetic field in Euclidean space-time.

3. Constant electric and magnetic fields on the torus are quantized.

4. There exists additional twist on the boundary for constant electric and magnetic fields on the lattice with periodical boundary conditions.

External Fields as Additional Parameters of the Theory

Magnetic Moments and Electric Polarizabilities from lattice QCD

Extracting Nucleon Magnetic Moments and Electric Polarizabilities from Lattice QCD in Background Electric Fields. W. Detmold, B.C. Tiburzi, A. Walker-Loud, Phys.Rev.D81:054502,2010. **And references therein**

The main idea is the calculation of the mass shift due to the background fields, *e.g.* for spin ½ baryons in the external *electric* field

$$M \to M + \frac{1}{2}E^2\left(4\pi\alpha_E - \frac{\mu^2}{4M^3} + \ldots\right),$$

for spin ½ baryons in the external *magnetic* field

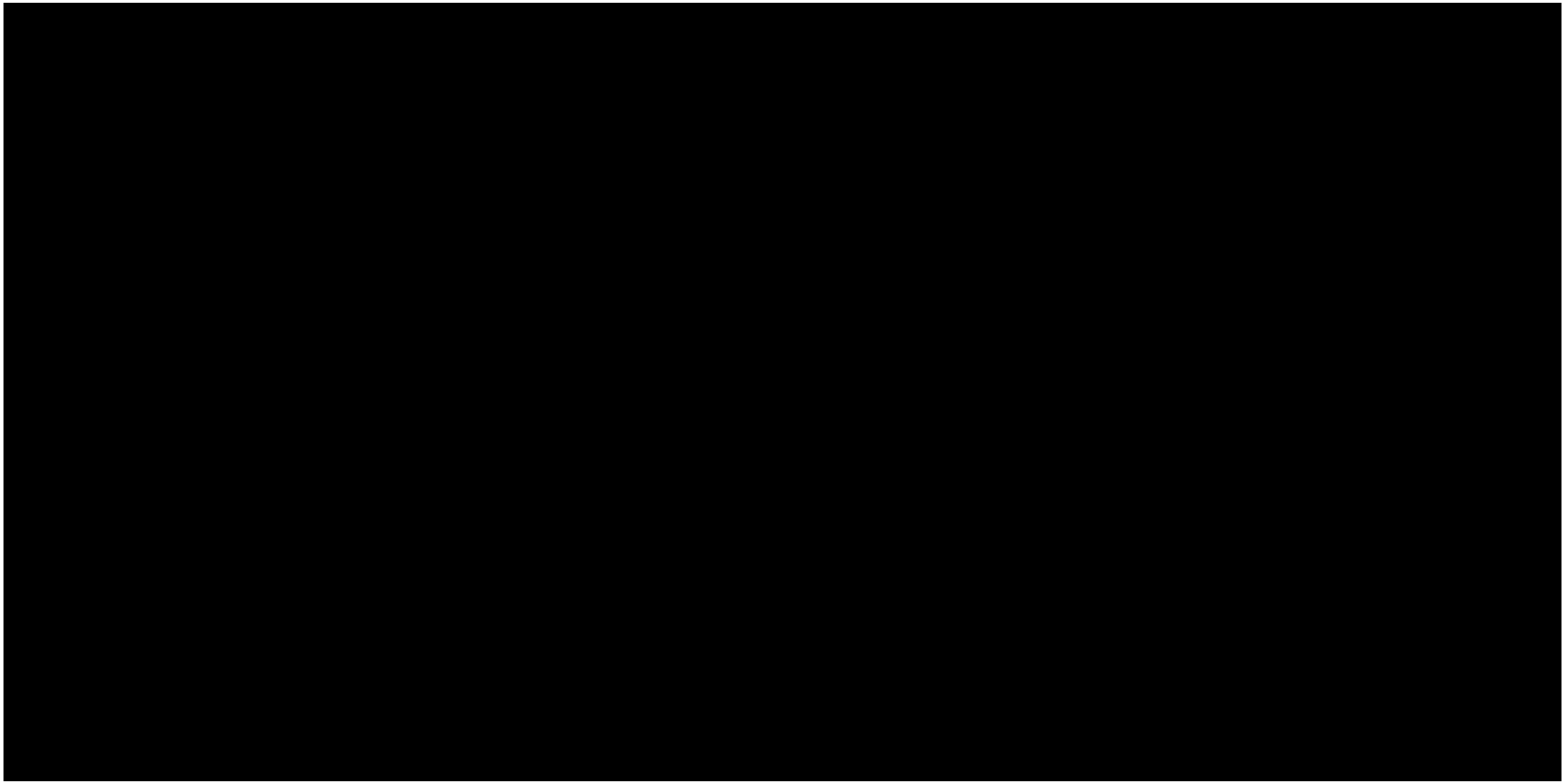
Magnetic Moments of Negative-Parity Baryons from Lattice QCD. Frank X. Lee, Andrei Alexandru , PoS LATTICE2010 (2010) 148 e-Print: arXiv:1011.4325 [hep-lat]. **And references therein**

$$M \to M \pm \mu B,$$

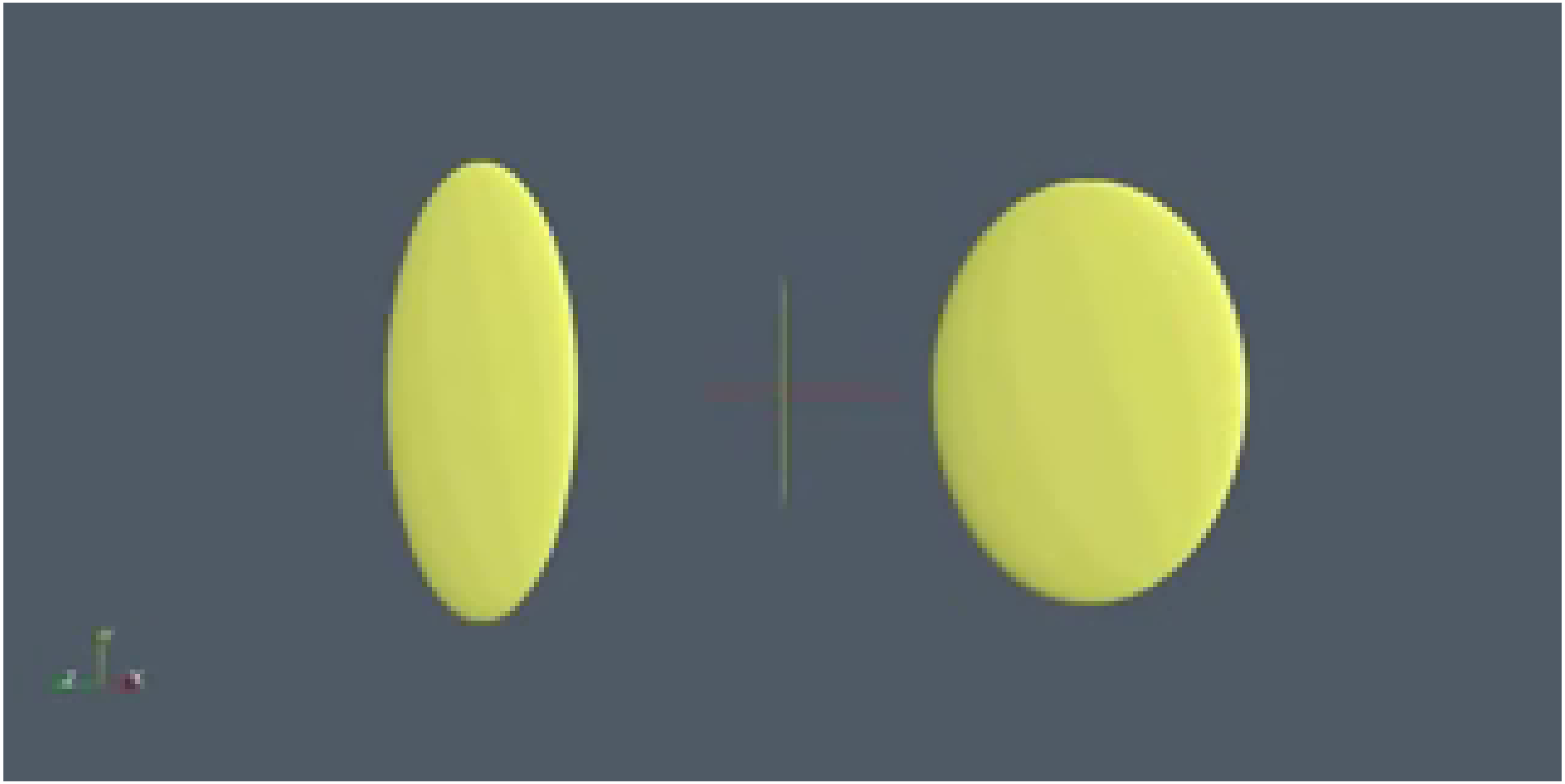
I's very interesting, but I will not speak about all that

Very Strong Magnetic Fields in Heavy Ion Collisions

Very strong magnetic fields in heavy ion collisions

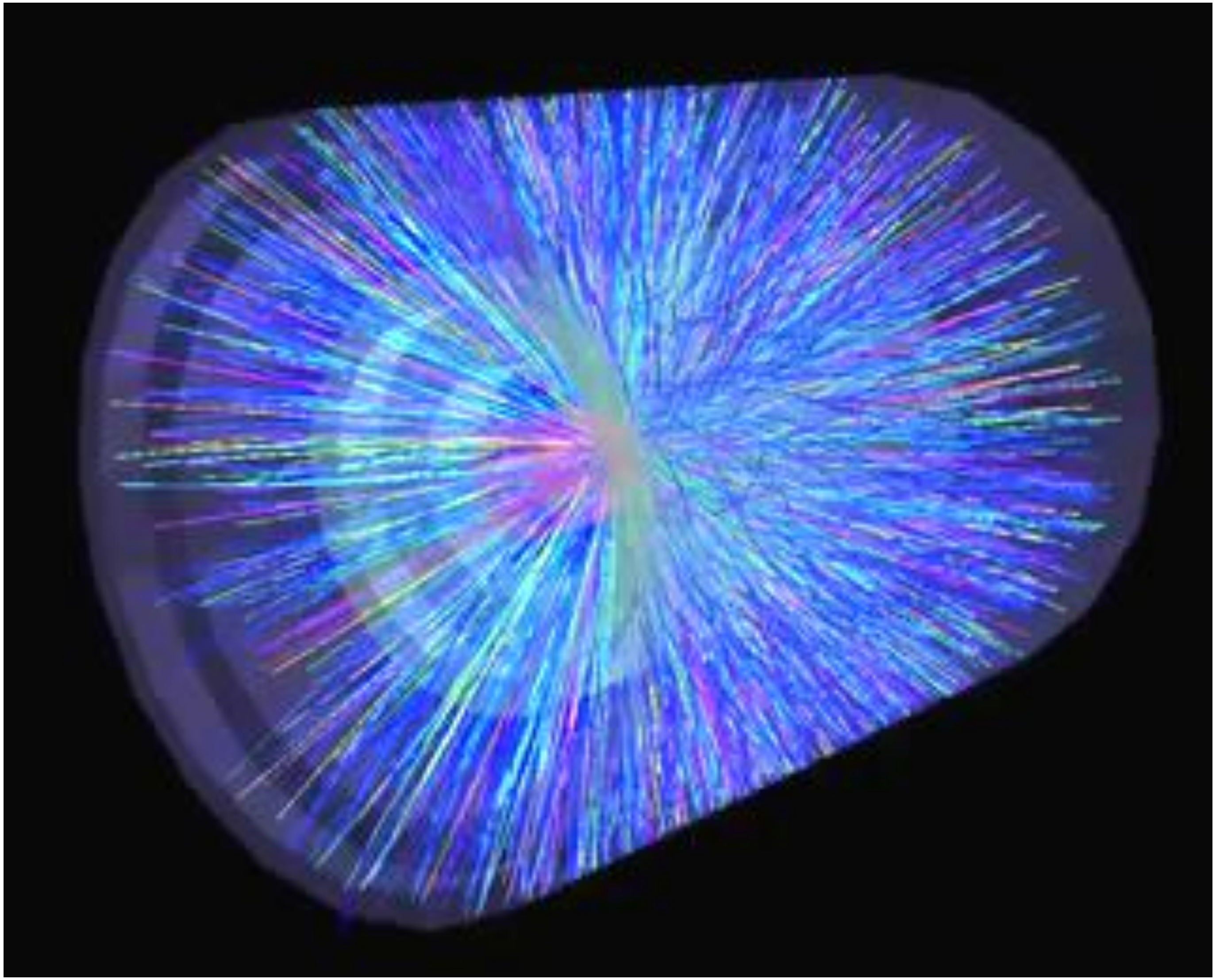


Very strong magnetic fields in heavy ion collisions



Very strong magnetic fields in heavy ion collisions

First collisions of Pb+Pb (Lead+lead) seen by the ALICE experiment at LHC



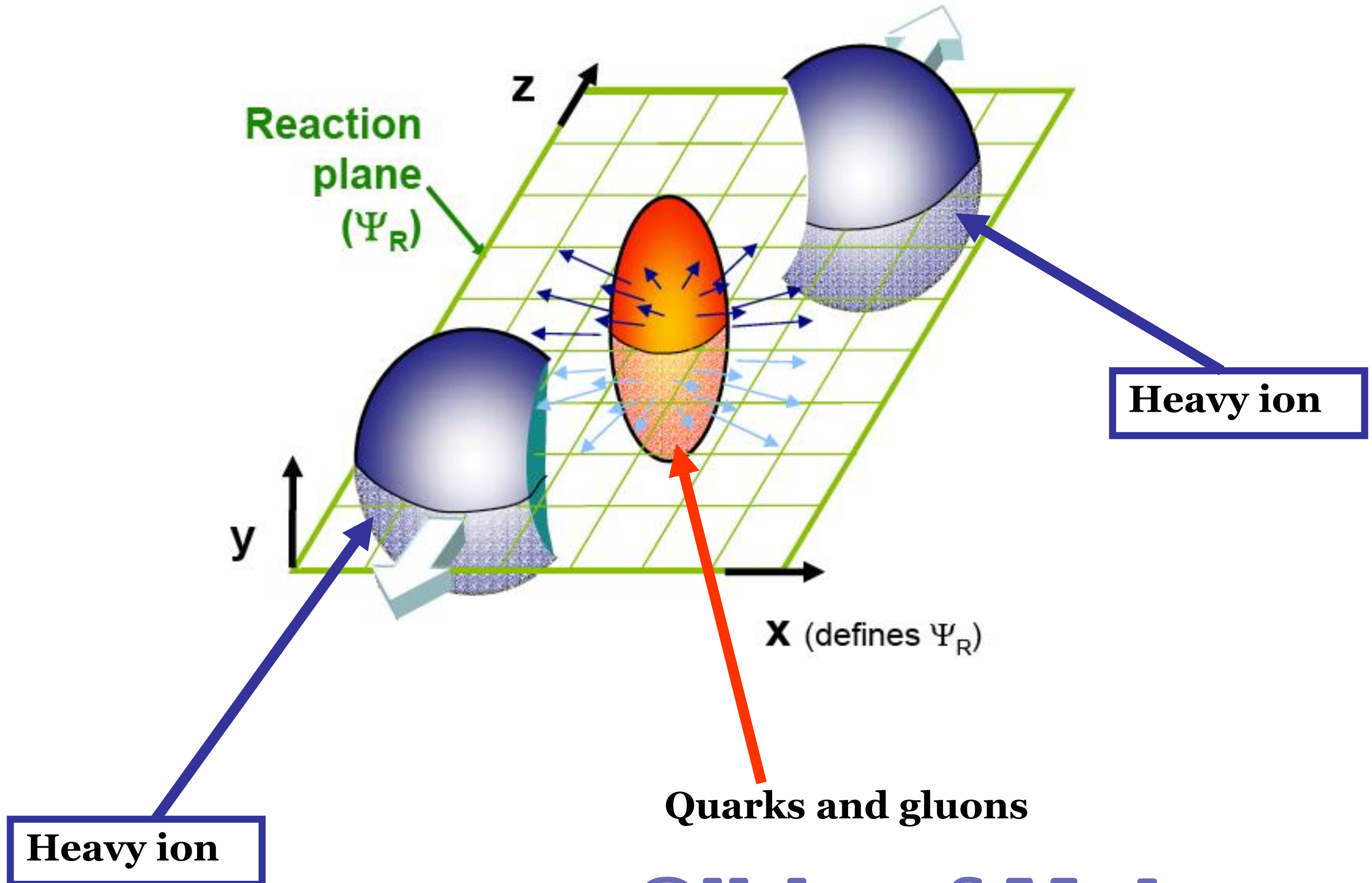
1) QG plasma is thermalized

2) Viscosity of QG plasma is very small

3) Multiplicity is very high

Magnetic fields in non-central collisions

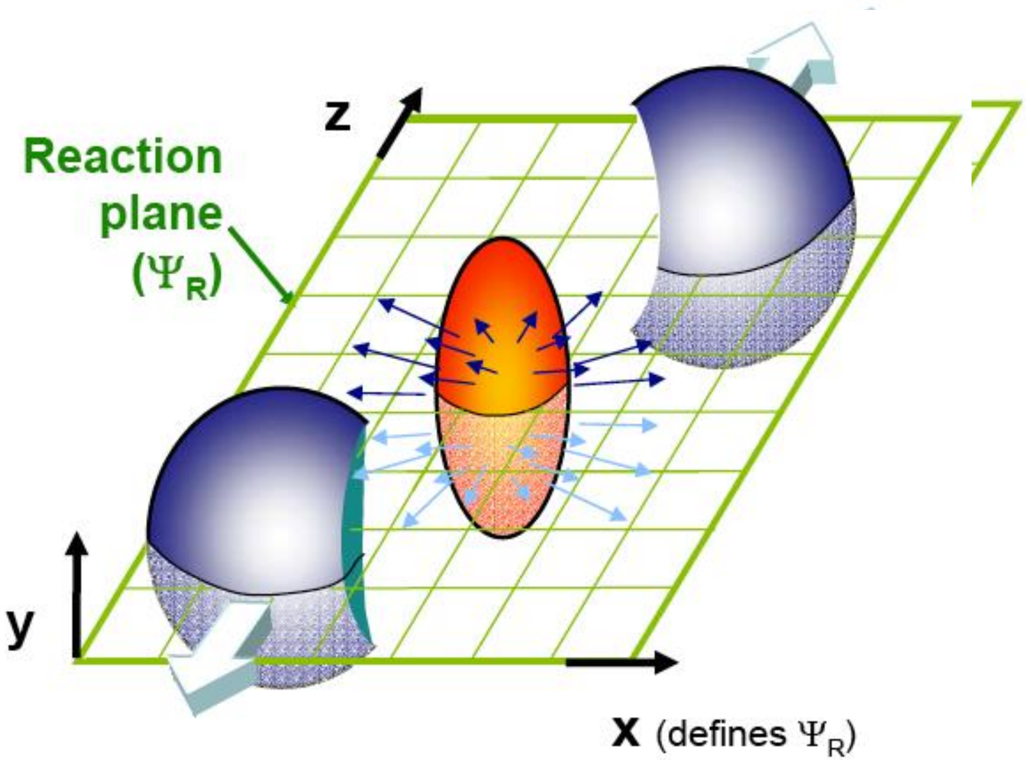
[Fukushima, Kharzeev, Warringa, McLerran '07-'08]



Slide of McLerran

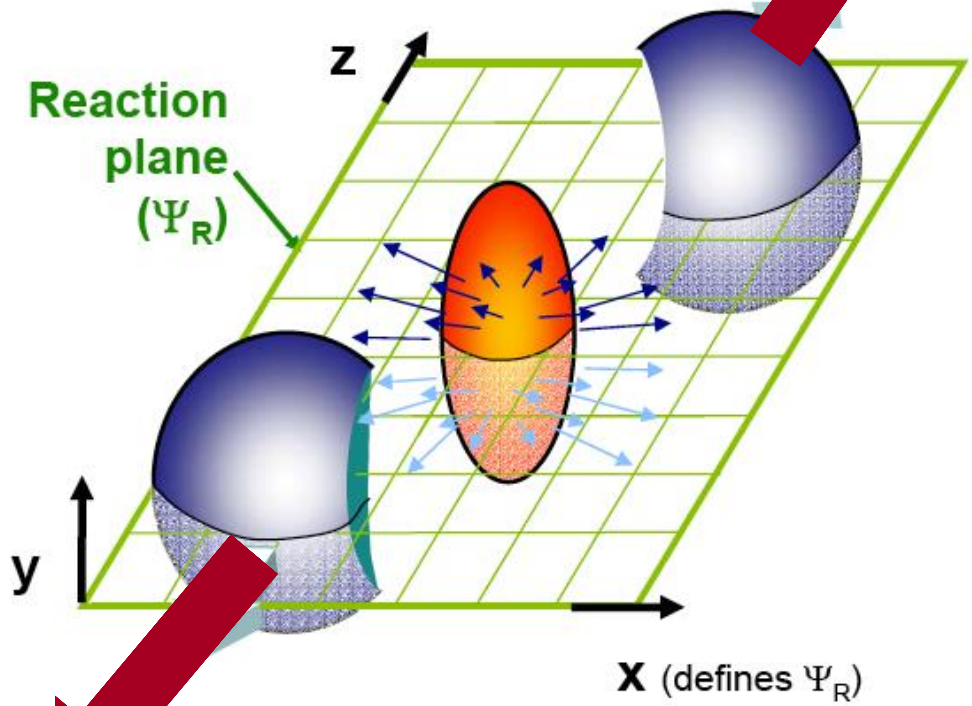
Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran ’07-’08]



[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008), URL http://arxiv.org/abs/0808.3382.
[2] D. Kharzeev, R. D. Pisarski, and M. H. G.Tytgat, Phys. Rev. Lett. 81, 512 (1998), URL http://arxiv.org/abs/hep-ph/9804221.
[3] D. Kharzeev, Phys. Lett. B 633, 260 (2006), URL http://arxiv.org/abs/hep-ph/0406125.
[4] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A 803, 227 (2008), URL http://arxiv.org/abs/0711.0950.

Magnetic fields in non-central collisions



Charge is large
Velosity is high

Thus we have two very big currents

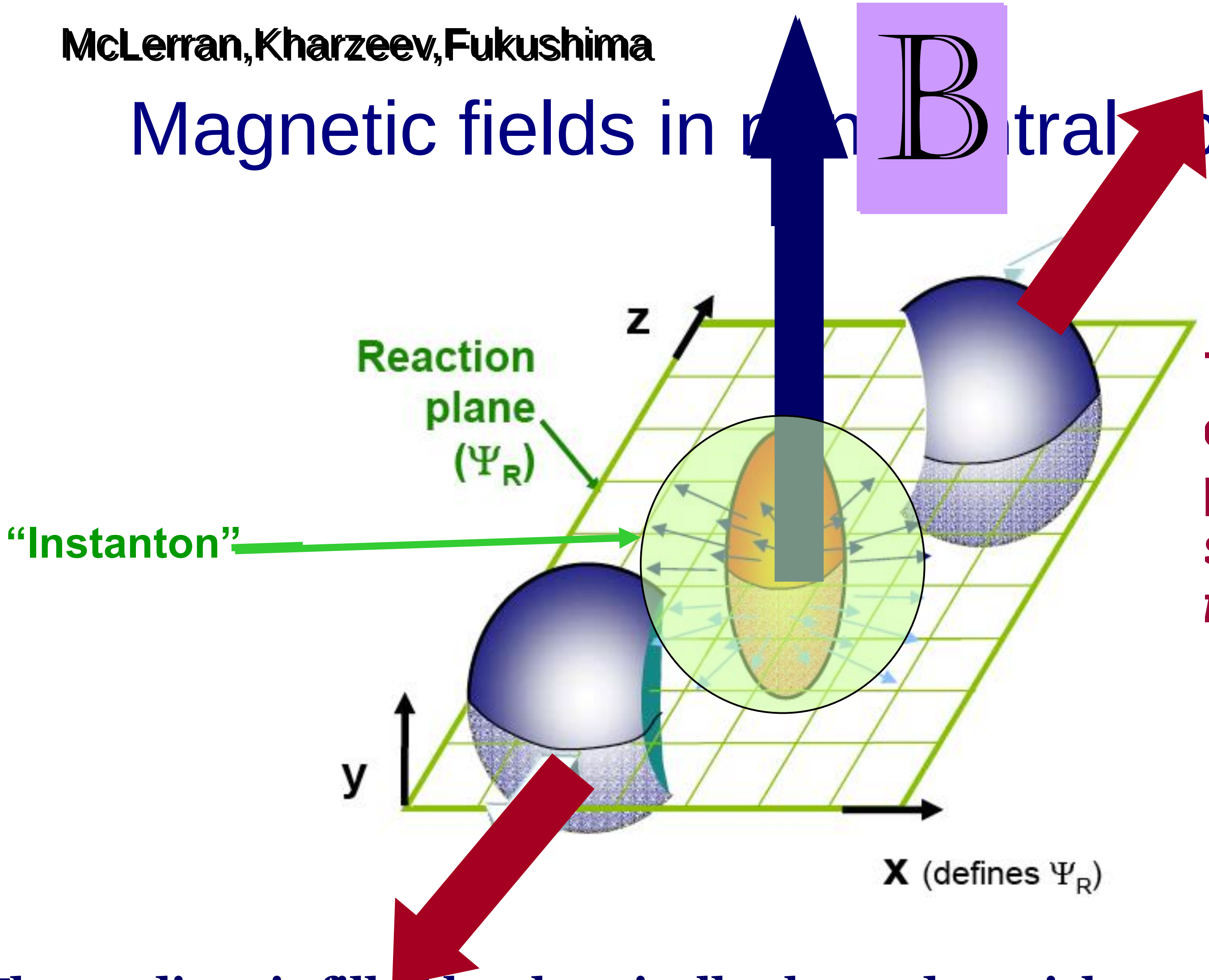
The medium is filled by electrically charged particles

Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

McLerran,Kharzeev,Fukushima

Magnetic fields in non-central collisions



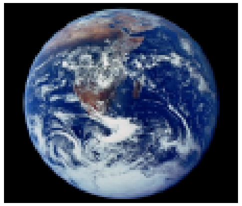
Two very big currents produce a very *strong magnetic field*

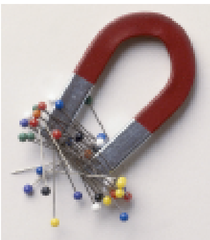
The medium is filled by electrically charged particles

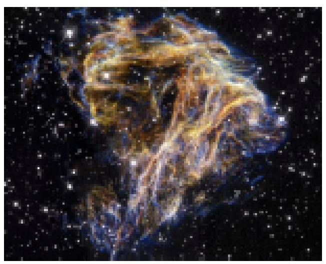
Large orbital momentum, perpendicular to the reaction plane

Large magnetic field along the direction of the orbital momentum

Comparison of magnetic fields



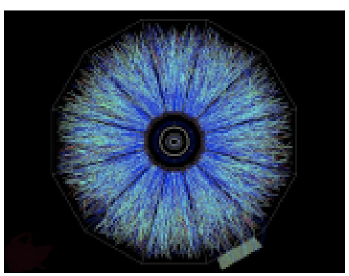




The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html

Slide of D. Kharzeev



At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau=0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

In heavy ion collisions magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda_{QCD}^2$$

Magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda^2_{QCD}$$

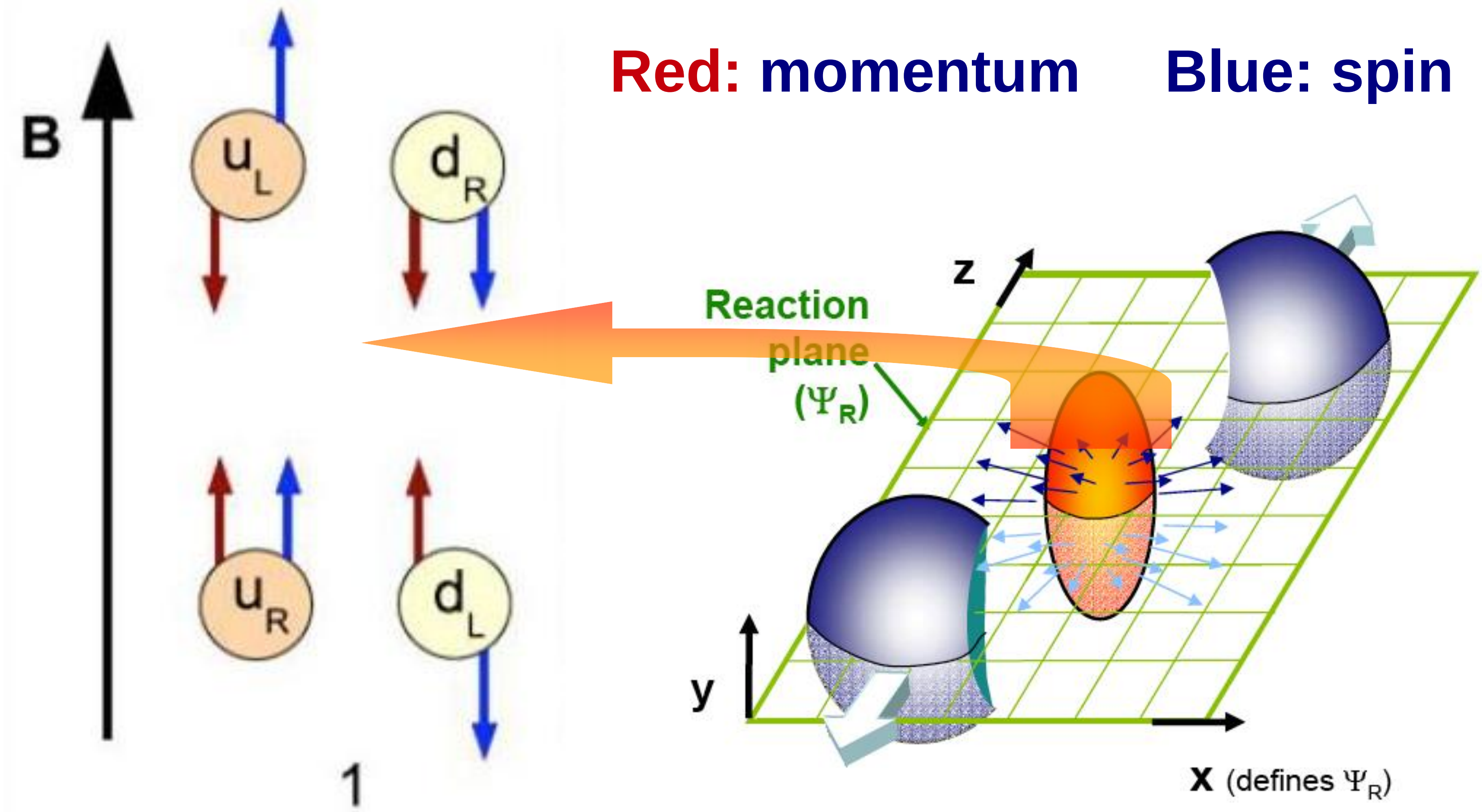
We expect the influence of magnetic field on strong interaction physics

Chiral Magnetic Effect (CME)

Chiral Magnetic Effect by **Fukushima, Kharzeev, Warringa, McLerran**

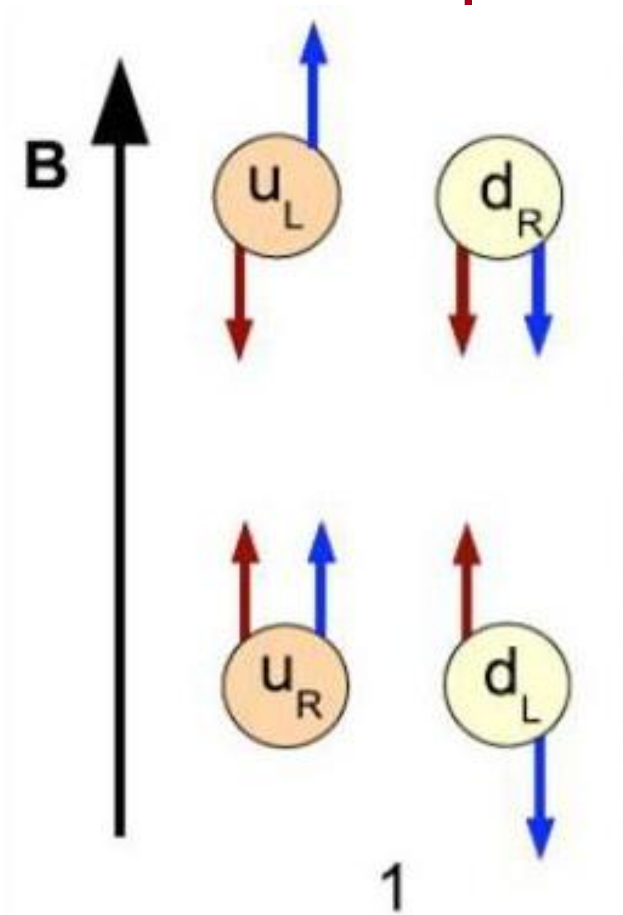
1. Massless quarks in external magnetic field.

Red: momentum **Blue: spin**

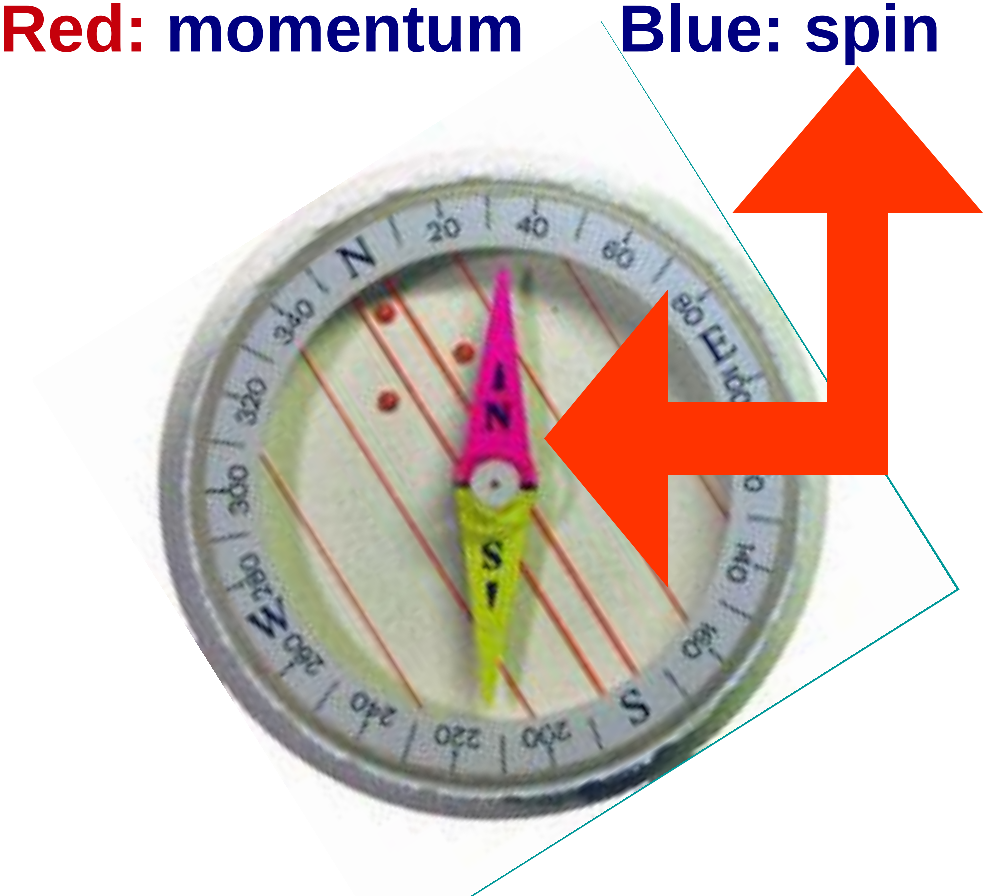


Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

1. Massless quarks in external magnetic field.

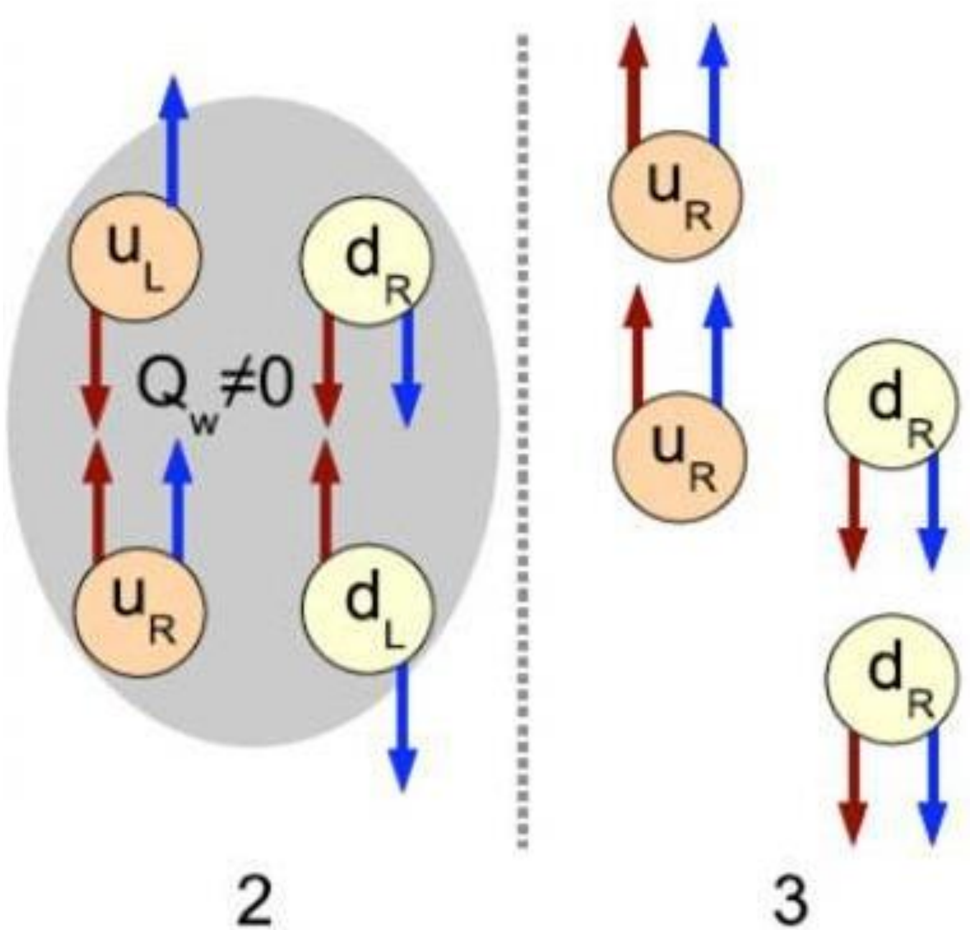


Red: momentum **Blue: spin**



Chiral Magnetic Effect by **Fukushima, Kharzeev, Warringa, McLerran**

2. Quarks in the instatnton field.



Red: momentum
Blue: spin

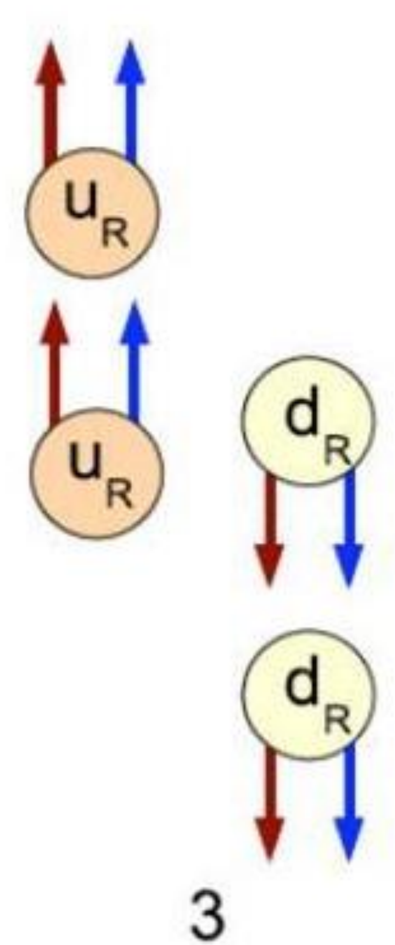
Effect of topology:

$u_L \rightarrow u_R$

$d_L \rightarrow d_R$

Chiral Magnetic Effect by
Fukushima, Kharzeev, Warringa, McLerran

3. Electric current along magnetic field



Red: momentum
Blue: spin

Effect of topology:

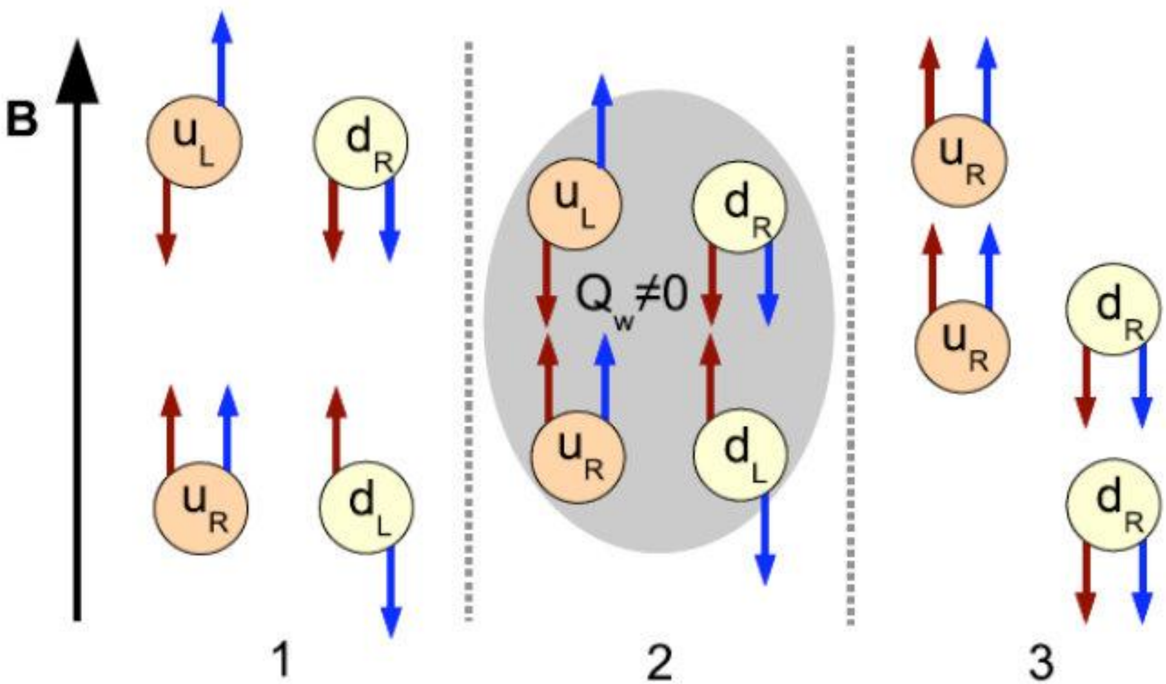
$u_L \rightarrow u_R$

$d_L \rightarrow d_R$

u-quark: q=+2/3
d-quark: q= - 1/3

Chiral Magnetic Effect by **Fukushima, Kharzeev, Warringa, McLerran**

3. Electric current is along magnetic field
In the *instanton* field



Red: momentum
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Effect of topology:
$u_L \to u_R$
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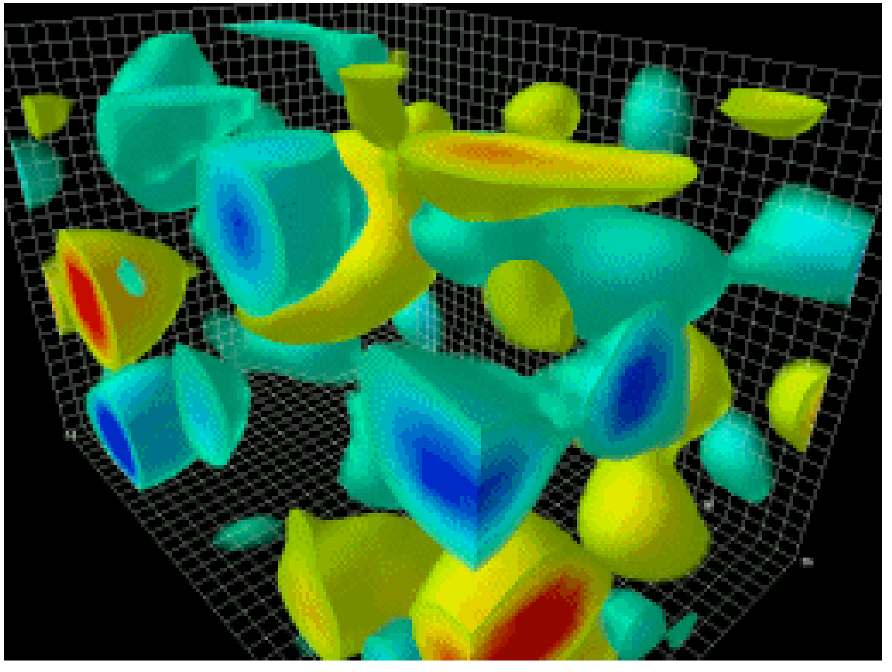
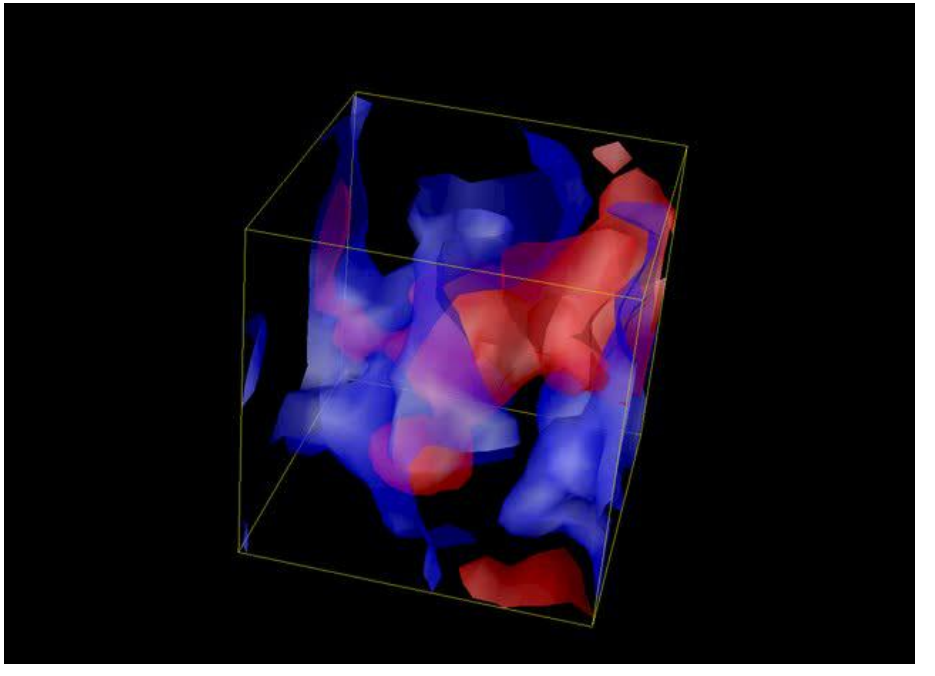
3D time slices of topological charge density, lattice calculations



D. Leinweber

Topological charge density after vacuum cooling



P.V.Buividovich,
T.K. Kalaydzhyan, M.I. Polikarpov

Fractal topological charge density without vacuum cooling

Summary

1. In heavy ion noncentral collisions the very strong magnetic fields can be generated

2. The interference between strong and electromagnetic interactions can produce new physical effects (CME)

Very Strong Magnetic Fields in Lattice Calculations

Magnetic forces are of the order of strong interaction forces

$$eB \approx \Lambda^2_{QCD}$$

We expect the influence of magnetic field on strong interaction physics

The effects are nonperturbative,

and we use

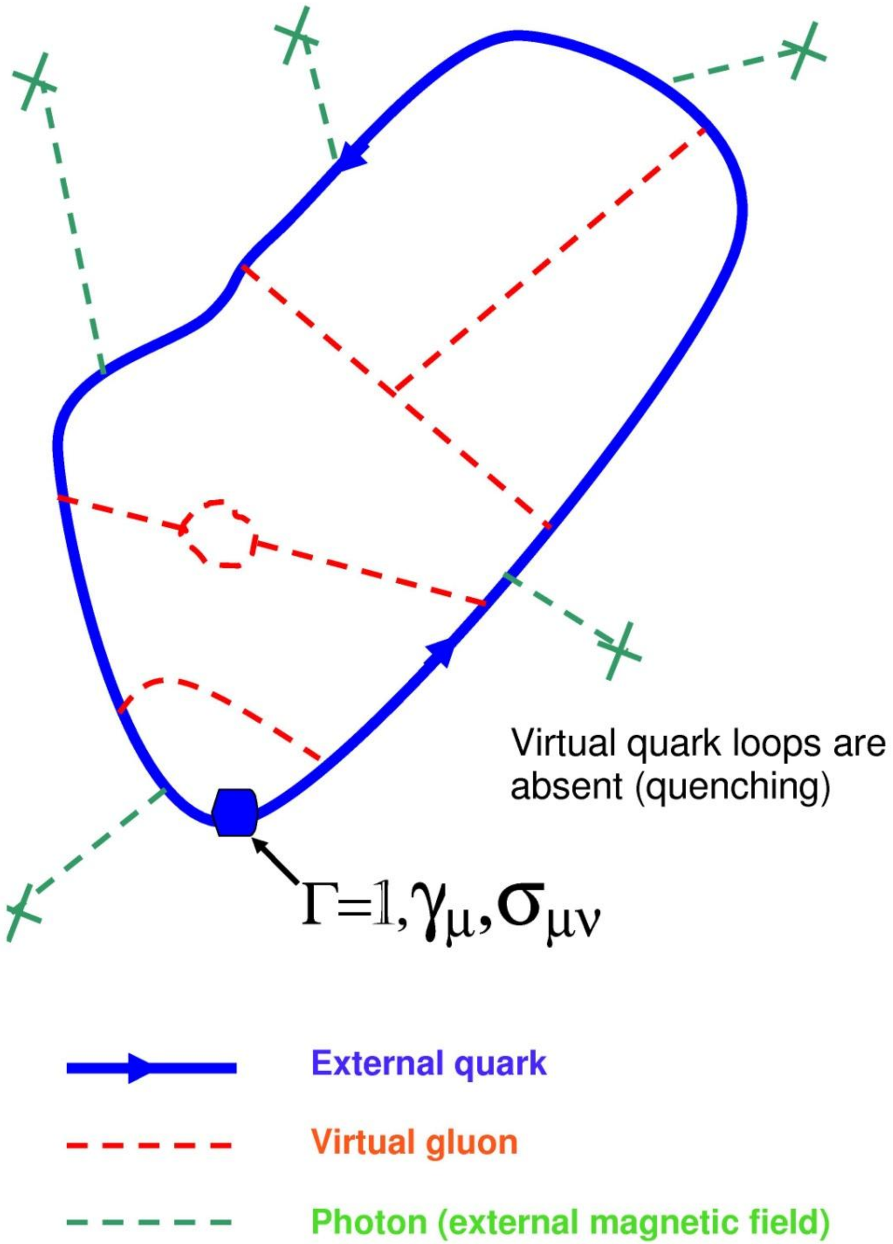
<u>Lattice Calculations</u>

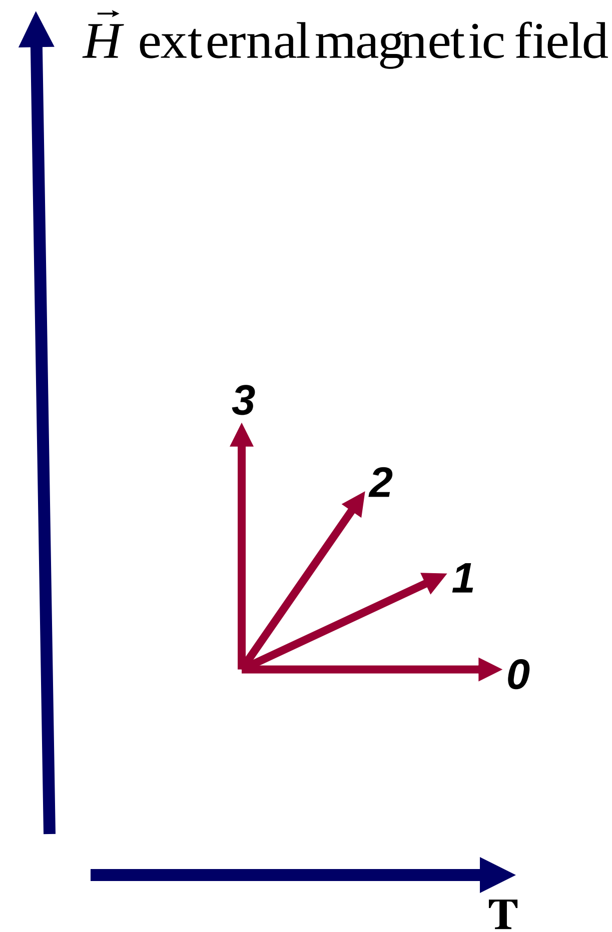
ITEP lattice group publications on gluodynamics with strong magnetic fields

P.V.Buividovich, V.V. Braguta, M.N.Chernodub, T.K. Kalaydzhyan, D.E. Kharzeev, O.V. Larina, E.V.Luschevskaya, M.I. P.

arXiv:1104.3767, arXiv:1011.3001, arXiv:1011.3795, arXiv:1003.2180, arXiv:0910.4682, arXiv:0909.2350, arXiv:0909.1808, arXiv:0907.0494, arXiv:0906.0488, arXiv:0812.1740

We calculate $<\overline{\psi}\,\Gamma\psi>;\ \Gamma=1,\gamma_{\mu},\sigma_{\mu\nu}$ **in the external magnetic field and in the presence of the vacuum gluon fields We consider SU(2) gauge fields and quenched approximation**

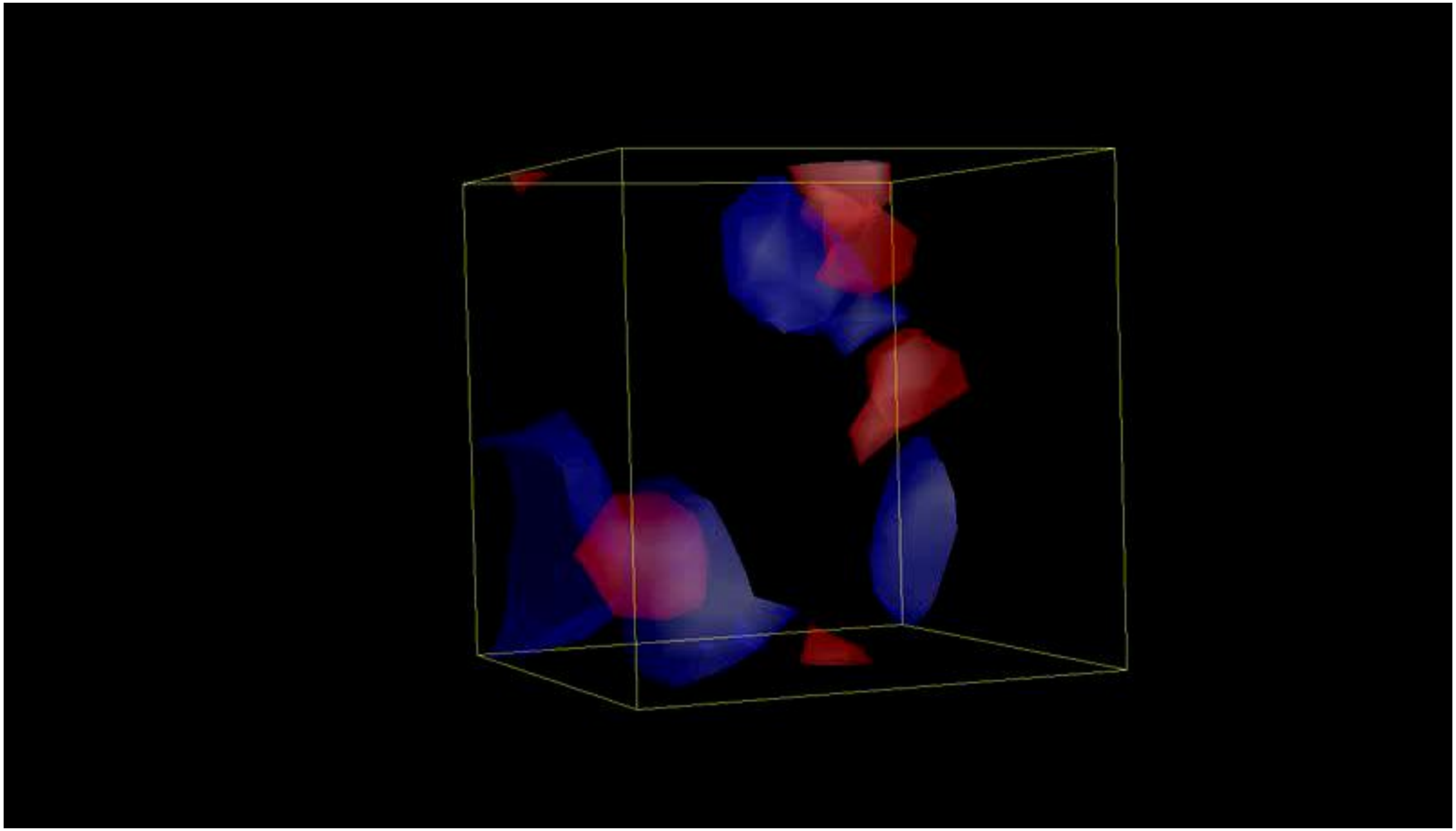




Quenched vacuum, overlap Dirac operator, external magnetic field

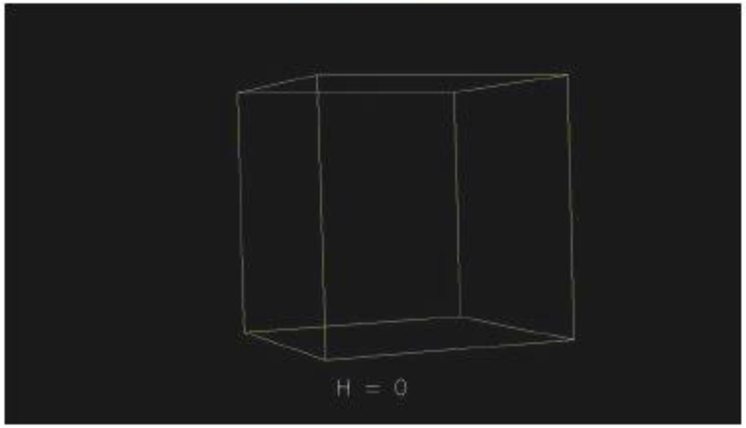
$$eB = \frac{2\pi k}{L^2}; eB \geq (250\ Mev)^2$$

Density of electric charge in the vacuum vs. magnetic field

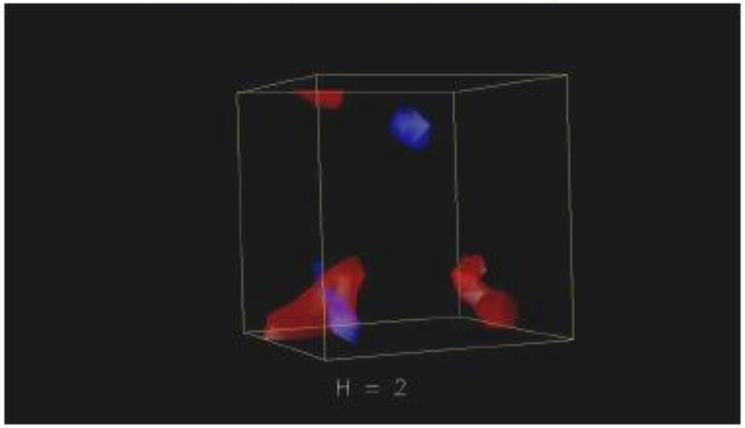


Density of the electric charge vs. magnetic field, 3D time slices

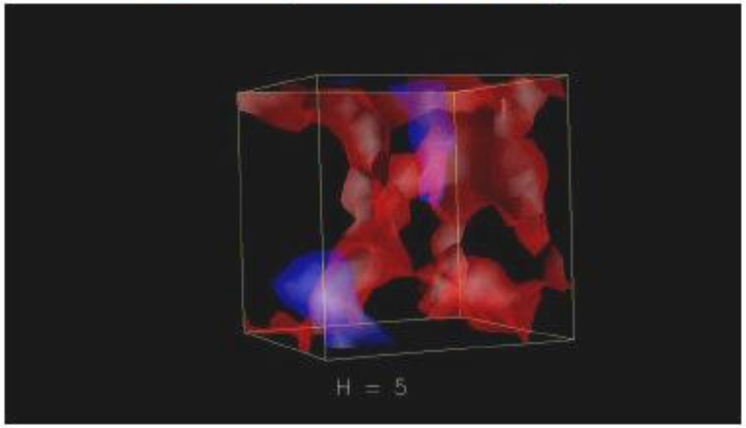
$B = 0$



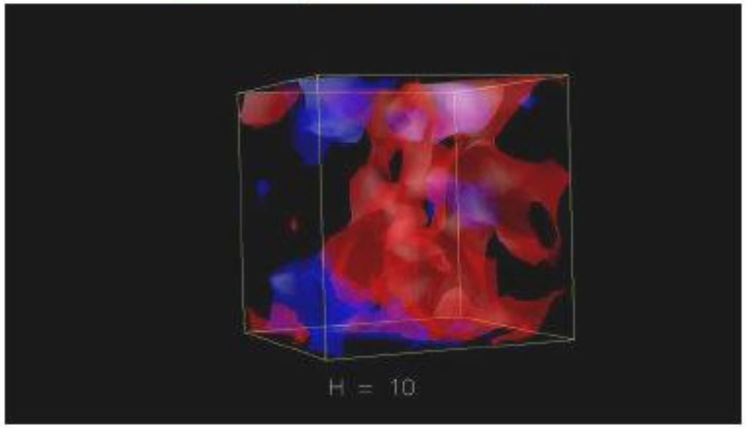
$B = (500\,\mathrm{MeV})^2$



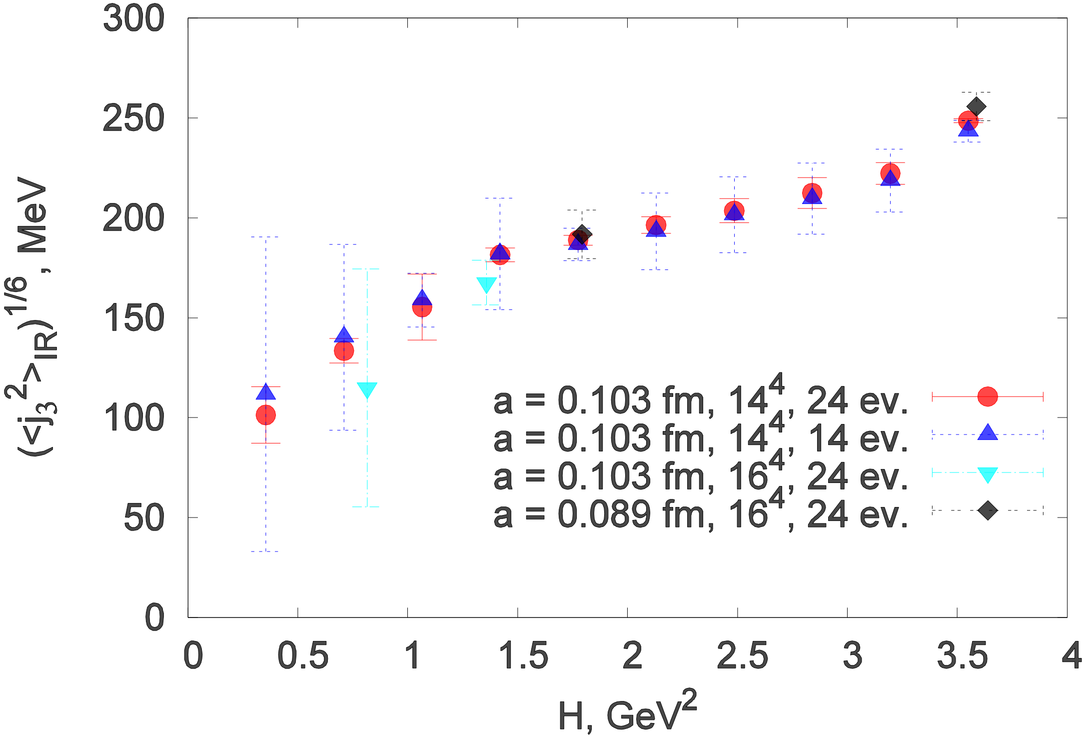
$B = (780\,\mathrm{MeV})^2$



$B = (1.1\,\mathrm{GeV})^2$



Chiral Magnetic Effect on the lattice, numerical results T=0



Regularized electric current:

$$< j_3^2 >_{IR} = < j_3^2(H,T) > - < j_3^2(0,0) >, \quad j_3 = \bar{\psi}\gamma_3\psi$$

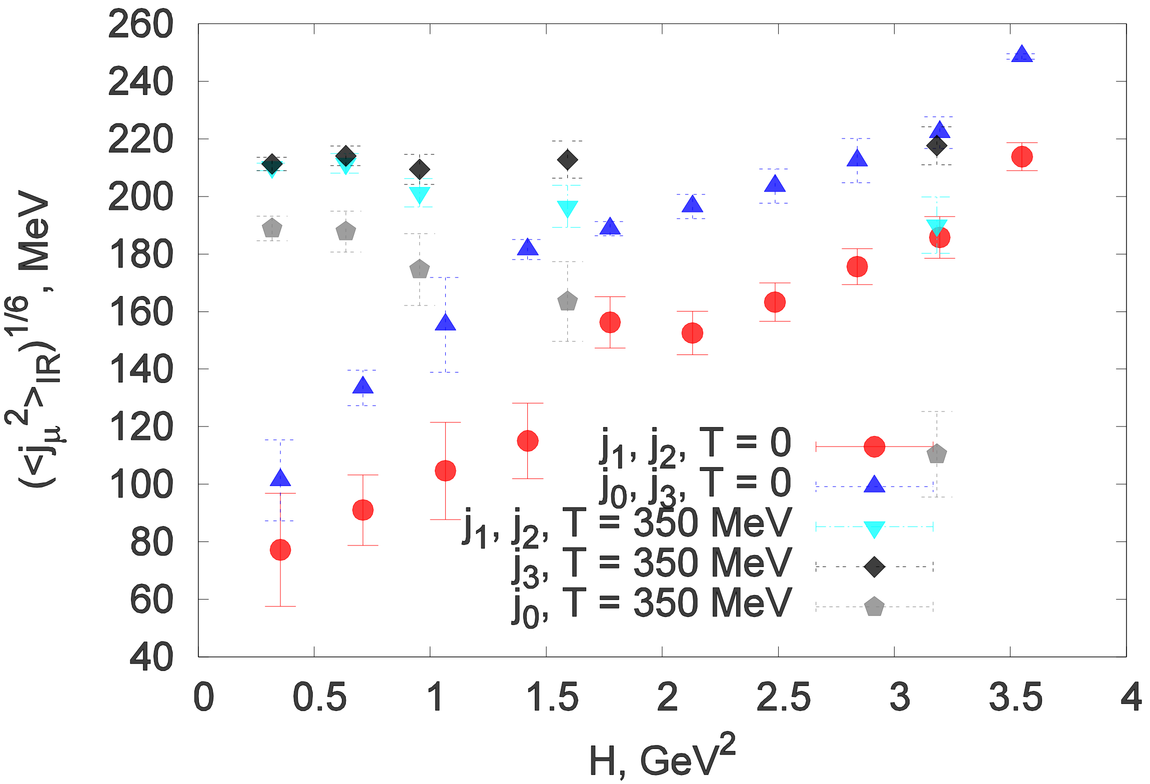
Chiral Magnetic Effect on the lattice, numerical comparison of results near Tc and nearzero

$T = 0$

$F_{12} \neq 0$

$< j_1^2 > = < j_2^2 >$

$< j_3^2 > = < j_0^2 >$



$T > 0$

$F_{12} \neq 0$

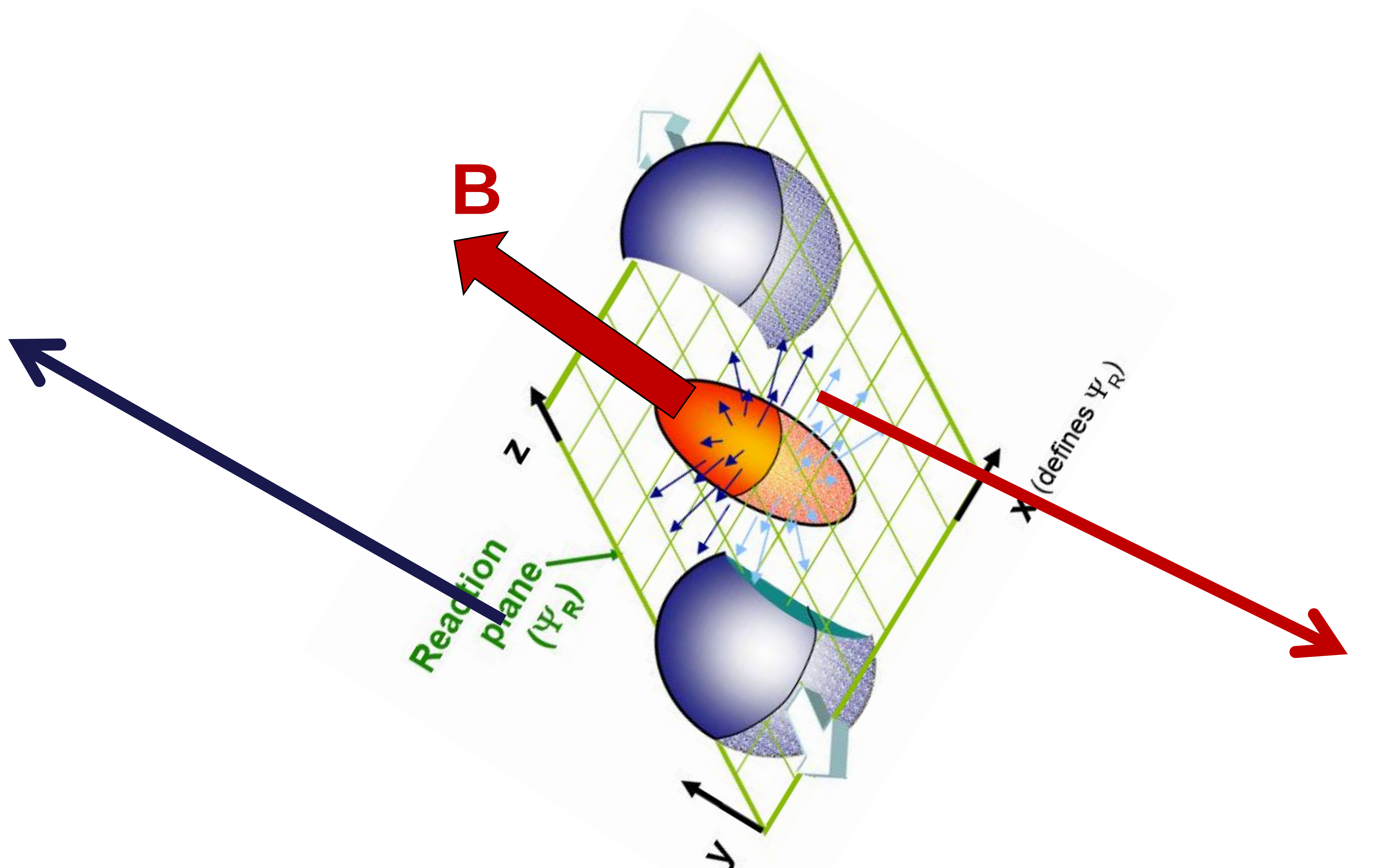
$< j_1^2 > = < j_2^2 >$

$< j_3^2 > \neq < j_0^2 >$

Regularized electric current:

$$< j_i^2 >_{IR} = < j_i^2(H,T) > - < j_i^2(0,0) >, \quad j_i = \overline{\psi} \gamma_i \psi$$

We really observe two opposite flows of negative and positive particles



Magnetic Field Induced Conductivity of the Vacuum

Qualitative definition of conductivity, ♦

$$< j_\mu(x) j_\nu(y) > = C + A \cdot \frac{\exp\{-m|x-y|\}}{r^\alpha}$$

$$j_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$$

$$\sigma \propto C$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

- Conductivity (Kubo formula)

$$G_{ij}(\tau) = \int\limits_0^{+\infty} \frac{dw}{2\pi} K(w,\tau)\, \rho_{ij}(w),$$

Maximal entropy method

$$K(w,\tau) = \frac{w}{2T} \frac{\cosh\left(w\left(\tau - \frac{1}{2T}\right)\right)}{\sinh\left(\frac{w}{2T}\right)},$$

$$G_{ij}(\tau) = \int d^3\vec{x} \langle\, j_i\left(\vec{0},0\right) j_j(\vec{x},\tau)\,\rangle$$

Magnetic Field Induced Conductivity of the Vacuum

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

- Conductivity (Kubo formula)

For weak constant *electric* field

$$< j_i > = \sigma_{ik} E_k$$

Magnetic Field Induced Conductivity of the Vacuum
Calculations in SU(2) gluodynamics

$$\langle\, \bar{q}\,(x)\,\gamma_i q\,(x)\;\; \bar{q}\,(y)\,\gamma_j q\,(y)\,\rangle$$

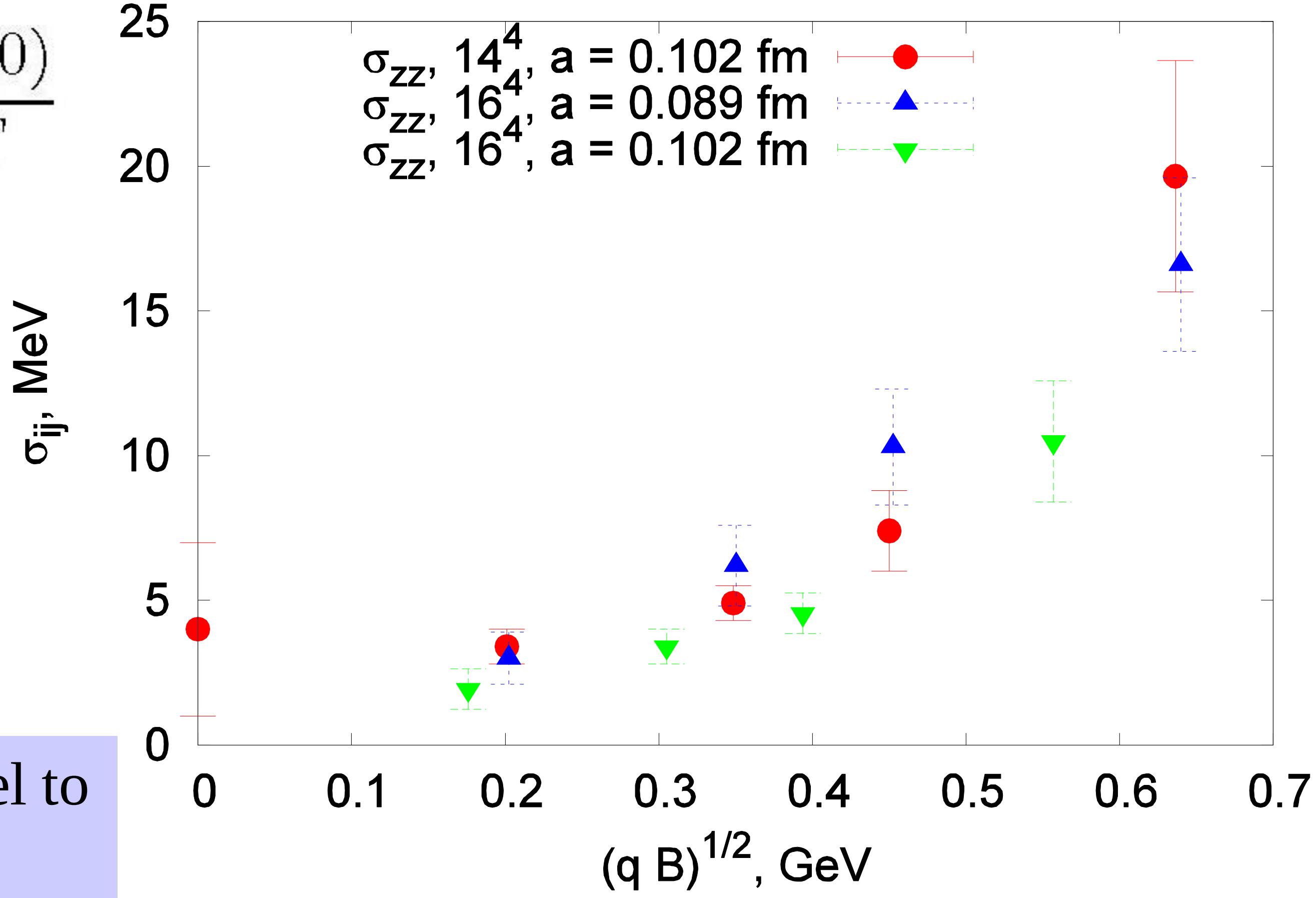
$$= \int \mathcal{D}A_\mu\, e^{-S_{YM}[A_\mu]}\, \mathrm{Tr}\,\left(\frac{1}{\mathcal{D}+m}\,\gamma_i\,\frac{1}{\mathcal{D}+m}\,\gamma_j\right)$$

We use overlap operator + Shifted Unitary Minimal Residue Method *(Borici and Allcoci (2006))* to obtain fermion propagator

$$G_{ij}\,(\tau) = \int d^3\vec{x}\langle\, j_i\left(\vec{0},0\right) j_j\,(\vec{x},\tau)\,\rangle$$

Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T/Tc=0.45*

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

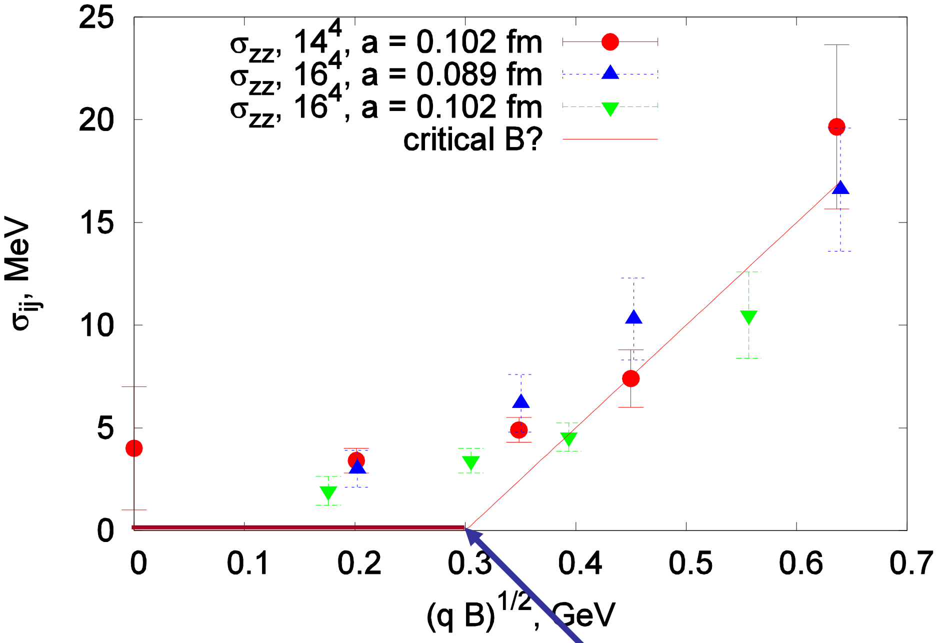


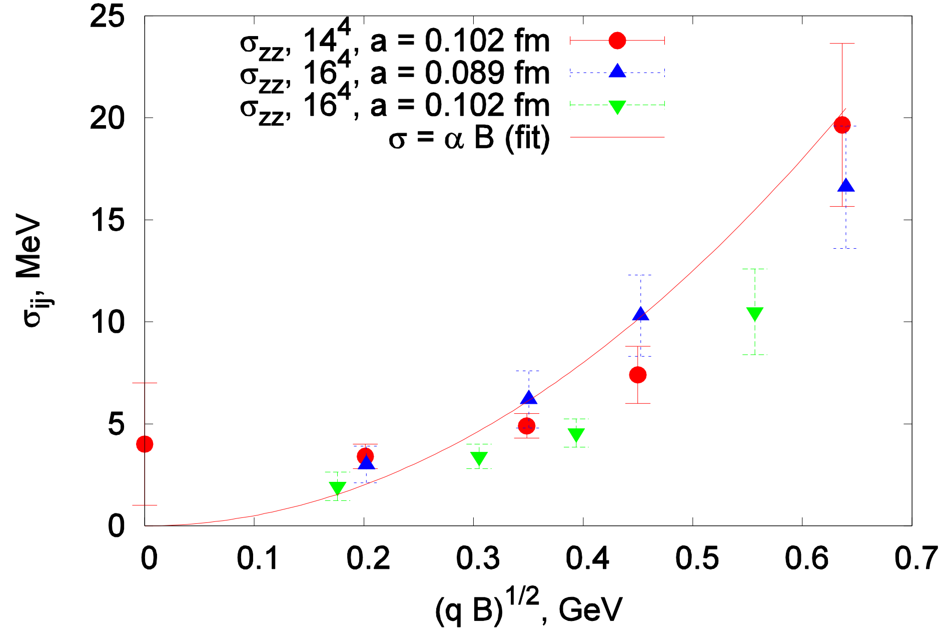
$\vec{B}$ is parallel to 0Z axis

Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T/Tc=0.45*

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

At T=0, B=0 vacuum is insulator

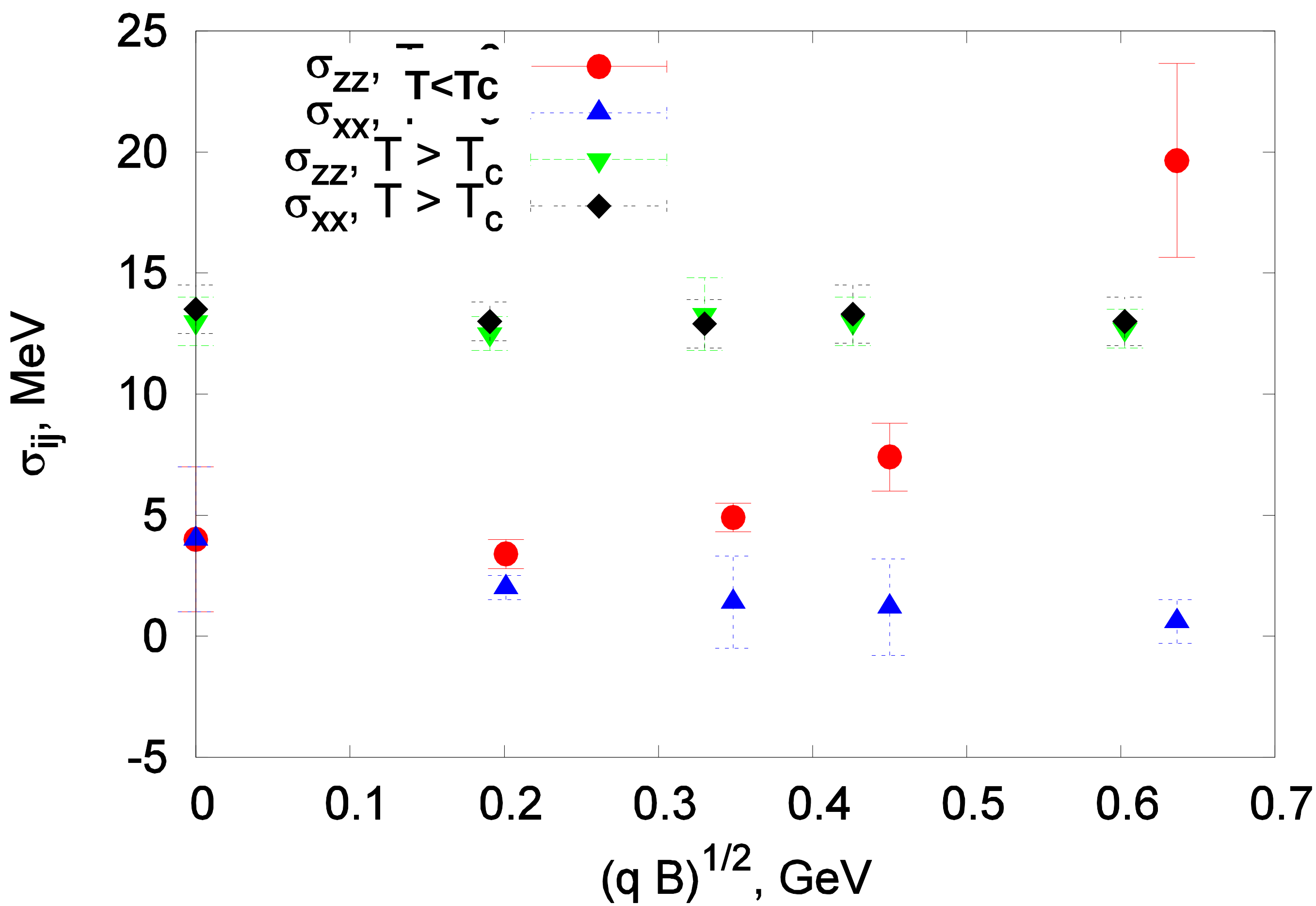




Critical value of magnetic field?

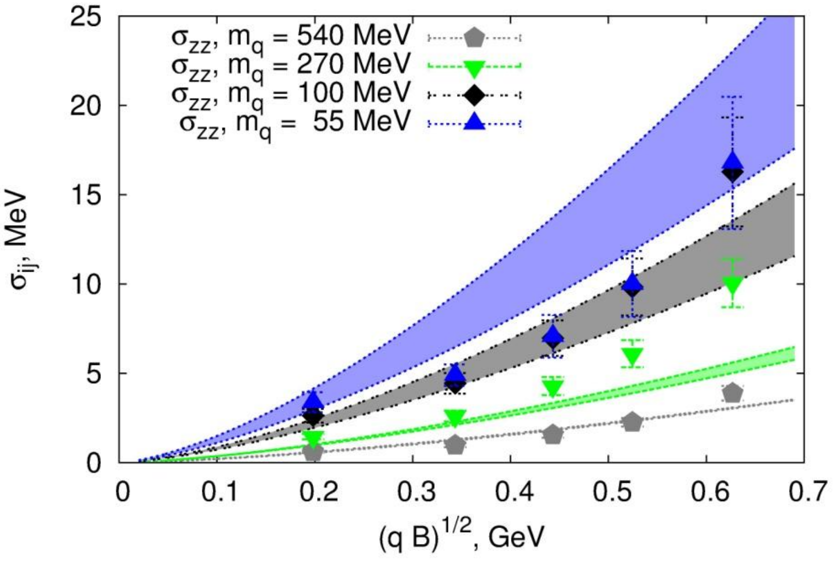
Calculations in SU(2) gluodynamics, conductivity along magnetic field at *T<Tc, T>Tc*

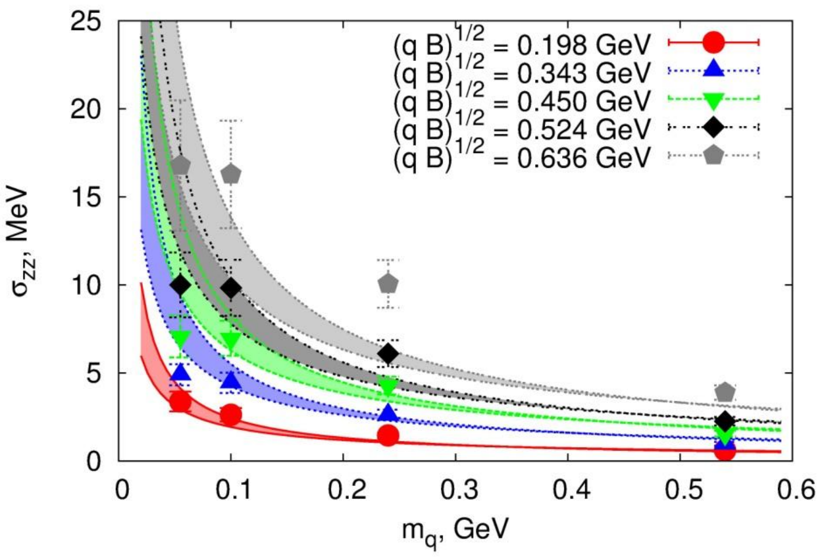
$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$



$\vec{H}$ is parallel to $0Z$ axis

Calculations in SU(2) gluodynamics, conductivity at *T/Tc=0.45*, variation of the quark mass





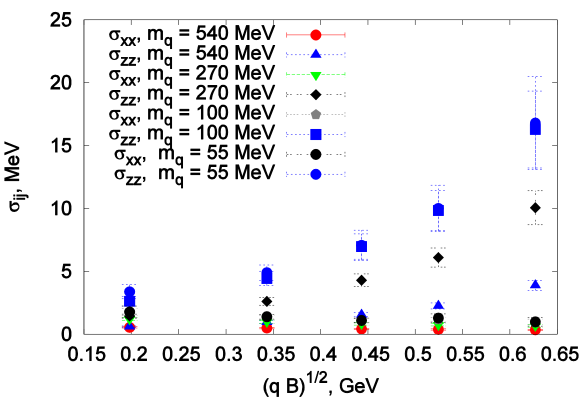
$$\sigma_{zz}\left(m_q, qB\right) \sim m_q^{-\alpha}\left(|qB|\right)^{\beta}$$

$$\alpha \approx 1$$

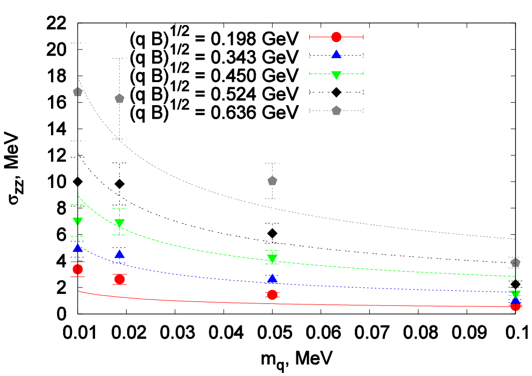
$$\beta \approx 1$$

Calculations in SU(2) gluodynamics, conductivity at *T/Tc=0.45*, variation of the quark mass and magnetic field

$$\sigma_{ij} \propto \frac{B_i B_j}{B m_q} \propto \frac{B_i B_j}{B m_\pi^2}$$



Why?



1.3 Dilepton emission rate

L. D. McLerran and T. Toimela, Phys. Rev. D 31, 545 (1985),
E. L. Bratkovskaya, O. V. Teryaev, and V. D.Toneev, Phys. Lett. B 348, 283 (1995)

$$\frac{R}{V} = -4e^4 \int \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} L^{\mu\nu}(p_1, p_2) \frac{\rho_{\mu\nu}(q)}{q^4}, (7)$$

where p_1 and p_2 are the momenta of the leptons, $q = p_1 + p_2$, m is their mass and $L^{\mu\nu} = \left(\left(p_1 \cdot p_2 + m^2\right)\eta^{\mu\nu} - p_1^\mu p_2^\nu - p_2^\mu p_1^\nu\right)$ is the dilepton tensor. If the electric conductivity is nonzero in the direction of the magnetic field, for sufficiently small p_1, p_2 one has $\rho_{ij}(q) \approx \rho_{ij}(0) \sim \sigma_{ij} \sim B_i B_j / |B|$, and hence

$$\sigma_{ij} = \frac{\rho_{ij}(0)}{4T}$$

$$\frac{R}{V} \sim \int \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} \frac{d^3 p_1}{(2\pi)^3 \, 2E_1} \frac{(p_1 \cdot B)(p_2 \cdot B)}{|B|}. \qquad (8)$$

- There should be more soft dileptons in the direction ***perpendicular*** to magnetic field

$$\frac{d\sigma}{dp_1 dp_2} \propto \frac{|B|}{m_\pi} \sin^2\theta$$

θ is the angle between the spatial momentum of the leptons and the magnetic field, in the center of mass of dilepton pair

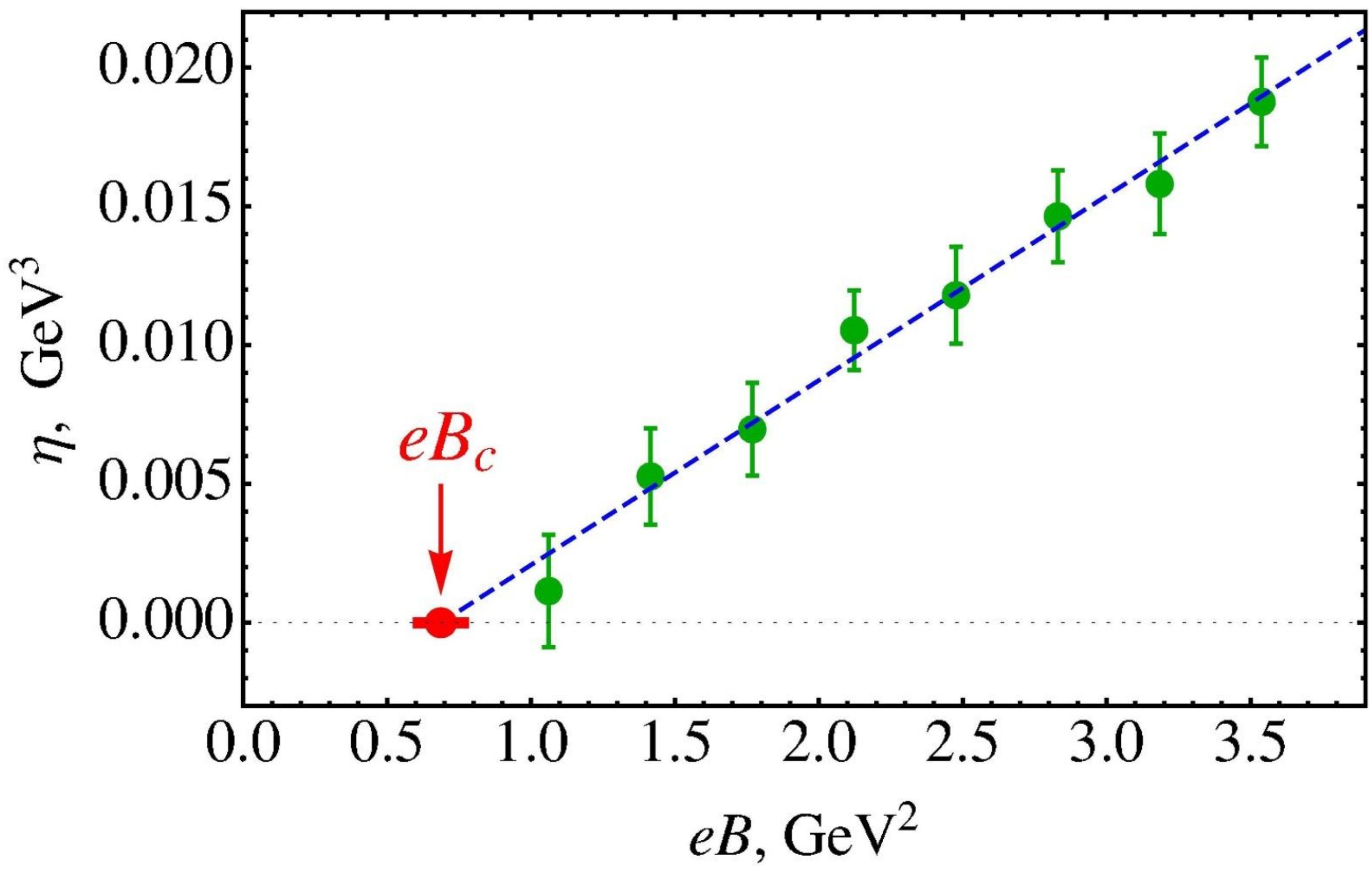
Superconductivity of the Vacuum

Can nothing be a superconductor and a superfluid?
M.N. Chernodub arXiv:1104.4404

Superconductivity of the vacuum can exist at T=0

Model explanation: Superconductivity is due to the condensation of the □ mesons

Preliminary lattice results



Condensate of the charged ❐-mesons vs value of the magnetic field

CME Summary

a) Visualization

b) Large fluctuations of the electric current in the direction of the magnetic field

c) Conductivity of the vacuum in the direction of the magnetic field at T>0

d) Dilepton emission rate

e) Superconductivity of the vacuum at T=0

Other effects induced by magnetic field

1. Chiral symmetry breaking

2. Magnetization of the vacuum

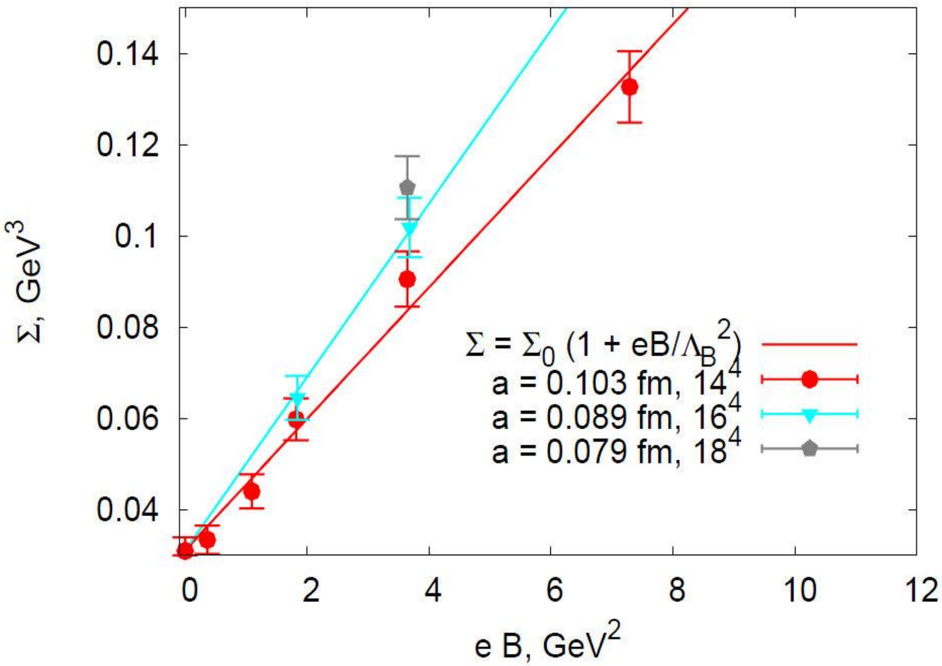
3. Electric dipole moment of quark along the direction of the magnetic field

1. Chiral condensate in QCD

$$\Sigma = - < \bar{\psi} \psi >$$

$$m_\pi^2 f_\pi^2 = m_q < \bar{\psi} \psi >$$

Chiral condensate vs. field strength, SU(2) gluodynamics



$$\Sigma = \Sigma_0 \left(1 + \frac{eB}{\Lambda_B^2}\right)$$

- Our value for Λ_B:

$$\Lambda_B^{\rm fit} = (1.41 \pm 0.14 \pm 0.20)\,\mathrm{GeV}$$

- χPT result:

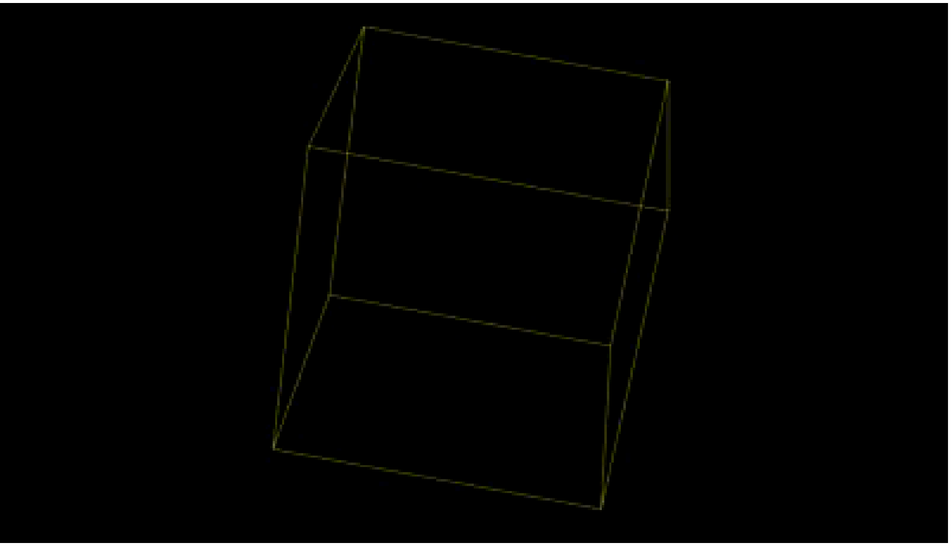
$$\Lambda_B^{\chi PT} = 1.96\ GeV \quad (F_\pi = 130\ MeV - real\ world)$$
$$\Lambda_B^{\chi PT} = 1.36\ GeV \quad (F_\pi = 90\ MeV - quenched)$$

- Chiral condensate at $B = 0$: $\Sigma_0^{\rm fit} = [(310 \pm 6)\ \mathrm{MeV}]^3$

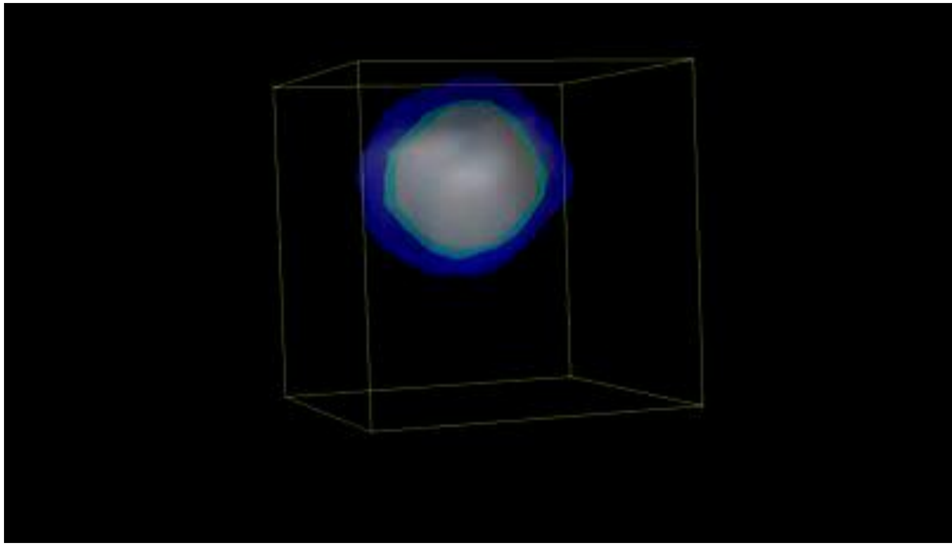
We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!

Localization of Dirac Eigenmodes

Typical densities of the nearzero eigenmodes vs. the strength of the external magnetic field



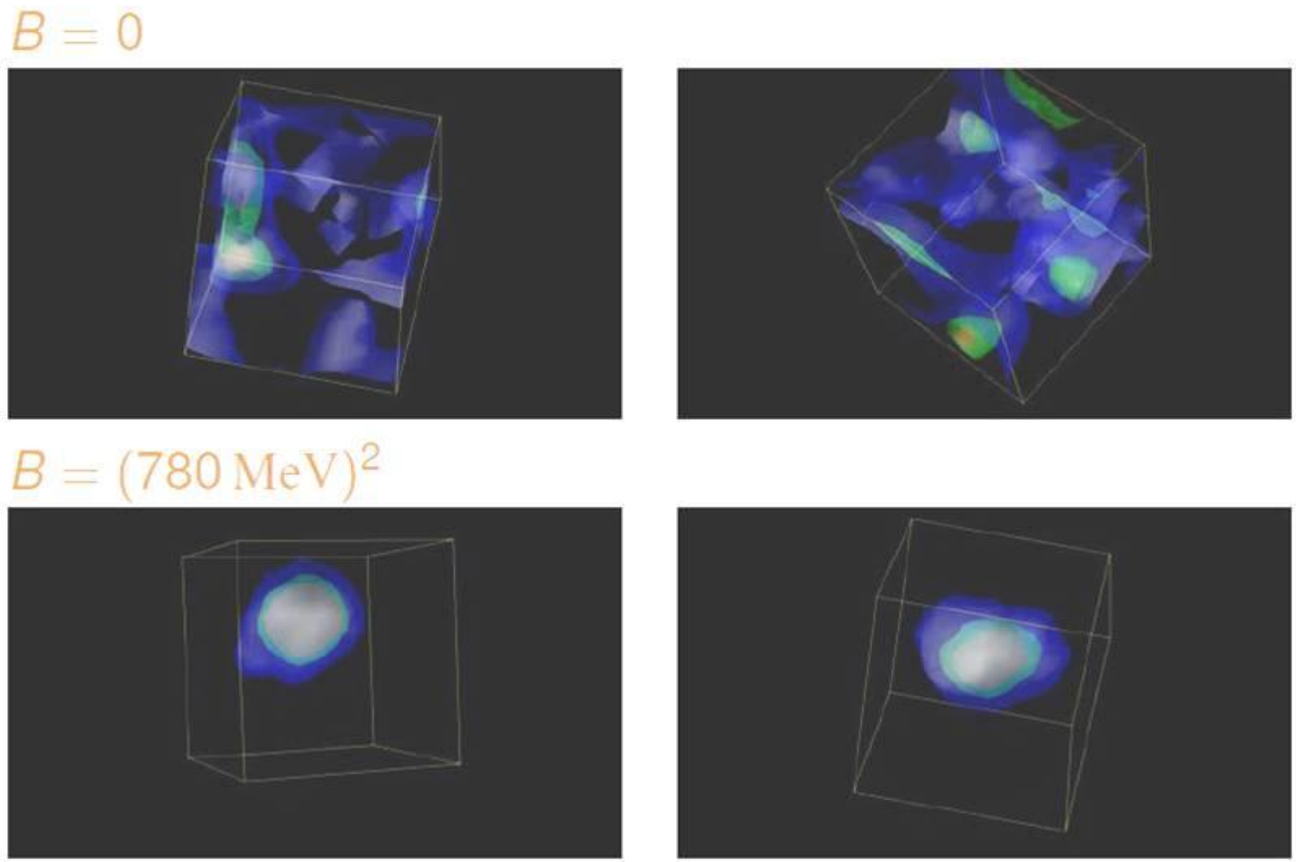
$B=0$



$B=(780Mev)^2$

Localization of Dirac Eigenmodes

Typical densities of the nearzero eigenmodes vs. the strength of the external magnetic field



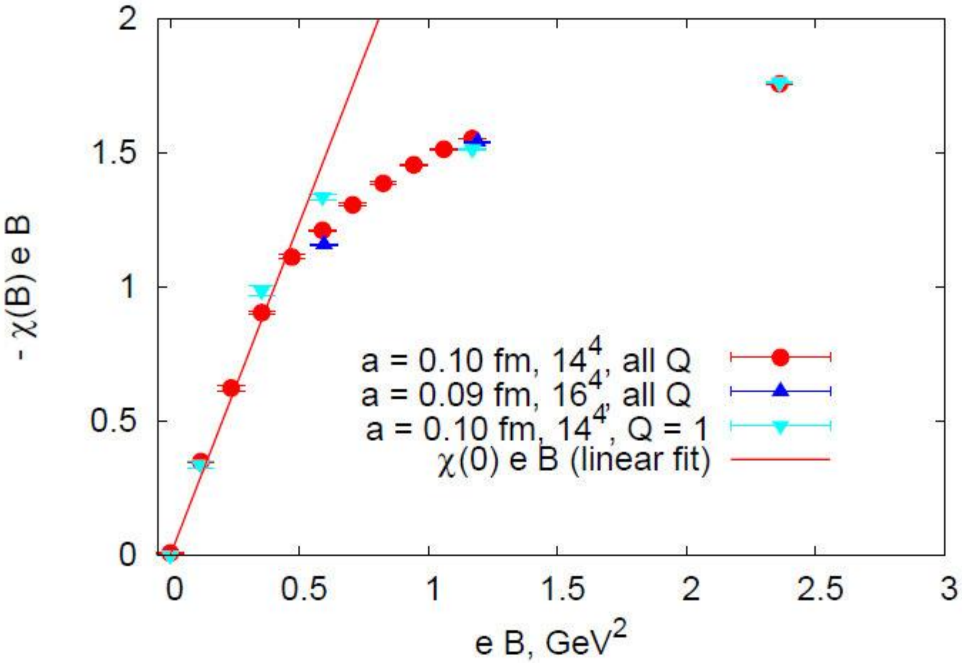
B=0

B=(780Mev)^2

Chiral condensate, simulations with dynamical quarks

- Massimo D'Elia, Francesco Negro, Swagato Mukherjee, Francesco Sanfilippo, **arXiv:1103.2080, PoS LATTICE2010:179,2010**.
- Michael Abramczyk, Tom Blum, and Gregory Petropoulos, R. Zhou, **arXiv:0911.1348**

2. Magnetization of the vacuum as a function of the magnetic field



Spins of virtual quarks turn parallel to the magnetic field

$$<\bar{\psi}\sigma_{\alpha\beta}\psi> = \chi <\bar{\psi}\psi> F_{\alpha\beta}$$

$$\sigma_{\alpha\beta} = \frac{1}{2i}[\gamma_\alpha, \gamma_\beta]$$

$<\bar{\psi}\psi>\chi = -46(3)Mev \leftrightarrow$ our result

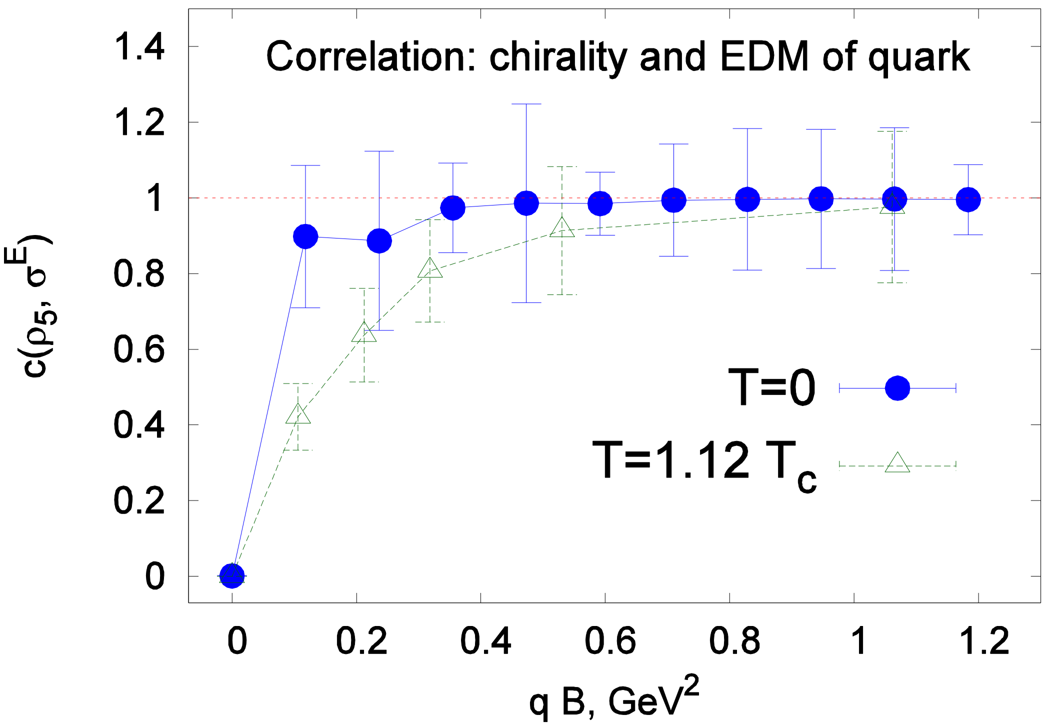
$<\bar{\psi}\psi>\chi \approx -50Mev \leftrightarrow$ QCD sum rules

(I.I. Balitsky, 1985, P. Ball, 2003.)

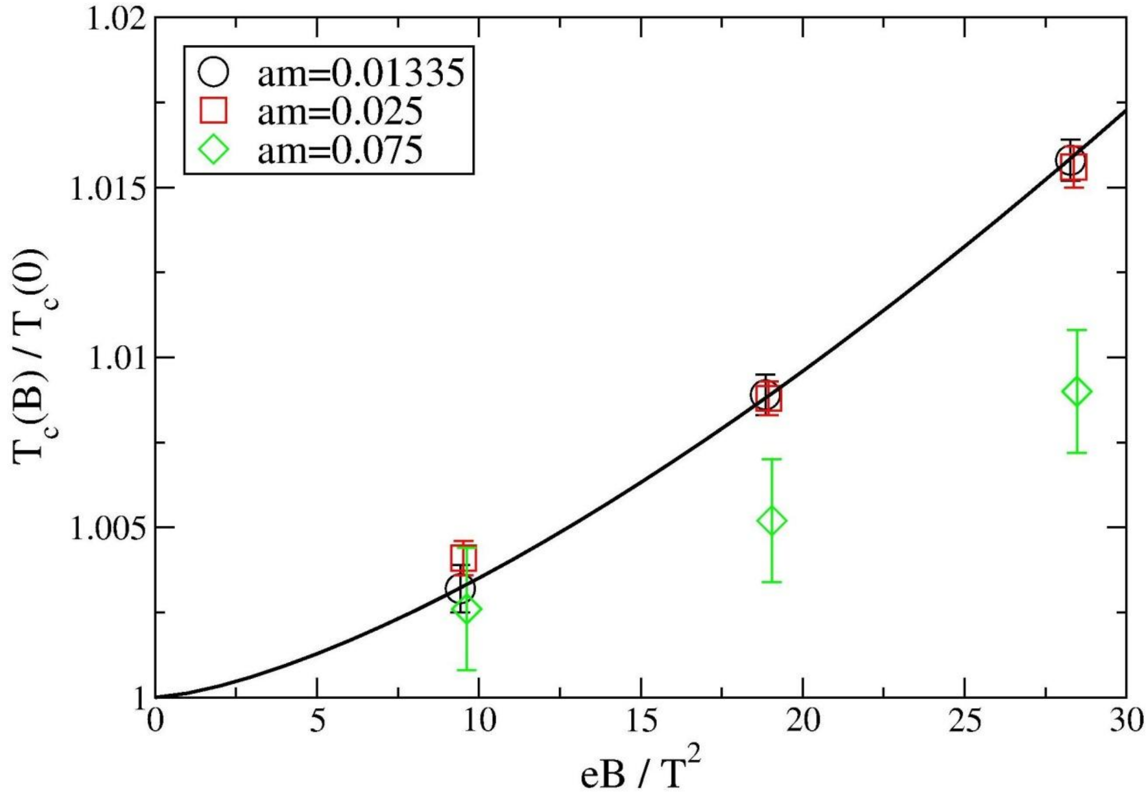
3. Generation of the anomalous quark electric dipole moment along the axis of magnetic field

- **Large correlation between square of the electric dipole moment**

$\sigma_{0i} = i\,\overline{\psi}[\gamma_0, \gamma_i]\psi$ **and chirality** $\rho_5 = \overline{\psi}\gamma_5\psi$



4. Deconfinement phase transition in the presence of magnetic field



Massimo D'Elia, Swagato Mukherjee and Francesco Sanfilippo (2011)

Summary

- Chiral condensate
- Magnetization of the vacuum
- Quark local electric dipole Moment
- B-T phase diagram

Systematic errors

- SU(2) gluodynamics instead of QCD
- Moderate lattice volumes
- Not large number of gauge field configurations
- In some cases we calculate the overlap propagator using summation over eigenfunctions:

$$\langle \bar{\Psi} \Sigma_{\alpha\beta} \Psi \rangle = 2m \langle \sum_{\lambda_k > 0} \frac{\psi_k^{\dagger}(x)\, \Sigma_{\alpha\beta}\, \psi_k(x)}{\lambda_k^2 + m^2} \rangle$$

Proposals for calculations

SU(3) gluodynamics

SU(2) with dynamical quarks

Lattice QCD

variation of T, H, m_q, a

$$< \bar{\psi}\sigma_{\alpha\beta}\psi > \quad < \bar{\psi}\psi > \quad < i\bar{\psi}[\gamma_0, \gamma_i]\psi \cdot \bar{\psi}\gamma_5\psi > \quad < j_3^2 >_{IR}$$

$$< j_\mu(x) j_\nu(y) >$$

- CME, vacuum conductivity
- Chiral condensate
- Magnetization of the vacuum
- Quark local electric dipole moment
- Dilepton pair angular distribution
- Shift of the phase transition

Proposals for calculations

- D. Kharzeev

$$\mu \leftrightarrow \mu_5 \qquad L_\mu = i\mu\bar{\psi}\gamma_0\psi + \mu_5\bar{\psi}\gamma_5\psi$$

$$\sigma \leftrightarrow \sigma_5 \qquad \sigma \to < j_\mu(x) j_\nu(y) > \qquad \sigma_5 \to < j_{5\mu}(x) j_{5\nu}(y) >$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks

Chiral Magnetic Effect (CME) + Chiral Separation Effect (CSE) =
= Chiral Magnetic Wave

Dmitri E. Kharzeev, Ho-Ung Yee, arXiv:1012.6026

Proposals for calculations

- D. Kharzeev

$$\mu = 0,\ \mu_5 \neq 0 \qquad L_\mu = \mu_5 \bar{\psi}\gamma_5\gamma_0\psi$$

$$\sigma_{5ij}(\mu_5, B, T)$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD (imaginary unity is absent)

Proposals for calculations

- D.T. Son, N.Yamamoto **Holography and Anomaly Matching for Resonances.** e-Print: **arXiv:1010.0718**

$$< j_\mu(-q)\, j^5{}_\nu(q) >= -\frac{q^2}{4\pi^2} P_\mu^{\alpha\perp}[P_\nu^{\beta\perp}\omega_T(q^2) + P_\nu^{\beta\parallel}\omega_L(q^2)]\tilde{F}_{\alpha\beta}$$

$$\omega_L(q^2) = \frac{2N_c}{q^2} \quad \Leftarrow \quad \text{no quantum corrections}$$

$$\omega_T(q^2) = \frac{N_c}{q^2} \quad \Leftarrow \quad \text{there are nonperturbative corrections}$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

New proposals for calculations

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$$\omega_L(q^2) = \frac{2N_c}{q^2} \quad \Leftarrow \quad \text{no quantum corrections}$$

$$\omega_T(q^2) = \frac{N_c}{q^2} \quad \Leftarrow \quad \text{there are nonperturbative corrections}$$

$$\omega_T(q^2) = \frac{N_c}{q^2} - \frac{N_c}{f_\pi^2}[< j_\mu^5 j_\mu^5 > - < j_\mu j_\mu >]$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

Proposals for calculations

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$$< j_\mu(-q)\, j^5{}_\nu(q) > = -\frac{q^2}{4\pi^2} P_\mu^{\alpha\perp}[P_\nu^{\beta\perp}(\frac{N_c}{q^2} - \frac{N_c}{f_\pi^2}[< j^5_\mu j^5_\mu > - < j_\mu j_\mu >])$$

$$+P_\nu^{\beta\parallel} \frac{2N_c}{q^2}]\tilde{F}_{\alpha\beta}$$

$$P_\mu^{\alpha\perp} = \eta_\mu^\alpha - \frac{q_\mu q^\alpha}{q^2}, \quad P_\mu^{\alpha\parallel} = \frac{q_\mu q^\alpha}{q^2}$$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

Proposals for calculations

- **D.T. Son, N.Yamamoto** ⟹ $< \bar{\psi}\gamma_{\mu}\psi(x)\,\bar{\psi}\gamma_{\mu}\gamma_5\psi(y) >$

- **D. Kharzeev** ⟹ $< \bar{\psi}\psi(x)\,\bar{\psi}\gamma_5\psi(y) >$

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

Proposals for calculations

- L. McLerran "Nonsymmetric condensate"

Calculate $<\bar{\psi}\psi(x)\,\bar{\psi}\psi(y)>$ parallel and perpendicular to the field. Chiral condensate may depend on the direction!

SU(2), SU(3) gluodynamics, SU(2) with dynamical quarks, QCD

Lattice simulations with magnetic fields, status

1. Chiral Magnetic Effect

1.1 CME on the lattice

1.2 Vacuum conductivity induced by magnetic field

1.3 Quark mass dependence of CME (+ talk of P. Buividovich)

1.4 Dilepton emission rate (+ talk of P. Buividovich)

2. Other effects induced by magnetic field

2.1 Chiral symmetry breaking

2.2 Magnetization of the vacuum

2.3 Electric dipole moment of quark along the direction of the magnetic field

Lattice simulations with magnetic fields, future

1. **Calculations of "OLD" quantities in *SU(3), SU(2)* with dynamical quarks, QCD. Decreasing systematic errors in *SU(2)* calculations**

2. **Calculation of "NEW" physical quantities**