SUPERSYMMETRIC BLACK HOLES

$D=4$

1) • CLASSICAL BH SOLUTIONS
   • BH THERMODYNAMICS:
     AREA LAW, WALD'S LAW

   • CHARGED BH AS SUPERSYMMETRIC SOLITONS ; $N=2$ SUSY IN $D=4$

2) • SUPERSYMMETRIC BHS IN $D=4$, $N=2$ SUPERGRAVITY WITH R^2-TERMS
   • ATTRACTOR MECHANISM
   • THEIR THERMODYNAMIC ENTROPY
   • CLOAKING OF SINGULARITIES DUE TO R^2-TERMS

3) • MICROSCOPIC DERIVATION OF BH ENTROPY IN STRING THEORY
     SUPERSYMMETRIC BHS

   • CORRESPONDENCE PRINCIPLE BETWEEN BHS AND STRINGS

   • MICROSCOPICALLY COMPUTE ENTROPY OF ELECTRICALLY CHARGED BHS $= S_{THERMO}$ ✓

   • OSV - CONJECTURE

LITERATURE

REVIEWS

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SUPERSYMMETRIC BLACK HOLES

GENERAL RELATIVITY (1915)

→ BLACK HOLE SOLUTIONS

THEORETICAL CONSTRUCTS?

NO! ASTROPHYSICAL MEASUREMENTS

SHOW: BLACK HOLES ARE

UBIQUITOUS !

HUBBLE CENSUS 1997:

EVERY LARGE GALAXY HAS ONE

SUPERMASSIVE BLACK HOLE

HTTP://HUBBLESITE.ORG/NEWSCENTER/NEWSDESK/ARCHIVE/RELEASES/1997/01

FOR INSTANCE: GALACTIC CENTER OF MILKY WAY CONSISTS OF A SUPERMASSIVE BLACK HOLE WITH MASS

$$M = 3.6 \times 10^6 \, M_\odot$$

BLACK HOLES HAVE HUGE ENTROPY:

$$M = M_\odot \Rightarrow S_{MACRO} = 10^{19}\, S_{SUN}$$

ENTROPY IS MEASURE FOR NUMBER N OF INTERNAL STATES (MICROSTATES) OF THE BH: $S_{MICRO} = k_B \ln N$

THESE CANNOT BE SEEN BY EXTERNAL OBSERVER: MACROSTATE OF THE BH, I.E.

$$(M, Q, J)$$

QUESTION: WHAT ARE MICROSCOPIC DOF RESPONSIBLE FOR THE HUGE ENTROPY OF THE BH?

STRING THEORY: CAPABLE OF PROVIDING ANSWER TO THIS QUESTION!

STROMINGER + VAFA
HEP-TH / 9601029

RESTRICTIONS: SO FAR, DETAILED COMPUTATIONS OF S_{MICRO} IN STRING THEORY ONLY POSSIBLE FOR SUPERSYMMETRIC BLACK HOLES.

WHERE ARE THE STATES OF A BH ?

→ MATHUR, HEP-TH/0401115

HERE: BHS IN $D=4$

VARIOUS TYPES :

• SIMPLEST : SCHWARZSCHILD BH

FIRST EXACT SOLUTION TO GR

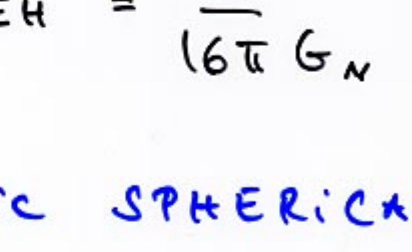
$$S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, R$$

STATIC SPHERICALLY SYMMETRIC SOL. TO

$$R_{\mu\nu} = 0$$

ONE-PARAMETER SOLUTION (M)

GRAVITATIONAL FIELD OF POINT MASS M



SCHWARZSCHILD RADIUS

OUTER

EVENT HORIZON : R_s, SEMIPERMEABLE WALL; NOTHING CAN COME OUT (CLASSICALLY)

$$R_S = \frac{2 M G_N}{c^2}$$

EARTH: $R_S = 9$ mm

LINE ELEMENT IN ISOTROPIC COORD.

$$ds^2 = - H^{-1} dt^2 + H \left(dr^2 + r^2 d\Omega_2 \right)$$

$$d\Omega_2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

$$H = 1 + \frac{R_S}{r}$$, EVENT HORIZON AT $r = 0$

$$\Delta_{3D, \text{FLAT}} H = 0$$ (COVERS REGION OUTSIDE HORIZON)

$r = 0$: 2-SPHERE WITH RADIUS R_S AND AREA

$$A = 4\pi R_S^2 = 16\pi \frac{M^2 G_N^2}{c^4}$$

$c = 1$ IN THE FOLLOWING

- REISSNER-NORSTROM BH

STATIC SPHERICALLY SYMMETRIC SOL TO

EINSTEIN + MAXWELL

$$8\pi S = \int d^4x \sqrt{-g} \left(\frac{1}{2G_N} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

EQS OF MOTION :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 2 G_N \left(F_{\mu\rho} F^{\rho}{}_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right)$$

LINE ELEMENT IN SPHERICAL COORDINATES

$$ds^2 = - e^{2f(r)} dt^2 + e^{-2f(r)} dr^2 + r^2 d\Omega_2$$

$$F_{tr} = - q/r^2 \quad , \quad F_{\theta\phi} = p \sin\theta$$

$$e^{2f(r)} = 1 - \frac{2MG_N}{r} + \frac{(q^2+p^2)G_N}{r^2}$$

3-PARAMETER SOLUTION (M, q, p)

CHARGED DYONIC BH.

NOTE: $q = p = 0 \Rightarrow$ SCHWARZSCHILD BH

REWRITE

$$e^{2f} = 1 - \frac{2MG_N}{r} + \frac{Q^2 G_N}{r^2} = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)$$

WHERE $\quad Q^2 = q^2 + p^2$

$$r_\pm = M G_N \pm \sqrt{M^2 G_N - Q^2}\,\sqrt{G_N}$$

THREE CASES :

- $M > Q/\sqrt{G_N} \quad \Rightarrow$ 2 HORIZONS

r_- CAUCHY HORIZON
r_+ EVENT HORIZON

- $M = Q/\sqrt{G_N} \quad \Rightarrow r_+ = r_-$

EXTREMAL REISSNER-NORDSTROM BH

→ SUPERSYMMETRIC SOLUTION TO $N=2$ SUGRA (EINSTEIN + MAXWELL + SUSY)

- $M < Q/\sqrt{G_N} \quad \Rightarrow$ NO HORIZON, NAKED SINGULARITY

→ DISCARD (UNPHYSICAL)

EXTREMAL RN-BH: $r_+ = r_- = Q\sqrt{G_N}$

ISOTROPIC COORDINATES: $r = r_+ + R$

$$\Rightarrow ds^2 = -H^{-2} dt^2 + H^2 \left(dR^2 + R^2 d\Omega_2 \right)$$

$$H = 1 + \frac{r_+}{R} = 1 + \frac{\sqrt{q^2+p^2}}{R}\sqrt{G_N}$$

EVENT HORIZON AT $R=0$:
2-SPHERE WITH RADIUS $r_+ = Q\sqrt{G_N}$
AND AREA $A = 4\pi Q^2 G_N$

COMPARE SCHWARZSCHILD BH WITH EXTREMAL RN-BH:

IN ISOTROPIC COORDINATES

$$ds^2 = -H^{-p} dt^2 + H^p \left(dR^2 + R^2 d\Omega_2 \right)$$

$$\Delta_{3,\text{FLAT}} H = 0 \quad \Rightarrow \quad H = 1 + \frac{R_{\text{EVENT}}}{R}$$

EVENT HORIZON AT $R=0$

SCHWARZSCHILD: $p=1$, $A = 16\pi M^2 G_N^2$

EXTREMAL RN: $p=2$, $A = 4\pi Q^2 G_N$

IMPORTANT DIFFERENCE

NEAR-HORIZON LIMIT ($R \to 0$)
OF EXTREMAL RN-BH:

BERTOTTI-ROBINSON-GEOMETRY,
CONFORMALLY FLAT:

$$ds^2 = \underbrace{-\left(\frac{R}{Q\sqrt{G_N}}\right)^2 dt^2 + \left(\frac{Q\sqrt{G_N}}{R}\right)^2 dR^2}_{ADS_2}$$

$$+\, Q^2 G_N \underbrace{d\Omega_2}_{S_2}$$

MAXIMALLY SYMMETRIC SPACES:

$$ADS_2 = \frac{SO(2,1)}{SO(1,1)} \quad , \quad S^2 = \frac{SO(3)}{SO(2)}$$

$$R_\mu{}^\mu = 0 \quad \text{SINCE} \quad T_\mu{}^\mu = 0$$

EXTREMAL

RN

$R = \infty$ FLAT M_4 ——— $R = 0$ $ADS_2 \times S_2$

SUPERPOSITION OF EXTREMAL BLACK HOLES:

$$H = 1 + \frac{Q}{R}\sqrt{G_N} \longrightarrow 1 + \sum_{I=1}^{N} \frac{Q_I \sqrt{G_N}}{|\vec{x} - \vec{x}_I|}$$

$$\Delta H = 0$$

$\cdot\ Q_I = (q_I, p^I)$
$\vec{x}_I$

MULTI-CENTER SOLUTION; STATIC CONFIGURATION, HORIZONS AT $\vec{x}_I$; $M_I = Q_I / \sqrt{G_N}$

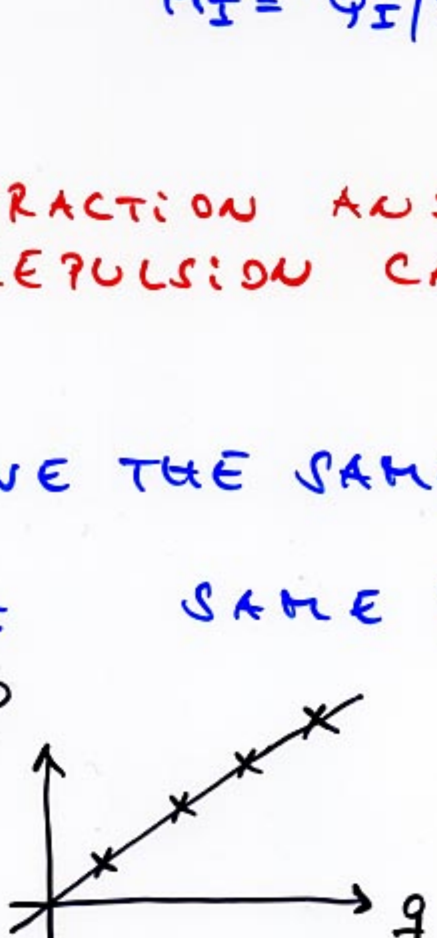
$\vec{x}_I$: ARBITRARY

GRAVITATIONAL ATTRACTION AND ELECTROMAGNETIC REPULSION CANCEL PRECISELY

ELECTRIC : q_I HAVE THE SAME SIGN

DYONIC : $q_I + i p_I$ SAME PHASE

$\Leftrightarrow p^I q_J - p^J q_I = 0$



BLACK HOLE THERMODYNAMICS

CLASSICALLY: BH IS BLACK

QUANTUM MECHANICALLY:

EMITS BLACKBODY RADIATION AT A TEMPERATURE

$$k_B T_H = \hbar \frac{\kappa_s}{2\pi}$$

HAWKING TEMPERATURE

κ_s : SURFACE GRAVITY

CALCULATION: QFT IN SPACETIME WITH HORIZON, INDEPENDENT OF DETAILS OF THE DYNAMICS OF GRAVITY THEORY

$$(\mathcal{L} = R, R + R^2, \cdots)$$

ESTIMATE κ_s USING NEWTONIAN LAWS OF GRAVITATION: SCHWARZSCHILD

AT HORIZON, THE GRAVITATIONAL FORCE IS

$$|\vec{g}| = \frac{G_N M}{R_s^2} = \frac{c^4}{4 G_N M} = \kappa_s$$

$$R_s = \frac{2 G_N M}{c^2}$$

CORRECT VALUE!

SMALL FOR VERY MASSIVE OBJECTS

SCHWARZSCHILD : $k_B T_H = \dfrac{\hbar c^3}{8\pi G_N M}$ (*)

REISSNER-NORSTROM :

$$\kappa_S \underset{\substack{G_N=1\\ c=1}}{\overset{}{\underset{\uparrow}{=}}} \frac{\sqrt{M^2-Q^2}}{2M\left(M+\sqrt{M^2-Q^2}-Q^2\right)}$$

EXTREMAL: $M^2 = Q^2 \Rightarrow \kappa_S = 0 \Rightarrow T_H = 0$

STABLE

NEAR-EXTREMAL : $M^2 = Q^2 + \delta$

COOLS DOWN TO $T_H = 0$

USING (*), CALCULATE MACROSCOPIC (THERMODYNAMIC) ENTROPY OF SCHWARZSCHILD BH :

FIRST LAW OF THERMODYNAMICS

$$dE = T\,dS$$

ENERGY OF BLACK HOLE : $E = Mc^2$

$$\Rightarrow \quad dE = c^2 \, dM = T_H \, dS = \frac{\hbar c^3}{8\pi G_N M k_B} \, dS$$

$$\Rightarrow \quad \frac{1}{k_B} \, dS = \frac{4\pi G_N}{\hbar c} \, dM^2$$

$$\Rightarrow \quad \frac{1}{k_B} \, S_{MACRO} = \frac{4\pi G_N}{\hbar c} \, M^2$$

WHERE WE ASSUME $S_{MACRO} = 0$ FOR $M = 0$

AREA OF HORIZON

$$A = 4\pi R_S^2 = 16\pi \frac{G_N^2 M^2}{c^4}$$

$$\Rightarrow S_{MACRO} = k_B \frac{A}{4 \ell_P^2} \quad , \quad \ell_P^{-2} = \frac{c^3}{\hbar G_N}$$

AREA LAW OF BEKENSTEIN + HAWKING

APPLIES TO OTHER BHS AS WELL,

REISSNER-NORDSTROM

HUGE ENTROPY! $R_S^2 \gg \ell_P^2$ TINY

ASTROPHYSICAL-SIZE BH

COMPARE : $k_B = \hbar = c = 1$

$$S_{MACRO} = \frac{A}{4 G_N}$$

$$= \begin{cases} 4\pi M^2 G_N & \text{SCHWARZSCHILD} \\ \pi Q^2 & \text{EXTREMAL RN} \end{cases}$$

INDEPENDENT OF G_N !

STRING THEORY : $G_N \sim g_s^2 l_s^2$

$\Rightarrow$ ENTROPY OF EXTREMAL RN

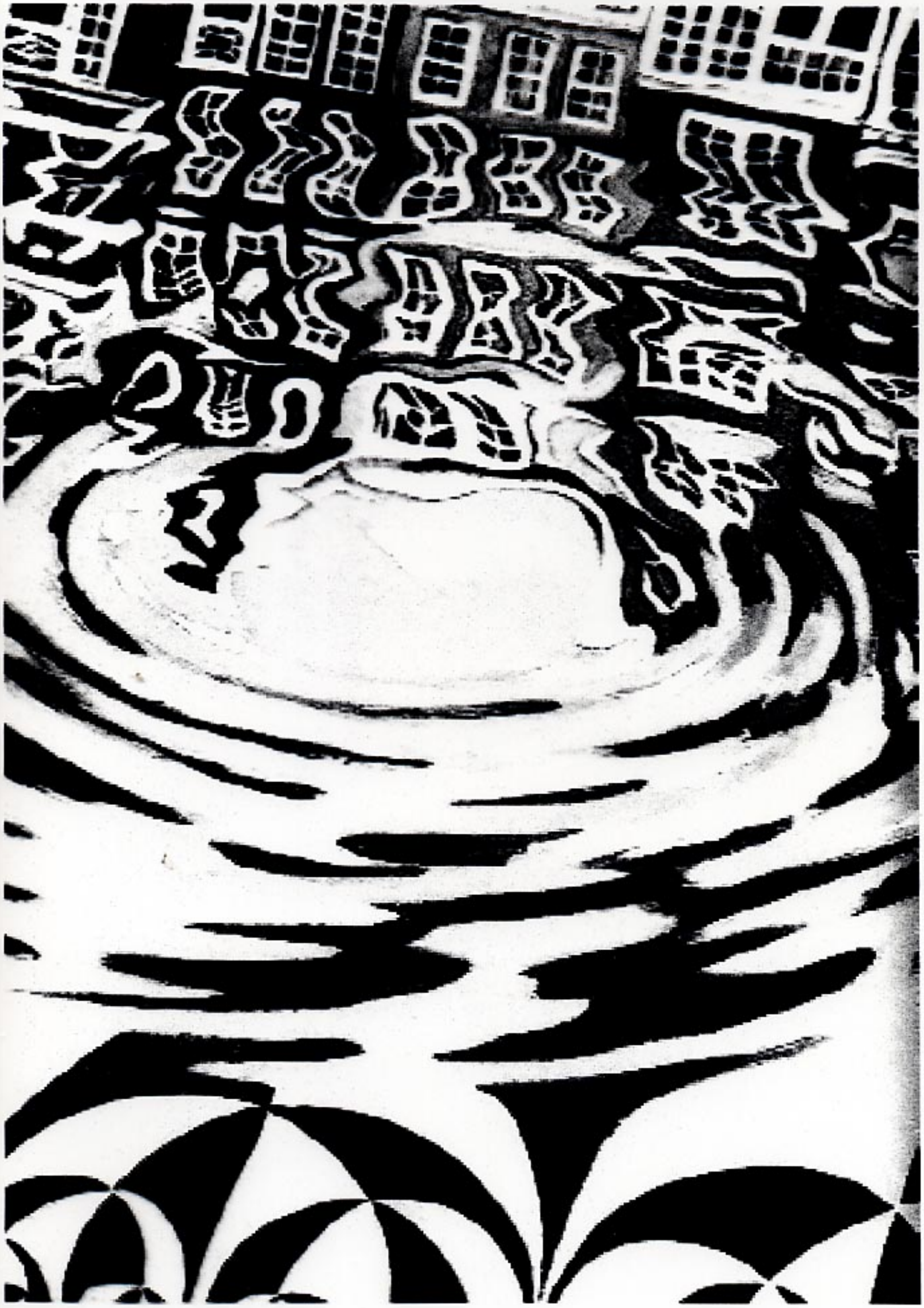
INDEPENDENT OF g_s !

2 PHASES :

a) MACROSCOPIC PHASE $r_s \gg l_s$

$\Rightarrow Q g_s \gg 1$

b) MICROSCOPIC PHASE $Q g_s \ll 1$



gs~1

gs
~0

THERMODYNAMIC ENTROPY OF BHS
IN EINSTEIN GRAVITY + MATTER
AREA LAW OF BEKENSTEIN + HAWKING

$$S_{MACRO} = k_B \, \frac{1}{4} \, \frac{A}{\ell_P^2}$$

GEOMETRIC QUANTITY EVALUATED ON EVENT HORIZON:

$$A = \int_{S_2} d^2x \, \sqrt{h} = \int_{S_2} d\phi \, d\theta \sin\theta \, r_+^2$$

EFFECTIVE SUPERGRAVITY THEORY FROM STRING THEORY:

$$\mathcal{L} = R + R^2 + \ldots + \mathcal{L}_{MATTER}$$

WHAT IS THE OBJECT S_{MACRO} SUCH THAT

$$\delta M = \frac{\kappa_S}{2\pi} \, \delta S$$

WALD : GR-QC/9307038

BLACK HOLE ENTROPY IS NOETHER CHARGE.

EFFECTIVE LAGRANGIAN FROM STRING THEORY

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, R_{\mu\nu\rho\sigma} ; \phi, \nabla\phi)$$

ANY POWER, BUT NO DERIVATIVE

WALD FINDS FOR STATIC BLACK HOLES

$$S_{MACRO} = 2\pi \int_{HORIZON} d^2x \sqrt{h} \, \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \, \epsilon_{\rho\sigma} \, \epsilon_{\mu\nu}$$

BINORMAL OF HORIZON; $\neq 0$ IN NORMAL DIRECTIONS

STATIC BH

$\epsilon_{tr} \neq 0$

Ex: $8\pi \mathcal{L} = -\frac{1}{2} R$

$$8\pi \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} = -\frac{1}{2} g^{\mu[\rho} g^{\sigma]\nu}$$

$$\Rightarrow S_{MACRO} = 2\pi \int_{HORIZON} d^2x \sqrt{h} \left(-\frac{1}{16\pi} \underbrace{\epsilon^{\mu\nu} \epsilon_{\mu\nu}}_{=-2}\right)$$

$$= \frac{1}{4} A \quad ✓$$

$\uparrow \, k_B = \ell_P = 1$

COMMENTS:

1) WALD'S DERIVATION ASSUMES $T \sim \kappa_S \neq 0$. NO PROBLEM WITH $\kappa_S \to 0$ IN S_{MACRO}

2) WALD IS POWERFUL RECIPE FOR COMPUTING S_{MACRO}. IN FULL AGREEMENT WITH STRING THEORY!

EINSTEIN + MAXWELL + $N=2$ SUSY

$\Rightarrow$ $N=2$ SUPERGRAVITY

$$(e_\mu{}^a \, , \, \psi_{\mu\alpha}^{i} \, , \, A_\mu^I) \quad , \; I=0$$

VIERBEIN   2 GRAVITINI   GRAVIPHOTON

$$\mathcal{L} = \frac{1}{2} R + F_{\mu\nu}^2 + \dots$$

COUPLE ADDITIONAL $N=2$ ABELIAN VECTOR MULTIPLETS TO $N=2$ SUGRA

$N=2$ ABELIAN VECTORMULTIPLET :

$$(A_\mu^A \, , \, X^A_c \, , \, \dots) \quad , \; A=1,\dots,n$$

$\Rightarrow$ COMBINE $\quad A_\mu^I \quad , \; I=0,\dots,n$

X^I $\leftarrow$ ONE MORE

WILSONIAN $N=2$ LAGRANGIAN ENCODED IN HOLOMORPHIC HOMOGENEOUS FUNCTION $F(X)$ OF DEGREE 2

$$X \to \lambda X \quad , \quad F(X) \to F(\lambda X) = \lambda^2 F(X)$$

CONSTRAINT:

$$\mathcal{L} = -\frac{\Delta}{2} R - \frac{i}{4}\left(\mathcal{N}_{IJ}(X)\, F^{+I}_{\mu\nu} F^{\mu\nu +J} - \text{c.c.} \right) + \dots$$

$$\Delta = i\left[\bar{X}^I F_I(X) - \bar{F}_I(X) X^I \right] = 1$$

$$F_I = \frac{\partial F(X)}{\partial X^I}$$

EXAMPLES :

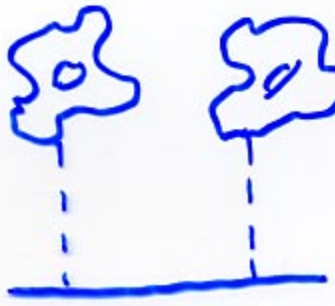
1) PURE $N=2$ SUGRA

$$F(X) = -\frac{i}{4}\left(X^0\right)^2$$

$$\Rightarrow \Delta = |X^0|^2 = 1 \qquad U(1)\text{-PHASE}$$

2) STRING THEORY COMPACTIFICATIONS

$$D=10 \longrightarrow D=4$$



$$F(X) = D_{ABC}\,\frac{X^A X^B X^C}{X^0}$$

$D=4$

LARGE VOLUME LIMIT

SUPERSYMMETRIC CHARGED BLACK HOLE SOLUTIONS TO $N=2$ SUPERGRAVITY + n ABELIAN $N=2$ VECTORMULTIPLETS:

$$X^I \,, \quad I=0,\dots,n$$

RESCALED VARIABLES Y^I, $U(1)_R$ INVARIANT

LINE ELEMENT (ISOTROPIC COORDINATES)

$$ds^2 = -e^{2g}\,dt^2 + e^{-2g}\,d\vec{x}^{\,2}$$

$$e^{-2g} = i\left[\bar{Y}^I F_I(Y) - Y^I \bar{F}_I(\bar{Y})\right]$$

ATTRACTOR EQUATIONS

$$\begin{pmatrix} Y^I - \bar{Y}^I \\ F_I(Y) - \bar{F}_I(\bar{Y}) \end{pmatrix} = i \begin{pmatrix} H^I \\ H_I \end{pmatrix}$$

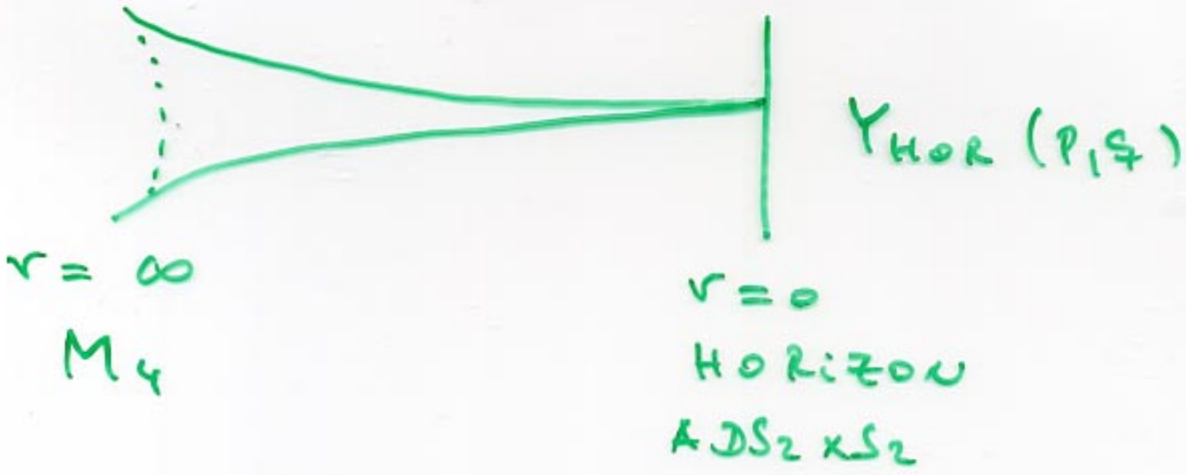
HARMONIC FUNCTIONS SC

$$H^I = h^I + \frac{p^I}{|\vec{x}|} \,, \quad H_I = h_I + \frac{q_I}{|\vec{x}|}$$

GAUGE FIELDS

$$F^I_{tm} = -\nabla_m\left[e^{2g}\,(Y+\bar{Y})^I\right]$$

SOLUTION ENTIRELY DETERMINED IN TERMS OF (H^I, H_I) THROUGH ATTRACTOR EQUATIONS:



ENTROPY:

$$S_{MACRO}(p,q) = \frac{A}{4G_N} = \pi\,|Z|^2$$

$$|Z|^2 = p^I F_I(Y_{HOR}) - q_I Y^I_{HOR}$$

Ex: PURE $N=2$ SUPERGRAVITY $F = -i\dfrac{(Y^0)^2}{4}$

$Y^0 \rightarrow H^0, H_0$; SET $H^0 = 0$

$\Rightarrow Y^0 = \bar{Y}^0 = -H_0$

$\Rightarrow e^{-2g} = (H_0)^2$

COMMENT: $|Z|^2 \neq 0$ FOR DYONIC BLACK HOLES IN STRING THEORY

NOW: ADD HIGHER-CURVATURE INTERACTIONS.

A SPECIAL SUBSET HAS BEEN COMPUTED IN STRINGTHEORY:

$$\mathcal{L} = R + \Big[F^{(1)}(X)\, C^2_{\mu\nu\rho\sigma} + \sum_{g=2}^{\infty} F^{(g)}(X)\, \underbrace{C^2_{\mu\nu\rho\sigma}}_{\text{WEYLTENSOR}}\, \underbrace{(T_{ab})}_{\text{SELFDUAL LORENTZTENSOR}}^{2(g-1)} + \text{c.c.} \Big]$$

$$+ \mathcal{L}_{MATTER}$$

ENCODED IN

$$F(X, T^2) = \sum_{g=0}^{\infty} F^{(g)}(X)\, T^{2g} = F^{(0)}(X) + F^{(1)} T^2 + \ldots$$

RESULTING ENTROPY (WALD'S LAW):

$$(X, T^2) \rightarrow (Y, \Upsilon)$$

$$S_{MACRO} = \pi \Big[\underbrace{|Z|^2}_{\text{AREA LAW}} - 256\, \mathrm{Im}\, F_\Upsilon \Big]$$

$$F_\Upsilon = \frac{\partial F(Y, \Upsilon)}{\partial \Upsilon} \;(= F^{(1)} + \ldots) \qquad \Upsilon = -64 \text{ AT HORIZON}$$

AT HORIZON, $Y_{HOR} = Y_{HOR}(q, p)$ THROUGH ATTRACTOR MECHANISM, WHICH ALSO HOLDS IN PRESENCE OF R^2-TERMS.

APPLICATIONS :

1) SUBLEADING CORRECTIONS

→ CAPTURED BY S_{MICRO} ? ✓

2) HETEROTIC STRING THEORY ON $K3 \times T_2$: $F^{(0)}(Y) = -\frac{Y^1}{Y^0} Y^a \eta_{ab} Y^b$

$$F^{(1)} = i c_1 S \,, \quad c_1 = -1/64 \,,$$

$$S = -i \frac{Y^1}{Y^0} = \frac{1}{g_s^2} + i\theta$$

WEAK COUPLING ($\mathrm{Re}\, S \to \infty$)

ELECTRICALLY CHARGED BLACK HOLE

$$\Rightarrow S_{MACRO} = \pi \sqrt{-512\, c_1\, q_e^2}$$

$$\to 0 \quad \text{FOR} \quad c_1 \to 0$$

CLOAKING OF NAKED SINGULARITY DUE TO R^2-TERMS

DABHOLKAR, KALLOSH, MALONEY
0410076

$c_1 = 0$ $\qquad$ $c_1 \neq 0$

MICROSCOPIC INTERPRETATION :

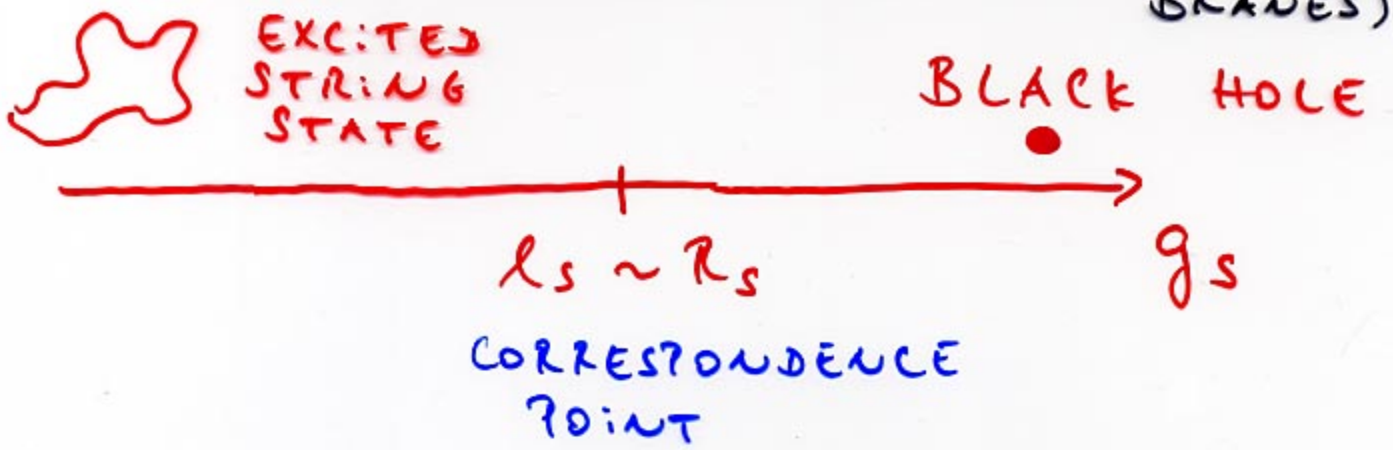
CHARGED EXCITATION OF PERTURBATIVE HETEROTIC STRING (DABHOLKAR, 0409148)

THE CORRESPONDENCE PRINCIPLE

(SUSSKIND; HOROWITZ AND POLCHINSKI)

9309145 9612146

SAYS: BLACK HOLES 1:1 - CORRESPONDENCE WITH STRING STATES (STRINGS/BRANES)



HETEROTIC BH WITH ELECTRIC CHARGES (m, w) CORRESPONDS TO CHARGED BPS STATE $|m, w\rangle$ OF PERTURBATIVE HETEROTIC STRING.

RECALL SOME ELEMENTS OF BOSONIC STRING THEORY :

$$\partial^i \partial_i X^\mu(\sigma, \tau) = 0$$

$\Rightarrow$ p^μ CM MOMENTUM

$a_m^\mu = \frac{1}{\sqrt{m}} \alpha_m^\mu$ OSCILLATORS

UPON QUANTISATION:

$$a_m^\mu |0\rangle = 0 \quad , m > 0$$

$$a_{-m}^\mu = a^{+\,\mu}_m \qquad \text{CREATION OPERATORS}$$

PHYSICAL STATES OSCILLATE IN TRANSVERSE DIRECTIONS , $i = 1, \dots, 24$

STATES $\quad (a_1^+)^{m_1} (a_2^+)^{m_2} \cdots |0\rangle$

NUMBER OPERATOR $\hat{N}$

$$\hat{N} = \sum_{m=1}^{\infty} m \, a^{+\,i}_m \, a^i_m$$

$$\hat{N} |\psi\rangle = N |\psi\rangle$$

LEVEL NUMBER $\quad N = \sum_{i=1}^{24} \sum_{k=1}^{\infty} k \, m_k^i$

OCCUPATION NUMBERS

FOR FIXED N, HAVE MANY STATES WITH $\hat{N}$ EIGENVALUE EQUAL TO N

NOW: ASSUME THAT $X^{25}(\sigma, \tau)$ COMPACT

X^{25} $\Rightarrow p^{25} = \frac{m}{R}$, $m \in \mathbb{Z}$

$\Rightarrow$ ELECTRIC CHARGE FROM $D=4$ POINT OF VIEW

IN ADDITION STRING CAN WIND AROUND X^{25} !

$$X^{25}(\sigma + 2\pi, \tau) = X^{25}(\sigma, \tau) + 2\pi \underset{\uparrow \text{WINDING}}{\omega} R$$

ω: ELECTRIC CHARGE IN $D=4$

IN HETEROTIC STRING THEORY, CHARGED BPS STATES HAVE TO SATISFY LEVEL MATCHING:

$$N - 1 = m\omega \quad \Rightarrow |m, \omega, m_L^i\rangle$$

DISTRIBUTE LEVEL N EXCITATIONS BETWEEN OSCILLATORS IN 24 TRANSVERSE DIRECTIONS TO X^{25}:

$$\# \simeq e^{4\pi\sqrt{m\omega}}$$

$$\Rightarrow S_{MICRO} \underset{\substack{\uparrow \\ k_B = 1}}{=} \ln \# = 4\pi\sqrt{m\omega}$$

$$= \underset{\uparrow\, R + R^2}{S_{MACRO}} \,!$$

OUTLOOK :

1) WHERE ARE THE STATES OF A BLACK HOLE ?

MATHUR / 0401115

2) OSV - CONJECTURE 0405146

S_{MACRO} = LEGENDRE OF TOPOLOGICAL FREE ENERGY $\sim \mathrm{Im}\, \mathcal{F}(Y, \gamma)$

$e^{S_{MICRO}}$ = INVERSE LAPLACE OF $e^{\mathcal{F}_{TOP}}$

INVERSE LAPLACE $\neq$ LEGENDRE

3) NEW CONNECTION BETWEEN D=4 AND D=5 BPS BLACK HOLES

STROMINGER ET AL, 0503217

$\Rightarrow$ MICROSCOPICS MATCH !

DVV = SV