Dark energy: from phantom cosmology to modified gravity

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Cosmic acceleration.

EOS $p = w\rho$

$w = -1$ cosm. constant

$w < -1$ phantom

$w > -1$ quintessence $(-1, -\frac{1}{3})$.

$w = -1$ phantom divide.

Dark energy models:

Phenomenological:

$$p = w\rho, \quad \text{negative } w$$

Generalized EOS

$$p = -\rho + f(\rho)$$

Implicit

$$f_1(\rho, p) = 0$$

Oscillating:

$$p = w(t)\rho$$

Modified gravity DE:

a). Modifying action:

$$L = R + f(R, R_{\mu\nu}^2, \Box R, \ldots)$$

Example

$$L = R + \frac{\mu}{R} + b R^2 \quad \text{(consistent).}$$

b). Modifying FRW eq.

$$H^2 + F(\dot{H}) = \frac{8\pi G}{3}\rho$$

c). Modification of ~~scalar~~ spinor (DE)-gravity.

- 0'' -

Phantom DE.

$$\mathcal{L} = \frac{1}{\varkappa^2}R - \frac{\epsilon}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \tilde{V}(\varphi)$$

$\gamma = -1$ (phantom).

$$H = \frac{h_0}{t_s - t}$$ Big Rip?

Crossing phantom divide DE.

$$p = -\rho - f(\rho)$$

$$a(t) = a_0\left(\frac{t}{t_s - t}\right)^n,$$

$$H = n\left(\frac{1}{t} + \frac{1}{t_s - t}\right),$$

$$f(\rho) = \pm\frac{2\rho}{n}\left\{1 - \frac{4n}{t_s}\left(\frac{3}{\varkappa^2\rho}\right)^{1/2}\right\}^{1/2}$$

Holographic DE.

$\rho_{DE} \simeq \frac{3c^2}{\varkappa^2 L_\Lambda^2}$, L_Λ - IR Cut-off

First FRW Eq.

$$H = \frac{c}{L_\Lambda}$$

Particle horizon L_p, future horizon L_f

$$L_p \equiv a\int_0^t \frac{dt}{a}\,, \quad L_f \equiv a\int_t^\infty \frac{dt}{a}$$

Identifying L_Λ with L_p or L_f $\Rightarrow$

$$\frac{d}{dt}\left(\frac{1}{aH}\right) = \pm\frac{c}{a}$$

Solution $a = a_0 t^{h_0}$, $h_0 = 1 \mp c$.

Generalization.

$$L_\Lambda = L(L_p, L_f)\ ?$$

For finite span t_s of universe life L_f is not well-defined.

$$L_f \to \tilde{L}_f \equiv a\int_t^{t_s}\frac{dt}{a} = a\int_0^\infty \frac{da}{Ha^2}$$

$$L_\Lambda = L_\Lambda(L_p, \tilde{L}_f, t_s)$$

Examples:

$$\frac{L_\Lambda}{c} = \frac{2t_s\left(\frac{L_p+\tilde{L}_f}{\pi t_s}\right)^2}{\left\{1+\left(\frac{L_p+\tilde{L}_f}{\pi t_s}\right)^2\right\}^2} \Rightarrow$$

$$\Rightarrow \quad a = a_0\sqrt{\frac{t}{t_s-t}}$$

E. Elizalde, S. Nojiri, S.D.O., P. Wang, hep-th 0502082, PRD

Other choices.

Cosm. constant DE. Decaying vacuum cosmology.

Phantom dark energy universe and Big Rip.

$$S = \frac{1}{\varkappa^2}\int d^4x\sqrt{-g}\left(R - \frac{\gamma}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right)$$

$\gamma < 0$ $(\gamma = -1)$ phantom:

or negative kinetic energy negative V.

Spatially-flat FRW Universe

$$ds^2 = -dt^2 + a^2(t)\sum_{i=1,2,3}(dx^i)^2$$

The calculation of energy density gives:

$$\rho_\varphi = \frac{\gamma}{2}\left(\frac{d\varphi}{dt}\right)^2 + V(\varphi).$$

For $V=0$, $\gamma<0$, ρ_φ is NEGATIVE!

$$\rho_\varphi = \frac{\gamma}{2}\dot\varphi^2 - V(\varphi).$$

Equations of motion:

$$\text{(FRW)}: \frac{6}{\varkappa^2}H^2 = \rho_\varphi ,$$

$$0 = -\gamma\left(\frac{d^2\varphi}{dt^2} + 3H\frac{d\varphi}{dt}\right) - V'(\varphi)$$

Solutions: (exponential V)

$$H = -\frac{\gamma\varkappa^2}{4(t_s - t)}, \quad \varphi = \varphi_0 \ln\left|\frac{t_s - t}{t_1}\right|$$

$$H = \frac{\dot a}{a}, \quad t_1^2 = -\frac{\gamma\varphi_0^2\left(1 - \frac{3\gamma\varkappa^2}{4}\right)}{2V_0}, \quad t_s - \text{integration constant}$$

Then

$$a(t) = a_0 \left| \frac{t_s - t}{t_1} \right|^{+\gamma \frac{\varkappa^2}{4}}$$

For phantom ($\gamma < 0$), $a(t)$ grows up to infinity at $t = t_s$, this is

BIG RIP singularity.

In general,

$$\frac{d\rho_\varphi}{dt} = -3\gamma H \left(\frac{d\varphi}{dt}\right)^2$$

It is positive if

$\gamma < 0$, $H > 0$, $\dot{\varphi} \neq 0$, then

energy density GROWS!

Big Rip occurs due to the rapid increase of the energy density!

With $\bar{V}(\varphi) = 0$, no singularity.

Matter is dust: example.

$\bar{V} = 0$, $\rho_d = \frac{\rho_0}{a^3}$, $(\rho_0 > 0)$

$\frac{d\varphi}{dt} = \frac{c}{a^3}$ (solution of φ-equation)

FRW eq.:

$$\frac{6}{\varkappa^2} H^2 = \frac{\gamma c^2}{2a^6} + \frac{\rho_0}{a^3} \Longrightarrow$$

$$a^3(t) = -\frac{\gamma c^2}{2\rho_0} + \frac{9\varkappa^2}{4\rho_0^2}(t - t_s).$$

Basically, $\rho_\varphi \leq \bar{V}(\varphi)$ for phantom.
If $\bar{V}(\varphi)$ is bounded from above by maximum $\bar{V}_m$, ρ_φ does not grow infinitely when $\varphi \to -\infty$.

$\bar{V}(\varphi) \to \bar{V}_m$ (constant), when $\varphi \to -\infty$

$$0 = -\gamma\left(\frac{d^2\varphi}{dt} + 3H\frac{d\varphi}{dt}\right)$$

$$\rho_\varphi = \frac{\gamma c^2}{2a^6} + \bar{V}_m$$

FRW equation:

$$\frac{6}{\varkappa^2}H^2 = \frac{\gamma c^2}{2a^6} + \bar{V}_m$$

First term could be neglected for large universe,

$$H^2 \to \frac{\bar{V}_m}{6\varkappa^2} \quad \text{(de Sitter)}$$

According to estimations less than $10-15 \times 10^9$ years is left before Big Rip.

Finite-time future singularity (Barrow model).

Even at strong energy condition

$$\rho > 0, \quad \rho + 3p > 0$$

Big Rip is possible!

J. Barrow, gr-qc 0403084.

Indeed, FRW eqs.

$$H^2 = \frac{\kappa^2 \rho}{6}, \qquad H = \frac{\dot{a}}{a}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2(\rho + 3p)}{12}.$$

Simple transformation:

$$\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2},$$

$$\dot{H} + H^2 = -\frac{\kappa^2}{12}(\rho + 3p) = -\frac{\kappa^2}{12}\left(\frac{6}{\kappa^2}H^2 + 3p\right)$$

$$\Downarrow$$

$$\begin{cases} p = -\frac{2}{\kappa^2}\left(2\dot{H} + 3H^2\right), \\ \rho = \frac{6H^2}{\kappa^2} \end{cases} \quad \text{FRW eqs.}$$

Assumption: $H(t) = \tilde{H}(t) + A'|t_s - t|^\alpha$

For α-negative, the singularity is pole; for α-positive, not integer, cut-like singularity.

Let us consider $0<\alpha<1$, then $t\sim t_s$

$$\rho \sim \frac{6}{\varkappa^2}\tilde{H}(t_s)$$

$$p \sim \pm \frac{4A'\alpha}{\varkappa^2}|t_s-t|^{\alpha-1}$$ – divergent!

+ when $t<t_s$.

$$w = \pm\frac{2}{3}\frac{A'\alpha|t_s-t|^{\alpha-1}}{\tilde{H}(t_s)^2}$$

$\rho_m>0$, $\rho_m+3p_m>0$ fulfilled.

this is

Barrow model:

$$a(t) = A + Bt^q + C(t_s-t)^n$$

$A>0$, $B>0$, $q>0$, $t_s>0$,

$C=-At_s^{-n}$, $t<t_s$, $2>n>1$.

when $t\to t_s$:

$$\frac{1}{a}\frac{d^2a}{dt^2}\to+\infty.$$

The correspondence is

$$A' = -\frac{Cn}{A+Bt_s^q}, \quad \alpha = n-1.$$

When $-1<\alpha<0$,

$$\rho = \frac{6A'^2}{\varkappa^2}|t_s-t|^{2\alpha}, \quad p \sim \pm \frac{4A'\alpha}{\varkappa^2}|t_s-t|^{\alpha-1}$$

and

$$w = \pm \frac{2\alpha}{3A'}|t_s-t|^{-\alpha-1}$$

Divergence at $t=t_s$.

w is positive, say, for $A'<0$, $t<t_s$.

<u>Big Rip again.</u>

For $\alpha=-1$ or $\alpha<-1$ even more strong singularities but w could be mainly negative there. Weak energy condition maybe violated.

Classes of singularity.

I (Big Rip) $t \to t_s$, $a \to \infty$, $\rho \to \infty$, $|p| \to \infty$

II (Sudden)

$t \to t_s$, $a \to a_s$, $\rho \to \rho_s$, $|p| \to \infty$

III $t \to t_s$, $a \to a_s$, $\rho \to \infty$, $|p| \to \infty$,

IV. $t \to t_s$, $a \to a_s$, $\rho \to 0$, $|p| \to 0$

HD derivatives of H diverge.

EOS:

$$p = -\rho - f(\rho),$$

DEC:

$\rho \geq 0, \quad \rho \pm p \geq 0$

SEC:

$\rho + 3p \geq 0, \quad \rho + p \geq 0$

S. Nojiri, S. D. Odintsov and S. Tsujikawa, hep-th 0501025.

QG escape of Big Rip.

$$L = \frac{1}{\varkappa^2}\left(R - \frac{\gamma}{2} g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \bar{V}(\varphi)\right).$$

With approach to Big Rip, curvatures grow. QG era comes back (typical energies grow).
The one-loop EA maybe found:

$$W^{(1)} = -\frac{1}{2}\int d^4x\sqrt{-g}\,\ln\frac{|R|}{\mu^2}\Big\{\frac{5}{2}\bar{V}^2 - \gamma\bar{V}'^2 +$$

$$+\frac{1}{2}V''^2 + \left[\frac{\gamma}{2}V - 2V''\right]\varphi_{,\mu}\varphi^{,\mu} -$$

$$-\left[\frac{13}{3}V + \frac{\gamma}{12}\bar{V}''\right]R + \frac{43}{60}R^2_{\alpha\beta} + \frac{1}{40}R^2 -$$

$$-\frac{\gamma}{6}R\varphi_{,\mu}\varphi^{,\mu} + \frac{5}{4}\left(\varphi_{,\mu}\varphi^{,\mu}\right)^2\Big\}.$$

For large energies (near Big Rip) $|W^{(1)}| > |L|$. $W^{(1)}$ is dominant.
For instance, for exponential $\bar{V}(\varphi) = \bar{V}_0 e^{-2\frac{\varphi}{\varphi_0}}$
The solution $R = \frac{12}{e^2}$ (dS) exists!

Asymptotic dS Universe occurs!

Modified Gravities as Dark Energy

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Simplest modified gravity:

$$S = \frac{1}{\varkappa^2}\int d^4x\sqrt{-g}\left(R - \frac{\mu^4}{R}\right). \quad (1)$$

$R \to 0$, $\mathcal{L} \sim -\frac{\mu^4}{R} \to \infty$,

Acts like effective cosm. constant!

Cosmic acceleration?

Gravitational alternative for

DARK ENERGY?

no conflict with GR:

at intermediate curvature $\mathcal{L} \sim R$.

More choices:

$\frac{1}{R}\ln R$, $\ln R$, $\frac{1}{R^m}(\ln R)^n$,

$\frac{1}{\sin R}$, ...

Equivalent to (1):

$$S = \frac{1}{\varkappa^2}\int d^4x\sqrt{-g}\left(R - 2\mu^2 A + A^2 R\right) \quad (2)$$

Choosing $e^{2\varphi} = 1 + A^2$, then

$$g_{\mu\nu} \to e^{-2\varphi} g_{\mu\nu}$$

$$S = \frac{1}{\varkappa^2}\int d^4x\sqrt{-g}\Big(R - 6g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \mp 2\mu^2 e^{-4\varphi}\sqrt{1 - e^{2\varphi}}\Big) \quad (3)$$

(1) Equival. (3)

Sudden singularity in $f(R)$ theory.

M. Abdalla, S. Nojiri and S.D. Odintsov,
hep-th 0409177, CQG 2005

$$f(R) = R - \gamma R^{-n}$$

$-1 < n < -\frac{1}{2}$ (effective phantom).

$w < -1$.

R, H diverges at $t = t_s$

Sudden singularity!

consistent modified gravity

$$f(R) = R - \gamma R^{-n} + \eta R^2$$

at late times $R \to \text{const}$,

dS phase!

a). pass solar system tests, observable Newton limit
b) dark energy dominance is consequence of expansion
c). no instabilities, no cosmic doomsday.

S. Nojiri and S.D. Odintsov,
hep-th 0307288, PRD68(2003)123512.

M-/string inspired action:

$$\mathcal{L} = R + c_1 \alpha' e^{-2\varphi} \mathcal{L}_2 + c_2 \alpha'^2 e^{-4\varphi} \mathcal{L}_3 + \dots$$

where

$$\mathcal{L}_2 = G = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta},$$

$$\mathcal{L}_3 = \Omega_3 + R^{\mu\nu}_{\alpha\beta} R^{\alpha\beta}_{\lambda\rho} R^{\lambda\rho}_{\mu\nu},$$

Ω_3 - Euler density ($\sim R^3$ terms).

$c_1, c_2 = (0, 0), (\frac{1}{8}, 0), (\frac{1}{4}, \frac{1}{48})$
type II, heterotic, bosonic.

Starting action:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\varkappa^2} R - \frac{\gamma}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) + f(\varphi) G \right\}$$

$\gamma = \pm 1$. $\gamma = -1$ (phantom).

$f(\varphi) \sim$ const, $S \rightarrow$ scalar-tensor gravity

FRW Universe:

$$ds^2 = -dt^2 + a^2(t)\sum_{i=1}^{3}(dx^i)^2$$

Gravitational eqs. (t,t)

$$0 = -\frac{3}{\varkappa^2}H^2 + \frac{\gamma}{2}\dot{\varphi}^2 + \bar{V}(\varphi) - 24\dot{\varphi}f'(\varphi)H^3$$

φ - eq.

$$0 = -\gamma(\ddot{\varphi} + 3H\dot{\varphi}) - \bar{V}'(\varphi) + 24f'(\varphi)(\dot{H}H^2 + H^4)$$

Anzatz: $\bar{V} = \bar{V}_0 e^{-\frac{2\varphi}{\varphi_0}}$, $f = f_0 e^{\frac{2\varphi}{\varphi_0}}$, $a \sim a_0 t^{h_0}$

$$\Downarrow$$

$$H = \frac{h_0}{t}, \quad \varphi = \varphi_0 \ln\frac{t}{t_1}, \quad h_0 > 0,$$

$$H = -\frac{h_0}{t_s - t}, \quad \varphi = \varphi_0 \ln\frac{t_s - t}{t_1}, \quad h_0 < 0$$

Eqs. of motion:

$$\bar{V}_0 t_1^2 = \frac{1}{\varkappa^2(1+h_0)}\left\{3h_0^2(1-h_0) + \frac{\gamma\varphi_0^2\varkappa^2}{2}(1-5h_0)\right\},$$

$$\frac{48 f_0 h_0^2}{t_1^2} = -\frac{6}{x^2(1+h_0)}\left(h_0 - \frac{\gamma \varphi_0^2 x^2}{2}\right),$$

Eff. EOS parameter

$$w = -1 + \frac{2}{3h_0},$$

if $h_0<0$ ($h_0>0$), $w<-1$ ($w>-1$)

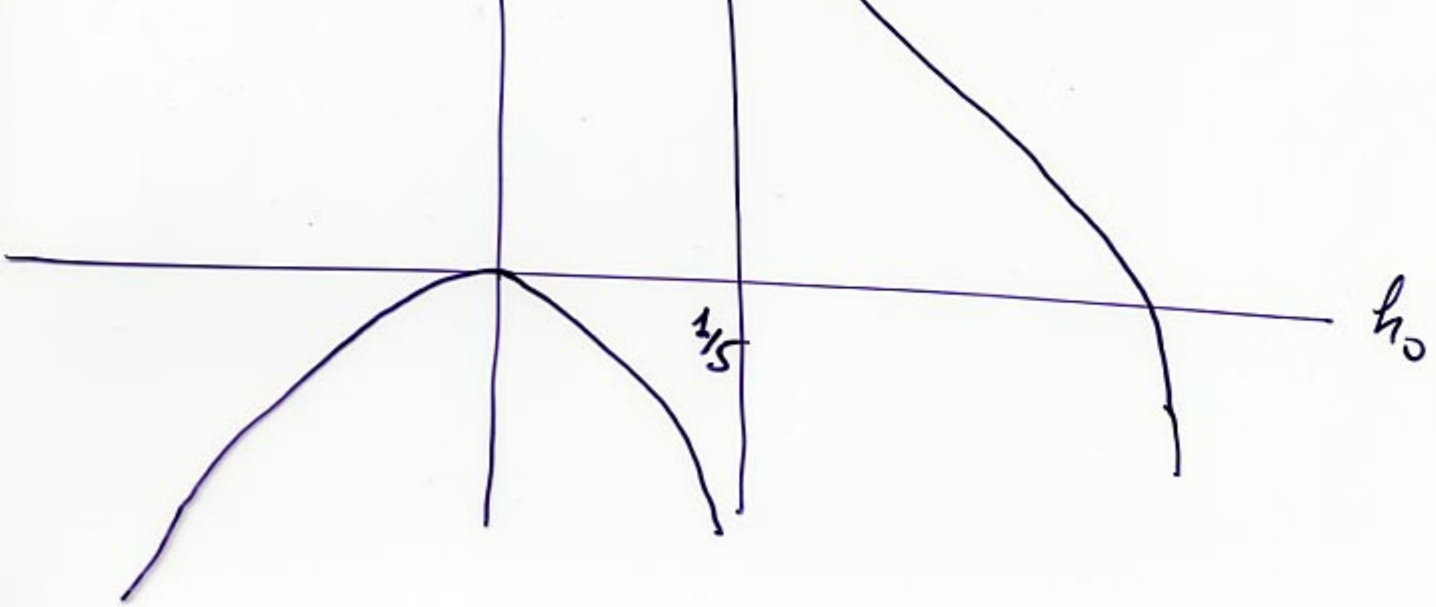
Even if $\gamma>0$, when $h_0<-1$, $V_0>0$

Special case

$$\varphi_0^2 = -\frac{6h_0^2(1-h_0)}{\gamma(1-5h_0)x^2}, \quad \bar{V}(\varphi)=0$$

Condition of reality φ_0:

$\frac{1}{5} < h_0 < 1$, $\gamma=1$ or $h_0>\frac{1}{5}$ or $h_0 \geq 1$.



$V_0 = 0$, $\gamma = 1$ $\frac{1}{5} < h_0 < 1$ - effective matter

$\gamma = -1$, three solutions:

$h_0 < 0$, phantom (also, $\gamma = 1$, $V \neq 0$).

$h_0 > 1$, quintessence

$0 < h_0 < \frac{1}{5}$, eff. matter

1 Example:

$h_0 = -\frac{80}{3} < -1$, $w = -1{,}025$

2 Example:

string, $\gamma = 1$, $V = 0$

$h_0 = 0{,}22$ (solution)

$w = 1.99$ (eff. matter)

no acceleration

3 Example:

$\varphi = \varphi_0$, $H = H_0$ - constants

$$H_0^2 = -\frac{e^{-\frac{2\varphi_0}{\varphi_0}}}{8 f_0 x^2}, \quad dS.$$

Late-time asymptotics.

$$V = V_0 e^{-\frac{2\varphi}{\varphi_0}}, \quad f = f_0 e^{\frac{2\varphi}{\alpha\varphi_0}} \quad (\alpha > 1).$$

potential dominates at small R

GB term at large R

a). $\gamma > 0$, ω is time-dependent, but no accelerating solution

b) $\gamma < 0$, Big Rip singularity (one possibility).

GB term terminates phantom phase, no Rip!

Non-linear matter-gravity as asymptotic dark energy.

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{\varkappa^2} R + \left(\frac{R}{\mu^2}\right)^{\alpha} L_d \right\}$$

Choice:

$$L_d = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

FRW Universe, $\varphi = \varphi(t)$.

φ - Eq. :

$$\dot{\varphi} = q\, a^{-3} R^{-\alpha}$$, q is integration constant.

Hence,

$$R^{\alpha} L_d = \frac{q^2}{2a^6 R^{\alpha}}$$

$> \frac{1}{\varkappa^2} R$ for small R, $\alpha > -1$

$> \frac{1}{\varkappa^2} R$ for large R, $\alpha < -1$.

Dark Energy grows to asymptotic dominance with decrease of curvature.

tt - gravitational Eq.:

$$a(t) = a_0 t^{\frac{\alpha+1}{3}}$$, $a_0 = a_0(q, \alpha, \mu)$.

Acceleration if $\alpha > 2$.

$\alpha < -1$, shrinking Universe (changing direction of time, $t \to t_s - t$ one has accelerating, expanding universe).

t_s is Rip time (singularity).

Effective EOS parameter:

$$w = \frac{1-\alpha}{1+\alpha}$$

Hence, $\alpha < -1 \to$ effective phantom era occurs.

It is stable against perturbations!

Other form of action:

$$L = \frac{1}{\varkappa^2}\xi + \xi^\alpha L_d + \eta (R - \xi)$$

where ξ, η are auxiliary fields.

Varying over $\xi \Rightarrow \eta = \frac{1}{\varkappa^2} + \alpha \xi^{\alpha-1} L_d$

$$S = \int d^4x \sqrt{-g} \left\{ \eta R + \left(\frac{1}{\alpha} - 1\right)\left(\eta - \frac{1}{\varkappa^2}\right)^{\frac{1}{1-\alpha}} \times (\alpha L_d)^{\frac{1}{1-\alpha}} \right\}, \quad \alpha \neq 1.$$

$$\eta = e^{-\sigma}\ , \quad g_{\mu\nu} \to e^{\sigma} g_{\mu\nu},$$

$$S = \int d^4x \sqrt{-g} \Big\{ R - \frac{3}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \left(\frac{1}{\alpha} - 1\right)\left(e^{-\sigma} - \frac{1}{x^2}\right)^{\frac{1}{1-\alpha}} \left(\alpha \mathcal{L}_d (e^{\sigma} g_{\mu\nu}, \varphi)\right)^{\frac{1}{1-\alpha}} \Big\}$$

non-linear Einstein frame action.
(with two scalars)

If usual matter doesn't couple with φ directly, the equivalence principle is not violated.

Generalization.

$$\mathcal{L}_d = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \tilde{V}(\varphi)$$

As an example

$$\tilde{V}(\varphi) = \tilde{V}_0\, \varphi^{2 - \frac{2}{\alpha}} \quad , \alpha \neq 1\ , \alpha \neq -1 + 3h_0$$

Anzatz:

$$\varphi = \varphi_0 t^{\alpha}\ , \quad H = \frac{h_0}{t} \ (a = a_0 t^{h_0}) \quad \text{(I)}$$

For $\alpha = -1 + 3h_0 < 0$, only trivial solution $\varphi_0 = h_0 = 0$.

For $\alpha = -1 + 3h_0 > 0$, $V_0 \neq 0$, no solution exists.

For anzatz I with dominated second term in the action

$$h_0 = \frac{\alpha - 3}{3(\alpha - 2)} > 0, \text{ if } \frac{3}{2} < \alpha < 2.$$

Hence, quintessence dark energy when EOS parameter $w = \frac{\alpha - 1}{\alpha - 3}$ lies in $(-1, -\frac{1}{3})$ interval.

For $1 < \alpha < 2$, $w < -1$, effective phantom phase.

Important: current dark energy dominance is explained by universe expansion.

Generalization:

$$L = \frac{1}{\varkappa^2} R + f(R, R_{\mu\nu}^2, \ldots) L_d + L_m$$

Example:

$f \sim aR^{\alpha} + bR^{\beta}$, $\alpha < \beta$

small R – previous results.

large R, $a \sim t^{\frac{\beta+1}{3}}$, $w \sim \frac{1-\beta}{1+\beta}$

Unification of early time and late time acceleration.

Non-linear gravity-matter QFT.

$$L = \frac{1}{2\varkappa^2} R + \left(\frac{R}{M^2}\right)^{\alpha} \mathcal{L}_d ,$$

$$\mathcal{L}_d = \frac{1}{2}\varphi_{,\mu}\varphi^{,\mu} - \frac{\xi R}{2}\varphi^2 + \frac{\mu^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4$$

Non-renormalizable QFT.

Scalar Green function:

$$\sqrt{-g}\left[\left(\frac{R}{M^2}\right)^{\alpha}\left(\Box + \xi R - \mu^2 + \frac{\lambda}{2}\varphi^2\right) + \alpha\left(\frac{R}{M^2}\right)^{\alpha-1}\frac{R_{;\mu}}{M^2}\nabla^{\mu}\right] iG_d(x,x') = \delta^{(D)}(x-x'),$$

Solution exists at small or large curvature.

Finite one-loop EA maybe found.

$$L^{4D}_{eff} = L + \frac{\hbar}{128\pi^2}\left[\lambda^2(\varphi^4 - 6M_1^2\varphi^2)\left(\frac{\lambda M_2^2}{\varkappa^2(M_2^2)} - \frac{1}{6}\frac{\lambda_2^2 M_2^4}{\varkappa^4(M_2^2)}\right) + \varphi^2\varkappa^2\text{-terms} + \varphi^4\text{-terms}\right.,$$

$\varkappa$ - effective mass

T. Inagaki, S. Nojiri and S.D. Odintsov, ~~hep-th~~ gr-qc/0504054

Dynamical CC problem solution.

A. Dolgov and M. Kawasaki, astro-ph 0307442
S. Mukohyama and L. Randall, PRL 92, 2004, 211302.

Model:

$$\mathcal{L} = \frac{R}{2\varkappa^2} + \alpha_0 R^2 + \frac{(\varkappa^4 \partial_\mu \varphi \partial^\mu \varphi)^q}{2q\,\varkappa^4 f(R)^{2q-1}} - V(\varphi),$$

For small R, assumption

$$f(R) \sim (\varkappa^2 R^2)^m, \quad m > 0$$

$$V \sim V_0 (\varphi - \varphi_c)$$

$q > \frac{1}{2}$, factor of kinetic term is large.

φ does not reach φ_c.

V is small

Example:

$$f(R) = \beta R^2, \quad V = V_0 (\varphi - \varphi_c),$$

$$\alpha_0 = 0.$$

exact SOLUTION: R is small

$$a = a_0 t^{h_0}, \quad \varphi = \varphi_c + \frac{\varphi_0}{t^2}$$

Example: $h_0 < 0$,

$$a = a_0 (t_s - t)^{h_0} \quad \left(H = \frac{h_0}{(t_s - t)^{h_0 - 1}}\right),$$

$$\varphi = \varphi_c + \frac{\varphi_0}{(t_s - t)^2}$$

Solution: algebraic Eqs.

$$\varphi_0^2 = \frac{54\beta(-1+2h_0)^3 h_0^4}{\varkappa^2(12h_0^2 - 2h_0 - 1)},$$

$$V_0 = \pm \frac{3h_0 + 1}{\sqrt{6\varkappa^2(12h_0^2 - 2h_0 - 1)(-1+2h_0)}}$$

$$\varphi_0^2 > 0 \implies$$

$\beta > 0$, $\frac{1-\sqrt{13}}{12} < h_0 < \frac{1+\sqrt{13}}{12}$ or $h_0 \geq \frac{1}{2}$

$\beta < 0$, $h_0 < \frac{1-\sqrt{13}}{12}$ or

$$\frac{1+\sqrt{13}}{12} < h_0 \leq \frac{1}{2}.$$

Example: $h_0 = -\frac{1}{60}$, $w = -1{,}025$,

$$\varkappa V_0 = \pm 0{,}3887\ldots$$

For $h_0 > 0$, $R \sim t^{-2}$,

$\varphi \to \varphi_c$ but it doesn't reach φ_c in finite time.

$t \sim 10^{10}$ (Universe age) $\Rightarrow$

in $H \sim \frac{h_0}{t}$ $(h_0 > 0)$ or $\frac{h_0}{t_s - t}$ $(h_0 < 0)$

observed value of H maybe reproduced.

This explains the smallness of eff. CC:

$$\Lambda \sim H^2,$$

DYNAMICALLY, with late-time acceleration.

Palatini formulation

$\mathcal{L} = R$ $\quad g_{\mu\nu} \Rightarrow$ second-order DE

$g_{\mu\nu}, \Gamma^{\alpha}_{\beta\gamma} \Rightarrow$ first-order Eqs.

solution gives $\Gamma^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\beta\gamma}(g_{\mu\nu})$!

For other gravities (with matter) non-equivalence:

G. Allemandi, A. Borowiec, M. Francaviglia and S. Odintsov, hep-th 0504057.

$$\mathcal{L} = F(R) + f(R)\mathcal{L}_d + \mathcal{L}_{mat}(\Psi).$$

Qualitatively same results.

New proposals for gravitational DE:

$$\mathcal{L} = F(GB) + R + f(GB)\mathcal{L}_d + \mathcal{L}_{mat}.$$

? ?