Approaches to QCD phase diagram; effective models, strong coupling lattice QCD, and compact stars

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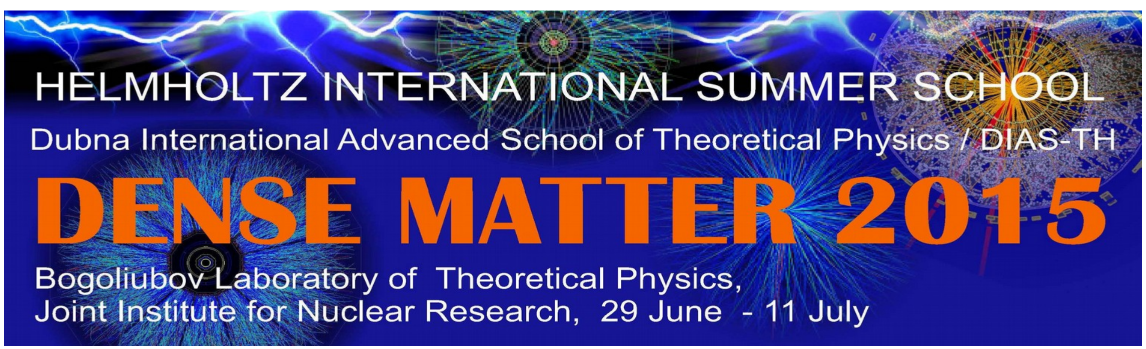
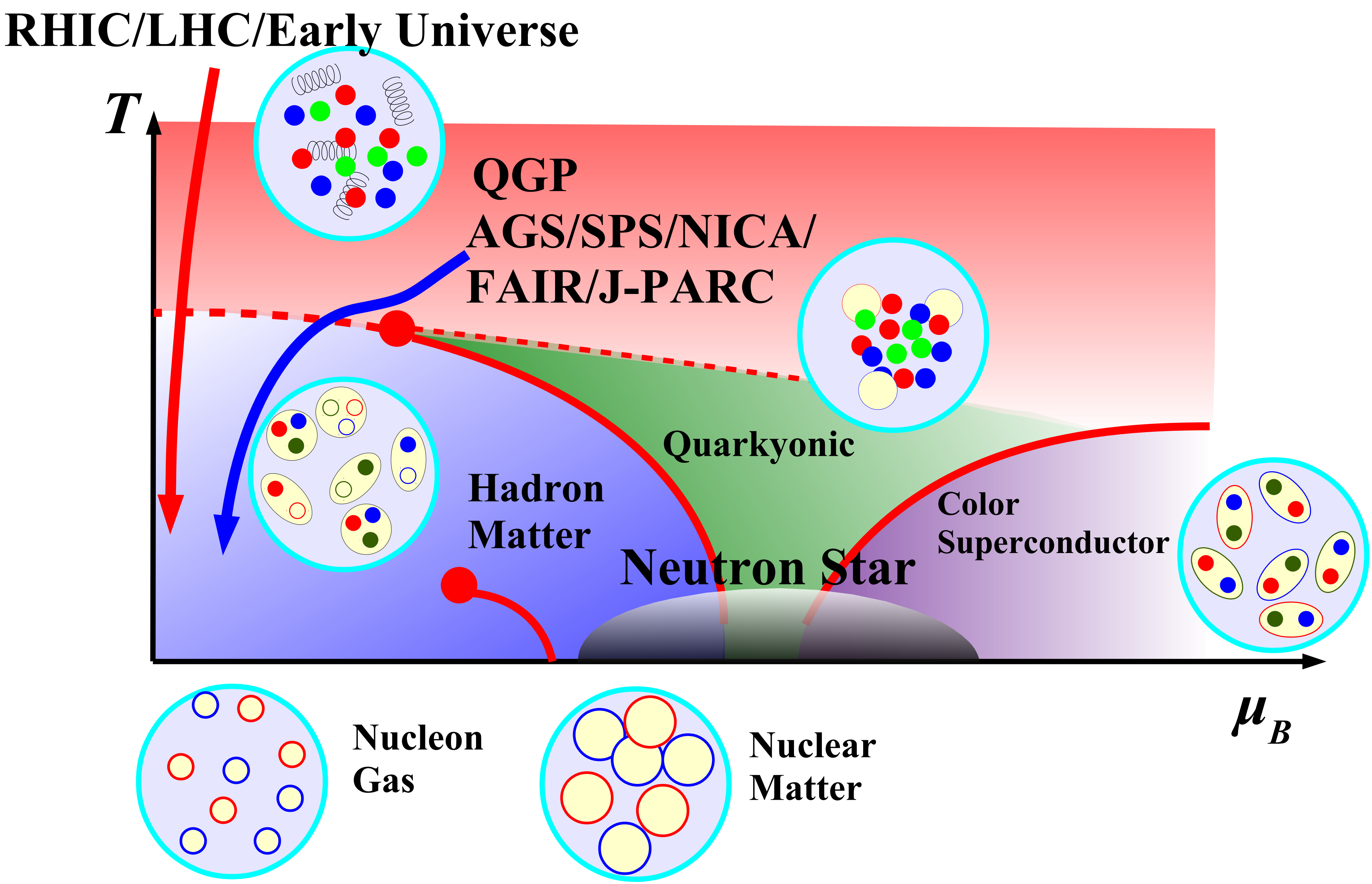


SQM DUBNA 2015

QCD Phase Diagram

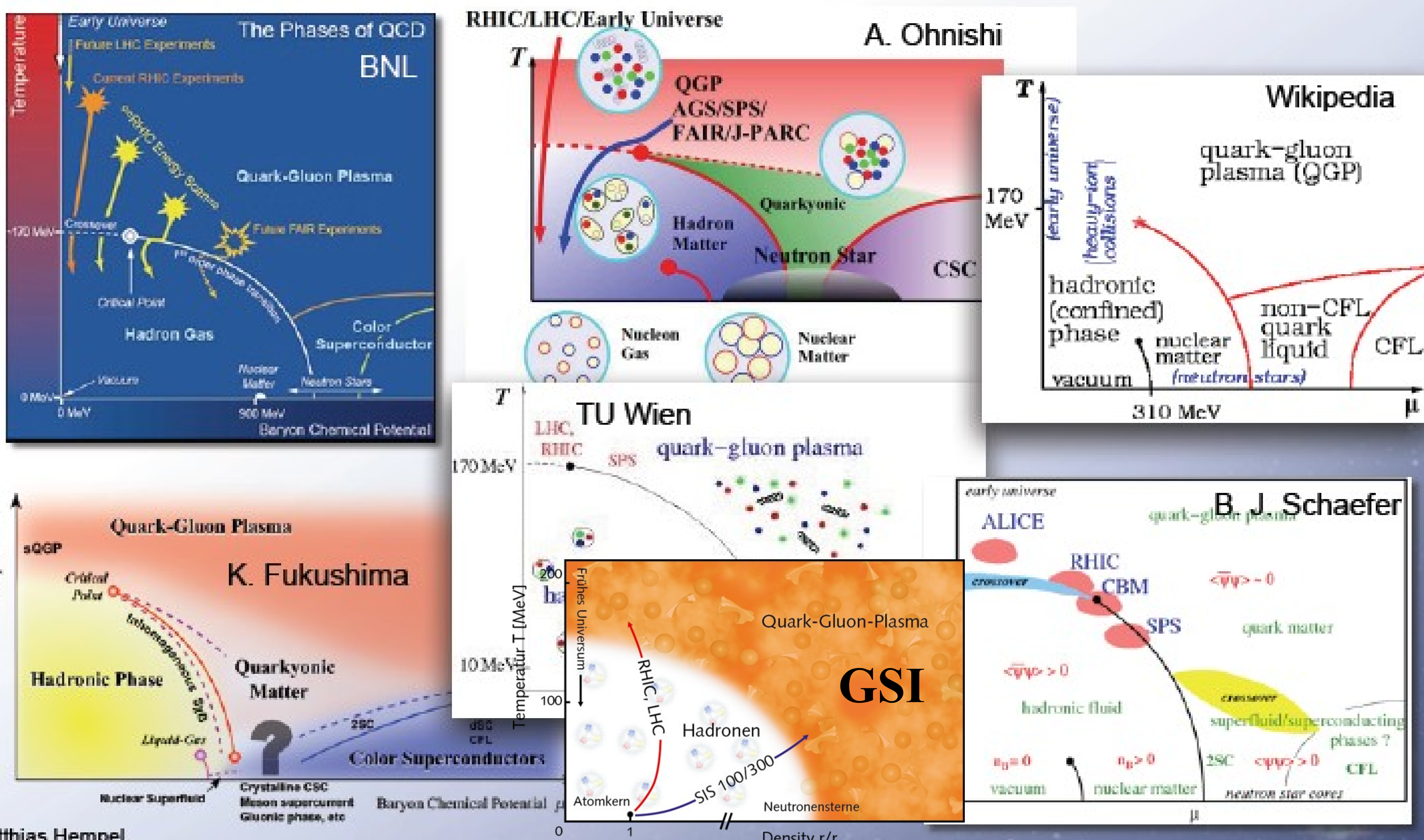


AO, PTPS 193('12)1

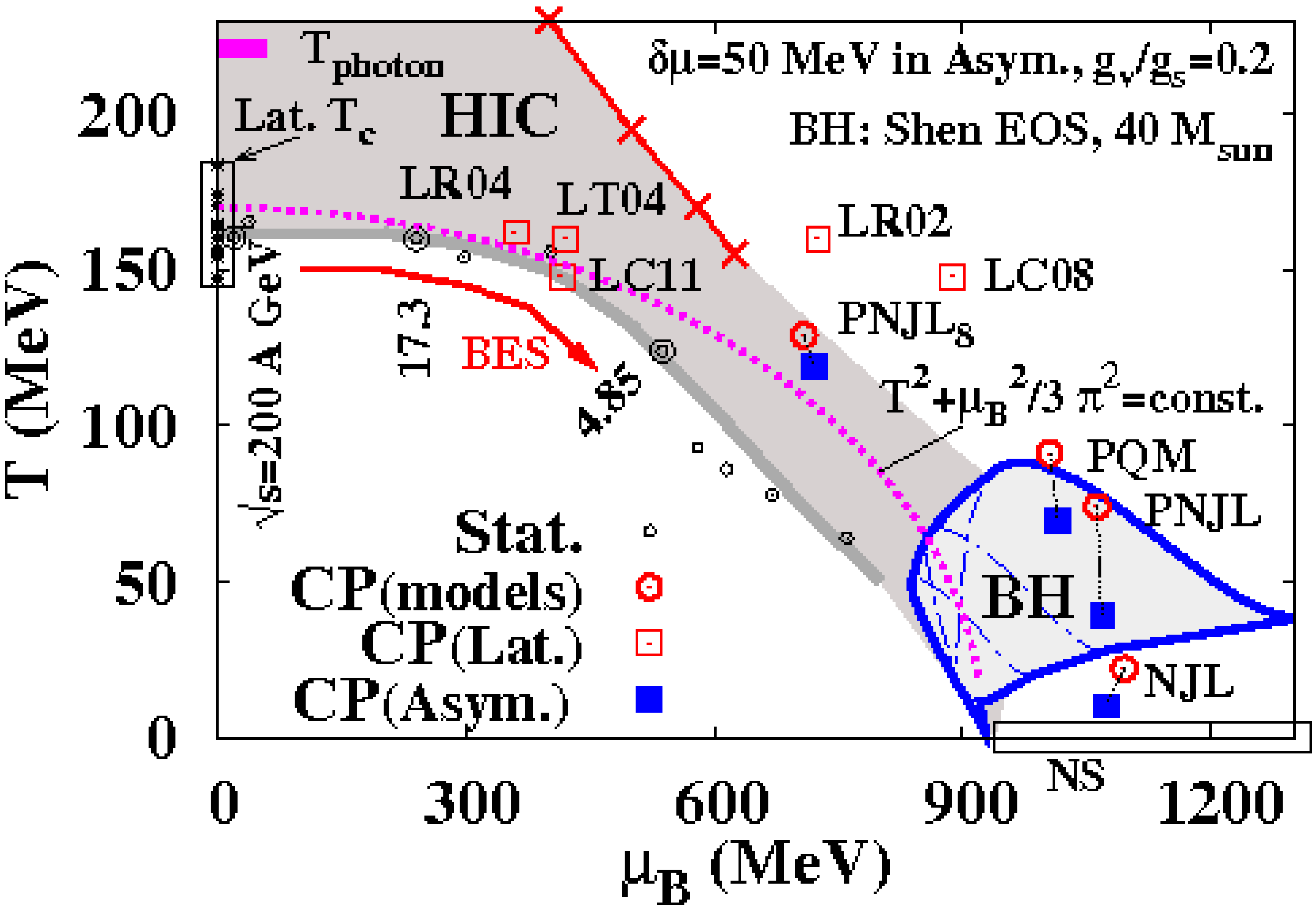
Introduction – QCD phase diagrams

by M. Hempel

- fundamental question: phase diagram of strongly interacting matter
- typical examples in T-μ, first order phase transitions (PT) as lines:



QCD phase diagram (Exp. & Theor. Studies)



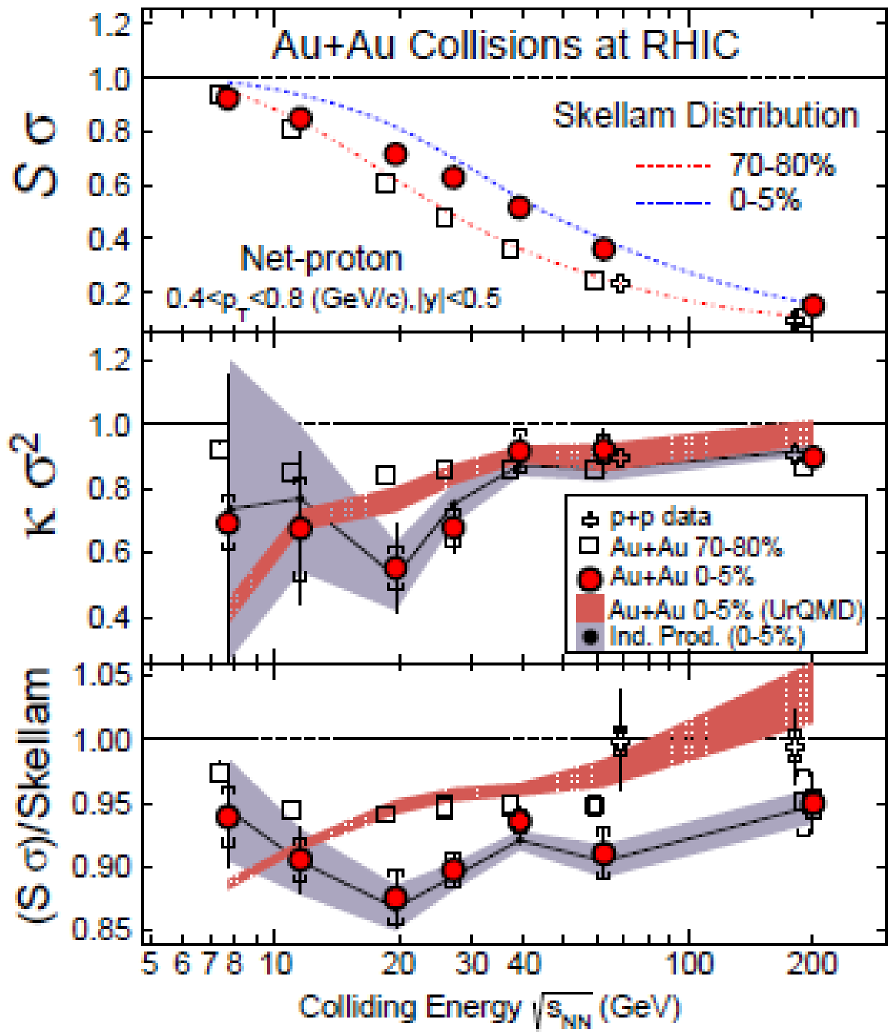
Hempel, Cleymans, Castorina, Randrup, and many others

QCD phase transition is not only an academic problem, but also a subject which would be measured in HIC or Compact Stars

AO, PTPS 193('12)1

Net-Proton Number Moments & Directed Flow

- Non-monotonic behavior of $\kappa\sigma^2$ and dv_1/dy. CP signal ?



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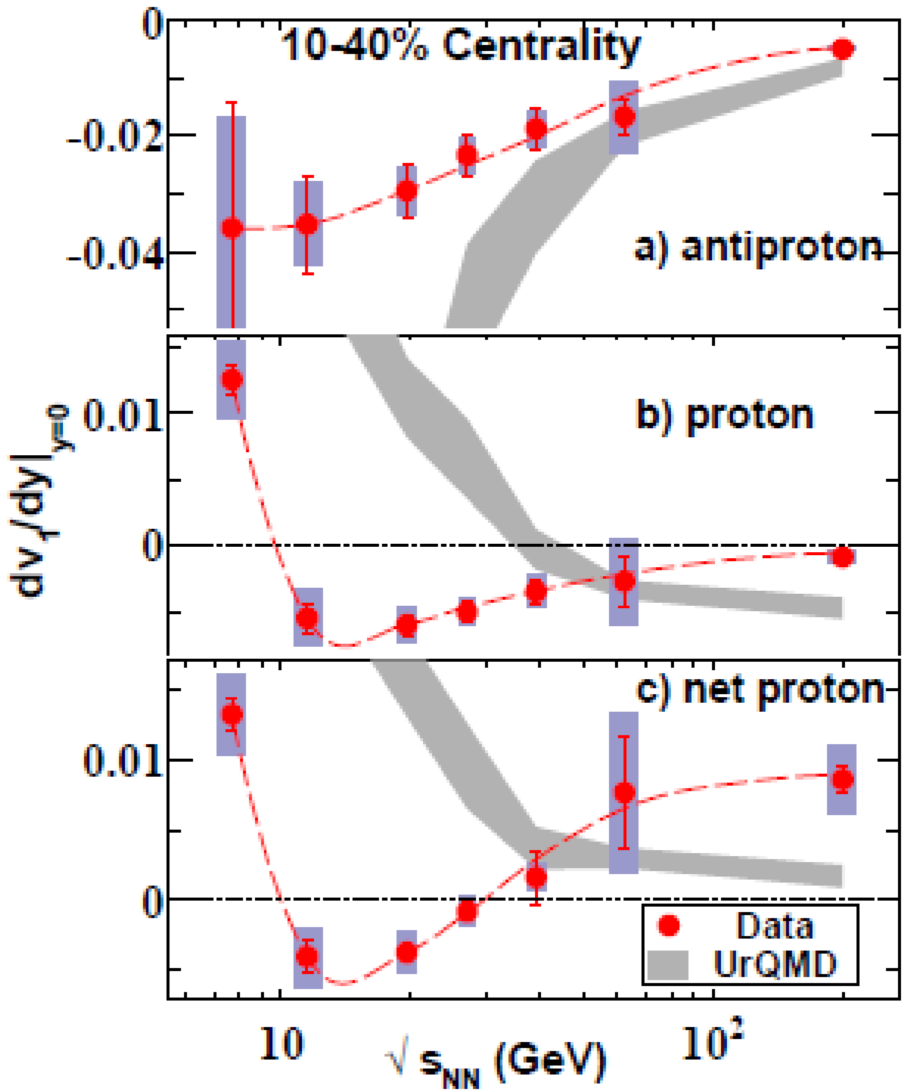
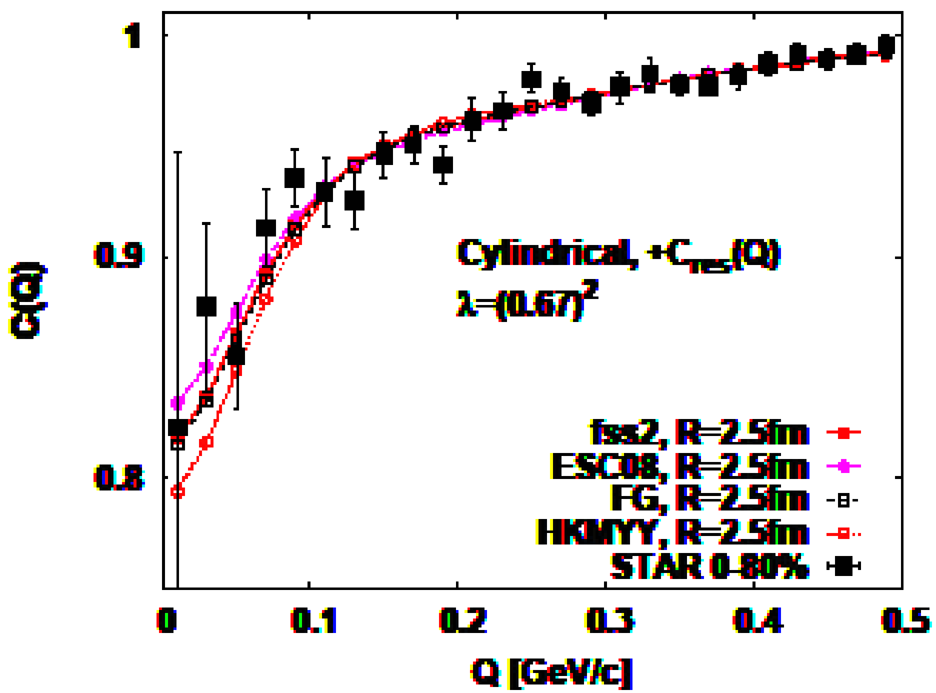
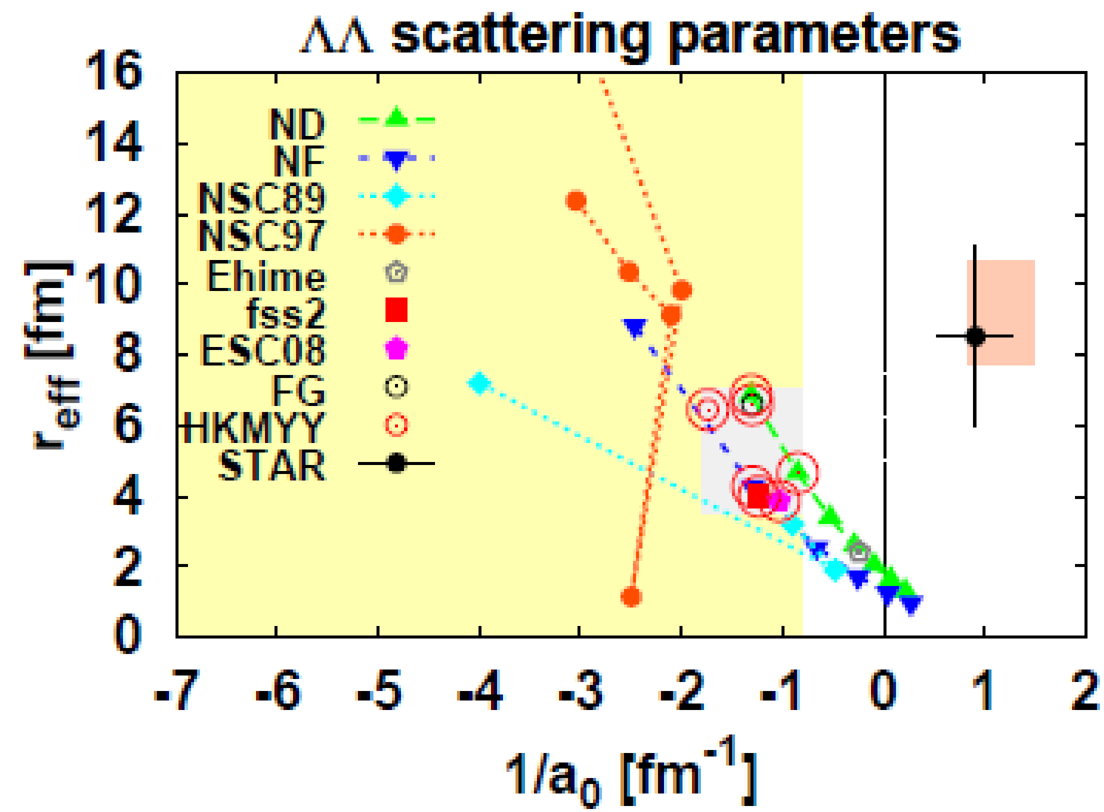


STAR Collab., PRL 112('14)162301.

ΛΛ interaction from ΛΛ correlation at RHIC

ΛΛ correlation with long. and transverse flow effects, Σ^0 feed down, and unknown long tail effects
→ Constraints on ΛΛ interaction





K.Morita, T.Furumoto, AO, PRC91('15)024916 [arXiv:1408.6682]
Data: Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301.

Physics of Dense Matter

- "Dense Matter" ($\rho_B > \rho_0$) and QCD phase diagram would be probed in heavy-ion collisions and compact star phenomena.
- Theoretical approaches to QCD phase diagram
  - Lattice QCD Monte-Carlo simulations (Sign problem)
  - Effective models (Lec.1, prediction is model dependent)
  - Approximation in LQCD, e.g. Strong-coupling lattice QCD
- Dense matter in compact star phenomena
  - Neutron Stars, Supernova, Black Hole formation,
    Binary Neutron Star Merger, ….
  - Key variable = Y_Q = Q(of hadrons) / B (Nuclear matter Y_Q=Y_e)
    → Phase diagram of isospin-asymmetric matter

Contents

- Lecture 1
  - Introduction to physics of QCD phase diagram
  - Spontaneous Chiral Symmetry Breaking in NJL
  - Restoration of Chiral Symmetry in NJL
  - Summary
- Lecture 2
  - Introduction
  - QCD Phase Diagram in Strong-Coupling Lattice QCD
  - Dense Matter in Compact Star Phenomena
  - Summary

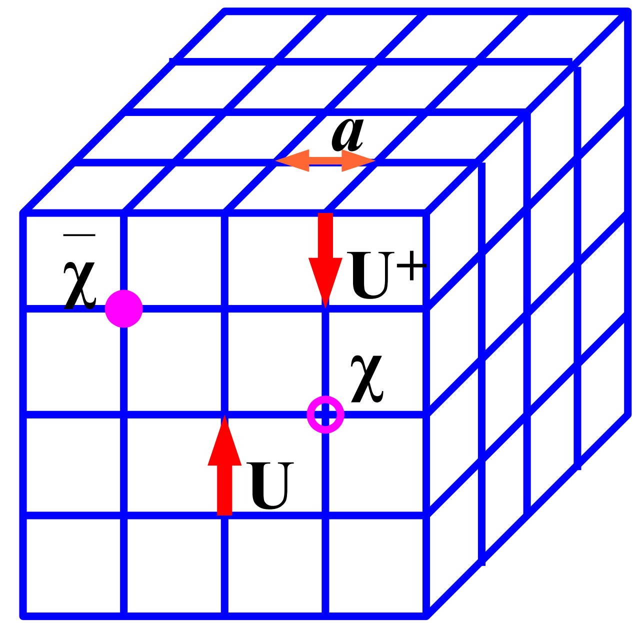
QCD phase diagram in strong-coupling lattice QCD

Lattice QCD

- **Space-time discretization of fields**
- **Quarks = Grassmann number on sites**

$$\chi_i \chi_j = -\chi_j \chi_i, \quad \int d\chi\, 1 = 0, \quad \int d\chi\, \chi = 1$$

$$\rightarrow \int d\chi_1 d\chi_2 \cdots d\bar{\chi}_1 d\bar{\chi}_2 \cdots \exp(\bar{\chi} D \chi) = det(D)$$



- **Gluons → Link variable**

$$U_\mu(x) = \exp\left[ig \int_x^{x+\hat{\mu}} dx\, A(x) \right] \sim \exp(i g A_\mu)$$

$$\int dU\, U_{ab} = 0, \quad \int dU\, U_{ab} U^+_{cd} = \delta_{ad}\delta_{bc}/N_c, \quad \int dU\, U_{ab} U_{cd} U_{ef} = \varepsilon_{ace}\varepsilon_{bdf}/N_c!$$

- **Gauge transf.**

$$\chi(x) \rightarrow V(x)\chi(x), \quad \bar{\chi}(x) \rightarrow \bar{\chi}(x) V^+(x),$$
$$U_\mu(x) \rightarrow V(x) U_\mu(x) V(x+\hat{\mu})$$
$$\bar{\chi}(x) U_\mu(x) \chi(x+\hat{\mu}) = \text{invariant}$$

Lattice spacing = a
→ Lattice unit: a=1

Lattice QCD action

- **Lattice QCD action (unrooted staggered fermion)**

$$L=\frac{1}{2}\sum_{x}\left[\bar{\chi}_x U_0(x) e^{\mu}\chi_{x+\hat{0}}-\bar{\chi}_{x+\hat{0}} U_0^{+}(x) e^{-\mu}\chi_x\right]$$

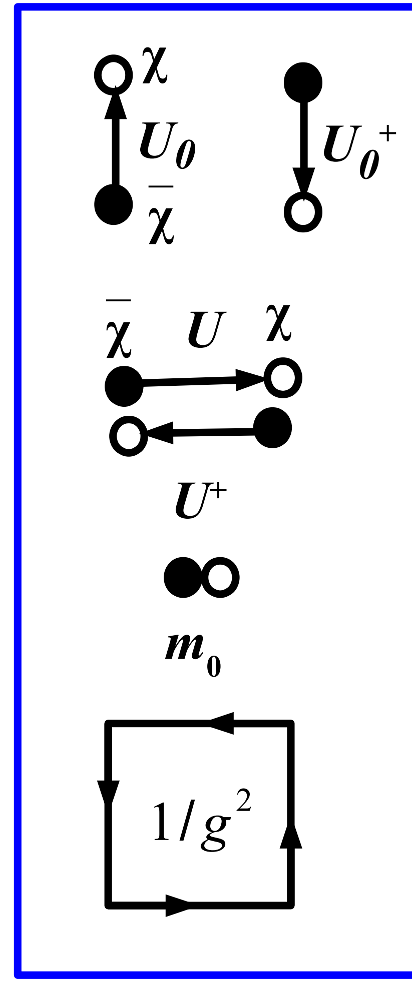
$$+\frac{1}{2}\sum_{x,j}\eta_j(x)\left[\bar{\chi}_x U_j(x)\chi_{x+\hat{j}}-\bar{\chi}_{x+\hat{j}} U_j^{+}(x)\chi_x\right]$$

$\rightarrow \chi\,(\partial + i\,gA)\,\chi$

$$+m_0\sum_x \bar{\chi}_x\chi_x$$

$$+\frac{2N_c}{g^2}\sum_{plaq.}\left[1-\frac{1}{N_c}\mathrm{Re\,tr}\,U_{\mu\nu}(x)\right]$$

Stokes theorem $\rightarrow$ rotation



- **Staggered sign factor (~ γ matrix)**
  $\eta_j(x)=(-1)^{**}(x_0+\ldots+x_{j-1})$

- **Chiral transf.**
  $\chi_x \rightarrow \exp[i\,\theta\varepsilon(x)]\,\chi_x$, $\varepsilon(x)=(-1)^{**}(x_0+x_1+x_2+x_3)$

χ quark (Grassmann #)
U link ~ exp(igA)

Sign problem in lattice QCD

- **Fermion determinant (= stat. weight of MC integral) becomes complex at finite μ in LQCD.**

$$Z = \int D[U, q, \bar{q}] \; \exp(-\bar{q} D(\mu, U) q - S_G(U))$$
$$= \int D[U] \; \mathrm{Det}(D(\mu, U)) \; \exp(-S_G(U))$$

$$[\gamma_5 D(\mu) \gamma_5]^+ = D(-\mu^*) \rightarrow [\mathrm{Det}(D(\mu))]^* = \mathrm{Det}(D(-\mu^*))$$
$$(\gamma_5 \;\; \text{hermiticity})$$

  - **Note: Euclidean $D = \gamma_\mu D_\mu + m - \mu\gamma_0$ (γ = Hermite, D_μ = anti-Hermite)**
  - **Fermion det. (Det D) is real for zero μ (and pure imag. μ)**
  - **Fermion det. is complex for finite real μ.**

- **Approximate methods:**
  - **Taylor expansion, Imag. μ , Canonical, Re-weighting, Fugacity expansion, Histogram method, Complex Langevin, Strong-coupling lattice QCD**

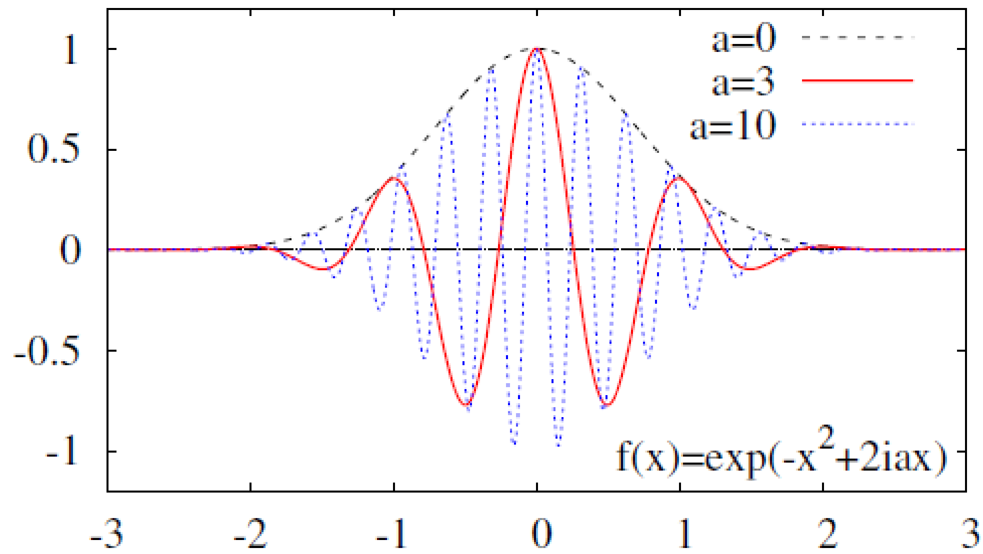
Sign Problem

- **Monte-Carlo integral of oscillating function**

$$Z=\int dx \exp(-x^2+2iax)=\sqrt{\pi}\exp(-a^2)$$

$$\langle O\rangle=\frac{1}{Z}\int dx\, O(x)\, e^{-x^2+2iax}$$

Easy problem for human is not necessarily easy for computers.



- **Complex phase appears from fluctuations of H and N.** ***de Forcrand***

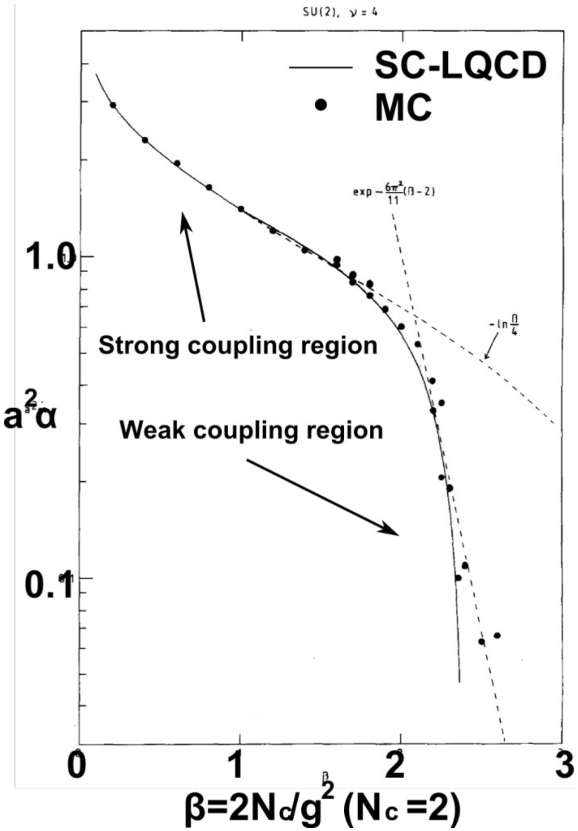
$$Z=\sum\langle\psi|\exp[-(H-\mu N)/T]|\psi\rangle=\sum\prod\langle\psi_\tau|\exp[-(H-\mu N)/(N_\tau T)]|\psi_{\tau+1}\rangle$$

→ **Description based on "Hadronic" (color singlet) action would be helpful to reduce fluctuations.**

→ **Strong coupling lattice QCD**

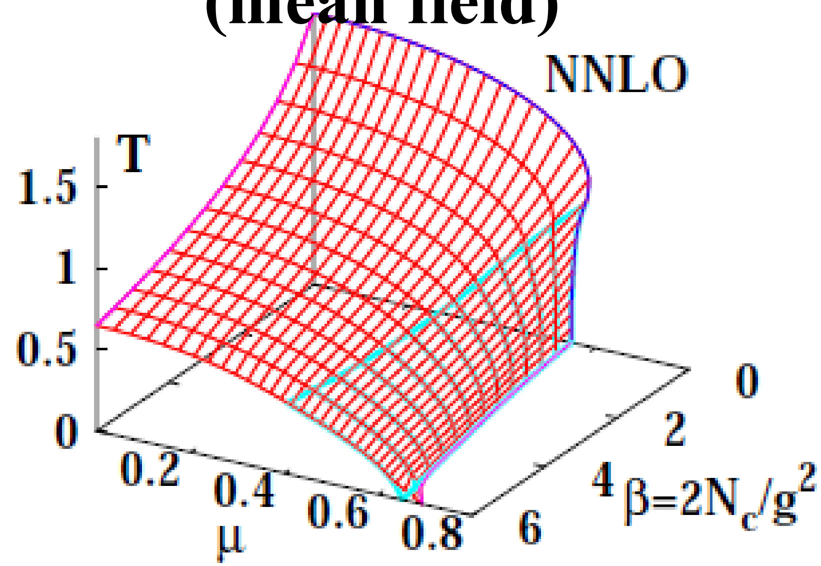
Strong Coupling Lattice QCD

Pure YM



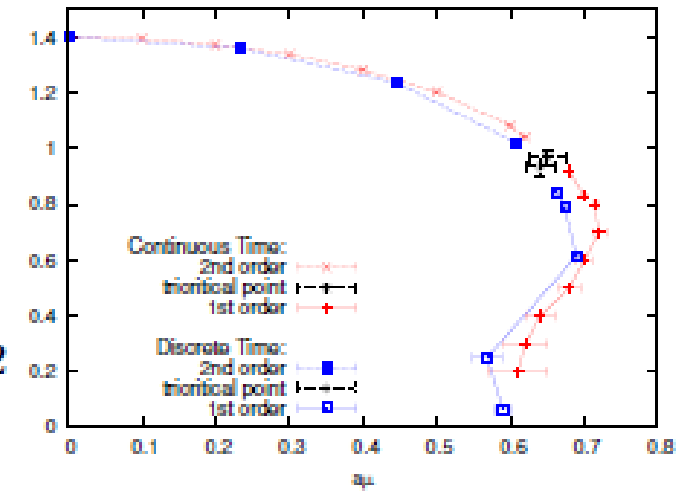
Wilson ('74), Creutz ('80), Munster ('80, '81), Lottini, Philipsen, Langelage's ('11)

Phase diagram (mean field)



Kawamoto ('80), Kawamoto, Smit ('81), Damagaard, Hochberg, Kawamoto ('85), Ilgenfritz, Kripfganz ('85), Bilic, Karsch, Redlich ('92), Fukushima ('03); Nishida ('03), Kawamoto, Miura, AO, Ohnuma ('07). Miura, Nakano, AO, Kawamoto ('09), Nakano, Miura, AO ('10)

Fluctuations



Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11), AO, Ichihara, Nakano ('12), Ichihara, Nakano, AO ('14), de Forcrand, Langelage, Philipsen, Unger ('14)

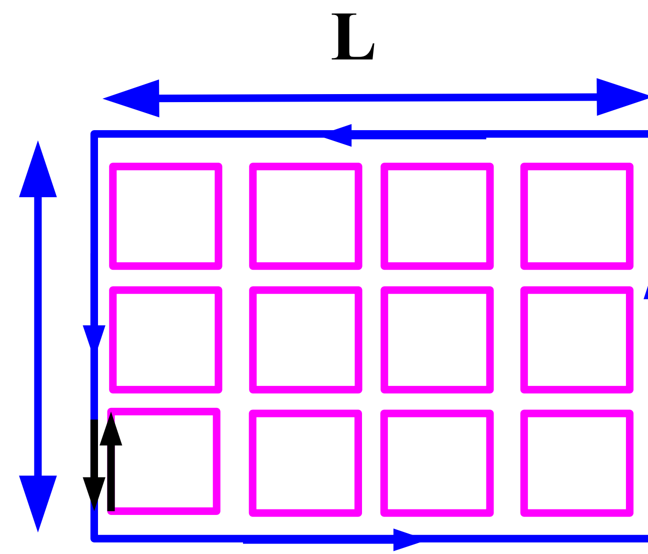
Area Law

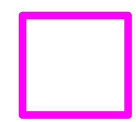
Wilson ('74), Creutz ('80), Munster ('80, '81)

- **Wilson loop in pure Yang-Mills theory**

$$\langle W(C=L\times N_\tau)\rangle = \frac{1}{Z}\int DU\, W(C)\exp\left[\frac{1}{g^2}\sum_P \mathrm{tr}(U_P+U_P^+)\right] = \exp(-V(L)N_\tau)$$

V(L)= heavy-qq pot.

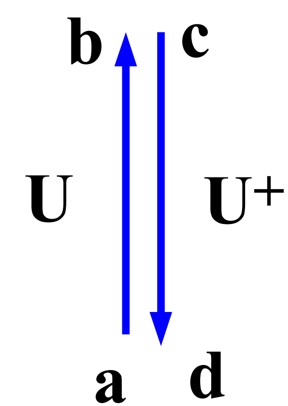




$= 1/N_c\, g^2$

- **One-link integral**

$$\int dU\, U_{ab} U^+_{cd} = \frac{1}{N_c}\delta_{ad}\delta_{bc}$$



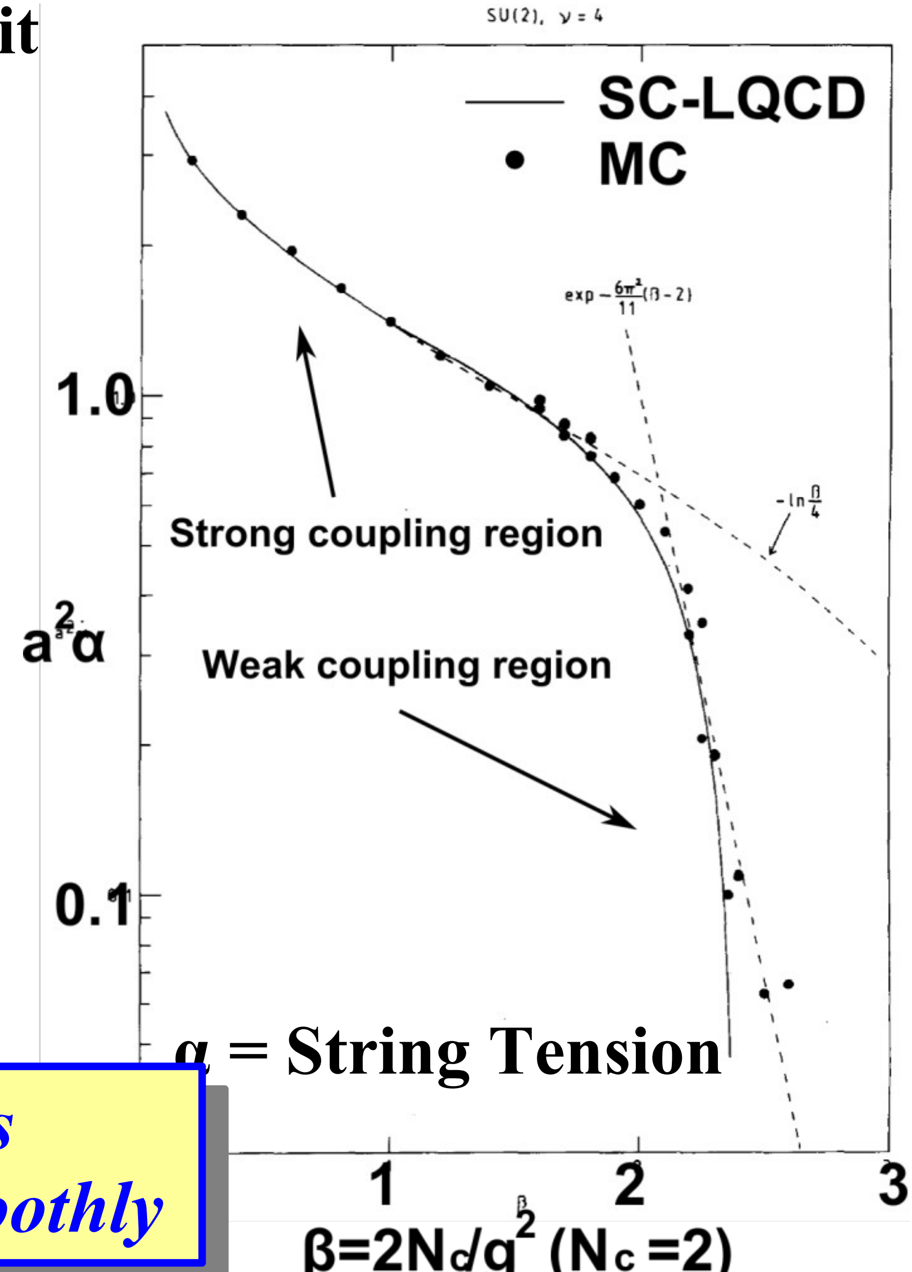
- **In the strong coupling limit**

$$\langle W(C)\rangle = N\left(\frac{1}{g^2 N}\right)^{L N_\tau} \rightarrow V(L) = L\log(g^2 N)$$

Linear potential between heavy-quarks
→ Confinement (Wilson, 1974)

Area Law

- Area law in the strong coupling limit
  Wilson ('74)
- Verification of the area law in Lattice MC simulation
  Creutz ('80)
- Strong coupling expansion to higher orders
  Munster ('80, '81),
  Lottini, Philipsen, Langelage ('11)
- Weak coupling region
  $\rightarrow g^2/4\pi = 1 / \beta_0 \log (q^2/\Lambda^2)$
  $\rightarrow a \sim 1/q \sim \exp(2\pi/g^2\beta_0)/\Lambda$



α = String Tension

Strong coupling expansion connects SCL and Weak coupling region smoothly

Strong Coupling Lattice QCD

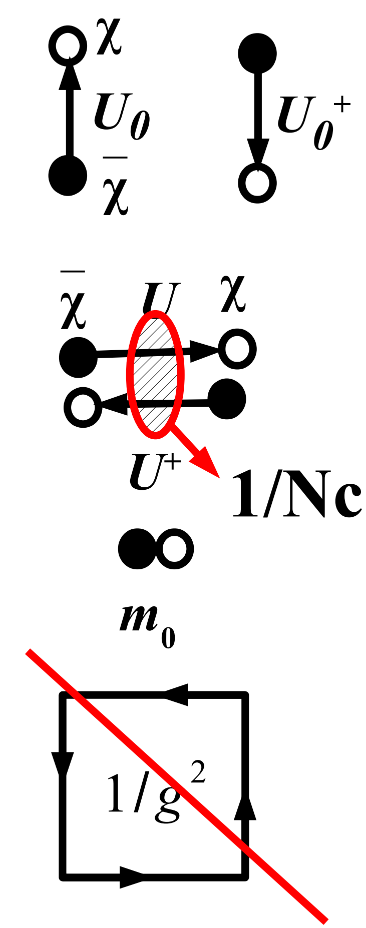
- **Strong coupling limit**
  Damgaard, Kawamoto, Shigemoto ('84)

$$S_{\rm SCL}=S_F^{(t)}-\frac{1}{4N_c}\sum_{x,j} M_x M_{x+\hat{j}}+m_0\sum_x M_x$$

$$\left(M_x=\bar{\chi}_x\chi_x\right)$$

  - **Integrate out spatial links using one-link formula, and pick up diagrams with min. quark numbers.**

$$\int dU\, U_{ab} U^+_{cd}=\delta_{ad}\delta_{bc}/N_c$$



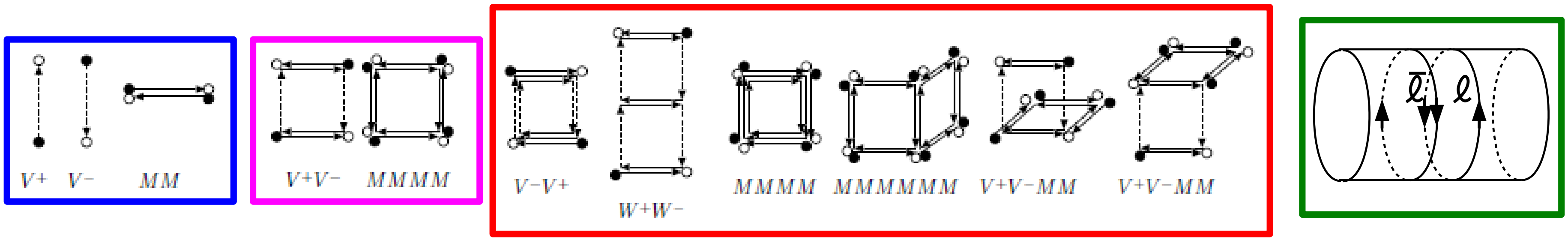
Lattice QCD in SCL
→ Fermion action with nearest neighbor four Fermi interaction

Finite Coupling Effects

- **Effective Action with finite coupling corrections**
  Integral of exp(-S_G) over spatial links with exp(-S_F) weight → S_{eff}

$$S_{\text{eff}} = S_{\text{SCL}} - \log \langle \exp(-S_G) \rangle = S_{\text{SCL}} - \sum_{n=1} \frac{(-1)^n}{n!} \langle S_G^n \rangle_c$$

$\langle S_G^n \rangle_c$=Cumulant (connected diagram contr.) *c.f. R.Kubo('62)*



$$S_{\text{eff}} = \frac{1}{2}\sum_x (V_x^+ - V_x^-) - \frac{b_\sigma}{2d} \sum_{x,j>0} [MM]_{j,x}$$

SCL (Kawamoto-Smit, '81)

$$+ \frac{1}{2}\frac{\beta_\tau}{2d} \sum_{x,j>0} [V^+V^- + V^-V^+]_{j,x} - \frac{1}{2}\frac{\beta_s}{d(d-1)} \sum_{x,j>0,k>0,k\neq j} [MMMM]_{jk,x}$$

NLO (Faldt-Petersson, '86)

$$- \frac{\beta_{\tau\tau}}{2d} \sum_{x,j>0} [W^+W^- + W^-W^+]_{j,x} - \frac{\beta_{ss}}{4d(d-1)(d-2)} \sum_{\substack{x,j>0,|k|>0,|l|>0 \\ |k|\neq j,|l|\neq j,|l|\neq |k|}} [MMMM]_{jk,x}[MM]_{j,x+\hat{l}}$$

$$+ \frac{\beta_{\tau s}}{8d(d-1)} \sum_{x,j>0,|k|\neq j} [V^+V^- + V^-V^+]_{j,x} \left([MM]_{j,x+\hat{k}} + [MM]_{j,x+\hat{k}+\hat{0}}\right)$$

NNLO (Nakano, Miura, AO, '09)

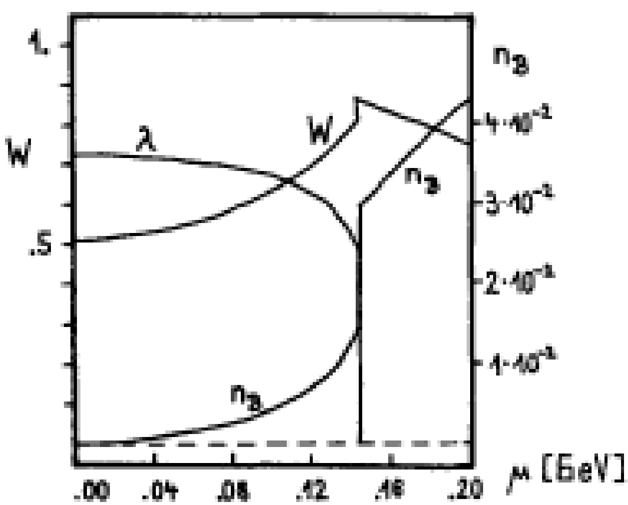
$$- \left(\frac{1}{g^2 N_c}\right)^{N_\tau} N_c^2 \sum_{\mathbf{x},j>0} \left(\bar{P}_{\mathbf{x}} P_{\mathbf{x}+\hat{j}} + h.c.\right)$$

Polyakov loop (Gocksch, Ogilvie ('85), Fukushima ('04)
Nakano, Miua, AO ('11))

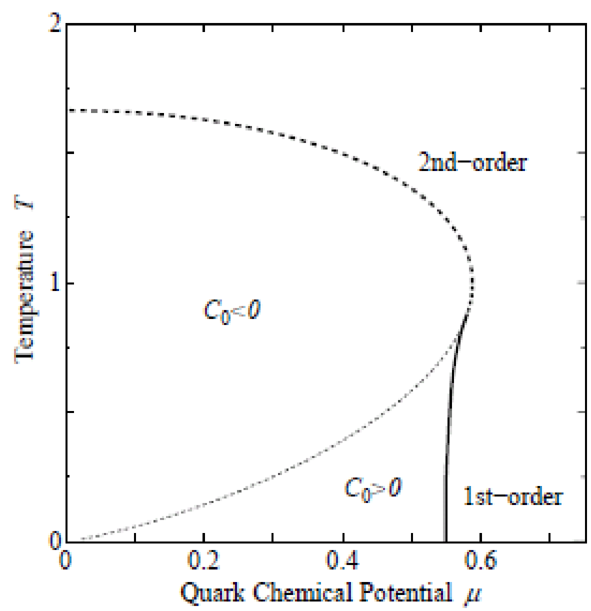
Phase diagram in SC-LQCD (mean field)

- **"Standard" simple procedure in Fermion many-body problem**
  - **Bonsonize interaction term (Hubbard-Stratonovich transformation)**
  - **Mean field approximation (constant auxiliary field)**
  - **Fermion & temporal link integral**

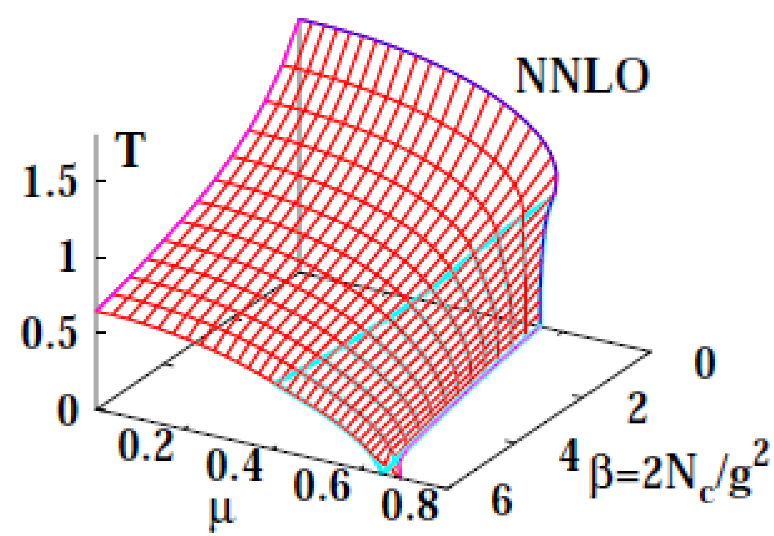
    Damgaard, Kawamoto, Shigemoto ('84); Ilgenfritz, Kripfganz ('85); Faldt, Petersson ('86); Bilic, Karsch, Redlich ('92); Fukushima ('04); Nishida ('04); Miura, Nakano, AO, Kawamoto ('09); Nakano, Miura, AO ('10, '11)



Ilgenfritz, Kripfganz ('85)



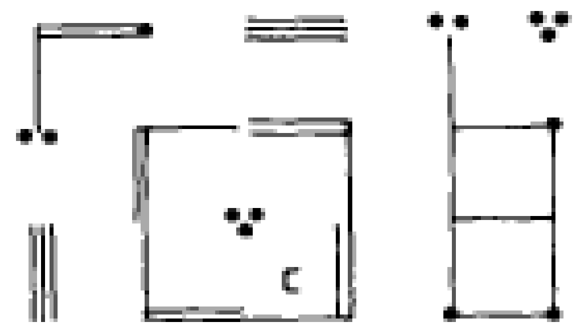
Fukushima ('04)



AO, Miura, Nakano, Kawamoto ('09)

SC-LQCD with Fluctuations

- **Monomer-Dimer-Polymer (MDP) simulation**
  Mutter, Karsch ('89), de Forcrand, Fromm ('10), de Forcrand, Unger ('11)
  - **Integrating out all links**
    → Z= weight sumof monomer, dimer, polymer configurations



$$Z(m,\mu) = \sum_{\{n_x,n_b,C_B\}} \prod_b \frac{(N_c - n_b)!}{N_c! n_b!} \prod_x \frac{N_c!}{n_x!}(2m)^{n_x} \prod_{C_B} w(C_B) \qquad w(C_B,\pm) = \varepsilon(C_B)\exp(\pm 3\ell L_t \mu)$$

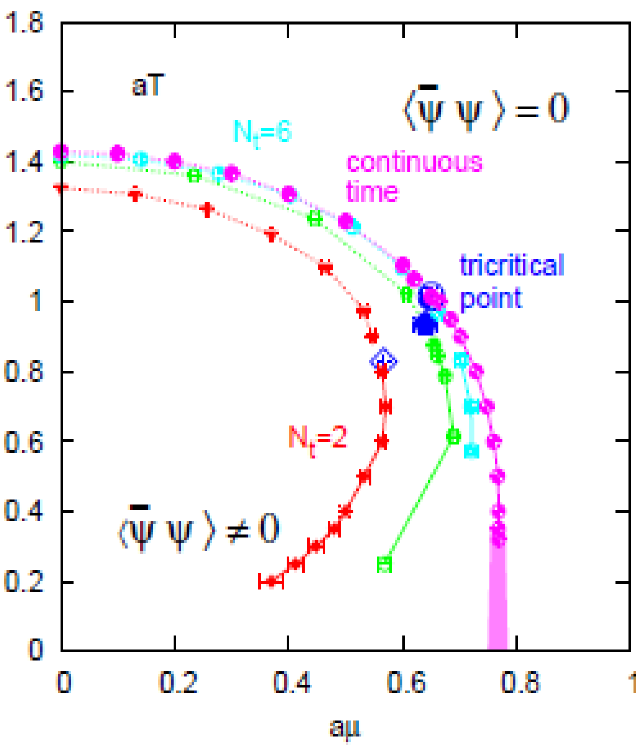
- **Auxiliary Field Monte-Carlo (AFMC) method**
  Ichihara, AO, Nakano ('14)
  - **Bonsonize the effective action, and MC integral over aux. field.**

$$S_{\text{eff}} = S_F^{(t)} + \sum_x m_x M_x + \frac{L^3}{4N_c} \sum_{\boldsymbol{k},\tau} f(\boldsymbol{k}) \left[|\sigma_{\boldsymbol{k},\tau}|^2 + |\pi_{\boldsymbol{k},\tau}|^2 \right]$$

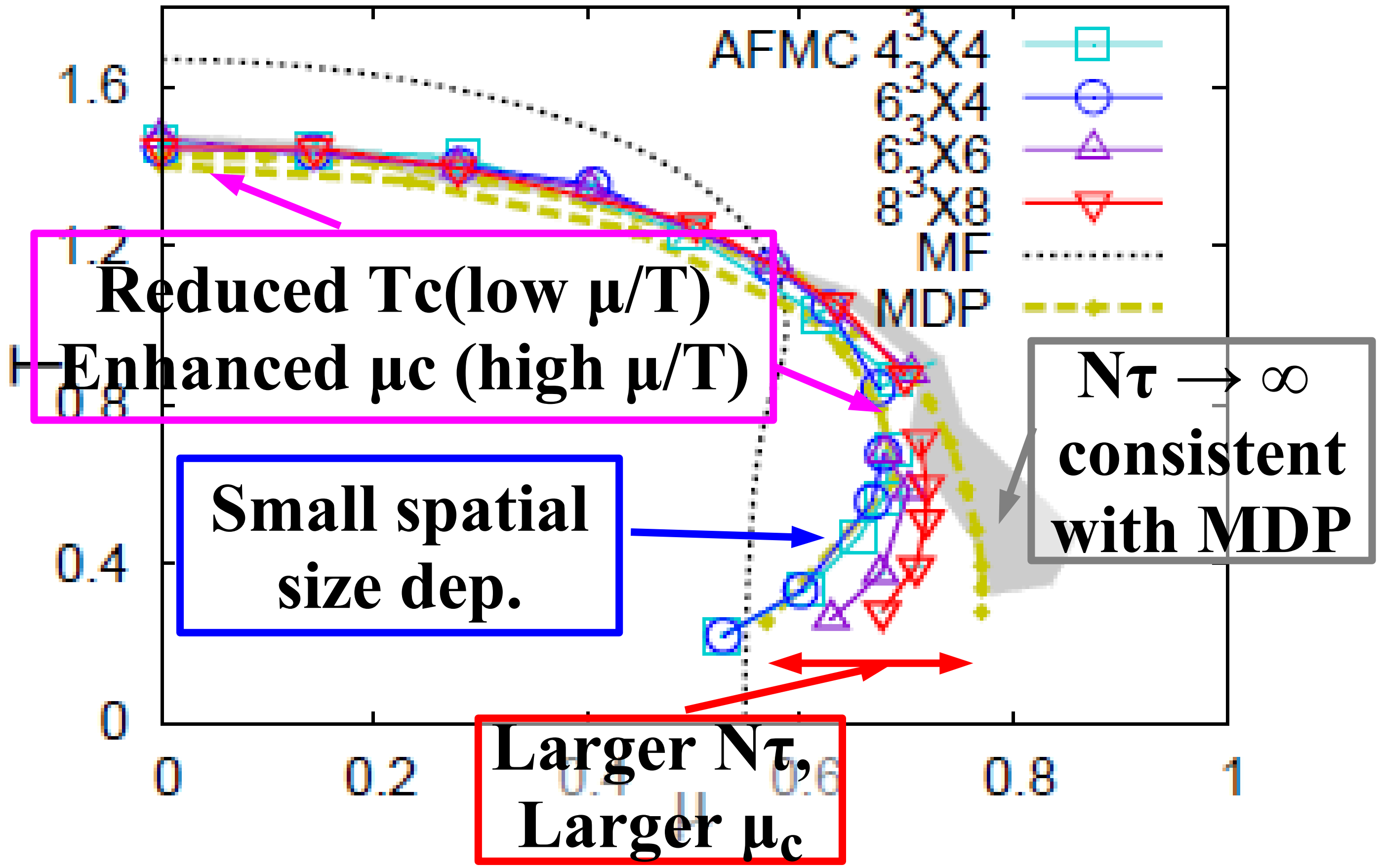
$$m_x = m_0 + \frac{1}{4N_c} \sum_j (\sigma + i\varepsilon\pi)_{x\pm\hat{j}}, \quad f(\boldsymbol{k}) = \sum_j \cos k_j, \quad \varepsilon = (-1)^{x_0+x_1+x_2+x_3}$$

Phase diagram

- **Phase diagrams in two independent methods (MDP & AFMC) agree with each other in the strong coupling limit.**
  → SCL phase diagram is determined !



de Forcrand, Fromm ('10), de Forcrand, Langelage, Philipsen, Unger ('14)



Ichihara, AO, Nakano ('14)

Cumulant Ratio: Phase transition signal ?

- **Cumulants** — **c.f. Kaczmarek**

$$\chi^{(n)} = \frac{\partial^n (P/T^4)}{\partial \hat{\mu}^n}, \quad \hat{\mu} = \mu_B / T$$

$$\chi^{(4)}/\chi^{(2)} = \kappa\sigma^2 \quad (\kappa : \text{kurtosis})$$

  - **$\kappa\sigma^2$ shows DOF at μ=0, and criticality at μ>0.**

- **Lattice MC at μ=0**
  Bazarov, .., Kaczmarek, et al.('14), Bellwied et al.('13), …. Gavai, Gupta ('05), Allton et al. ('05),

- **Lattice MC at μ>0 but large m_q**
  Jin, Kuramashi, Nakamura, Takeda, Ukawa ('15)

- **Scaling function analysis**
  Friman, Karsch, Redlich, Skokov ('11)

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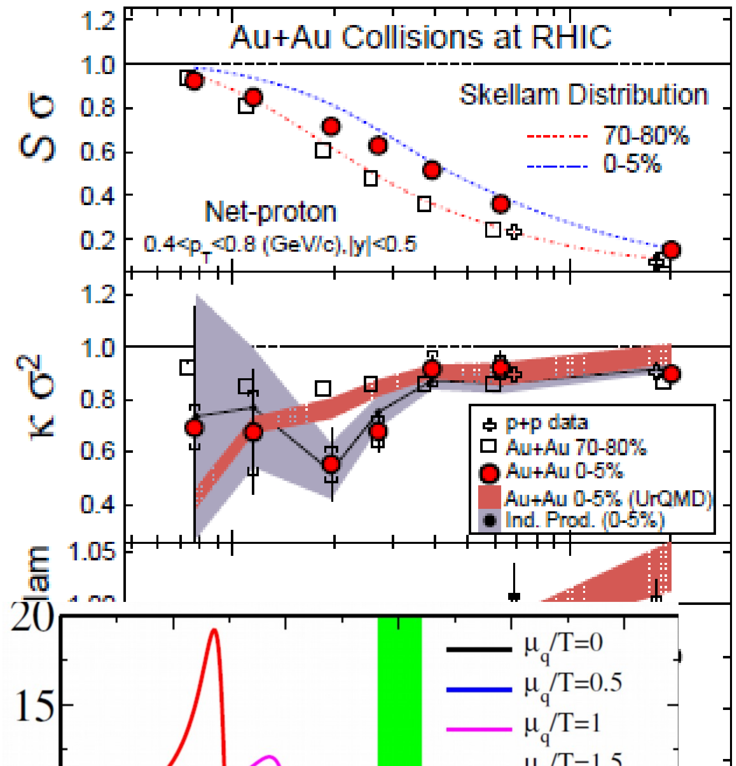
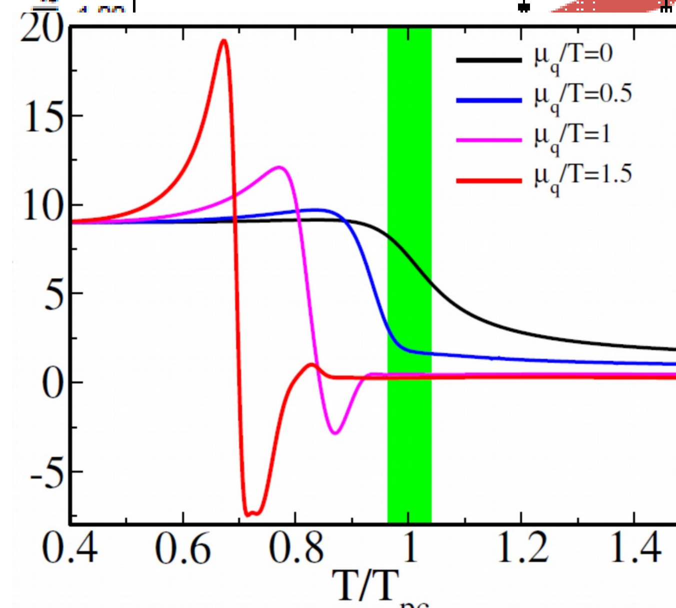
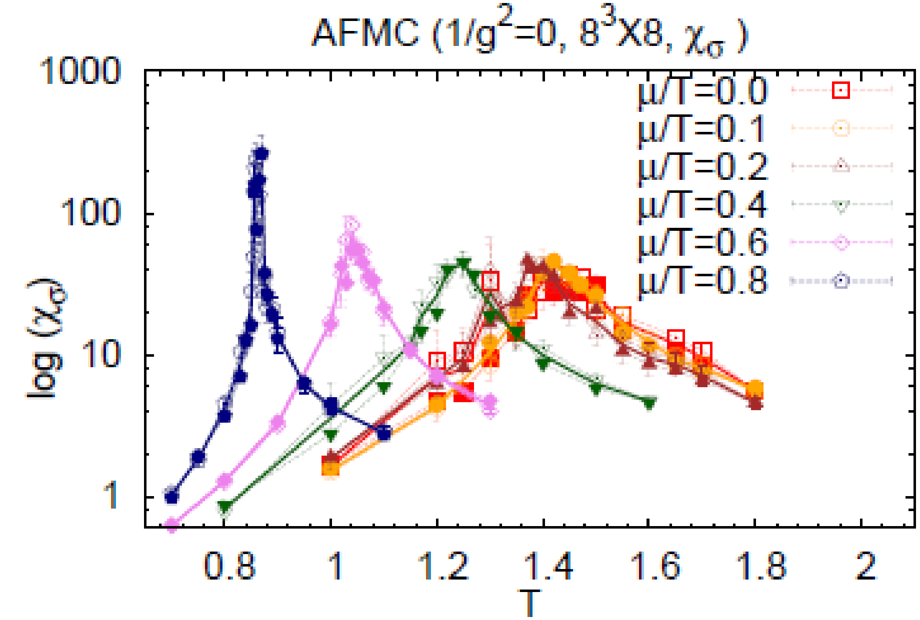




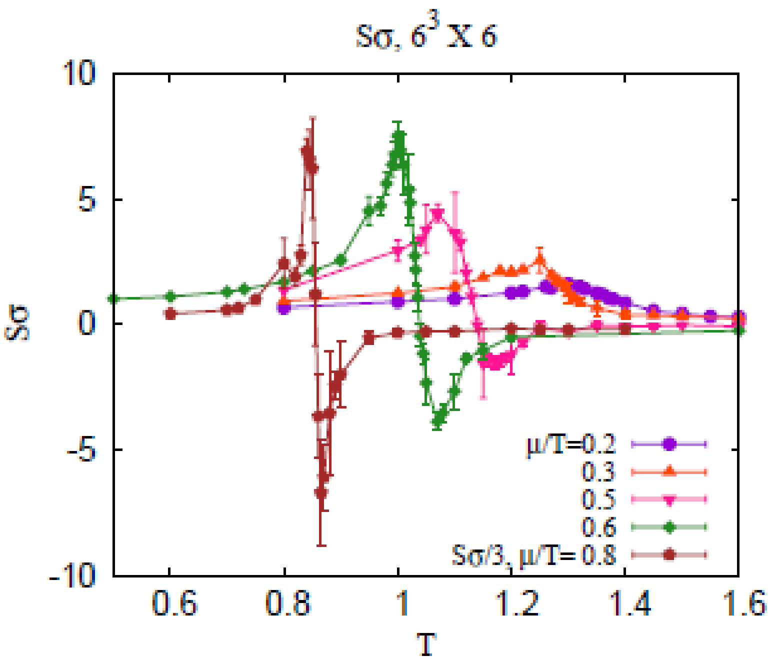
Friman et al. ('11)

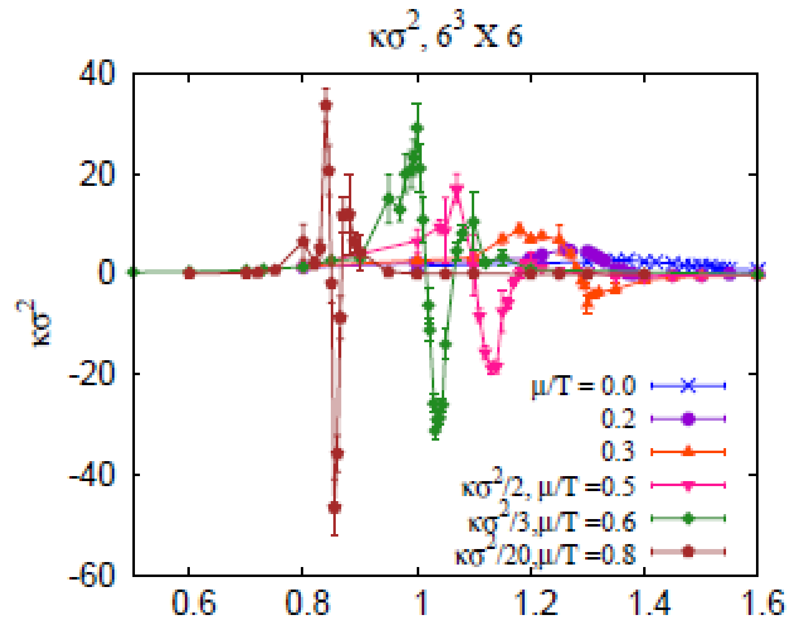
Susceptibilities, Skewness, Kurtosis, ...

- **Chiral susceptibility**
  → Divergent at V → ∞
- **Net baryon skewness**
  Sσ → +∞ from below
  - ∞ from above
- **Net baryon kurtosis**
  $\kappa\sigma^2$ → +-+ structure



Ichihara, AO, Nakano ('14)





Ichihara, Morita, AO, in prep.

Caveats

- One species of unrooted staggered fermion corresponds to Nf=4 in the continuum limit, and should show the first order phase transition at $\mu=0$. Second order transition shown here comes from O(2) chiral symmetry remaining also at coarse lattice spacing.
- We have worked in the leading order of 1/d expansion, where the MM term is assumed to remain finite at large spatial dim., d. Under this assumption, we quark field scales as $\chi \propto d^{-1/4}$, then terms with larger number of quarks such as spatial baryon hopping are suppressed. (MDP includes those terms.)
- Positive slope of the first order phase boundary comes from the saturated quark matter at high density, $\rho \sim$ Nc. In this case, entropy is carried by the holes rather than particles, and can be smaller in the high density phase. Thus the Clausius-Clapeyron relation is not violated.

$$P_H = P_Q \rightarrow \rho_H d\mu + s_H dT = \rho_Q d\mu + s_Q dT$$

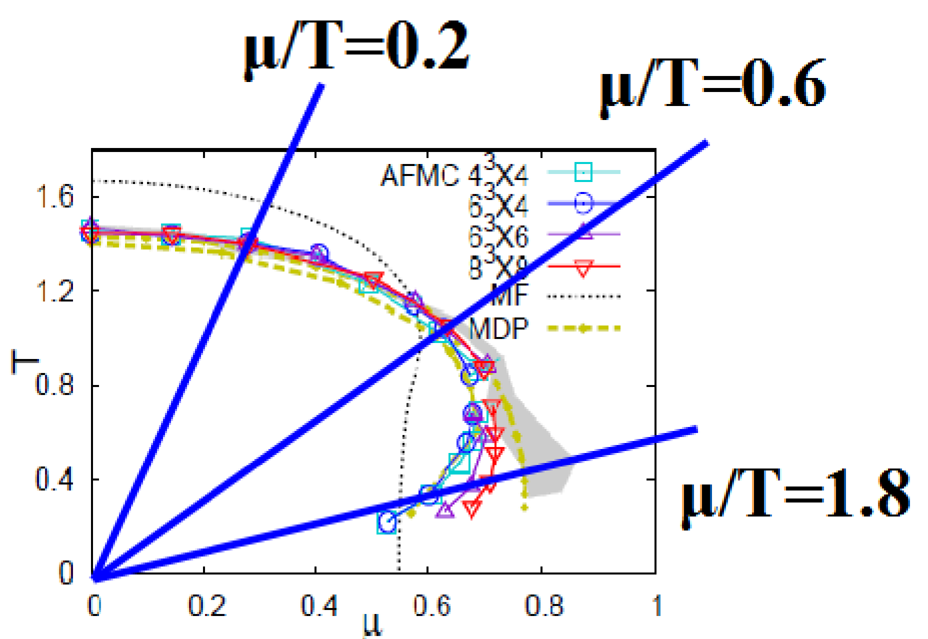
- The sign problem exists in SC-LQCD when fluctuations are included, but it is not very severe and $V \rightarrow \infty$ limit may be obtained.

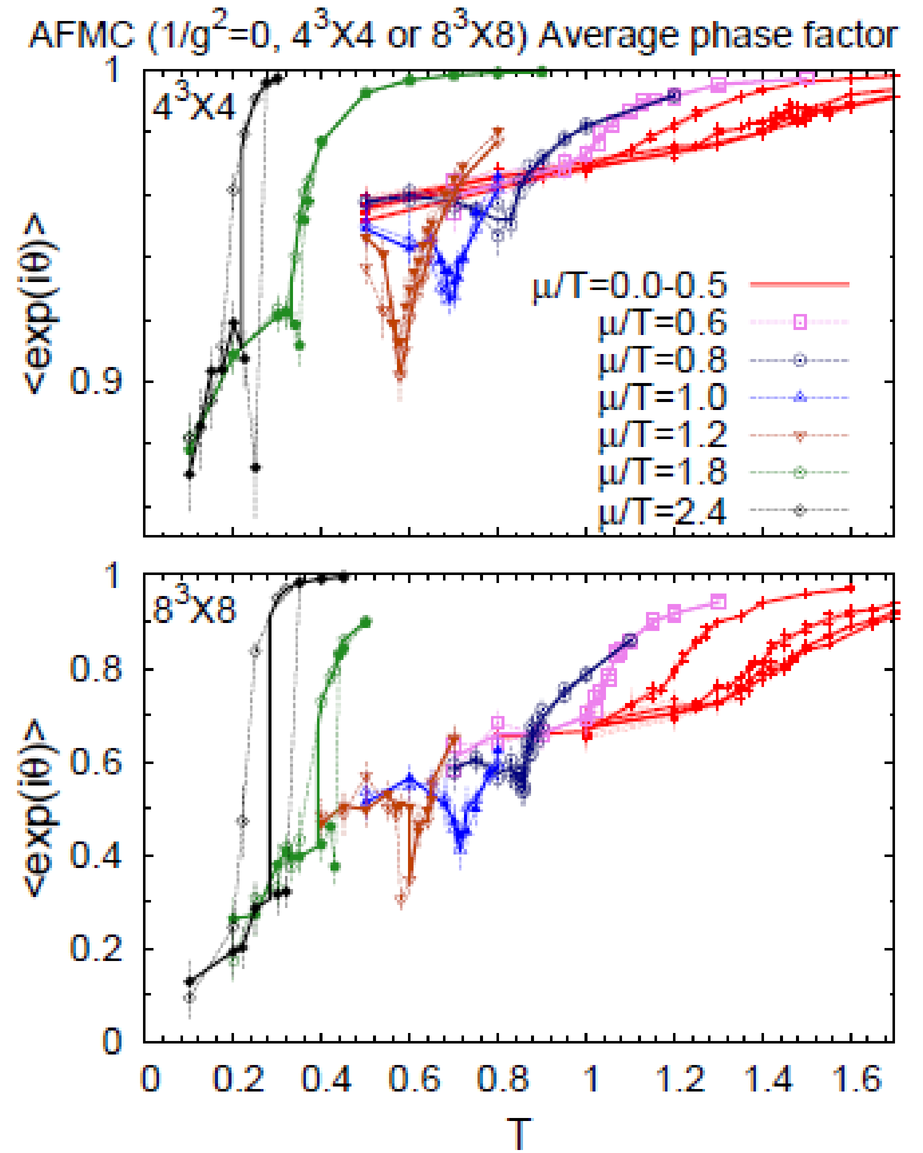
Average Phase Factor

- **Average phase factor = Weight cancellation**

$$\langle e^{i\theta}\rangle = Z_{\text{phase quenched}} \;/\; Z_{\text{full}}$$

- **AFMC results**
  - $\langle e^{i\theta}\rangle > 0.9$ on 4^4 lattice
  - $\langle e^{i\theta}\rangle > 0.1$ on 8^4 lattice

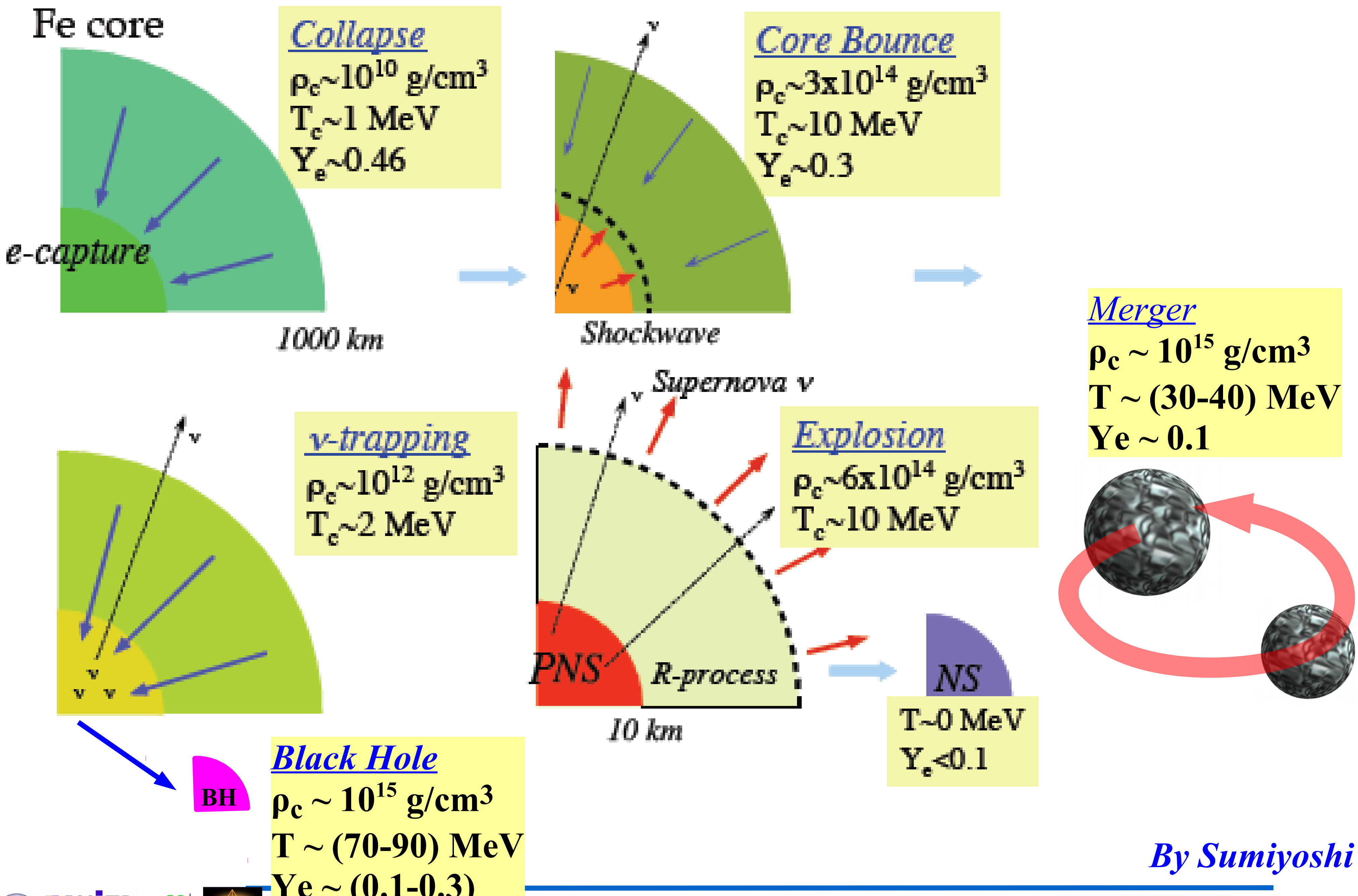




Ichihara, AO, Nakano ('14)

Dense Matter in Compact Star Phenomena

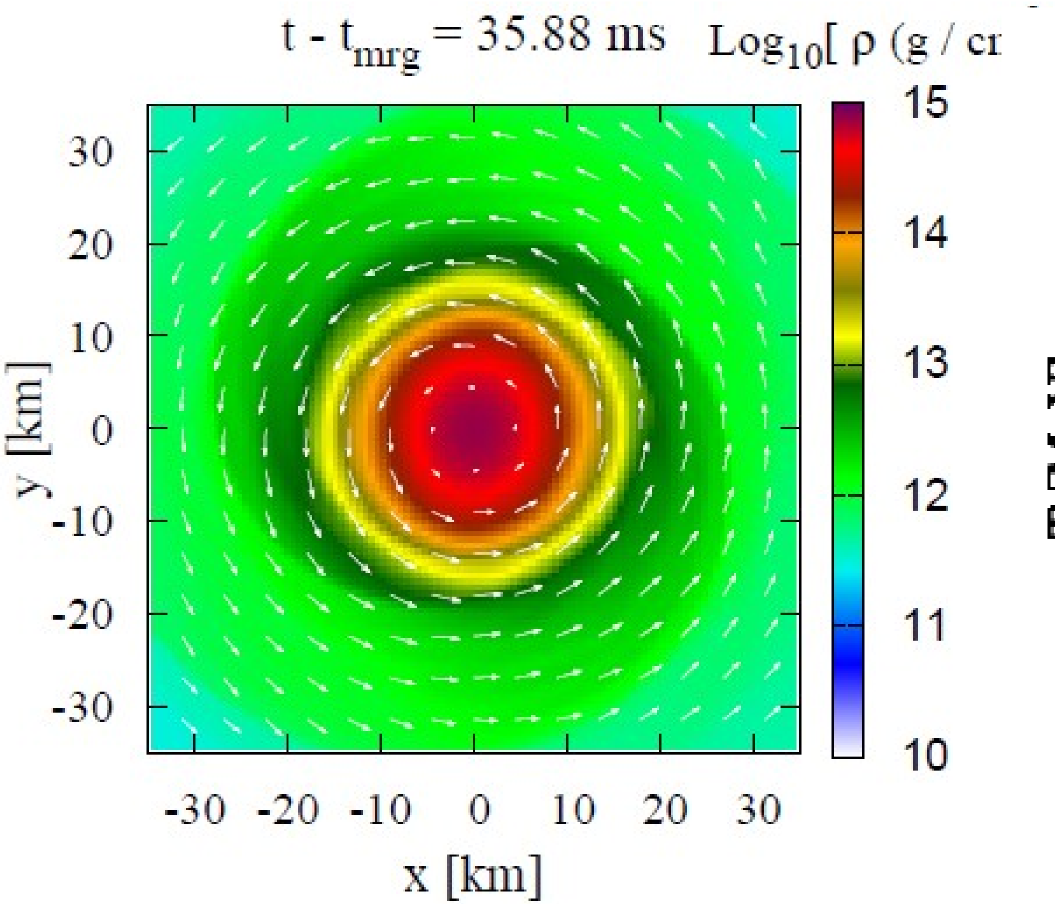
Gravitational Collapse of Massive Star

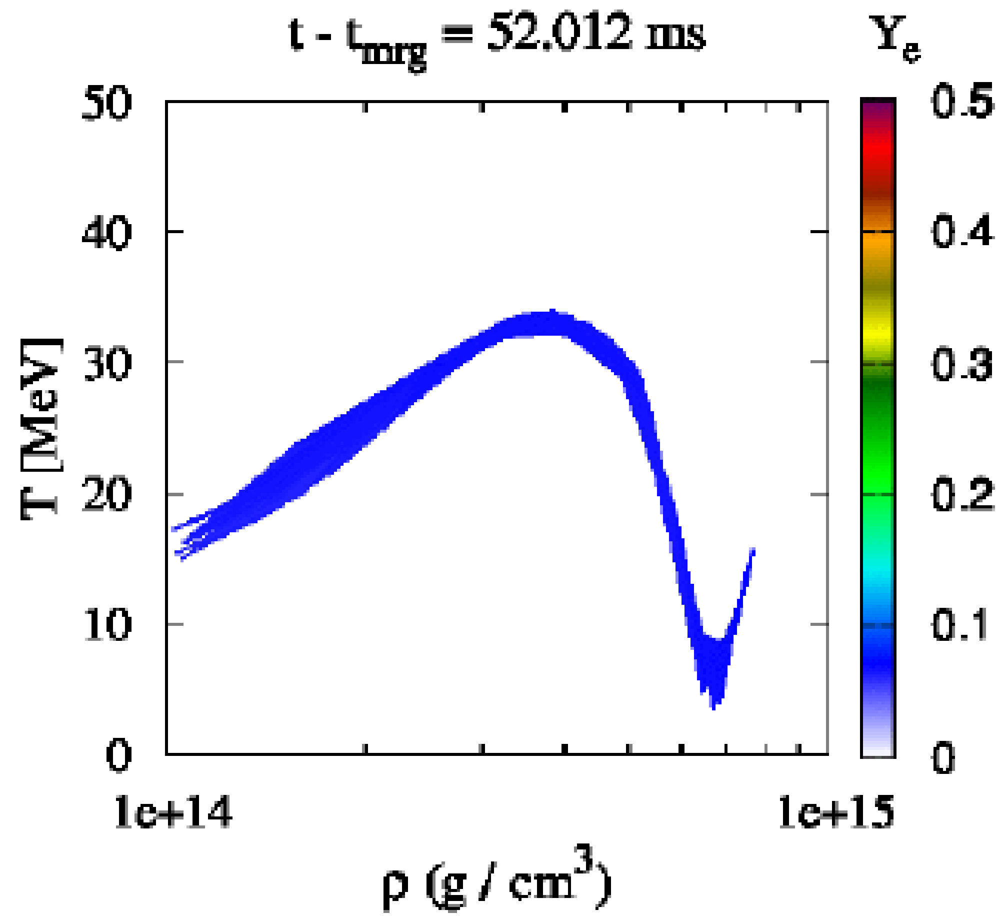


By Sumiyoshi

Binary Neutron Star Merger

- **T ~ 40 MeV, $\rho_B \sim 10^{15}$ g/cm^3 ~ 4 ρ_0 ($\rho_0 \sim 2.5 \times 10^{14}$ g/cm^3), Ye ~ 0.1**





Courtesy of K. Kiuchi
Data are from Y. Sekiguchi, K. Kiuchi, K. Kyotoku, M. Shibata, PRD91('15)064059.

Quark Matter in Compact Stars

- **Neutron Star**
  E.g. N. Glendenning, "Compact Stars";F. Weber, Prog.Part.Nucl.Phys.54('05)193
  - **Cold (T~0), Dense (ρ_B~5 ρ_0), Highly Asymmetric (Y_p ~ (0.1-0.2))**
- **Supernova** *T. Hatsuda, MPLA2('87)805; I. Sagert et al., PRL102 ('09) 081101.*
  - **Warm (T~20 MeV), Dense (ρ_B~1.8 ρ_0), mildly asym. (Y_p~ (0.3-0.4))**
- **Binary Neutron Star Merger**
  Sekiguchi, Kiuchi, Kyotoku, Shibata, PRD91('15)064059.
  - **Hot (T~30-40 MeV), Dense (ρ_B~(4-5) ρ_0), Highly Asymmetric (Y_p ~ (0.1-0.2))**
- **Dynamical black hole formation**
  K. Sumiyoshi, et al., PRL97('06) 091101;K.Sumiyoshi, C.Ishizuka, AO, S.Yamada, H.Suzuki, ApJL690('09),L43; Nakazato et al. ('10); Hempel et al. ('12); ...
  - **Hot (T~(70-90)MeV), Dense(ρ_B~(4-5)ρ_0), and Asymmetric (Y_p ~ (0.1-0.3))**

Comparison of conditions in NS, SN, and HIC

M. Hempel

	neutron stars	supernovae	heavy ion collisions
dynamic timescales	(d - yrs)	ms	fm/c
equilibrium	full	weak eq. only partly	only strong eq.
temperatures	0	0 - 100 MeV	10 - 200 MeV
charge neutrality	yes	yes	no
asymmetry	high	moderate	low
highest densities	$< 9\ \rho_0$	$< 2\text{-}4\ \rho_0$	$< 4\text{-}5\ \rho_0$

weak equilibrium

$\mu_i = B_i\mu_B + Q_i\mu_Q + L_i\mu_L$; $\mu_S=0$

charge neutrality:

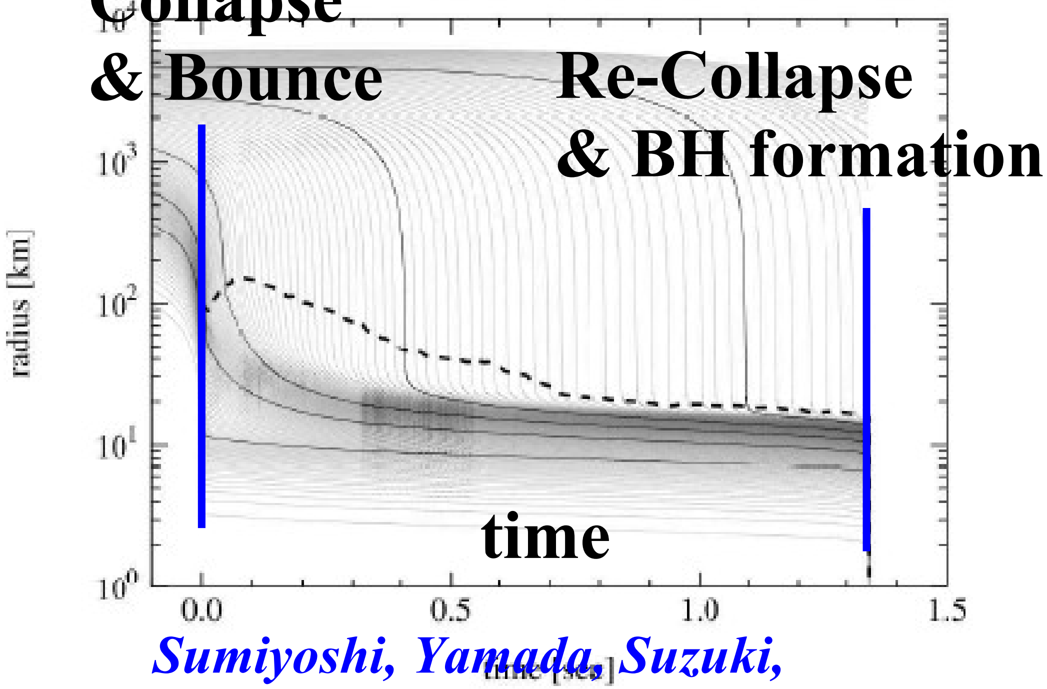
$Y_Q = Y_e + Y_\mu \Leftrightarrow n_Q = n_e + n_\mu$

- matter in SN: no weak equilibrium, finite temperature

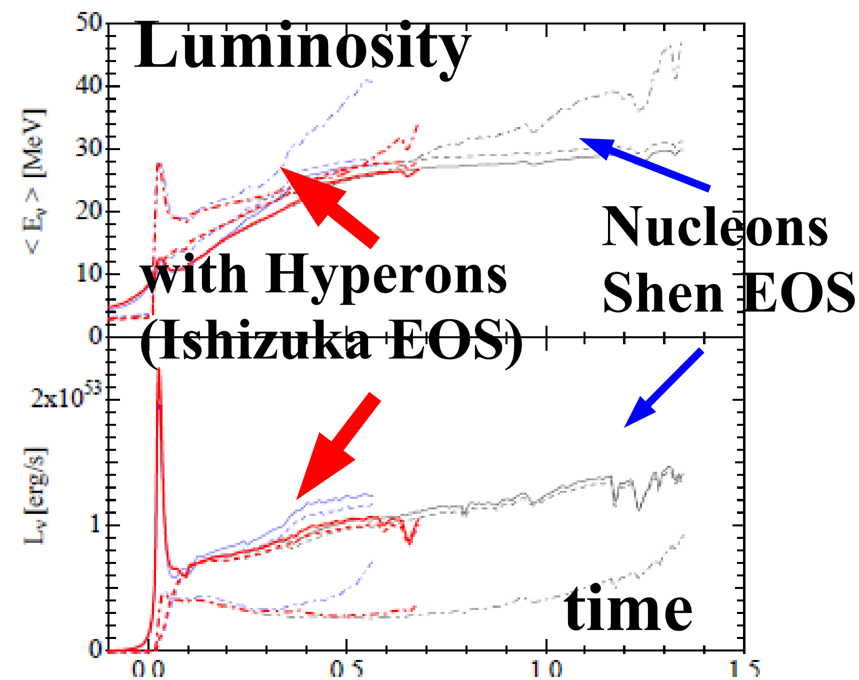
  → somewhere between cold neutron stars and heavy-ion collisions

Dynamical Black Hole Formation

- **Gravitational collapse of heavy (e.g. 40 $M_\odot$) progenitor would lead to BH formation.**
  - **Shock stalls, and heating by ν is not enough to take over strong accretion. → failed supernova**
  - **ν emission time ~ (1-2) sec w/o exotic matter.**
  - **emission time is shortened by exotic dof (quarks, hyperons, pions).**



Sumiyoshi, Yamada, Suzuki, Chiba, PRL 97('06)091101.

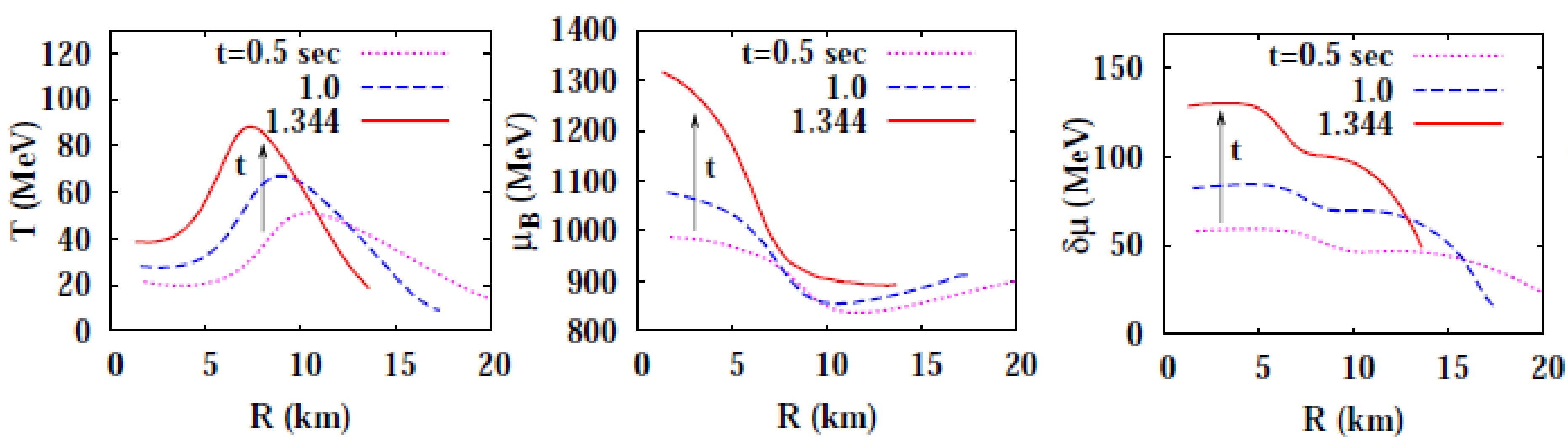


Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, ApJL 690('09)43.

Thermal Condition during BH formation

- **Quark-hadron and nuclear physicists are interested in (T, μ) !**
  - **Maximum T ~ 90 MeV (off-center)**
    (Heated by shock propagation)
  - **Maximum μ_B ~ 1300 MeV (center)**
  - **Maximum $\delta\mu = (\mu_n - \mu_p)/2$ ~ 130 MeV (center)**

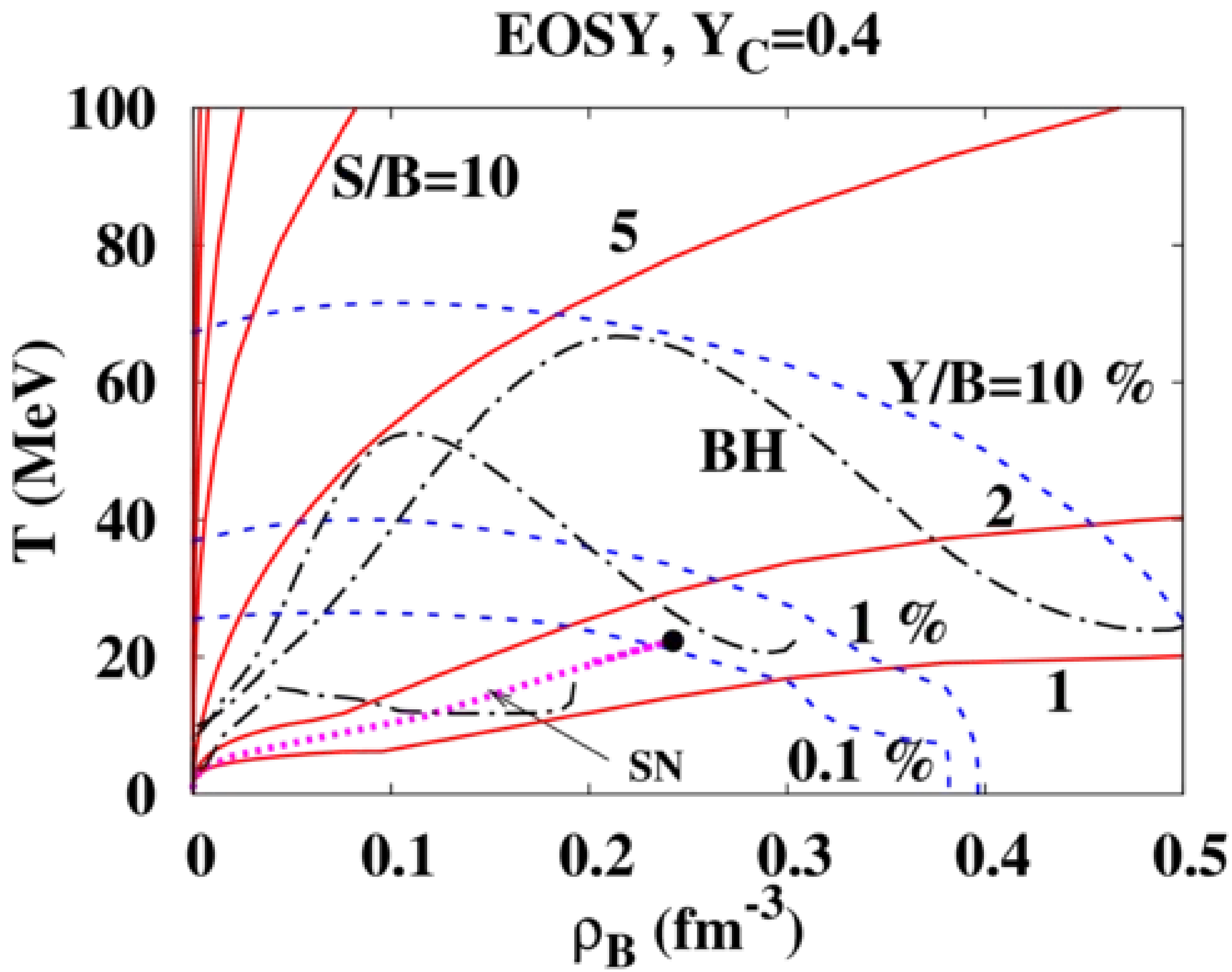
Can we reach CP ? What is the effects of δμ ?



Nucleon+leptons+photon (Shen EOS), 40 Msun

AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284

Thermal Condition during BH formation



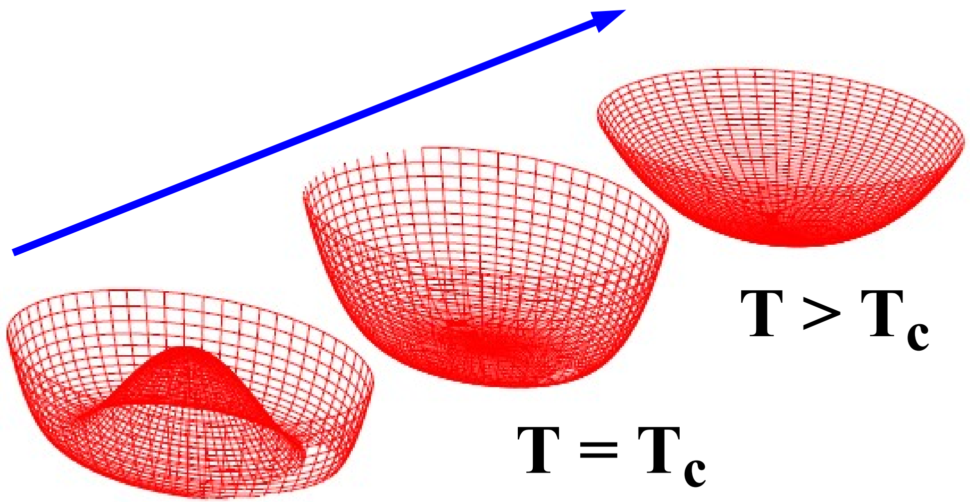
Ishizuka,AO, Tsubakihara, Sumiyoshi, Yamada, JPG 35('08) 085201;
AO et al., NPA 835('10) 374.

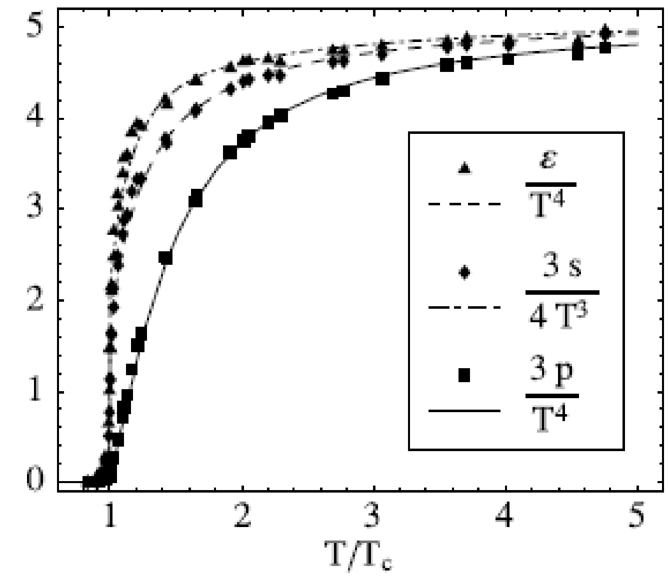
Chiral Effective Models

- **Chiral Effective models: NJL, PNJL, PQM**
  NJL=Nambu-Jona-Lasinio model,
  PNJL=Polyakov loop extended NJL,
  PQM=Pol. loop ext. Quark Meson model

Nambu, Jona-Lasinio ('61), Fukushima('03), Ratti, Thaler, Weise ('06), B.J.Schafer, Pawlowski, Wambach ('07); Skokov, Friman, E.Nakano, Redlich('10)

  - **Spontaneous breaking & restoration of chiral symmetry**
  - **Polyakov loop extension → Deconf. transitions**





Roessner et al.('07)

Chiral Effective Models (N_f=2)

- **Lagrangian (PQM, as an example)**

$$L=\bar{q}\left[i\gamma^\mu D_\mu - g_\sigma\left(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\right]q + \frac{1}{2}\partial^\mu\sigma\,\partial_\mu\sigma + \frac{1}{2}\partial^\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}$$

q-Pol. (D_μ) **quark-meson** ($\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}$)

$$-U_\sigma(\sigma,\pi) - U_\Phi(\Phi,\bar{\Phi})$$

chiral (U_σ) **Polyakov** (U_Φ)

$$F_{\rm eff}\equiv\Omega/V = U_\sigma(\sigma,\pi=0) + U_\Phi(\Phi,\bar{\Phi}) + F_{\rm therm} + U_{\rm vac}(\sigma,\Phi,\bar{\Phi})$$

particle exc. ($F_{\rm therm}$) **q zero point** ($U_{\rm vac}$)

- **Polyakov loop effective potential from Haar measure**

  U_Φ ~ -log (Haar Measure) **(Fit lattice data to fix parameters).**

- **Vector coupling is not known well → Comparison of g_V/g_S=0, 0.2**

$$L_V = -g_v\bar{q}\gamma_\mu\left(\omega^\mu + \boldsymbol{\tau}\cdot\boldsymbol{R}^\mu\right)q - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{2}m_\rho^2 R_\mu R^\mu$$

- **8 Fermi interaction**

T. Sasaki, Y. Sakai, H. Kouno, M. Yahiro ('10)

$$G_{\sigma 8}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]^2$$

Model Details

- **BH formation calculation**

  Sumiyoshi, Yamada, Suzuki, Chiba, PRL 97('06)091101.

  - **ν radiation 1D (spherical) Hydrodynamics**
  - **ν transport is calculated exactly by solving the Boltzmann eq.**
  - **Gravitational collapse of 40 $M_\odot$ star**
  - **Initial condition: WW95**
    S.E.Woosley, T.A.Weaver, ApJS 101 ('95) 181
  - **Shen EOS (npeμ)**
- **QCD effective models**
  - **NJL, PNJL, PNJL with 8 quark int., PQM**
  - **N_f=2**
  - **Vector coupling → G_V/G_s (g_V/g_s in PQM)=0, 0.2**

Isospin chemical potential

- **Isospin chemical potential δμ**

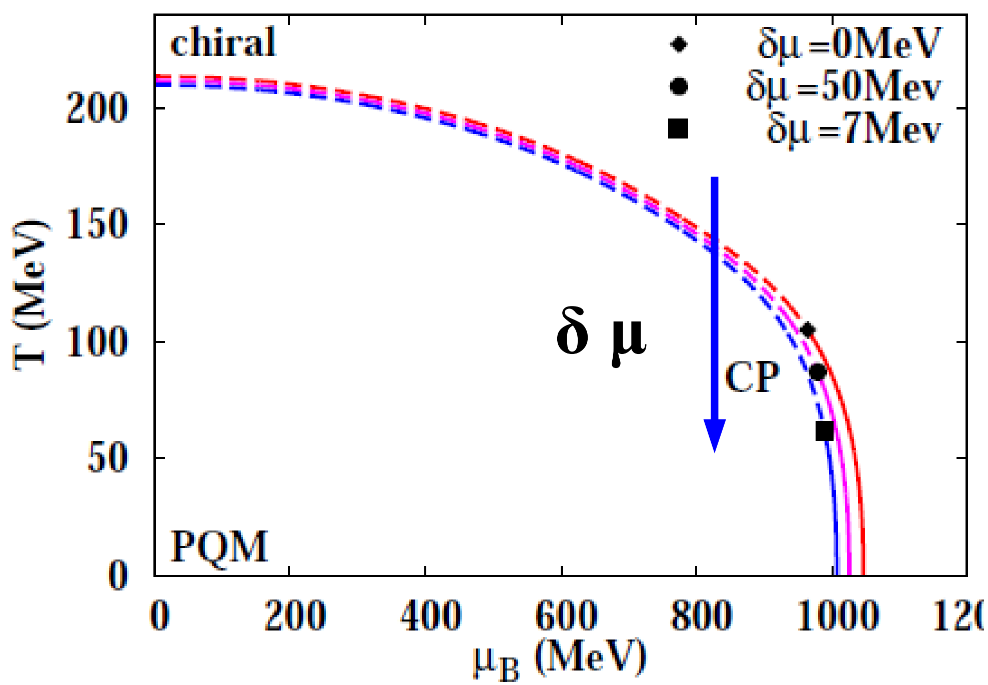
  $\delta\mu = (\mu_d - \mu_u)/2 = (\mu_n - \mu_p)/2 \rightarrow \mu_d = \mu_q + \delta\mu,\ \ \mu_u = \mu_q - \delta\mu$

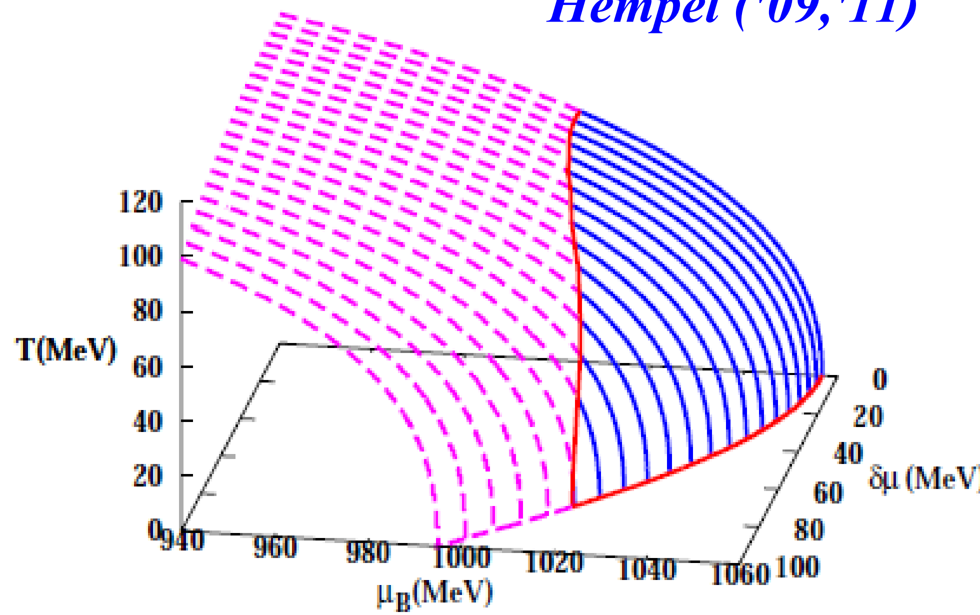
  - **Finite δμ → (Isospin) Asymmetric matter** $N_u \neq N_d$

    → Smaller "Effective" number of flavors

    → Weaker phase transition → smaller T_{CP}

c.f. Hempel's Lec.
Sagert, Pagliara, Schaffner-Bielich, Hempel ('09,'11)

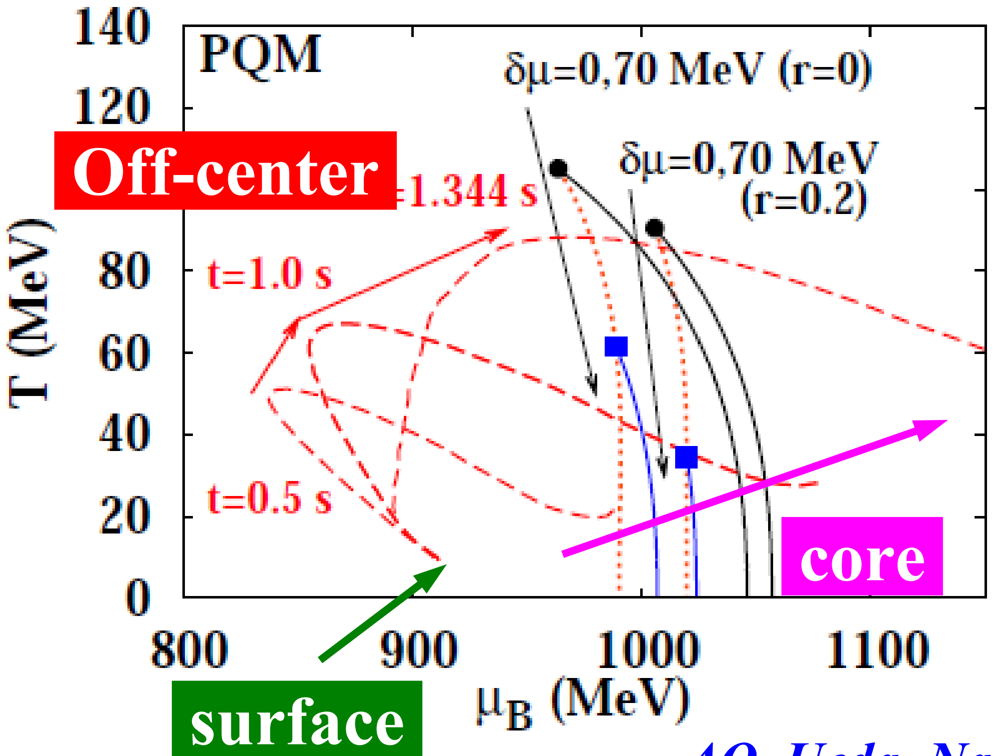


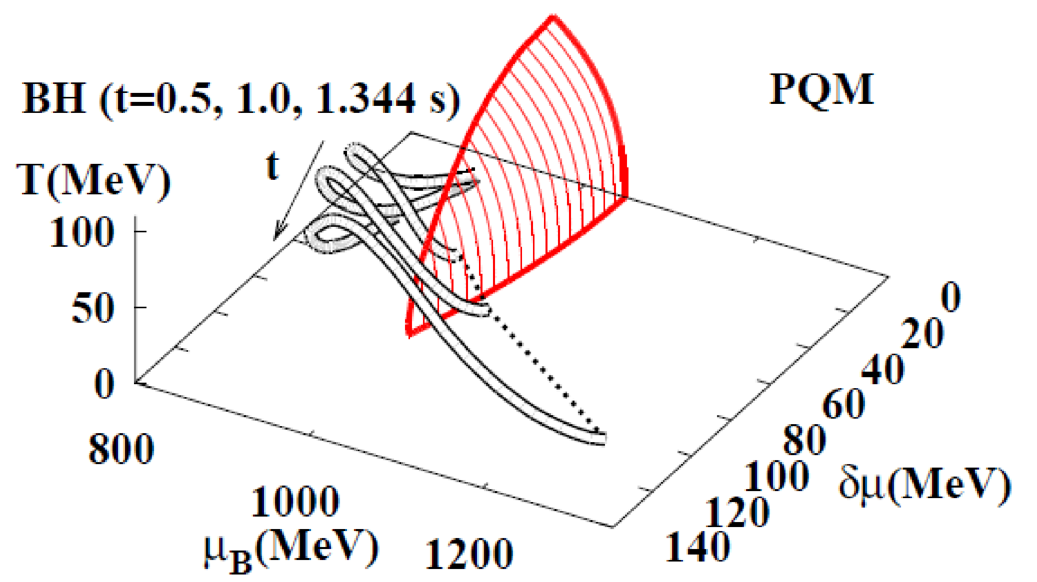


AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284
H. Ueda, T. Z. Nakano, AO, M. Ruggieri, K. Sumiyoshi, PRD88('13),074006

How is quark matter formed during BH formation ?

- **Highest μ_B just before horizon formation ~ 1300 MeV**
  > QCD transition μ (1000-1100 MeV)
  → *Quark matter is formed before BH formation*
- **Core evolves below CP, Off-center goes above CP**
  → *CP sweep*

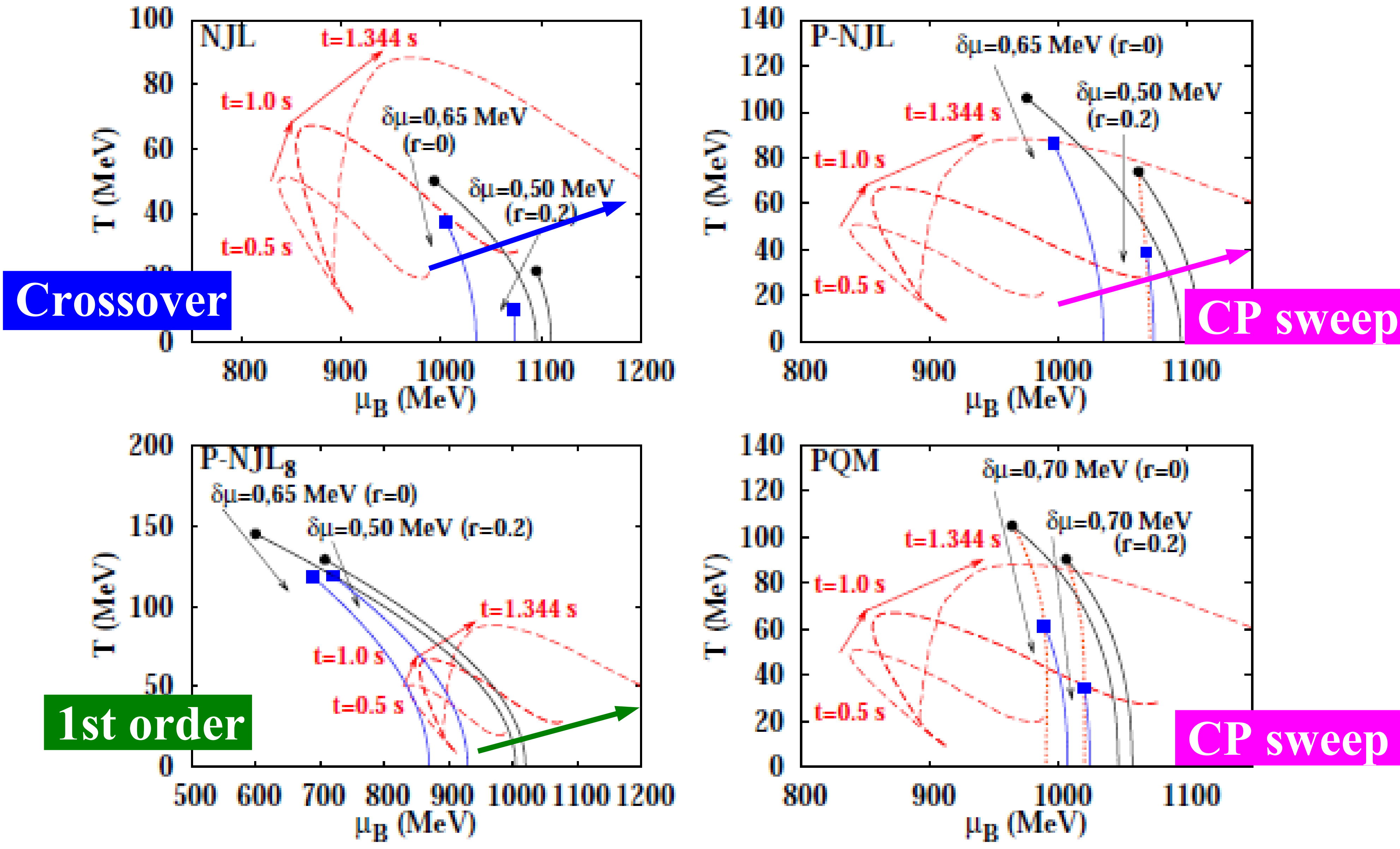




AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284

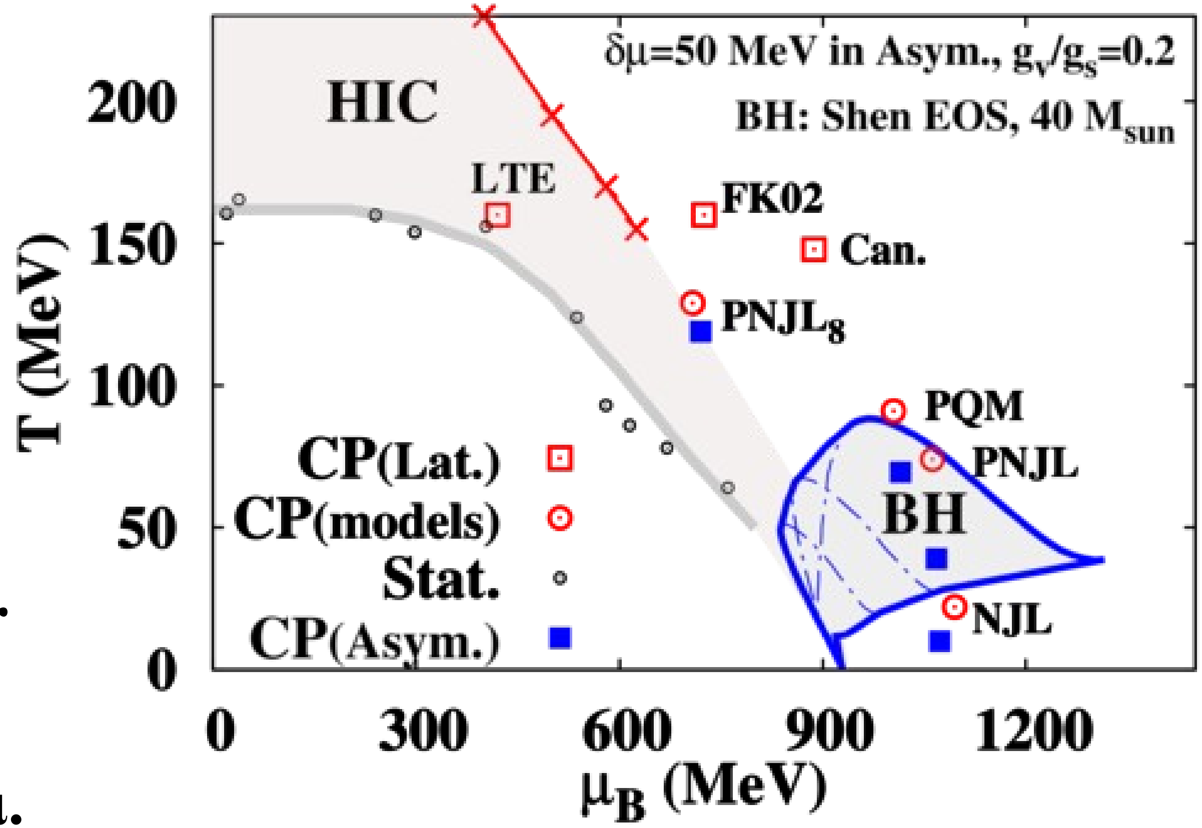
How is quark matter formed during BH formation ?

- **Model dependence to form quark matter → Three ways**



Probed Region of Phase Diagram during BH formation

- **CP location in Symmetric Matter**
  - **Lattice QCD** μ_{CP}=(400-900) MeV
  - **Effecitve models** μ_{CP}=(700-1050) MeV
- **CP in Asymmetric Matter (E.g. $\delta\mu$=50 MeV)**
  - **T_{CP} decreases at finite $\delta\mu$.** → **Accessible (T, μ_B) region during BH formation**

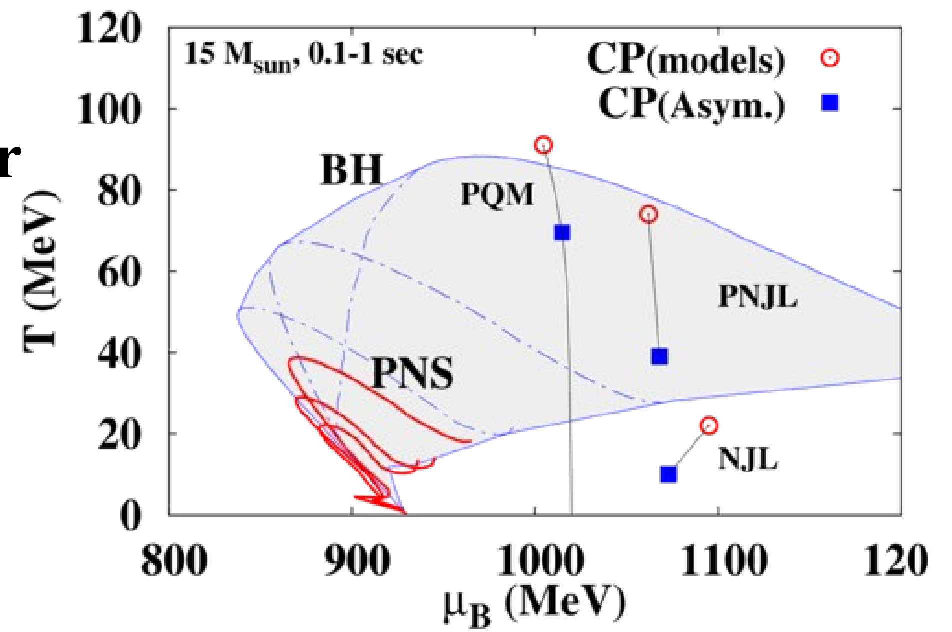


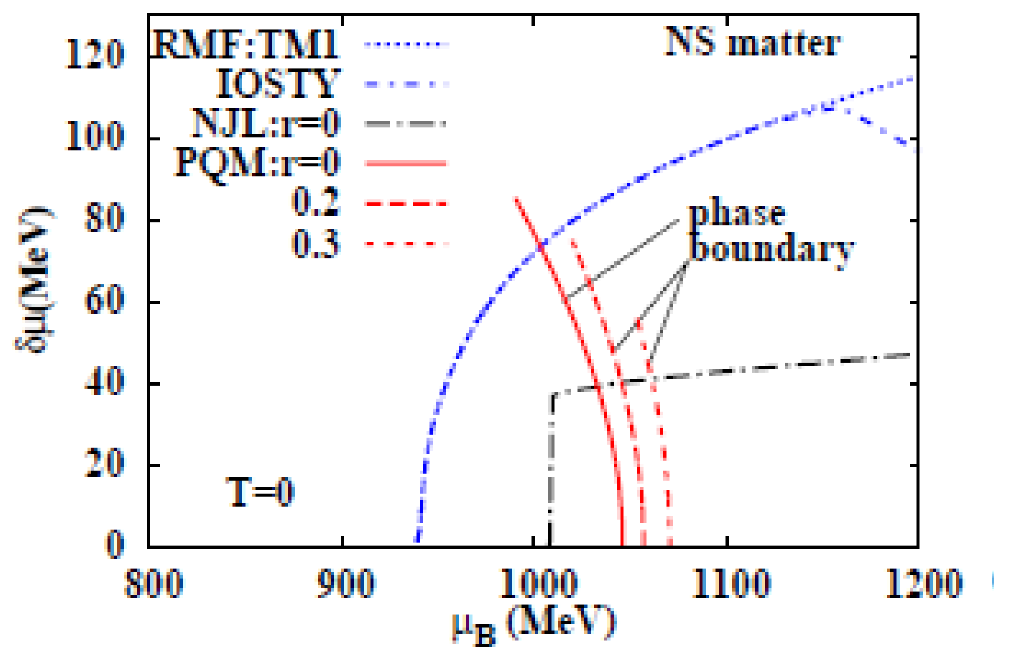
(Stephanov plot)

M.A.Stephanov, Prog.Theor.Phys.Suppl.153 ('04)139;
FK02:Z. Fodor, S.D.Katz, JHEP 0203 (2002) 014
LTE:S. Ejiri et al., Prog.Theor.Phys.Suppl. 153 (2004) 118;
Can: S. Ejiri, PRD78 (2008) 074507
Stat.:A. Andronic et al., NPA 772('06)167

How about Neutron Stars ?

- Contraction of Proto-Neutron Star
  - (T, μ_B) are not enough at 1 sec after bounce of 15 $M_\odot$ star collapse
  - Larger (T, μ_B) is expected in long time evolution (~20 sec) or heavier proto-neutron stars.
    K. Sumiyoshi et al. ApJ 629 ('05) 922;
    J. A. Pons et al., ApJ 513 ('99)780;
    J. A. Pons et al., ApJ 553 ('01) 382.
- Cold Neutron Star
  - max. $\delta\mu$~ 100 MeV
  - Possibility of cross over in NS

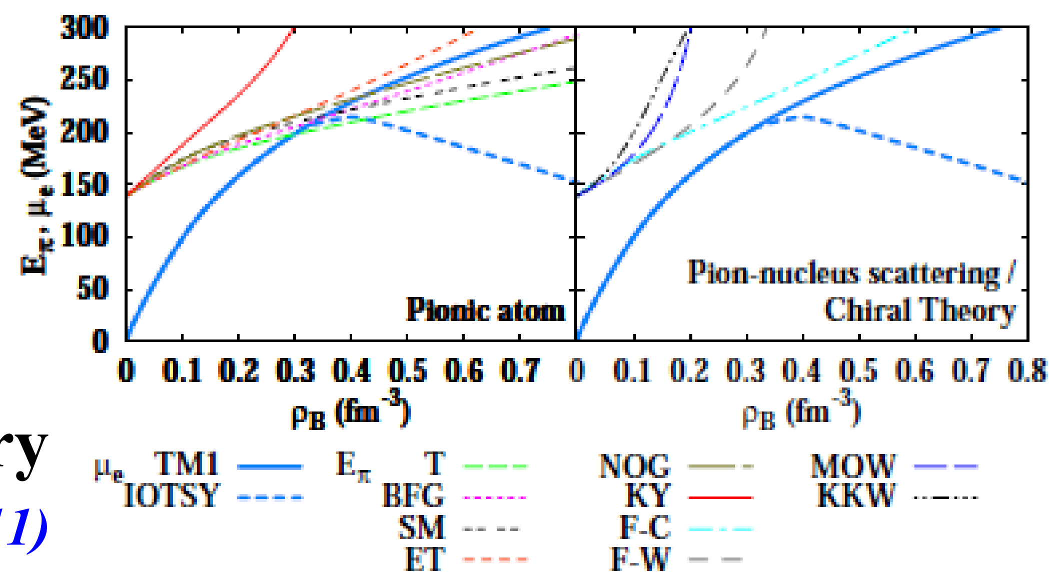




H. Ueda et al.('13)

Discussion

- **How can we observe the phase transition signal ?**
  - **ν spectrum ? Gravitational waves ?**
    Supernova: Second peak in ν & $\bar{\nu}$ emission
    Hatsuda('87), Sagert et al.('09)
- **How frequent do dynamical BH formation take place ?**
  - **Less frequent than SN (< 20 $M_\odot$), but should be in collapse of heavy stars (>40 $M_\odot$).**
    C.L.Fryer, ApJ 522('99)413; E.O'Connor, C.D.Ott, ApJ 730('11)70
- **Strangeness may reduce δμ in hadronic / quark matter**
  - **No s-wave π cond. in NS**
    AO, D. Jido, T. Sekihara, K. Tsubakihara, PRC80('09)038202.
- **Hadron-Quark EOS is necessary**
  E.g. Steinheimer,Schramm,Stocker('11)



Summary of Lecture 2

- While we have the sign problem in lattice QCD at finite μ, the phase diagram study is on going using various ideas.
  I have shown recent results based on the strong-coupling lattice QCD.
  - Smaller weight cancellation allow us to study phase transition at high density.
  - Phase diagram in the strong coupling limit has been confirmed. (Results from MDP and AFMC methods agree.)
  - Cumulant ratio would be interesting !
- Compact stars are also good laboratories of dense matter.
  - NS, SN, BH, BNSM → Dense, Cold/Hot, Isospin asymmetric matter
  - With the first order boundary (and CP) and isospin chem. pot, there are many ways of realizing phase transition in compact star phenomena.

Summary

- **Dense matter is "terra incognita", and there are many unsolved problems.**
- **In heavy-ion collisions at $\sqrt{s}$ = 5-10 A GeV, we expect formation of highest baryon density matter, whose density exceeds 5 ρ_0.**
  In equilibrium, this would be above the transition density.
- **In compact star phenomena, hydro simulations with hadronic matter EOS suggest the formation of dense matter (4-5 ρ_0, $\mu_B \sim$ 1300 MeV), which is above the transition density in many effective models.**
- **We need more experimental, observational, and theoretical works to explore dense matter.**