Lower-dimensional defects in lattice gluodynamics

Lattice QCD, Hadron Structure and Hadronic Matter

Lower-dimensional defects in lattice gluodynamics

V.I. Zakharov

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Motivation

Experiments at LHC do not see the "Standard Higgs" yet (masses from 145 to 450 GeV excluded)

Psychological pressure to consider alternatives to SM

Also, SM = pert. th, and perturbatively

\[ \delta M_H^2 \sim \alpha \Lambda_{UV}^2 \gg M_H^2 \]

Generically, non-perturbative alternatives to the SM are encouraged
Higgses and lattice

Often overlooked:
*Lattice is practically the only source of knowledge on non-perturbative, Higgs-like scalars*

**Alternative title of the lecture:** scalar fields in the lattice gluodynamics

**Conclusions:**
there are a few highly non-trivial observations on scalars on the lattice meeting theorists’ desires
but much more questions left open
Outline

Confinement and Scalar Fields
- Abelian case
- non-Abelian case

Confinement and strings
- Magnetic strings
- Scalars living on the strings

Deconfinement and scalars
- From 4d to 3d
- From magnetic monopoles to dyons
The lecture is rather theoretical. We will try to summarize some theoretical ideas closely related to interpretation of the lattice data but in fact independent. The theoretical developments took many years and the presentation is sketchy. No particular lattice data are quoted. However, close collaboration with the ITEP Lattice Group is gratefully acknowledged. No references here (turn to reviews for references)
Continuum theory

Abrikosov-Nielsen-Olesen vortex

Charged scalar coupled to electromagnetic field:

\[
S = \int d^4x \left( \frac{1}{4e^2} F_{\mu\nu}^2 + |D_\mu \phi|^2 + V(\phi) \right)
\]

where the potential

\[
V(\phi) = (|\phi|^2 - v^2)^2
\]

There exists a static solution with finite energy per unit of length called vortex

The pert. vacuum (\(\phi = 0\)) is restored along the axis of the vortex
Quantization of magnetic flux

The solution is:

\[ \phi = \phi(\rho)e^{i\theta}, \quad A_\theta = a(\rho) \]

with boundary conditions:

\[ \phi(\rho \to \infty) = v, \quad \phi(\rho = 0) = 0, \quad a(\infty) = \frac{1}{\rho} \]

The magnetic flux gets quantized:

\[ \int_0^{2\pi} d\theta \ A_\theta \cdot \rho = 2\pi \]

The flux is the same as for the Dirac magnetic monopole.
Confinement of the magnetic monopoles

The vortex can be closed or end up with magnetic monopoles. For a large separation $R$ of the monopoles

$$\lim_{R \to \infty} V_{M\bar{M}} = \sigma_{\text{Abrikosov}} \cdot R$$

where $\sigma_{\text{Abrikosov}}$ is energy of the vortex per unit length

$$\sigma_{\text{Abrikosov}} \sim V^2$$

in fact, physics of superconductors, well understood and checked.

Magnetic charges (monopoles) are confined if a charged scalar is condensed.
Dual superconductor model

To explain the confinement of quarks one interchanges electric and magnetic fields assuming

- In the physical vacuum, there is magnetically charged scalar condensed

\[ v_{magn} \neq 0 \]

Then

\[
\lim_{R \to \infty} V_{Q\bar{Q}} \sim v_{magn}^2
\]

and there is confinement of quarks

This idea dominates theory of confinement for 35 years

The whole issue is what are the magnetic degree of freedom, symmetry and so on
Mini-conclusion

So far we understood how confinement is related to scalars, in the Abelian case and classically.

However, in YM case there are no elementary scalars. Scalars should be made out of the glue.
Quantum, or lattice dual superconductor

Start with pure Abelian gauge field. In the continuum,

\[ S = \frac{1}{4e^2} \int d^4x (F_{\mu\nu})^2 \]

The vacuum is probed by external heavy electric charges. The definition of the confinement is:

\[ <W> = <P \exp \left( \int_C idx_\mu A_\mu \right) > \sim \exp \left( -\sigma \cdot R \cdot T \right) \]

In the lattice version there is confinement of charges at

\[ e^2 > e^2_{\text{crit}} \sim 1 \]

although there is no elementary scalar in the original action.

To understand need elements of quantum geometry.
A touch of quantum geometry:
Start with classical action for a particle of mass $M$

$$S_{cl} = M \cdot L$$

Quantum propagator is given by the path integral

$$D(x, x') = \Sigma_{paths} \exp (-S_{cl}(path))$$

To enumerate the paths, discretize space-time
that is introduce lattice spacing $a$ for pure theoretical needs
The probability to observe a particular path is equal to the product of the action factor for each link,

\[ \exp(-S_{\text{link}}) = \exp(-M \cdot a) \]

Since the number of steps is \( L/a \)

\[ W_{\text{particular trajectory}} = \exp\left(- (M \cdot a) \frac{L}{a}\right) \]

where \( a \) is the lattice spacing

Next, account for the fact that there different trajectories of the same length \( L \)
Action vs entropy

At each step of constructing trajectory we can choose one of 7 directions without changing the whole length. Total number is:

\[ N(L) \approx \exp( \ln 7 \cdot L/a) \]

As a result,

\[ W(L) \sim \exp(-M \cdot a + \ln 7)L/a \]

Two exponential factors pushing in opposite directions Analogy:

\[ (\text{free energy}) = (\text{energy}) - (\text{entropy}) \]

Both are divergent in the limit \( a \to 0 \)
One can rewrite the equations above as statement on mass renormalization. Namely, actual (renormalized) mass is

\[ m_{phys}^2 = \frac{const}{a} \left( M(a) - \frac{\ln 7}{a} \right) \]

Thus, we come back to the quadratic divergence in mass of a scalar particle.
Mini conclusion

Quantum geometry deals with geometrical objects changing directions on the lattice spacing scale and provides with an alternative to the standard field theory. The problem of the quadratic divergence in the mass is reproduced.
Abelian magnetic degree of freedom

How to match quantum geometry to physics.
Start with free Abelian gauge field

\[ S = \frac{1}{4e^2} \int d^4x (F_{\mu\nu})^2 \]

There is a singular solution, magnetic monopole

\[ H = \frac{r}{r^3} Q_M \]

where the magnetic charge \( Q_M \) is fixed in terms of charge \( e \) by the condition that the Dirac string is invisible.
Fine tuning in language of quantum geometry

The monopole mass

$$M_{\text{monopole}} = \frac{1}{8\pi} \int_0^\infty H^2 d^3 r$$

$$M_{\text{monopole}} = \frac{\text{const}}{e^2 a}$$

If

$$\frac{\text{const}}{e^2} = \ln 7 + O(m_{\text{phys}}^2 a^2)$$

then

$$m_{\text{monopole}}^2 \approx 0$$

Can get negative $m_{\text{phys}}^2$ once entropy enhancement prevails over suppression due to the action (singular monopole bare mass)
Symmetry breaking in geometrical terms

For $m_{\text{phys}}^2 > 0$ probability to have length $L$ trajectory

$$W(L) \sim \exp(-m_{\text{phys}}^2 L \cdot a)$$

and there exist only finite clusters of trajectories
If $m_{\text{phys}}^2 < 0$ then $L \to \infty$ is not suppressed
there appears an infinite cluster
Most beautiful, infinite cluster is dilute first:
Probability $\theta$ of a link to belong to the infinite cluster:

$$\theta(\text{link}) \sim |m_{\text{phys}}^2 a^2|^\gamma, \quad \gamma > 0$$

Emergence of the infinite cluster is the spontaneous symmetry breaking
Relation to field theory

On the lattice, monopole trajectories are defined as violations of the Bianchi identity:

\[ \partial_\mu \tilde{F}_{\mu \nu} \equiv j^{\text{monopole}}_\nu \]

One can measure length of monopole trajectories for each configuration

\[ < L_{\text{monopole}} > \equiv \rho_{\text{monopole}} V_{\text{tot}} \]

\[ < \phi^2_{\text{magn}} > = (\text{const}) \rho_{\text{monopole}} \cdot a \]

If we count only trajectories in infinite cluster

\[ < \phi_{\text{magn}} >^2 = (\text{const}) \rho_{\text{monopole}}^{\text{infinite}} \cdot a \]
Trading singularity for a scalar field

Classically (Higgs model) we have both electromagnetic field and charged scalar

Quantum mechanically, we start with free electromagnetic field, keep singular (monopole) solutions and see that these solutions can be traded for scalar field. Classically this is the same Higgs model
On the lattice, the monopole trajectories are uniquely determined in terms of quantum fields \( \{ A_\mu(x) \} \). All the theory predictions turn to be true. The role of the condensate \( \nu_{magn} \) is played by the infinite cluster of the monopole trajectories.
Quark confinement is non-Abelian \((L = 1/4g^2(G_{\mu\nu}^a)^2)\)
Would be nice to trade singularities of non-Abelian fields for a physical scalar field again
However, there is no direct generalization to non-Abelian case:

- no constant to tune: strong coupling runs, it is not a constant
- Monopoles are Abelian construct
- quite a few further reasons

We will come back to this later
Naive generalization

The idea is to replace the original, non-Abelian field configurations with the "closest" Abelian configurations. First, use gauge-fixing to minimize the functional

\[ R = \sum_{\text{lattice}} [(A_\mu^1)^2 + (A_\mu^2)^2] \]

where indices 1, 2 are color indices

Second, project non-Abelian component out:

\[ \tilde{A}_\mu^1,2 = 0, \quad \tilde{A}_\mu^3 \neq 0 \]

Define monopoles in projected fields:

\[ \partial_\beta \tilde{F}_{\beta\alpha} \equiv (j)_\alpha^{\text{monopole}} \]
Numerical, but beautiful findings

The monopoles defined in terms of projected fields exhibit remarkable features. E.g.:

- There exist both finite and infinite clusters of monopole trajectories.
  Removal of the infinite cluster destroys confinement.

- Simple scaling laws:

\[
\rho_{\text{monopole}}^{\text{infinite}} = (\text{const}) \Lambda_{QCD}^3
\]

This is highly nontrivial since \(\Lambda_{QCD}\) is related to the lattice spacing through 2-loop \(\beta\)-function.
Empirically, the lattice monopoles in Yang-Mills case look physical objects (know about \( \Lambda_{QCD} \))

The definition of the monopoles, however, looks rather arbitrary since there are many different Abelian projections. The definition is in pure lattice terms, non-local. No path to continuum theory visible.
Most beautiful of all

The total monopole density has two terms:

$$\rho_{\text{tot}}^{\text{monopole}} = \frac{(\text{const}) \Lambda_{QCD}^2}{a} + (\text{const})' \Lambda_{QCD}^3$$

and depends explicitly on the lattice spacing. The first term is due to finite (small) clusters.

In terms of matrix element:

$$\lim_{a \to 0} < \phi_{magn}^2 > \Lambda_{QCD}^2$$

and is exactly what we would like to have for Higgs

Lattice monopoles provide first example of dynamical solution of the hierarchy problem
Self-tuning

Matrix element $< \phi_{magn}^2 >$ does not depend on the measuring procedure. We can also clarify the mechanism on the UV scale. It turns out that non-Abelian action associated with the monopoles

$$S_{mon} \approx \ln 7 \frac{L}{a}$$

where $L$ is the length of trajectory. The magic $\ln 7$ is hit without any fine-tuning by hand in Yang-Mills theory self-tuning of monopoles is observed.
Monopoles and surfaces

An alternative way to express results
Monopoles cover densely a 2d surface
Just span smallest-area surface on the monopole trajectories and get

\[(Area)_{tot} \sim V_{tot} \Lambda_{QCD}^2\]

Actually these strings were defined and observed on the lattice as the so called center vortices. We skip this point.
Monopoles and strings

Surfaces in quantum geometry correspond to strings in the continuum limit. Quite obvious. Amusingly, for the string tension the result for $(\text{Area})_{\text{tot}}$ implies

$$T_{\text{string}} \sim \Lambda_{QCD}^2$$

Physics of monopoles, or scalars is physics of strings. Strings with physical tension in the vacuum seem to be observed.
For the monopole condensate we get

\[ < \phi_{\text{magn}} >^2 \sim \Lambda_{QCD}^3 \cdot a \]

that is, vanishing in the continuum limit
at the deconfinement phase transition
monopole trajectories become parallel to the (Euclidean) time and continue to ensure confinement in the 3d YM (time slice of the 4d volume)
Thoughts and doubts

- There can be no interesting singular fields because of the asymptotic freedom
- It is known that there is no fine-tuning procedure for surfaces (strings)

there are provisional answers to this particular questions (blackboard) but...
Further strategy

Imagine that by luck (!?) we did detect physical objects through particular projection. How to go further without having continuum theory definition of the object observed. A possibility: guess which physical object it could be and check the answers for observables which can be found (calculated) in the lattice
Educated guess: magnetic strings

Magnetic string, by definition are closed in vacuum and can be open on the so called ’t Hooft line. This can be checked on the lattice (blackboard). Fits well.
Educated guess next generation: holographic models

In modern language, strings living in extra dimensions are considered.

Cigar-shape geometry (blackboard)
Prediction: magnetic strings are topologically charged
Can be checked on the lattice (blackboard) and works again
Gukov-Witten operator

Integral over surface from:

\[ S = \alpha G^a_{\mu \nu} d\sigma_{\mu \nu} + \beta \tilde{G}^a_{\mu \nu} d\sigma_{\mu \nu} \]

Where the color field is rotated in the Abelian directions.

This could be the right answer for the lattice strings.
Mini conclusions

In recent years magnetic strings, as continuum-theory defects, were understood much better in holographic models. Some striking similarities to the lattice strings.

main problem: all observations are rather unique, no industry yet. therefore, it is not a mainstream yet.