Neutrino Processes in Neutron Stars

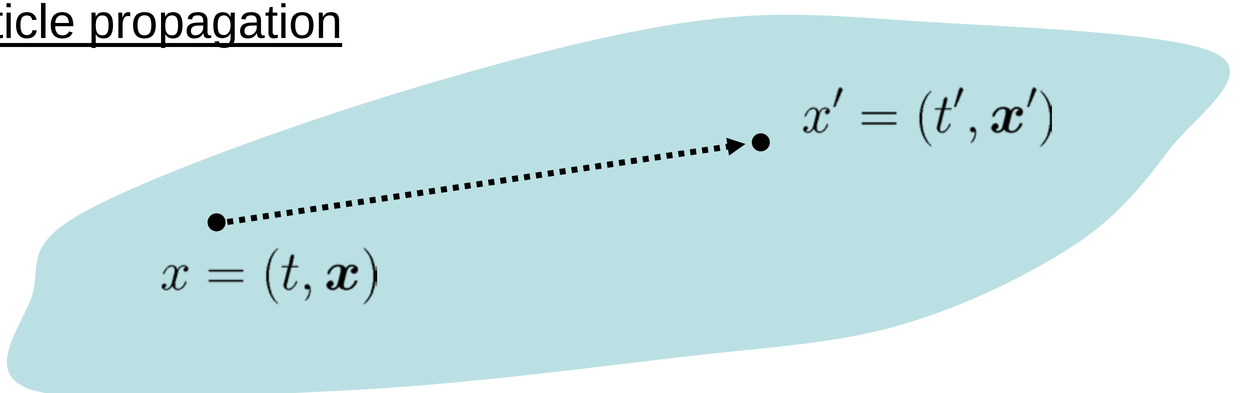
2. Inclusion of strong interactions

Green's functions

N-body system: wave function of the whole system $\Psi(x_1,x_2,\ldots,x_N)$ encodes the dynamics of all particles and is very complicated

Introduce the object which describes the dynamics of the reduced number of particles of interest

<u>one-particle propagation</u>



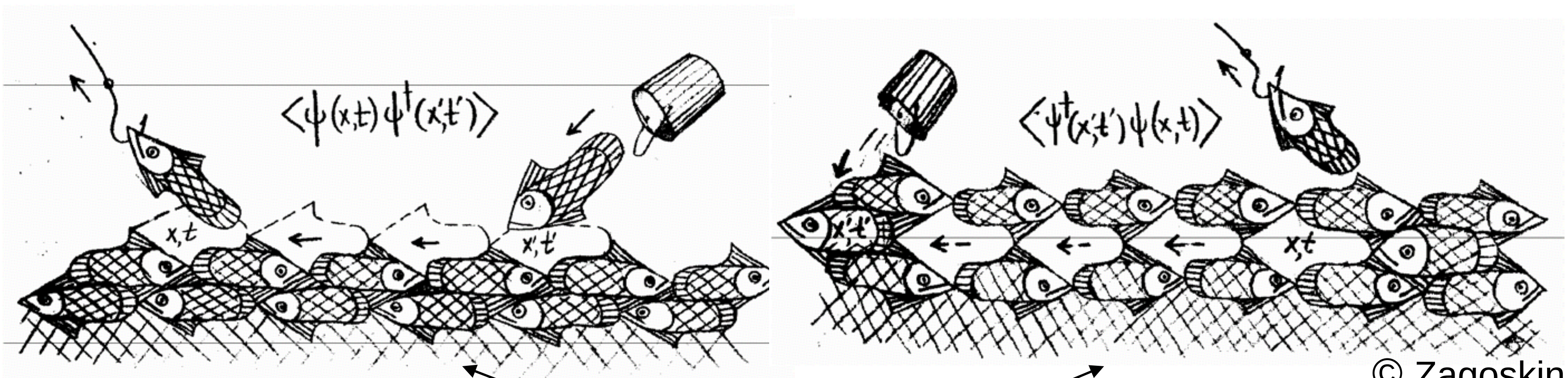
Amplitude of particle transition from a point (x,t) to a point (x',t')

$$\Psi(\boldsymbol{x}', t') = \int d\boldsymbol{x}\, G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t)\Psi(\boldsymbol{x}, t) \qquad t' > t$$

Green's function of non-interacting fermions

$$i\,G(\boldsymbol{x},t;\boldsymbol{x}',t') =< N|T\{\hat{\Psi}(\boldsymbol{x},t)\,\hat{\Psi}^{\dagger}(\boldsymbol{x}',t')\}|N >$$

$$=< N|\hat{\Psi}(\boldsymbol{x},t)\,\hat{\Psi}^{\dagger}(\boldsymbol{x}',t')|N > \theta_{t-t'} - < N|\hat{\Psi}^{\dagger}(\boldsymbol{x}',t')\,\hat{\Psi}(\boldsymbol{x},t)|N > \theta_{t'-t}$$



$$G(\epsilon,p) = \frac{\mathbf{1}-\mathbf{n_p}}{\epsilon-\epsilon_p+i\,0} + \frac{\mathbf{n_p}}{\epsilon+\epsilon^h_p-i\,0}$$

$T=0$

$$n_p = \theta(p_{\rm F}-p)$$

$$\epsilon^h_p = -\epsilon_p$$

$$G(\epsilon,\boldsymbol{p}) = \frac{1}{\epsilon-\epsilon_p+i\,\mathrm{sign}(\epsilon)}$$

$$\epsilon_p = \frac{p^2-p_{\rm F}^2}{2\,m}$$

Diagram technique

Ground state:

$$iG(x,y) = < N|\widehat{T}\{\widehat{\Psi}(x)\widehat{\Psi}^{\dagger}(y)\}|N > =< N|\widehat{S}^{-1}\widehat{T}\{\widehat{\Psi}_I(x)\widehat{\Psi}_I^{\dagger}(y)\}\widehat{S}|N >$$

in interaction picture: $$iG =< N|\widehat{T}\{\widehat{\Psi}_I(x)\widehat{\Psi}_I^{\dagger}(y)\}\widehat{S}|N >< \widehat{S}^{-1} >$$

transition from the ground state to the ground state under action of evolution operator

$$\widehat{S} = \widehat{T}\exp\left\{ -i\int_{-\infty}^{\infty}\widehat{V}_I(t)dt\right\}$$

time ordering

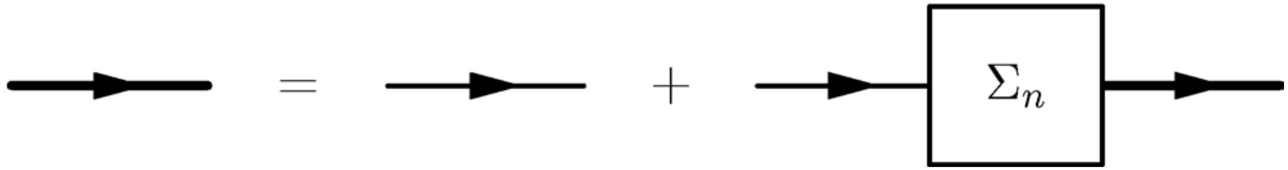
$$\widehat{V}_I(t) = e^{i\widehat{H}_0(\mu)\,t}\widehat{V}\,e^{-i\widehat{H}_0(\mu)t}$$

$$\widehat{H}_0(\mu) = H_0 - \sum_a \mu_a\widehat{N}_a$$

Only one type of Green's functions

Full Green's function

particle-line

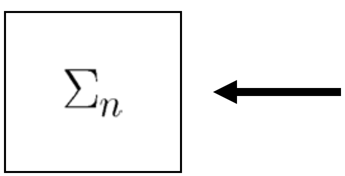


$$\widehat{G}_{\text{n.s.}} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}_{\text{n.s.}}\,\widehat{G}_{\text{n.s.}} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}_{\text{n.s.}}\right]^{-1}$$

diagonal in spin-space

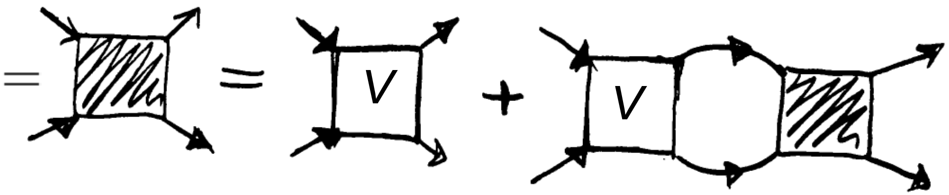
$$\hat{G}_0(\epsilon, \boldsymbol{p}\,) = \frac{\hat{\mathbf{1}}}{\epsilon - \boldsymbol{p}^{\,2}/2\,m_N + i\,0\,\text{sign}\,(\epsilon - \epsilon_{\text{F}})}$$

analogously for the hole-line

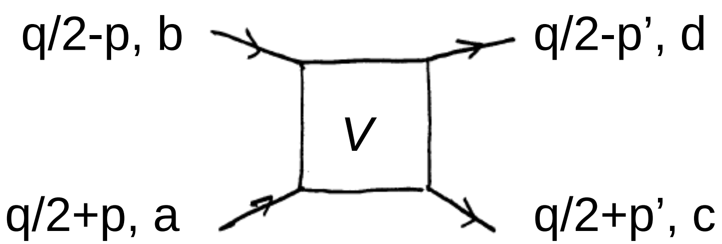


particle-particle, particle-hole hole-hole interactions

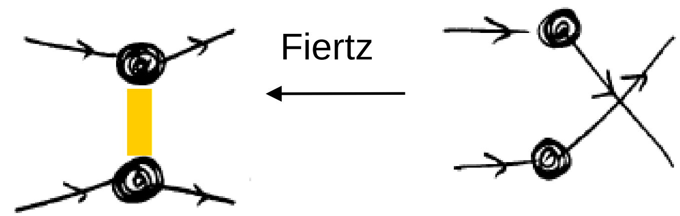
Particle-particle interaction

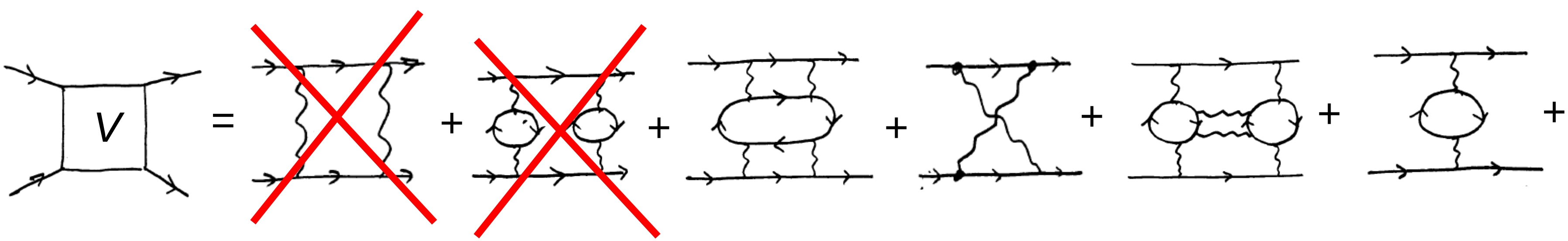
$-i\,T_{pp}(p,p';q) =$ 

two-particle irreducible interaction



$$\left[\widehat{V}(p,p',q)\right]_{cd,ab} = V_0(p,p',q)(i\sigma_2)_{dc}(i\sigma_2)_{ab} + V_1(p,p',q)(\boldsymbol{\sigma} i\sigma_2)_{dc}(i\sigma_2\boldsymbol{\sigma})_{ab}$$

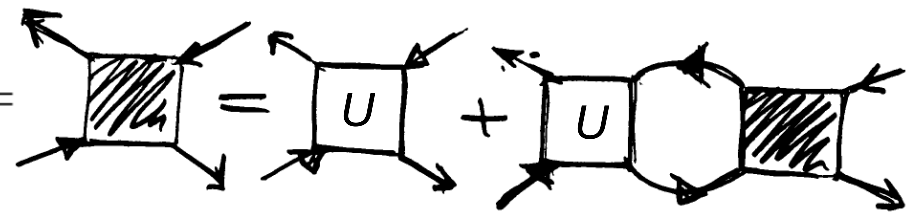


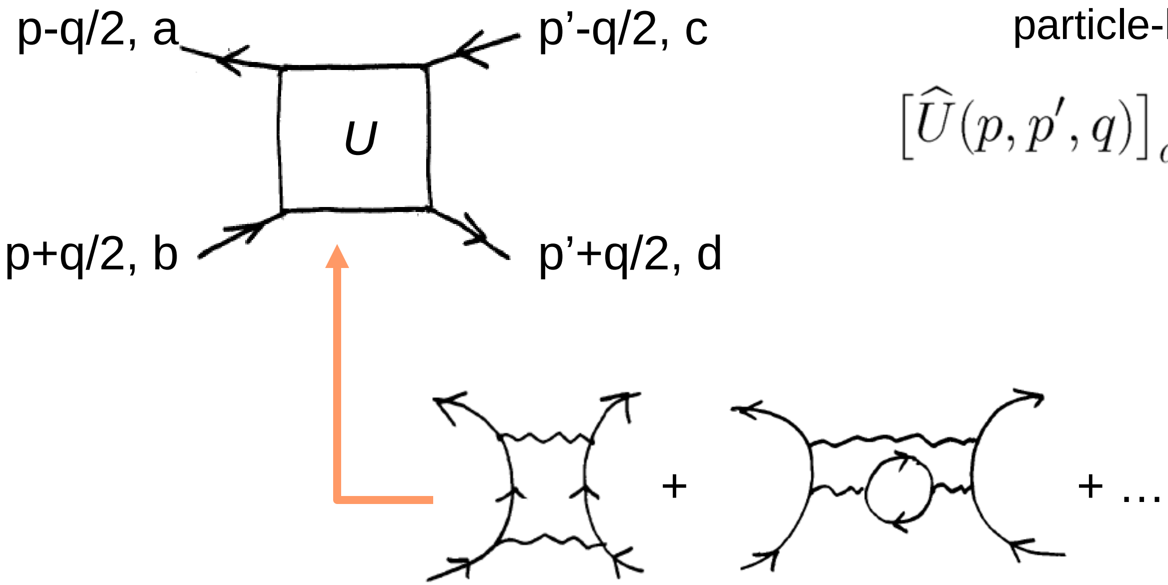


$$\widehat{T}_{\rm pp}(p,p',q) = \widehat{V}(p,p',q) + \int \frac{d^4p''}{(2\pi)^4\, i}\widehat{V}(p,p'',q)\,\widehat{G}(q/2+p'')\,\widehat{G}(q/2-p'')\,\widehat{T}_{\rm pp}(p'',p',q)$$

Particle-particle interaction

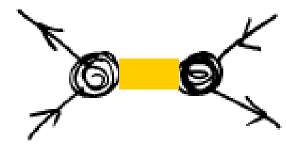
$$-i\,T_{\rm ph}(p,p';q) =$$



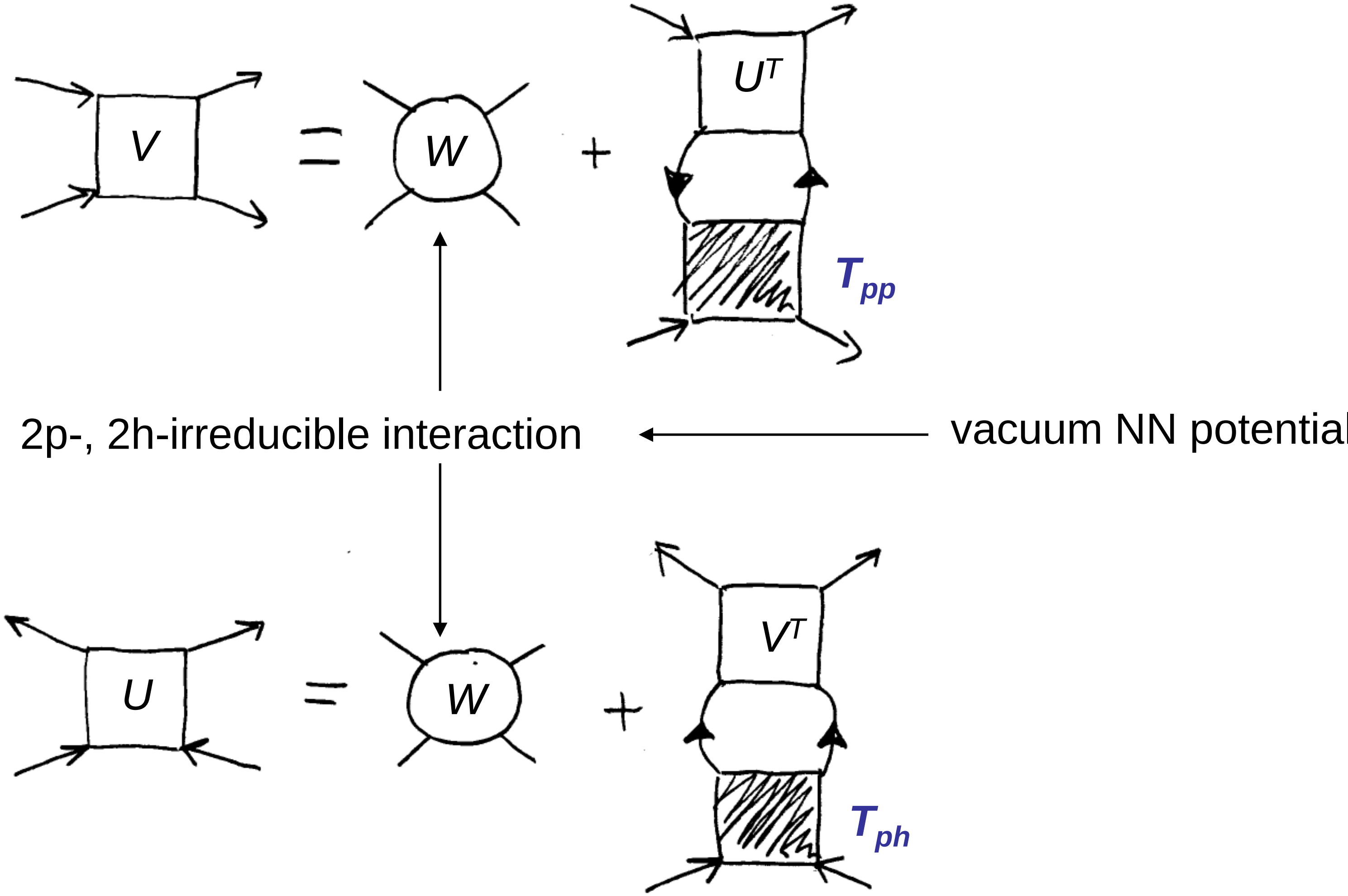


particle-hole irreducible interaction

$$\left[\widehat{U}(p,p',q)\right]_{dc,ab} = U_0(p,p',q)\,\delta_{dc}\delta_{ab} + U_1(p,p',q)\,\boldsymbol{\sigma}_{dc}\,\boldsymbol{\sigma}_{ab}$$

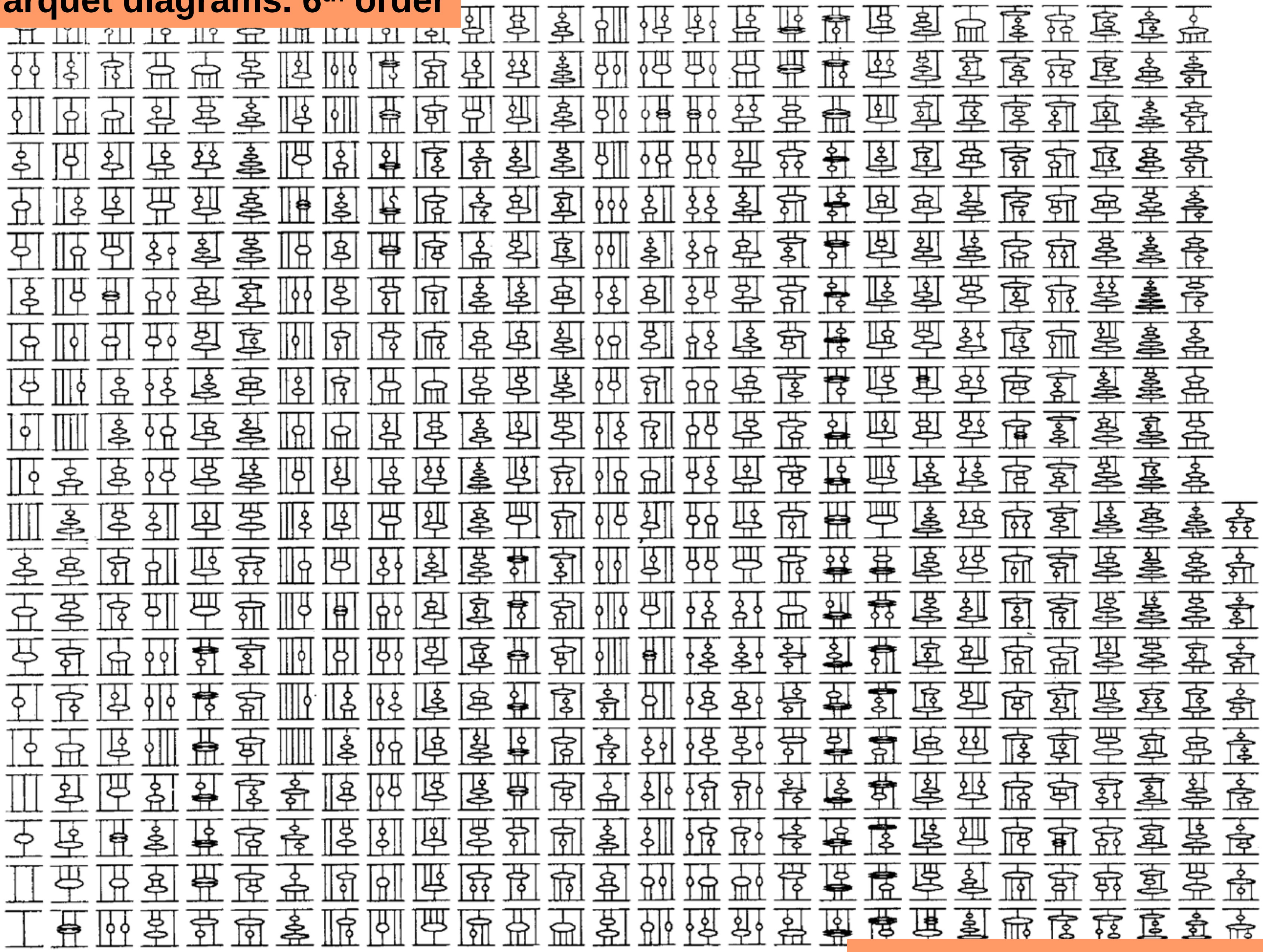


$$\widehat{T}_{\rm ph}(p,p',q) = \widehat{U}(p,p',q) + \int \frac{{\rm d}^4p''}{(2\pi)^4\,i}\widehat{U}(p,p'',q)\,\widehat{G}(q/2+p'')\,\widehat{G}^h(q/2-p'')\,\widehat{T}_{\rm ph}(p'',p',q)$$



[Wambach, Ainsworth,Pines NPA555]

Parquet diagrams. 6th order



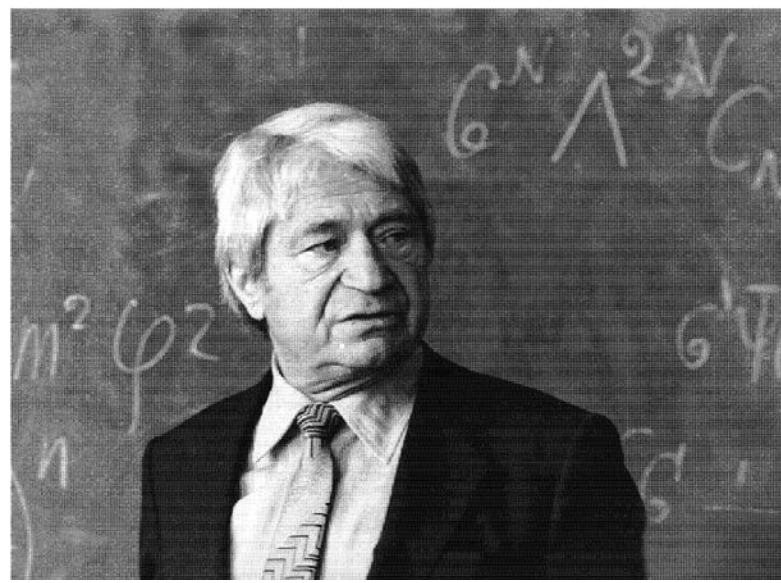
Jackson, Lande, Smith PR86

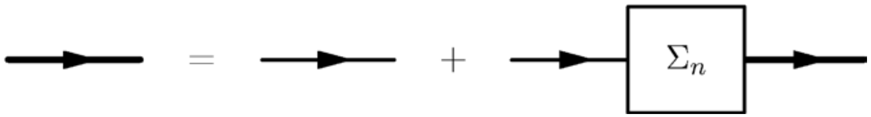
$$\frac{\delta E}{\delta n(p)} = \varepsilon(p)$$

$$\varepsilon(p) = \varepsilon^{(0)}(p) + \sum_{\boldsymbol{p}'} f(\boldsymbol{p}, \boldsymbol{p}')\, \delta n(\boldsymbol{p}')$$

NUCLEAR FERMI LIQUID

$$G(\epsilon, \boldsymbol{p}) = \frac{a}{\epsilon - \epsilon_p + i\,\gamma\,\epsilon^2\,\mathrm{sign}\epsilon} + G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$$





$$\widehat{G} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}\,\widehat{G} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}\right]^{-1}$$

$$T \ll \varepsilon_{\mathrm{F},n}, \varepsilon_{\mathrm{F,p}} \text{ and } \epsilon \sim \epsilon_{\mathrm{F}}\,,\ p \sim p_{\mathrm{F}}$$

pole residue

$$a^{-1} = 1 - \frac{\partial}{\partial \epsilon}\Sigma(\epsilon, 0, T)\Big|_{\epsilon \simeq \epsilon_{\mathrm{F}}}$$

$$G(\epsilon, \boldsymbol{p}) = \frac{a}{\epsilon - \epsilon_p + i\,\gamma\,\epsilon^2\,\mathrm{sign}\epsilon} + G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$$

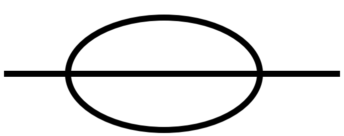
q.p. energy

q.p. width

$$\epsilon_p = \frac{p^2 - 2m_N\mu_N}{2\,m_N^*} \approx \frac{p^2 - p_{\mathrm{F}}^2}{2\,m_N^*} \approx v_{\mathrm{F}}\,(p - p_{\mathrm{F}})$$

$$\gamma = -\lim_{\epsilon \to 0} \mathrm{Im}\,\Sigma^R(\epsilon, p^2 = 2m_N\mu_N, T)/\epsilon^2$$

small for $T \ll \varepsilon_F$



q.p. effective mass

$$\frac{1}{m_N^*} = \frac{a}{m_N} + 2\,a\,\frac{\partial}{\partial p^2}\Sigma(\epsilon, \boldsymbol{p}, T)\Big|_{p=0, \epsilon \simeq \epsilon_{\mathrm{F}}}$$

$G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$ complicated background part

Fermi liquid approximation

particle-hole propagator .... for $q \to 0$

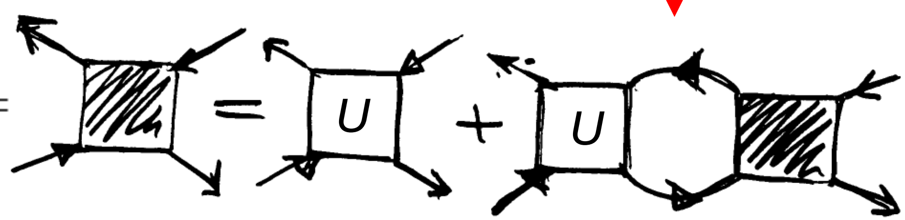
$$\boldsymbol{n} = \boldsymbol{p}/p$$

$$G(q/2+p)\,G^h(q/2-p) \simeq 2\pi i\, a^2\, \delta(\epsilon) \frac{v_\mathrm{F}\, \boldsymbol{q}\boldsymbol{n}}{\omega - v_\mathrm{F}\boldsymbol{q}\boldsymbol{n} + i\,0}\, \delta(p - p_\mathrm{F}) + B(p,q)$$

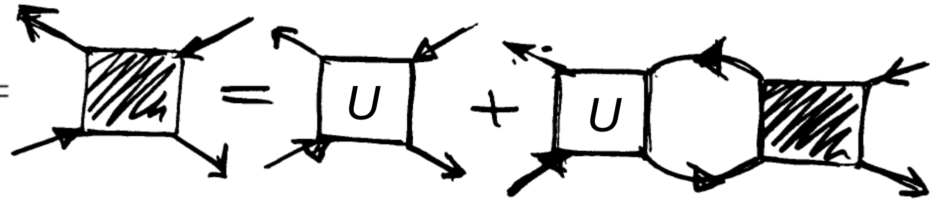
singular pole term

complicated background

$$-i\,T_\mathrm{ph}(p,p';q) =$$



$$-i\,T_{\rm ph}(p,p';q) =$$



for $|\boldsymbol{p}| \simeq p_{\rm F} \simeq |\boldsymbol{p}'|$ and $|\boldsymbol{q}\boldsymbol{p}| << \omega << \epsilon_{\rm F}$

$$\widehat{T}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}\,',q) = \widehat{\Gamma}^{\omega}(\boldsymbol{n},\boldsymbol{n}\,') - \int \frac{d\Omega_{p''}}{4\,\pi}\widehat{\Gamma}^{\omega}(\boldsymbol{n},\boldsymbol{n}\,')\,A(\boldsymbol{n},q)\,\widehat{T}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}\,',q)$$

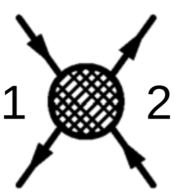
$$A(\boldsymbol{n},q) = a^2\,\frac{m^*\,p_{\rm F}}{\pi^2}\,\frac{v_{\rm F}\,\boldsymbol{q}\boldsymbol{n}}{\omega - v_{\rm F}\boldsymbol{q}\boldsymbol{n} + i\,0}$$

complicated dynamics is here:

$$\widehat{\Gamma}^{\omega}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}\,') = \widehat{U}(\boldsymbol{n},\boldsymbol{n}\,') - \int \frac{d^4p''}{(2\pi)^4\,i}\widehat{U}(\boldsymbol{n},\boldsymbol{n}\,')\,B(p,q=0)\,\widehat{\Gamma}^{\omega}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}\,')$$

parameterize

Landau-Migdal parameters

1  2 $= f_{12}(\boldsymbol{n},\boldsymbol{n}\,') + g_{12}(\boldsymbol{n},\boldsymbol{n}\,')\,\boldsymbol{\sigma_1}\boldsymbol{\sigma_2}$

extracted from experiment

$$= C_0 \left(f + f' \boldsymbol{\tau_1}\boldsymbol{\tau_2} + g\, \boldsymbol{\sigma_1}\boldsymbol{\sigma_2} + g'\, \boldsymbol{\sigma_1}\boldsymbol{\sigma_2}\, \boldsymbol{\tau_1}\boldsymbol{\tau_2}\right)$$

$$C_0 = \frac{\pi^2}{m_N p_{FN}(n_0)} \simeq 300 \text{ MeV fm}^3 \simeq 0.77 m_\pi^{-2} \text{ introduced for convenience}$$

$$f(\cos\theta_{pp'}) = \sum_l f_l\, P_l(\cos\theta_{pp'}) \qquad g(\cos\theta_{pp'}) = \sum_l g_l\, P_l(\cos\theta_{pp'}) \qquad \ldots$$

to be fitted to empirical information (nucleus properties)

effective mass $m^* = m\left(1 + \frac{2}{3} f_1\right)$

compressibility $K = 6\,\frac{p_F^2}{m^*}\,(1 + 2\, f_0)$

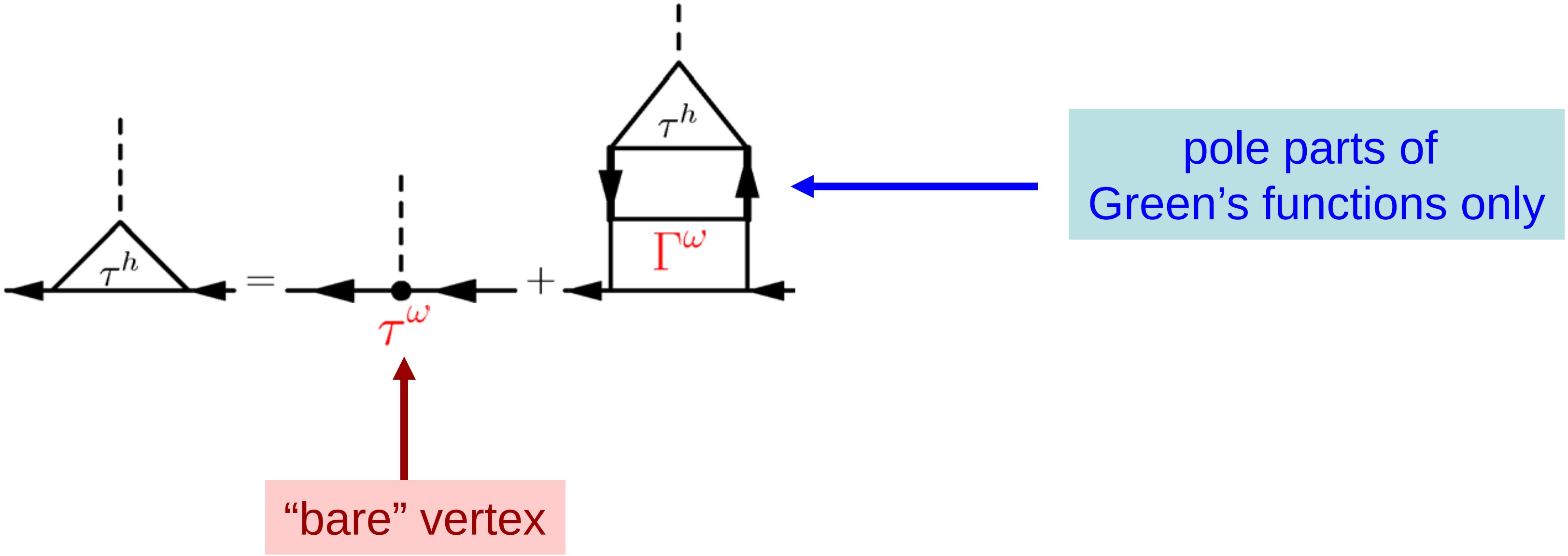
symmetry energy $E_{\rm sym} = \frac{1}{3}\,\frac{p_F^2}{2\, m^*}\left(1 + 2\, f_0'\right)$

[Saperstein,Fayans, et al. 1995, 1998]

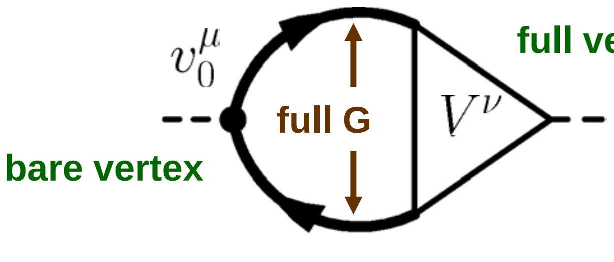
$$f \simeq 0,\ f' \simeq 0.5 - 0.6,\ g \simeq 0.05 \pm 0.1,\ g' \simeq 1.1 \pm 0.1$$

Fermi liquid approximation

Coupling of the external field to a particle



Vector current conservation



Current is conserved if $\Pi^{\mu\nu}\, q_\nu = 0$

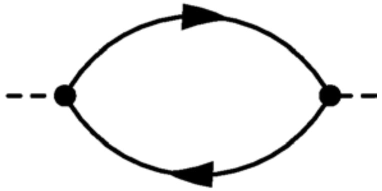
$$\Pi^{\mu\nu} \propto \int d^4p\, \mathrm{Tr}\left\{\gamma^\mu\, G(p+q/2)\, V^\nu(p,-q)\, G(p-q/2)\right\}$$

If the relation $q_\mu\, V^\mu(p,q) = G^{-1}(p+q/2) - G^{-1}(p-q/2)$ Is fulfilled

$$\Pi^{\mu\nu} q^\nu \propto \int d^4p\, \mathrm{Tr}\left\{\gamma^\mu \left[G(p-q/2) - G(p+q/2)\right]\right\} = 0$$

The Ward identities impose non-trivial relations between vertex functions and Green's functions, which synchronize any modification of the Green's function with a corresponding change in the vertex function.

in non-relativistic limit for free G and vertices: $\tau_0^\mu = (1, \boldsymbol{v})$ $\quad G(p) = (\epsilon - p^2/2m)^{-1}$



$$q \cdot \tau_0 = \omega - \boldsymbol{v}\boldsymbol{q} \equiv G_0^{-1}(p+q/2) - G_0^{-1}(p-q/2)$$

The Ward identity is fulfilled and the current is conserved

- **"Bare" vertices**

"bare" vertex after the Fermi-liquid renormalization $\tau_a^\omega = [1+\Gamma_0^\omega\,(G_+G_-)^\omega]\,\tau_a^0$

$$V_\mu^{nn} \approx g_V\,\chi_p^\dagger(p')\,(1,\boldsymbol{v})\,\chi_n(p)$$

$$A_\mu^{nn} \approx g_A\,\chi_p^\dagger(p')\,(\boldsymbol{\sigma}\cdot\boldsymbol{v},\boldsymbol{\sigma})\,\chi_n(p)$$

$$B = (G_+G_-)^\omega = \lim_{\boldsymbol{q}\to 0}\int\frac{2\,\mathrm{d}^4p}{(2\,\pi)^4\,i}\,G_+G_-$$

$$\tau_V^0 - \tau_A^0 = (V_\mu - A_\mu)\,l^\mu$$

weak interactions

$$\hat{\tau}_V^\omega = g_V\,(\tau_{V,0}^\omega\,l_0 - \boldsymbol{\tau}_{V,1}^\omega\,\boldsymbol{l}\,) \qquad \tau_{V,0}^\omega = \frac{e_V}{a}\,, \qquad \boldsymbol{\tau}_{V,1}^\omega = \frac{e_V}{a}\,\boldsymbol{v}$$

$$\hat{\tau}_A^\omega = -g_A\,(\boldsymbol{\tau}_{A,1}^\omega\boldsymbol{\sigma}\,l_0 - \tau_{A,0}^\omega\,\boldsymbol{\sigma}\boldsymbol{l}\,) \qquad \tau_{A,0}^\omega = \frac{e_A}{a}\,, \qquad \boldsymbol{\tau}_{A,1}^\omega = \frac{e_A}{a}\,\boldsymbol{v}$$

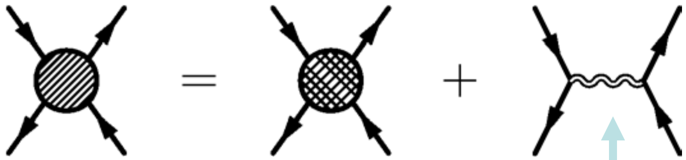
e_A $\quad e_V$ effective charges

$$e_V = 1 \qquad \omega\,\tau_{V,0}^\omega - \boldsymbol{q}\,\boldsymbol{\tau}_{V,1}^\omega = G^{(\mathrm{pole}),-1}(p+q/2) - G^{(\mathrm{pole}),-1}(p-q/2)$$

$e_A = 0.8$—0.95 experiment: Gamov-Teller transitions in nuclei $g_A^* \simeq 1$

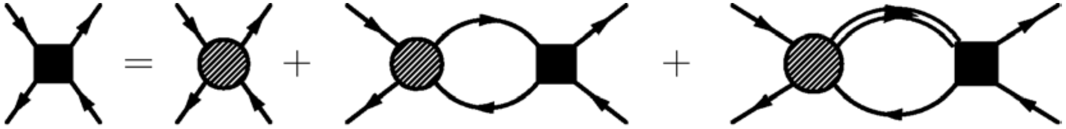
Fermi-liquid with pions

- explicit pionic degrees of freedom



pion with residual (irreducible in NN^{-1} and ΔN^{-1}) s-wave π N interaction and $\pi\pi$ scattering``

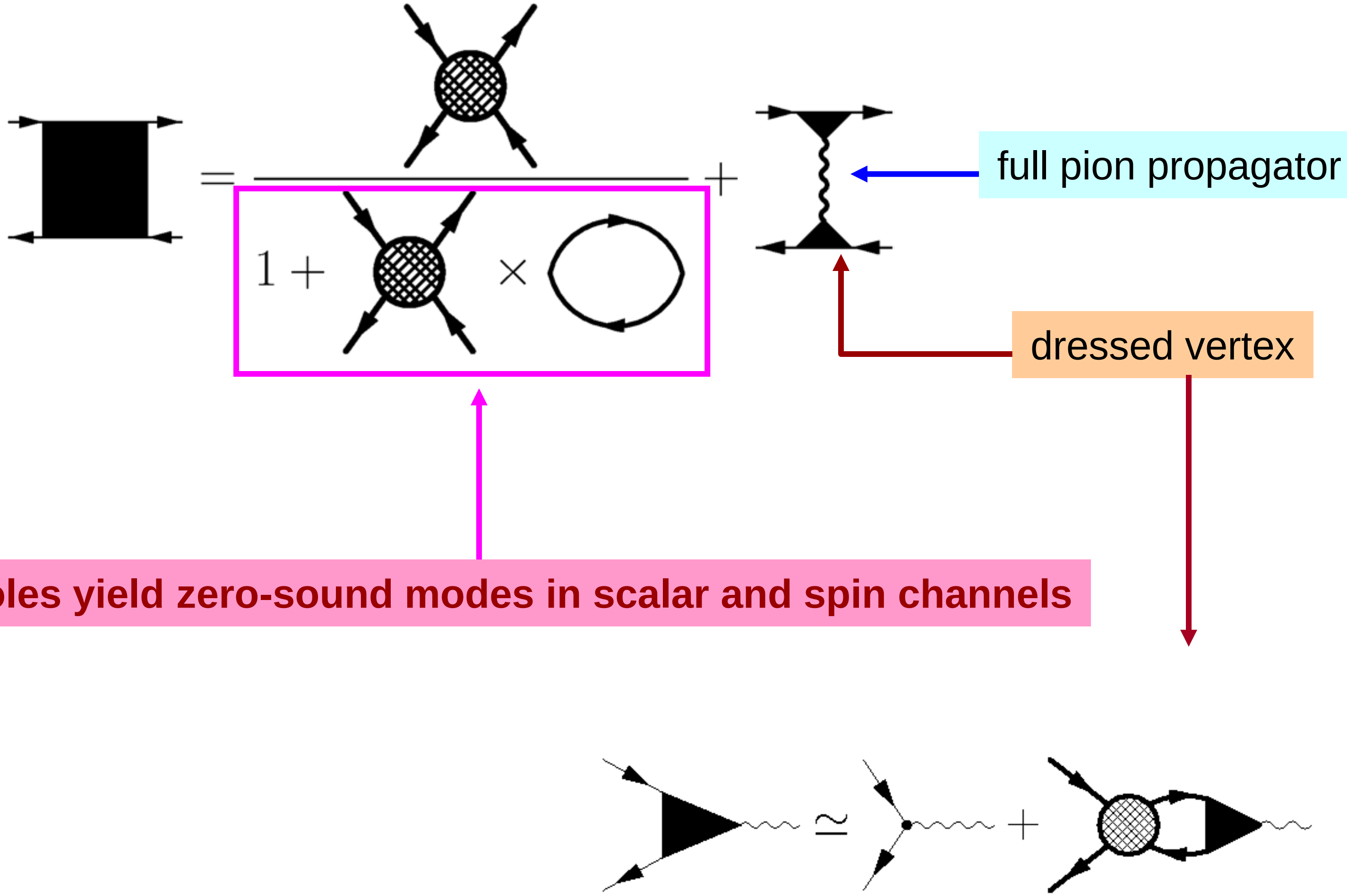
- explicit Δ degrees of freedom



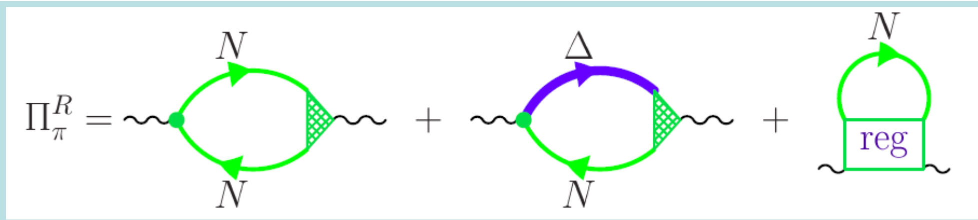
Part of the interaction involving Δ isobar is analogously constructed:

Resummed NN interaction

Graphically, the resummation is straightforward and yields:

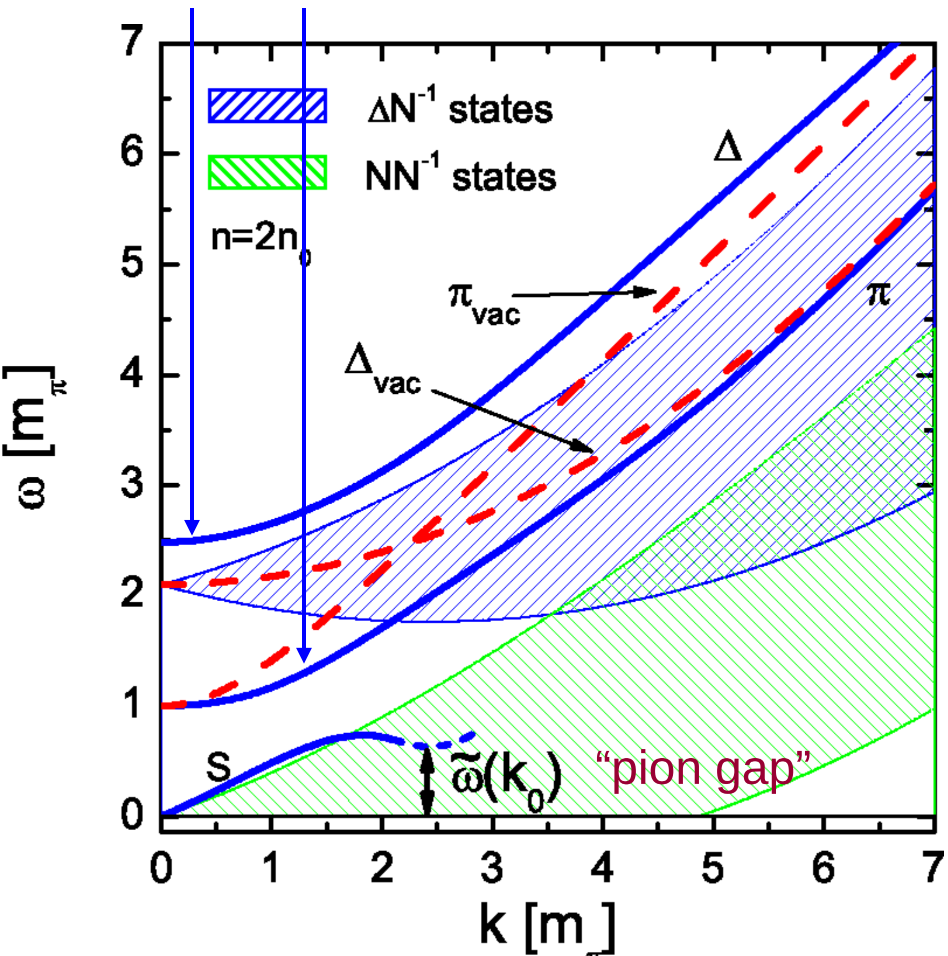


Pion modes in nuclear medium



$$A_\pi(\omega,\boldsymbol{k}) \approx \sum_{i=\pi,\Delta} \frac{2\pi\,\delta(\omega-\omega_i(\boldsymbol{k}))}{\left(2\omega - \frac{\partial \Pi^R}{\partial\omega}\right)\Big|_{\omega=\omega_i(\boldsymbol{k})}} + \frac{2\,\beta\,k\,\omega}{\widetilde{\omega}^4(k)+\beta^2\,k^2\,\omega^2}\,\theta(\omega < v_F\,k)$$

quasi-particle modes



pion propagator has a complex pole

$$D^{-1}(\omega,k) \simeq D^{-1}(0,k) + i\beta\,\omega$$

$$\beta = m_N^{*2}\,k\,f^2_{\pi NN}/\pi$$

$$\widetilde{\omega}^2(k) = -D^{-1}(0,k)$$

$$\omega \propto -i\widetilde{\omega}^2(k_{\rm min})/\beta$$

when $\quad \widetilde{\omega}^2(k_{\rm min} \simeq p_{\rm F}) < 0$

→ instability → pion condensation

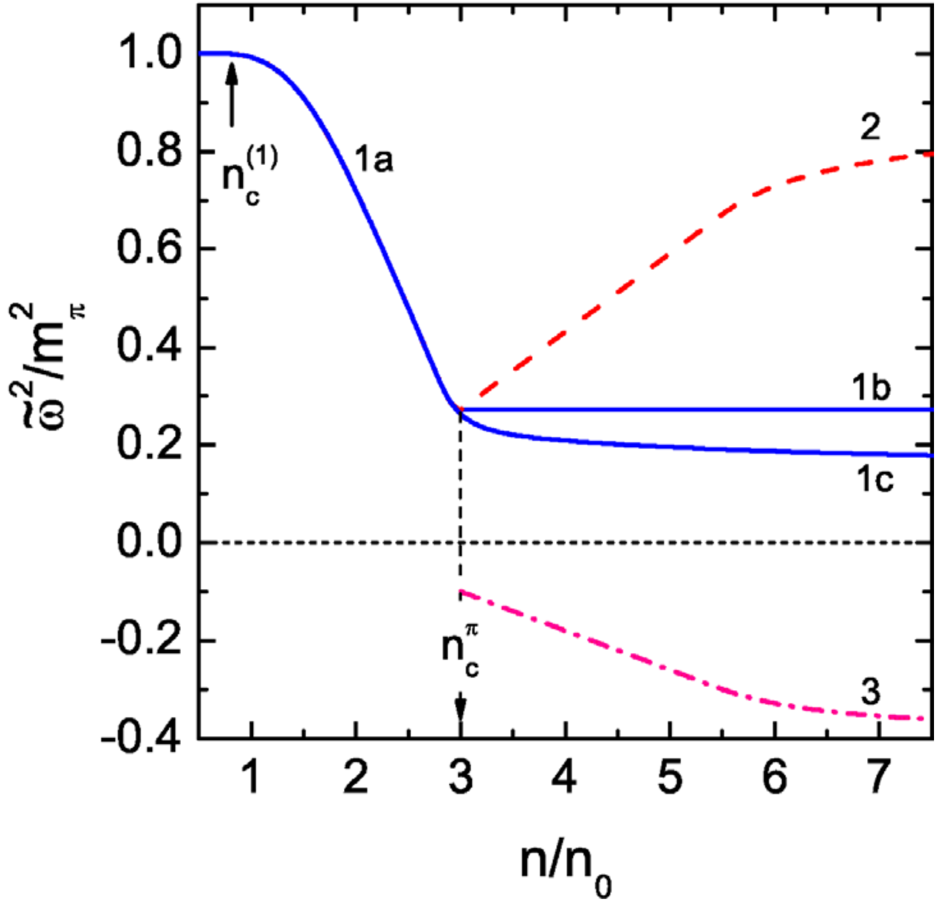
[A.B.Migdal et al, Phys. Rept. 192 (1990) 179]

• pion softenning

pion propagator for $\omega \ll k_0 v_F \quad k \sim k_0 \simeq p_F$

$$D^R_\pi(\omega, k) \simeq \frac{1}{-\tilde{\omega}^2 - \gamma\,(k-k_0)^2 + i\beta(k)\,\omega}$$

pion gap



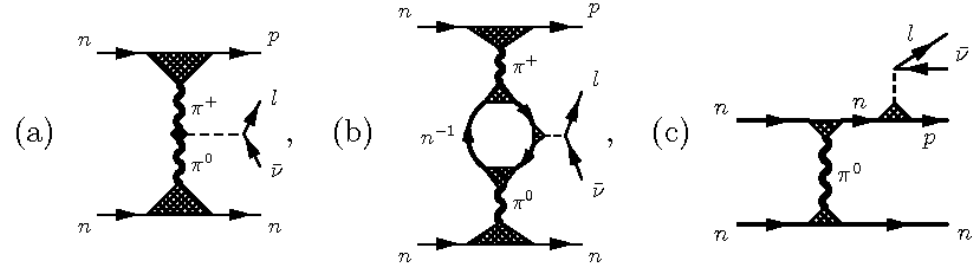
reconstruction of pion spectrum on top of the pion condensate

LM parameters increase with density
saturation of pion softenning
no pion condensate

amplitude of the pion condensate

pion softenning in neutrino production

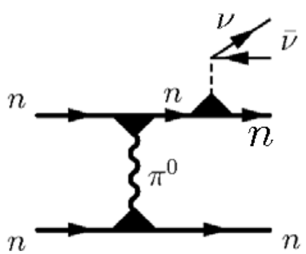
enhancement factors w.r.t. MU emissivity



medium MU reactions

$$F_{\rm MMU}(n) = 3\left(\frac{n}{n_0}\right)^{10/3}\frac{[\Gamma(n)/\Gamma(n_0)]^6}{(\widetilde{\omega}/m_\pi)^8}$$

medium bremsstrahlung reactions

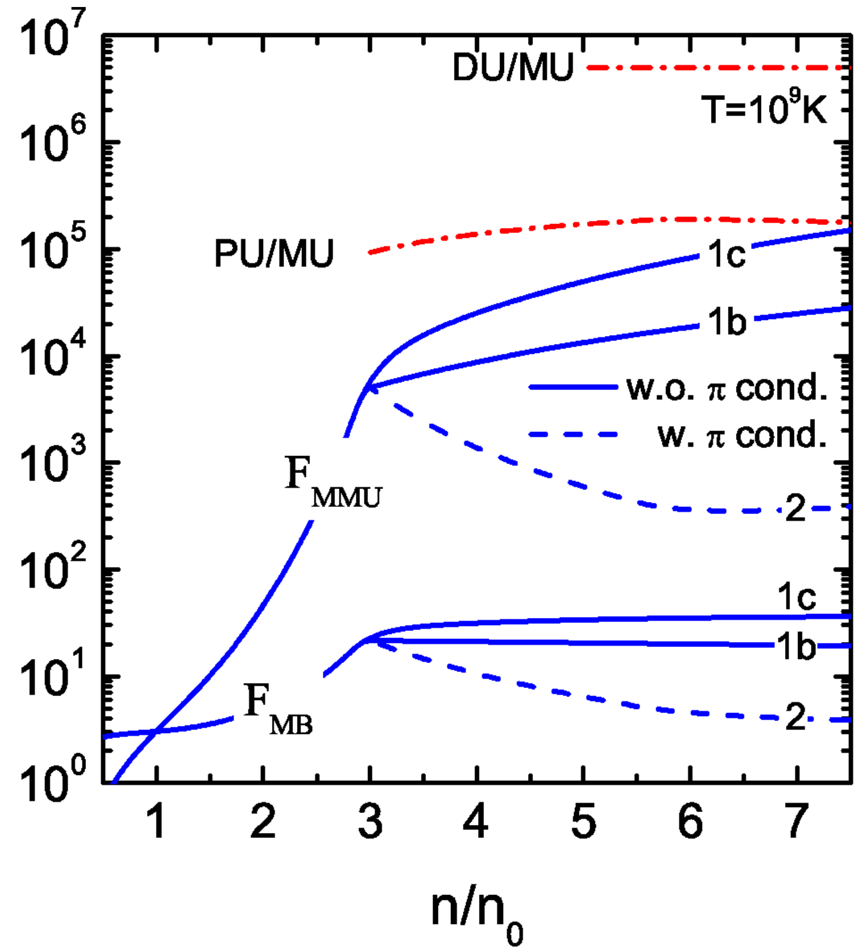


$+ \ldots$

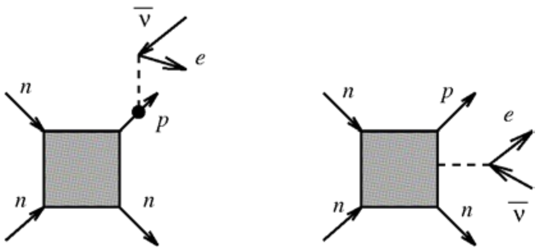
$$F_{\rm MB}(n) = 3\ \left(\frac{n}{n_0}\right)^{4/3}\frac{[\Gamma(n)/\Gamma(n_0)]^6}{[\omega^*(n)/m_\pi]^3}.$$

vertex correlation function

$$\Gamma(n) \simeq \frac{1}{1+1.6(n/n_0)^{1/3}}$$

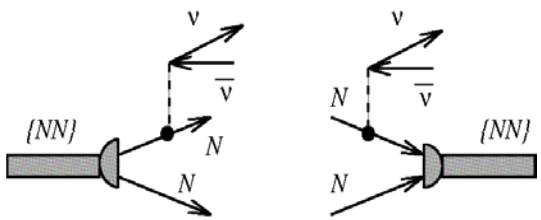


Basic reactions of neutron star cooling



medium Modified Urca (MMU)

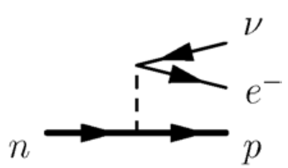
(depends strongly on the NN interaction)



pair formation-breaking process (PFB)

(operating at $T<T_c$ ~0.1 MeV)

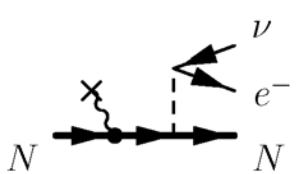
These processes can describe all three groups of the cooling data.



Direct Urca (DU) **much more efficient than MMU**

allowed only if the proton concentration is >11%,

for our EoS it corresponds to $M>1.8\ M_{sol}$



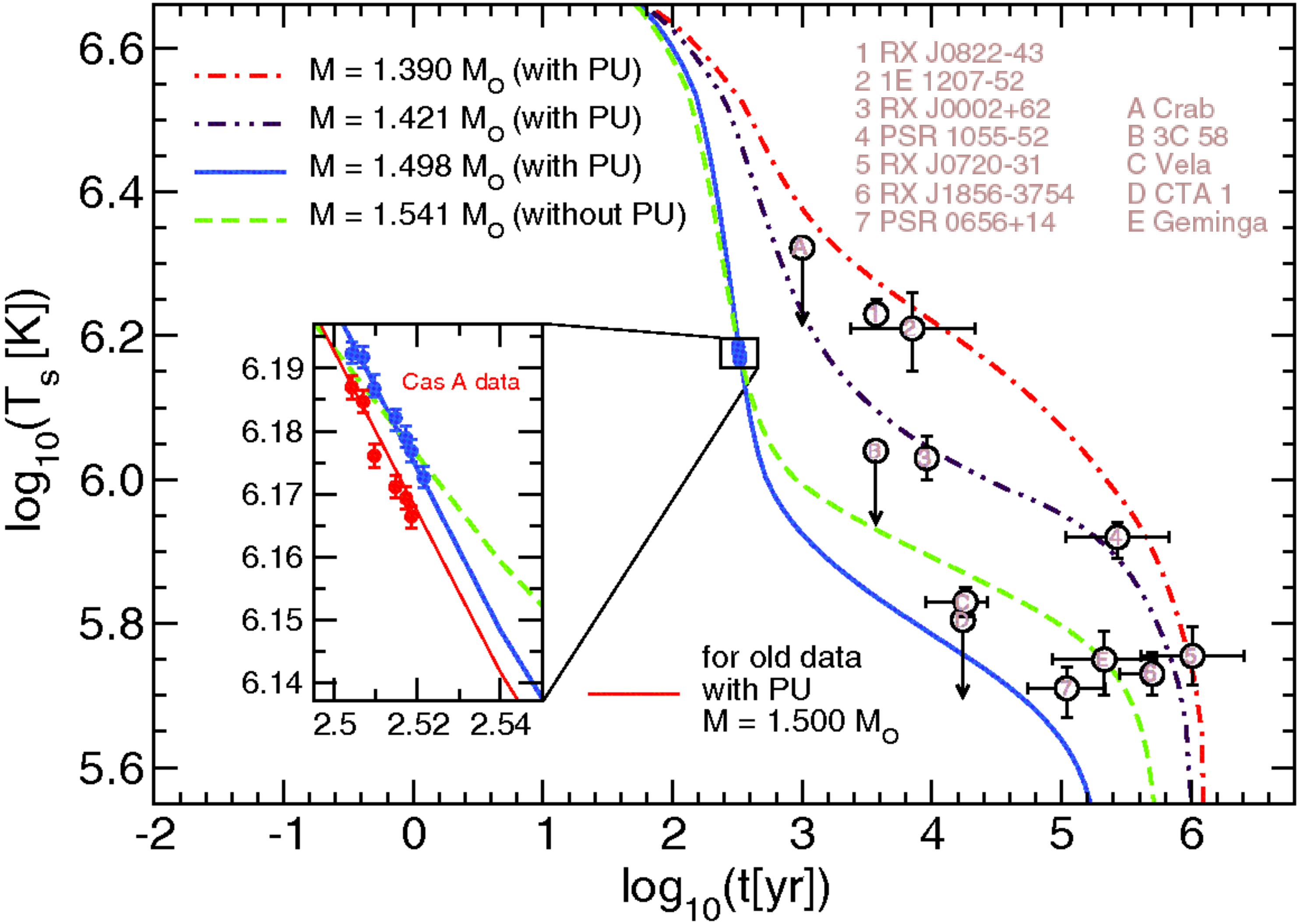
Processes of meson condensate (PU)

allowed only for $M>1.3\ M_{sol}$ for our Eos

much more efficient than MMU

Neutron star cooling

[Blaschke, Grigorian, Voskresensky PRC88 (2013)065805]



Neutrino Processes in Neutron Stars

3. Viscosity of neutron star matter and r-mode stability

Lepton shear viscosity

Lepton shear viscosity = electron + muon contribution $\eta_{e/\mu} = \eta_e + \eta_\mu$

low T, Fermi liquid results $\eta_l = \frac{1}{5} n_l p_{\mathrm{F},l}\, \tau_l$

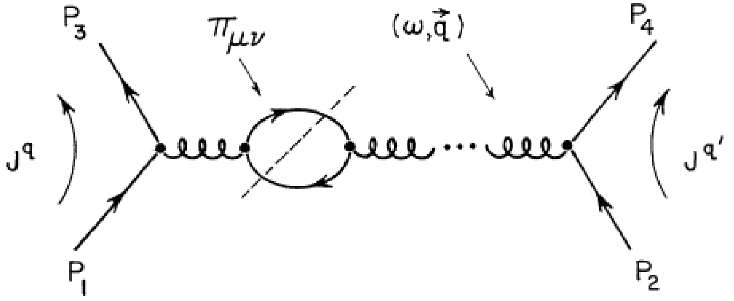
Lepton collision time τ_l is determine by lepton-lepton and lepton-proton collisions

Flowers and Itoh: $\eta_{e/\mu}^{(\mathrm{FI})} \approx \eta_e^{(\mathrm{FI})} = 4.2 \cdot 10^{17} \left[\frac{\mathrm{g}}{\mathrm{cm} \cdot \mathrm{s}}\right] \left(\frac{\rho}{\rho_0}\right)^2 T_9^{-2}$

Important role of the phonon modification (plasmon exchange)

for QCD plasma [Heiselberg, Pethick Phys. Rev. D 48 (1993) 2916]

[Shternin, Yakovlev, Phys. Rev. D 78 (2008) 063006]



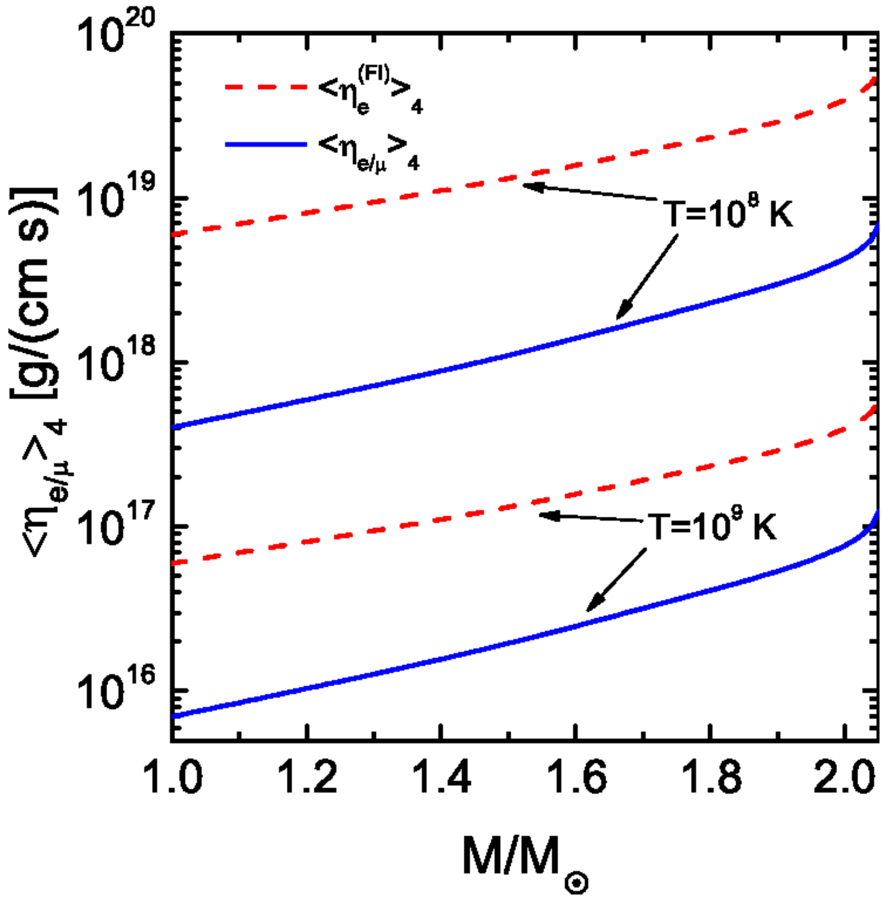
leading terms for small T

$$\widetilde{\eta}_e = 1.82 \cdot 10^{19} \left[\frac{\mathrm{g}}{\mathrm{cm} \cdot \mathrm{s}}\right] \left(\frac{n_p}{n_0}\right)^{\frac{14}{9}} \left(\frac{n_e}{n_p}\right)^2 \frac{T_9^{-\frac{5}{3}}}{(1+r)^{\frac{2}{3}}}$$

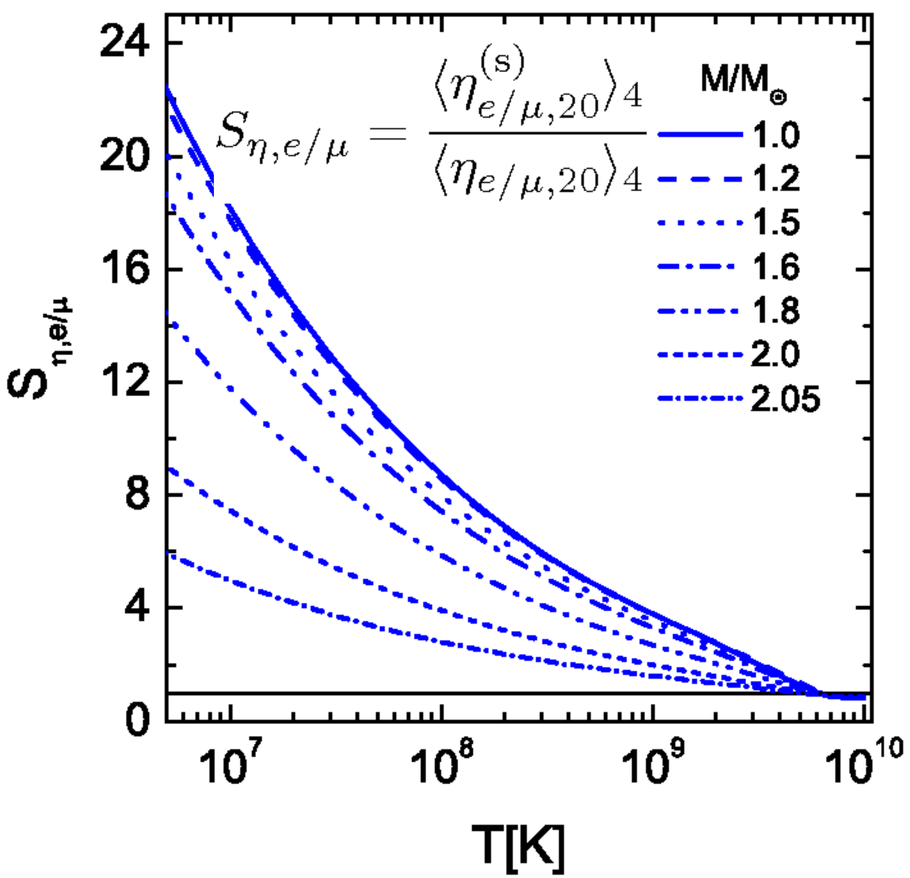
$$r = (p_{\mathrm{F,e}}^2 + p_{\mathrm{F},\mu}^2)/p_{\mathrm{F,p}}^2 \qquad \widetilde{\eta}_\mu = \left(\frac{n_\mu}{n_e}\right)^{\frac{5}{3}} \widetilde{\eta}_e$$

muon contributions are important

Lepton shear viscosity vs. neutron star mass



Effects of proton pairing on lepton shear viscosity



[Shternin, Yakovlev, Phys. Rev. D 78 (2008) 063006]

Nucleon shear viscosity

[Shternin, Yakovlev, Phys. Rev. D 78 (2008) 063006]

$$\eta_n = \frac{3 n_n p_{\mathrm{F},n}^2 m_N^2}{80 m_n^{*\,4} T^2 S_{nn}}$$

Fermi liquid result

effective NN cross section

$$S_{nn} = \frac{m_N^2}{16\pi^2} \int_0^1 dx' \int_0^{\sqrt{1-x'^2}} dx \frac{12\, x^2\, x'^2 Q_{nn}(q,q')}{\sqrt{1-x^2-x'^2}}$$

$$Q_{nn} = \frac{1}{4} \sum_{\mathrm{spin}} |M_{nn}|^2$$

FOPE: $S_{nn}^{\mathrm{FOPE}} \simeq \frac{3\, m_n^2 f_{\pi NN}^4}{40\pi} \simeq \frac{1.1}{m_\pi^2}$

MOPE: $S_{nn}^{\mathrm{MOPE}} = K\, S_{nn}^{\mathrm{FOPE}}$

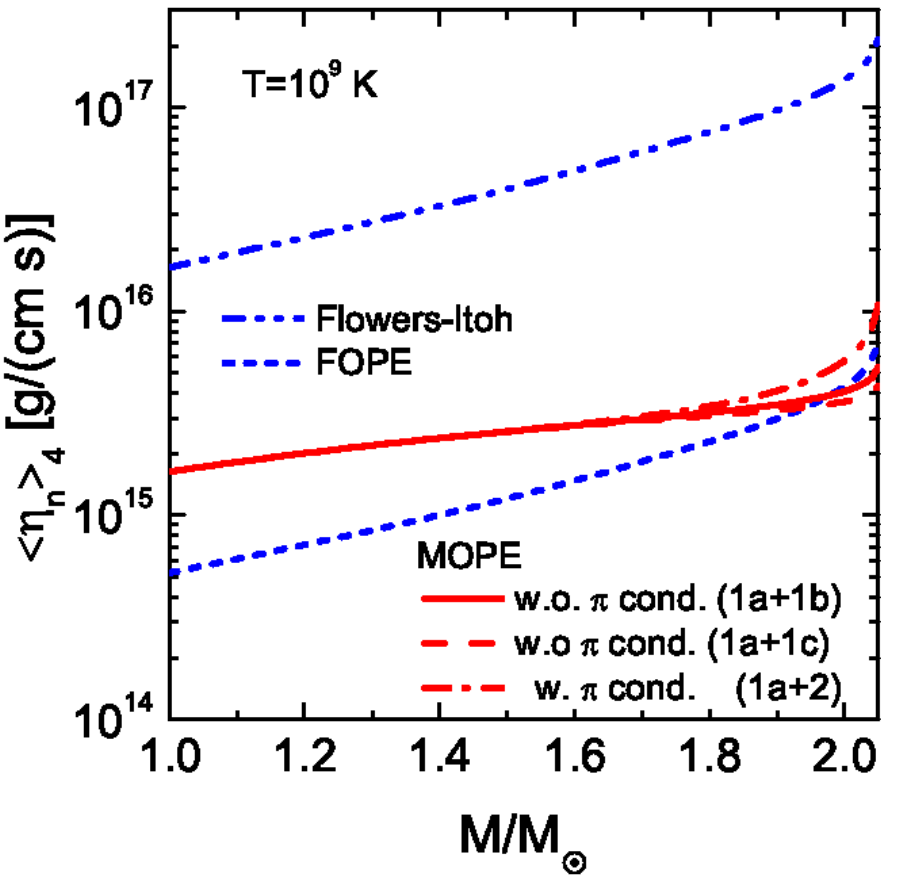
$$K = \frac{30\pi \Gamma^4 p_{\mathrm{F},n}^3}{128\gamma\, \widetilde{\omega}^3} \left[1 + \frac{2}{3} \frac{\widetilde{\omega}}{\gamma\, p_{\mathrm{F},n}} \right]$$

modification factor

$K(n = n_0) \simeq 0.3 \longleftrightarrow$ [Bacca et al, PRC80]

$K(n = 2.6 n_0) \simeq 1$

$K(n \geq 3 n_0) \simeq 2$



Phonon shear viscosity

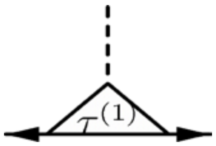
A shear viscosity induced by **phonon-phonon** interactions was discussed by Manuel and Tolos in [PRD84 (2011) 123007; PRD88 (2013) 043001].

These interactions may give a contribution to the resulting shear viscosity if $T_{cn} > T \gtrsim 10^9\mathrm{K}$ However, at such temperatures the bulk viscosity is dominant.

We consider the interaction of the phonon (Anderson-Bogoliubov) mode with neutrons

$$\eta_{\mathrm{ph}} = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\tau_{\mathrm{ph}}}{15T} \frac{(s v_{\mathrm{F},n})^2 q^2}{(e^{s v_{\mathrm{F},n} q/T} - 1)(1 - e^{-s v_{\mathrm{F},n} q/T})} = \frac{2\pi^2}{25} \frac{T^4}{v_{\mathrm{F},n}^3} \bar{\tau}_{\mathrm{ph}} \qquad s = 1/\sqrt{3}$$

From the Larkin-Migdal equation for anomalous vertex [E.K. Voskresensky, PRC77, PRC81]



$$\tilde{\tau}_{V,0} = \frac{-2\,\Delta\,\omega}{\omega^2 - \frac{1}{3} v_{\mathrm{F},n}^2 q^2 - i\omega\,\gamma_{\mathrm{ph}}(\omega, q)}\, \tau_{V,0}^{\omega}$$

bare p.-h. vertex

$$\gamma_{\mathrm{ph}}(q) = 1/\tau_{\mathrm{ph}} \approx \frac{2\pi}{3} v_{\mathrm{F},n} q e^{-\sqrt{\frac{3}{2}}\frac{\Delta}{T}}$$

The phonon lifetime $\bar{\tau}_{\mathrm{ph}} \simeq 5.9 \cdot 10^{-22} \frac{e^{\sqrt{\frac{3}{2}}\frac{\Delta}{T}}}{T_9}\ \mathrm{s}$ must be smaller than the balistic time

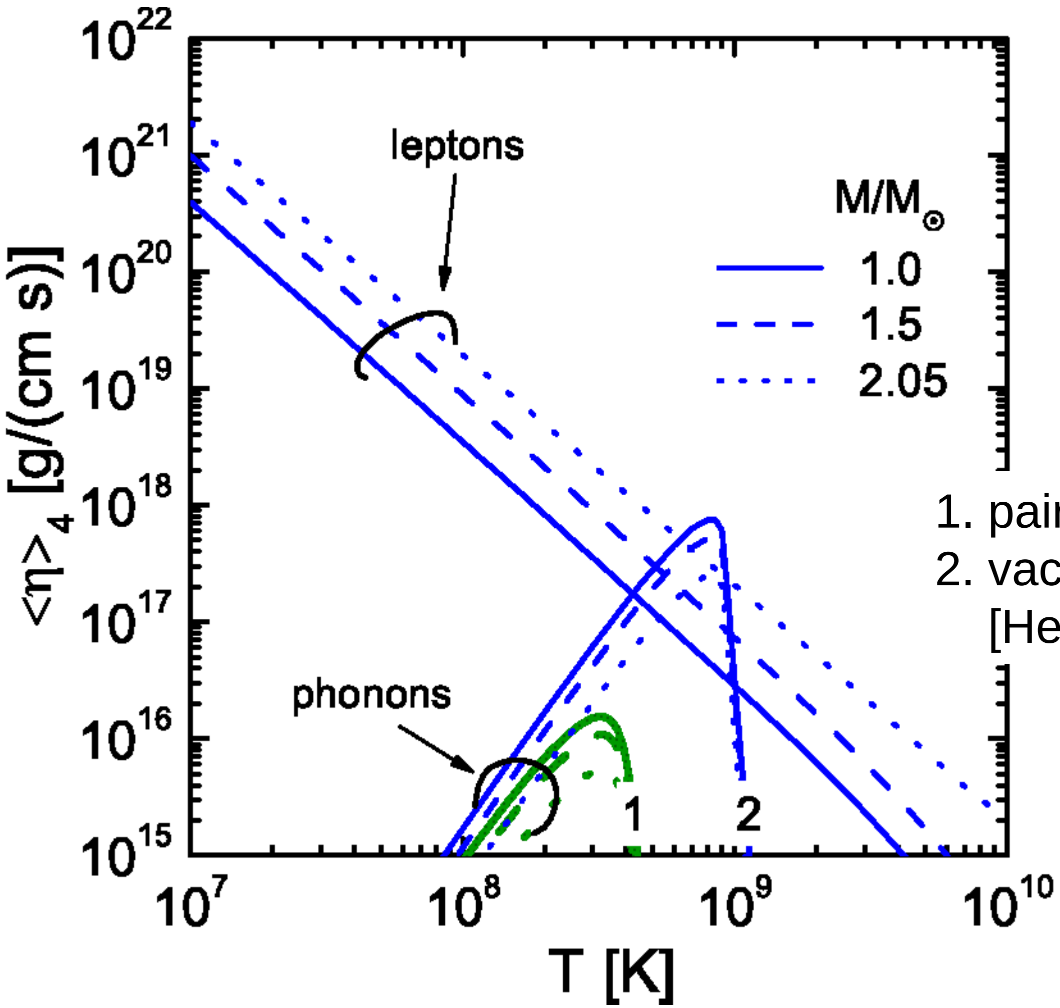
$$\tau_{\mathrm{bal}} \sim \frac{1\ \mathrm{km}}{s\, v_{\mathrm{F}}} \simeq 1.6 \cdot 10^{-5}\ \mathrm{s} \left(\frac{n_0}{n}\right)^{\frac{1}{3}} \frac{m_n^*}{m_N}$$

size of the region of the neutron pairing

$$\eta_{\rm ph} \simeq 2.1 \cdot 10^{23} \left[\frac{\rm g}{\rm cm \cdot s}\right] \left(\frac{n_0}{n}\right) \left(\frac{m_n^*}{m_N}\right)^3 T_9^4 \, \frac{\min\{\bar{\tau}_{\rm ph}, \tau_{\rm bal}\}}{\rm s}$$

$$\bar{\tau}_{\rm ph} \simeq 5.9 \cdot 10^{-22} \frac{e^{\sqrt{\frac{3}{2}}\frac{\Delta}{T}}}{T_9} \ {\rm s}$$

$$\tau_{\rm bal} \simeq 1.6 \cdot 10^{-5} \ {\rm s} \left(\frac{n_0}{n}\right)^{\frac{1}{3}} \frac{m_n^*}{m_N}$$



1. pairing gaps reduced in medium
2. vacuum pairing gaps [Hebeler, Schwenk, Friman]

Neutrino shear viscosity

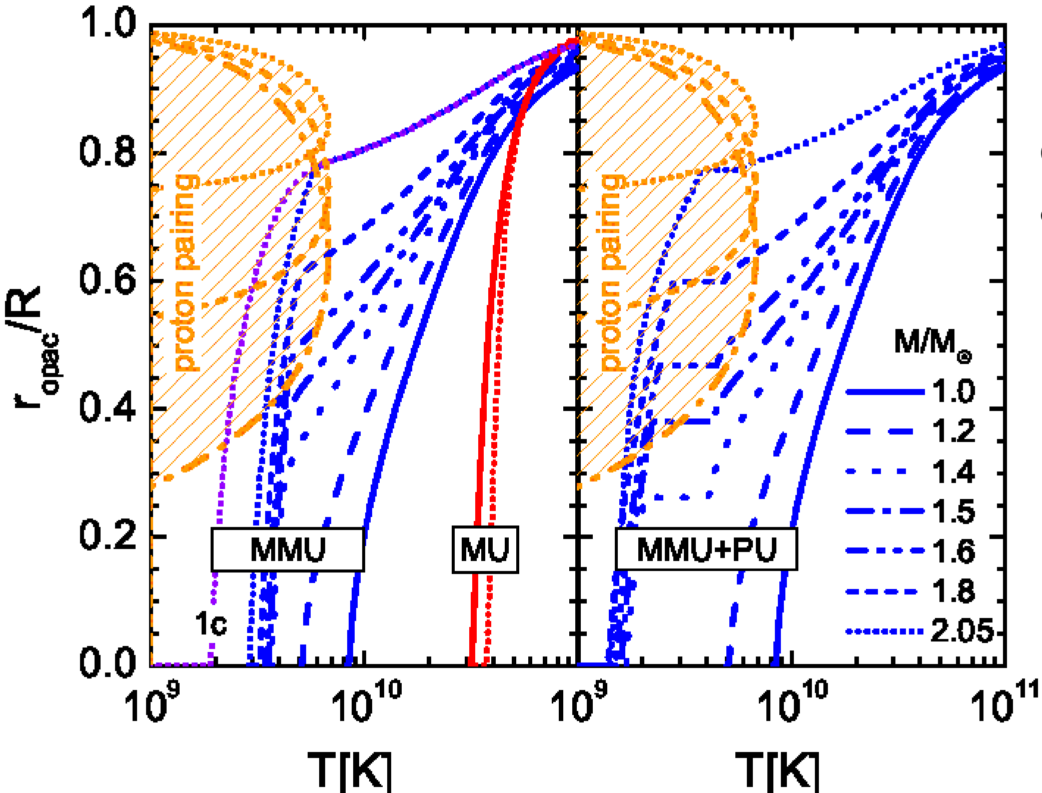
With the temperature increase the neutrino mean free path decreases and for sufficiently high temperatures neutrinos become trapped inside the neutron star interior.

$$\eta_\nu = 2\int \frac{2\mathrm{d}^3q}{(2\pi)^3}\frac{\tau_\nu}{15T}\frac{v_\nu^2\, q^2}{(e^{v_\nu q/T}+1)(1+e^{-v_\nu q/T})} = \frac{7\,\pi^2}{225\, v_\nu^3}T^4\bar{\tau}_\nu \qquad v_\nu = c$$

Neutrino mean free path is determined by inverse MMU and PU processes

$$\bar{\tau}_\nu \simeq \frac{8.7\ \mathrm{s}}{T_9^4\, F_{\mathrm{MMU}}(n)}\left(\frac{m_N}{m_N^*}\right)^4 \frac{(n_0/n_e)^{\frac{1}{3}}}{1+\chi_{\mathrm{PU}}(n,T)}$$

$$\eta_\nu \simeq \frac{3.08\cdot 10^{22}}{1+\chi_{\mathrm{PU}}(n,T)}\left[\frac{\mathrm{g}}{\mathrm{cm\cdot s}}\right]\left(\frac{n_0}{n_p}\right)^{\frac{1}{3}}\frac{(m_N/m_N^*)^4}{F_{\mathrm{MMU}}(n)}$$



weak T dependence

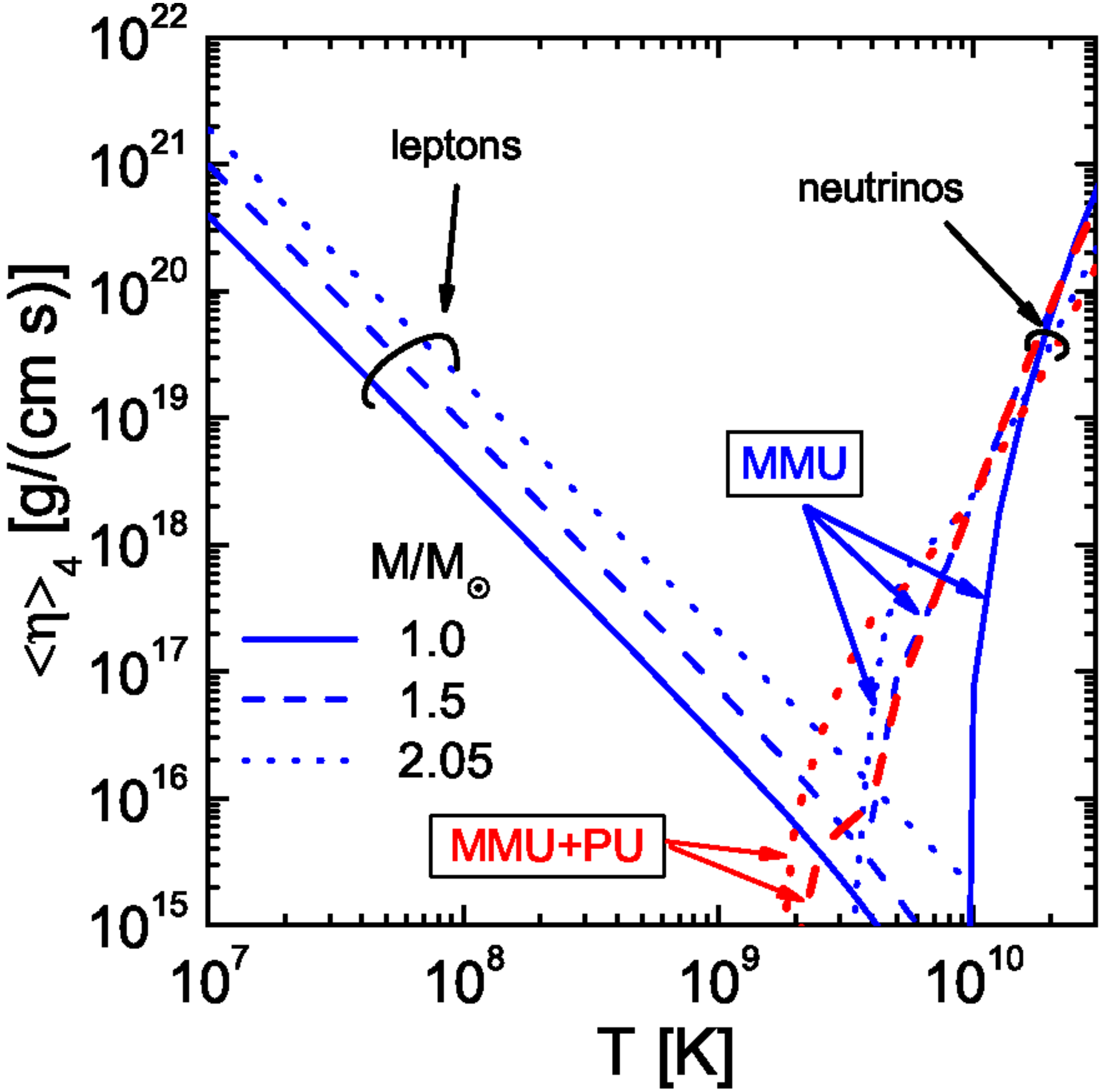
contributes only in regions where neutrinos are trapped

$$\eta_\nu^{(\mathrm{opac})}(r,T) = \eta_\nu(n(r))\,\theta(r_{\mathrm{opac}}-r)$$

opacity radius is determined as

$$v_\nu\,\bar{\tau}_\nu(n(r_{\mathrm{opac}}),T) = R - r_{\mathrm{opac}}$$

Lepton shear viscosity



Bulk viscosity

collisional

$$\zeta_{\rm coll} = \frac{m_N^{*3}/n_0}{162\pi^2}\,\tau\,T^4\left[\frac{n_0}{n}\right]^{1/3} F_0^2$$

[Sykes, Brooker, Ann. Phys. 56(1970) 1]

F_0 is the zeroth harmonics of the dimensionless scalar Landau-Migdal parameter, $F_0 \sim 1$

$\tau \sim m_\pi^2/(m_N^*\,T^2)$ nucleon relaxation time;

$$\zeta_{\rm coll} \sim 90\left[\frac{\rm g}{\rm cm\cdot s}\right] T_9^2\left[\frac{n_0}{n}\right]^{1/3} F_0^2$$

small contribution

Bulk viscosity

Energy dissipation of the mode: $\dot{E}_{\rm mode} = P\dot{V} - \epsilon_\nu$ ϵ_ν is neutrino emissivity

Energy of the mode decreases if the pressure depends on an order parameter, which variation is delayed with respect to the variation of the density [Mandelstam, Leontovich, ZhETF 7(1937)438]

soft mode [Sawyer PRD39, Haensel, Levenfish, Yakovlev A&A357, A&A372]

order parameter is $X_l = n_l/n$ *lepton concentration* $\delta\mu_l = \mu_n - \mu_p - \mu_l \neq 0$

$$\delta\dot{X}_l = -\frac{\delta X_l}{\tau_{X,l}} + n\frac{\partial\delta\mu_l}{\partial n}\,\delta n(t)$$

$$\zeta_{\rm s.m.} = n\frac{\langle P(n+\delta n(t), X_l + \delta X_l(t))\,\delta\dot{n}(t)\,\rangle_{\mathcal{P}}}{\langle(\delta\dot{n}(t))^2\rangle_{\mathcal{P}}}$$

relaxation time

$$\zeta_{\rm s.m.} \approx -\frac{\partial P}{\partial X_l}\frac{{\rm d}X_l}{{\rm d}n}\frac{n\tau_{X,l}}{1+\omega^2\,\tau_{X,l}^2}$$

$\langle\ldots\rangle_{\mathcal{P}}$ *average over the perturbation period*

$$\zeta_{\text{s.m.}} \approx -\frac{\partial P}{\partial X_l}\frac{\mathrm{d}X_l}{\mathrm{d}n}\frac{n\tau_{X,l}}{1+\omega^2\,\tau_{X,l}^2} \approx -\frac{\partial P}{\partial X_l}\frac{\mathrm{d}X_l}{\mathrm{d}n}\frac{n}{\omega^2\,\tau_{X,l}} \qquad \text{for} \quad \omega\tau_{X,l} \gg 1$$

$$\frac{1}{\tau_{X,l}} = \sum_{\mathrm{r}} \mathcal{R}^{[r]} = \sum_{\mathrm{r}} \frac{1}{\tau_{X,l}^{[\mathrm{r}]}}$$

***sum** over various processes changing* δX_l

$$\zeta_{\text{s.m.}} \approx \sum_{\mathrm{r}} \zeta_{\text{s.m.}}^{[r]}$$

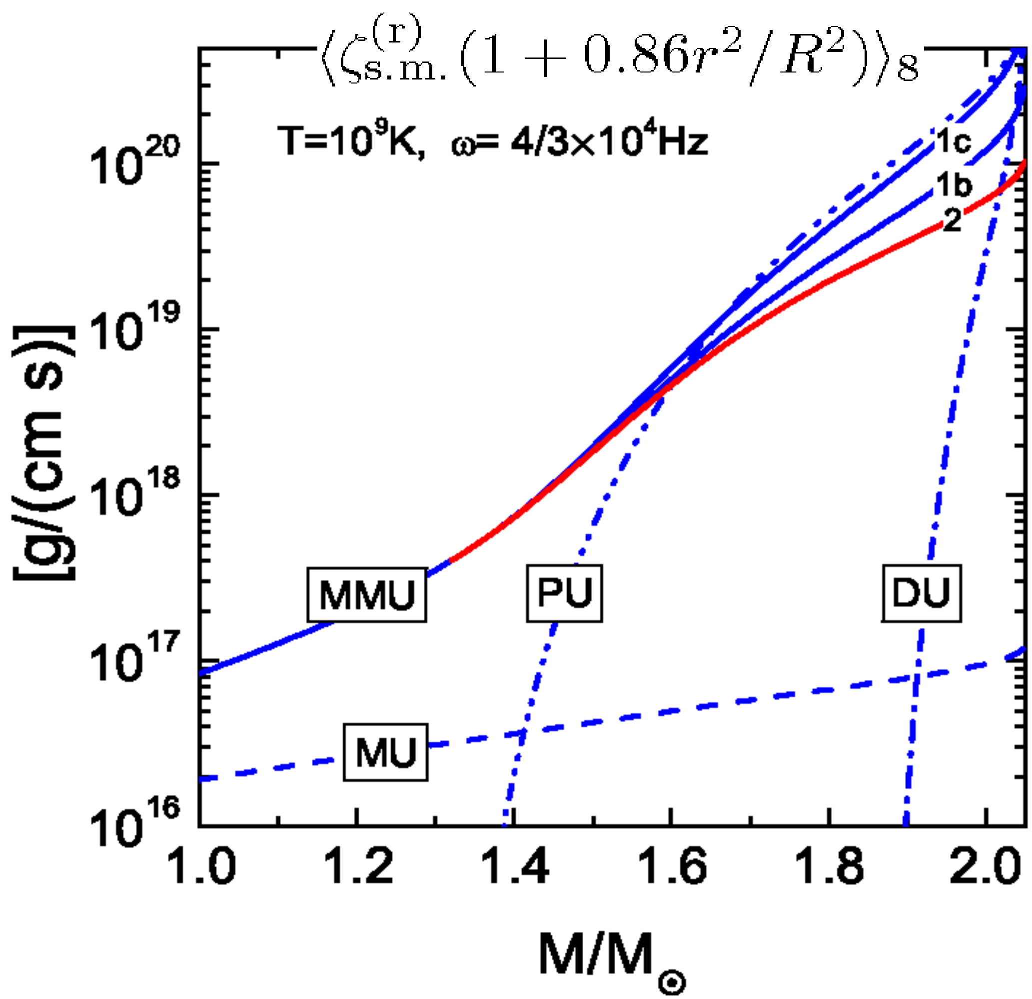
direct Urca (DU); $\zeta_{\text{s.m.}}^{[\mathrm{DU}]} \propto T^5$

modified Urca (MU)
[calculated with FOPE] $\zeta_{\text{s.m.}}^{[\mathrm{MU}]} \propto T^7$
or
medium MU (MMU)
[calculated with MOPE];

$$\zeta_{\text{s.m.}}^{[\mathrm{MMU}]} \propto F_{\mathrm{MMU}}(n) T^7$$

pion condensate Urca (PU)

$$\zeta_{\text{s.m.}}^{[\mathrm{PU}]} \propto T^5$$



Energy dissipation of the mode: $\dot{E}_{\rm mode} = P\dot{V} - \underline{\epsilon_\nu}$ ϵ_ν is neutrino emissivity

Meltzer, Thorne ApJ145(66)514; Hansen, Tsuruta, Can.J.Phys. 45 (67)2823;
Sa'd, Schaffner-Bielich 0908.4190

radiative viscosity $\zeta_{\rm rad} = -n^2 \frac{\langle \epsilon_\nu(n+\delta n(t), \delta\mu(t)) - \epsilon_\nu(n,0)\rangle_{\mathcal{P}}}{\langle (\delta\dot{n}(t))^2\rangle_{\mathcal{P}}} = \sum_r \zeta^{[r]}_{\rm rad}$

$$\zeta^{[r]}_{\rm rad} \approx \frac{3}{2}\zeta^{[r]}_{\rm s.m.} + \underline{\frac{n^2}{2\omega^2}\frac{\partial^2 \epsilon^{[r]}_\nu}{\partial n^2}}$$

reactions changing
the number of e,μ
Urca reactios [DU,MMU,PU]

νν production
Bremsstrahlung,
Pair-breaking formation

small contribution

Pairing effects on bulk viscosity

pairing suppression
factor for nucleon i=n,p

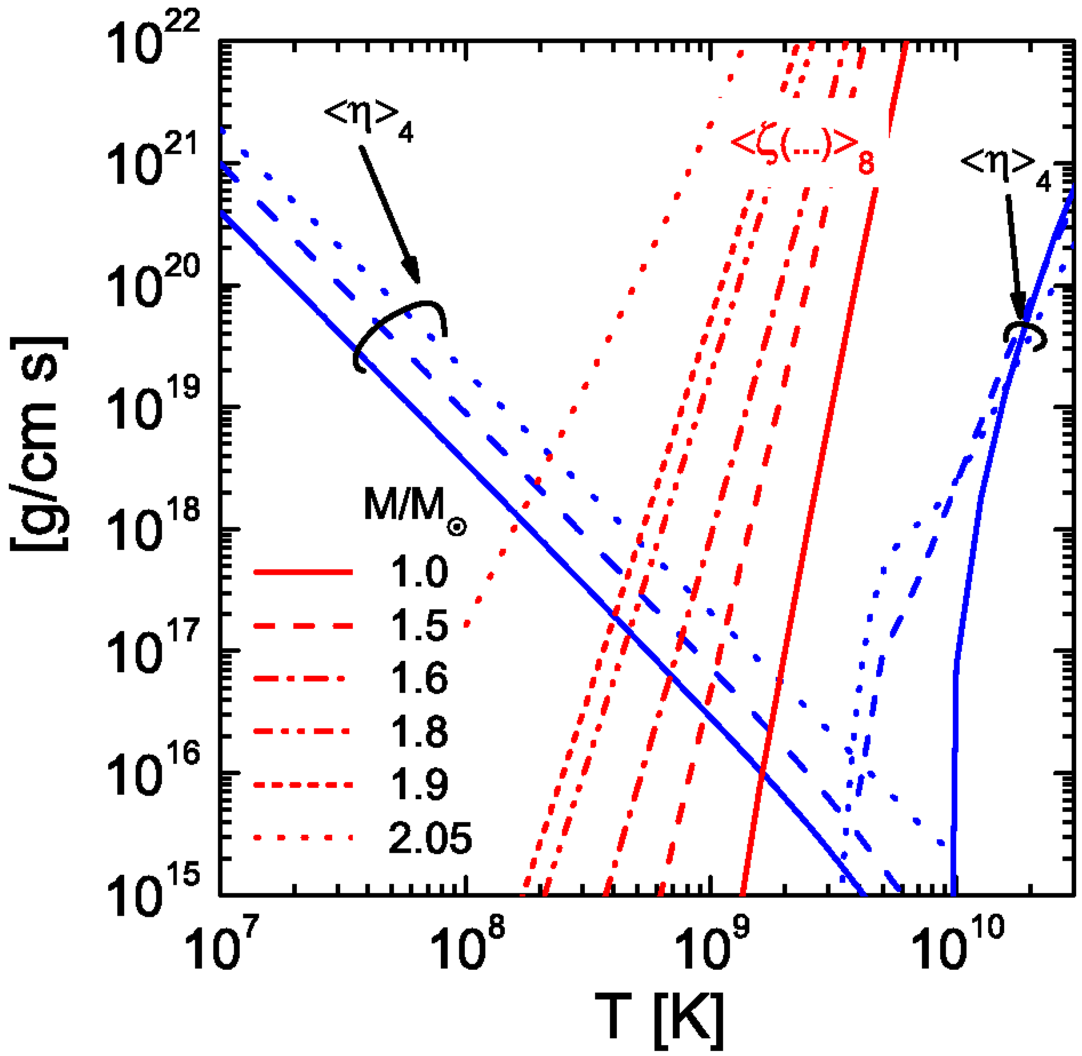
$$\xi_i = \exp(-\Delta_i/T)$$

$$\zeta^{\rm (DU/PU)(s)}_{{\rm s.m.},l} = \zeta^{\rm (DU/PU)}_{{\rm s.m.},l}\,{\rm min}[\xi_n, \xi_p]\,,$$

$$\zeta^{\rm (MU/MMU)i(s)}_{{\rm s.m.},l} = \zeta^{\rm (MU/MMU)i}_{{\rm s.m.},l}\,\xi_p\,\xi_i$$

$$T \ll \Delta_i$$

Shear and bulk viscosities. Results

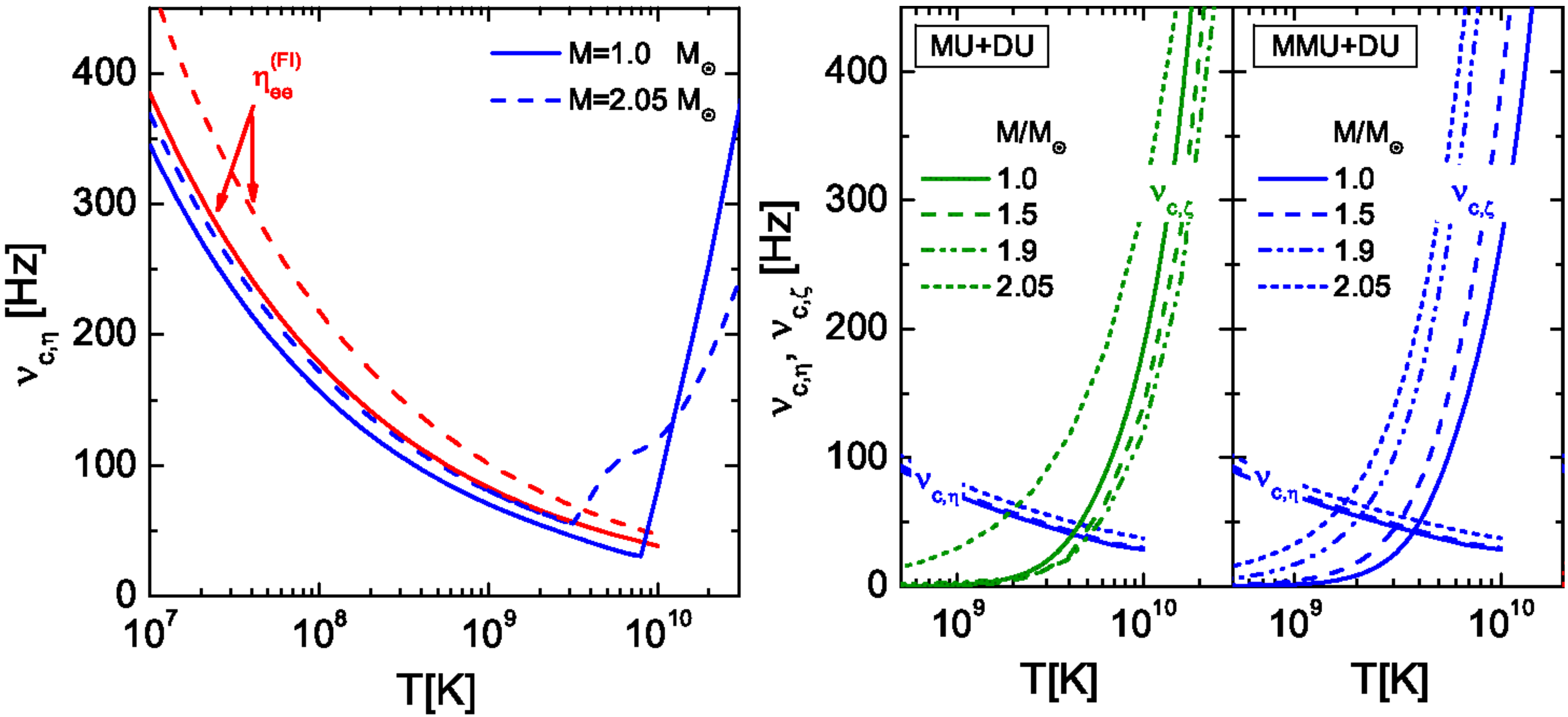


MMU+DU

R-mode stability window

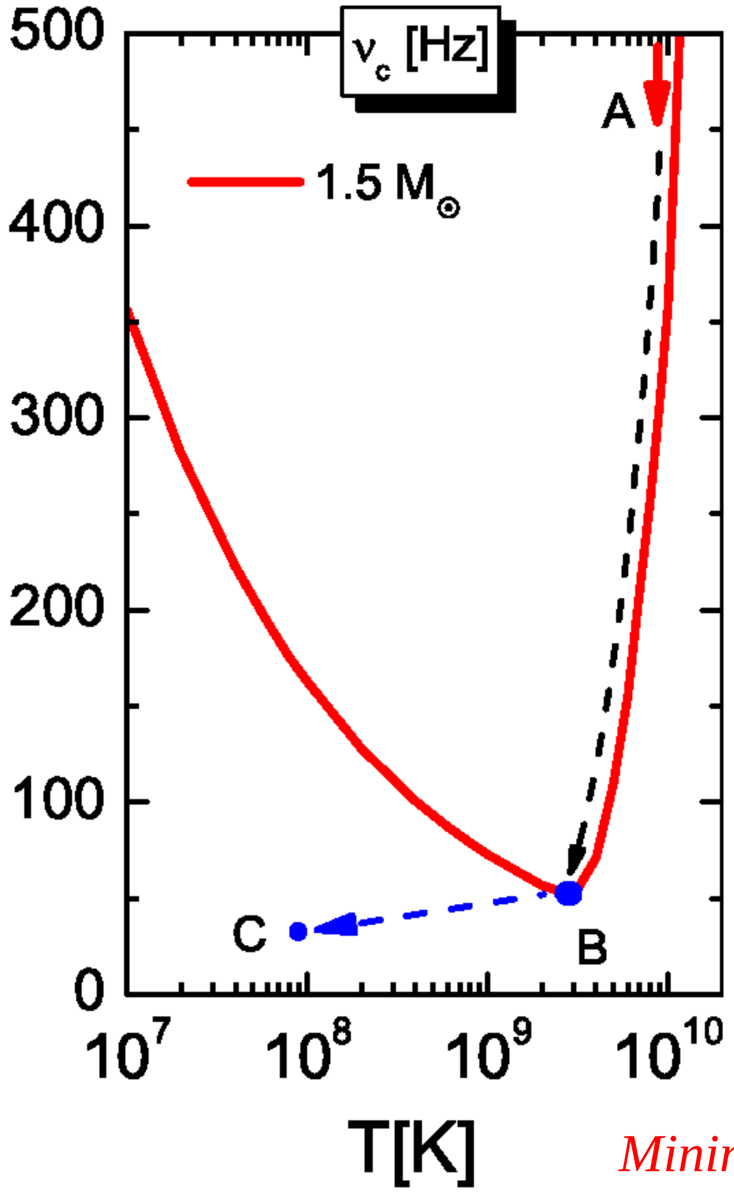
$$\tau_G^{-1}(\nu_c) = \tau_\eta^{-1}(\nu_c) + \tau_\zeta^{-1}(\nu_c) \quad \rightarrow \quad \nu_c = \nu_c(T)$$

$$\tau_G^{-1}(\nu_{c,\eta}) = \tau_\eta^{-1}(\nu_{c,\eta}) \rightarrow \nu_c^6 = \nu_{c,\eta}{}^6 + \nu_c^2\, \nu_{c,\zeta}{}^4 \leftarrow \tau_G^{-1}(\nu_{c,\zeta}) = \tau_\zeta^{-1}(\nu_{c,\zeta})$$



$\nu = \Omega/2\pi$

Young pulsars



1. star is born hot and rapidly rotating (point A)

2. cooling time (heat transport!) >> spin-down time

$$t_{\text{spin-down}} \sim 100\text{s}/a_{\max}^2 \nu_3^6$$

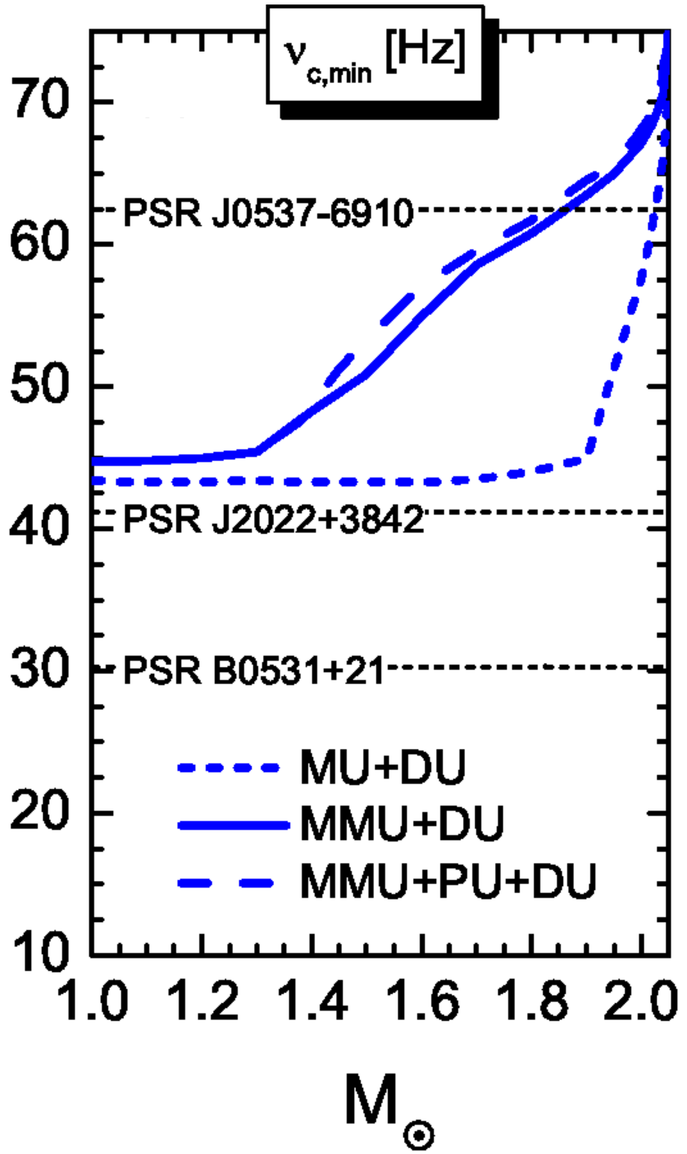
for max. r-mode amplitude $a_{\max} \sim 1$

3. star moves along line AB because of r-modes

4. line BC, cooling and magnetic breaking

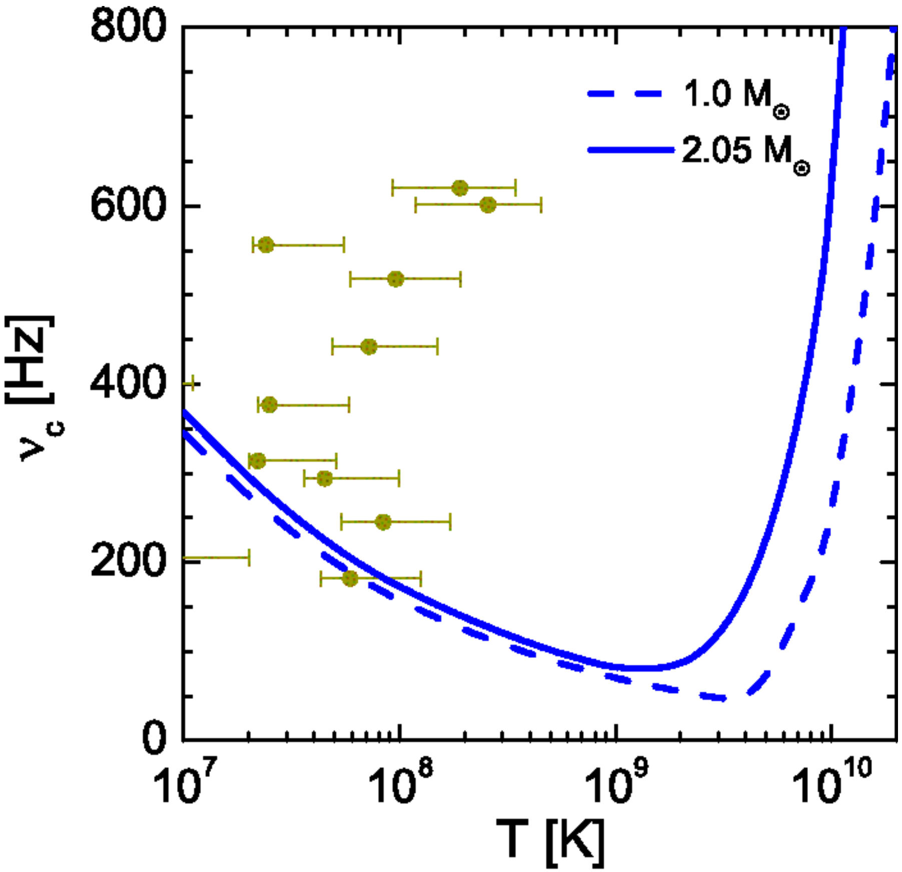
Minimum point B must be above 62 Hz (PSRJ0537-6910)

Minimum of the stability window



LMXB recycled pulsars

Rotation of LMXB pulsars cannot be explained. Shear viscosity is too small.



Alternative mechanisms

-differential drift in magnetic field
[Rezzolla, Lamb, Shapiro]
-weak reactions with hyperons +hyperon pairing
[Jones; Nayyar Owen]
-core-crust coupling
[Bildsten, Ushomirsky, Levin]
-saturation of r-mode amplitude at small values
[Arras, Bondaresku, Wasserman]
-non-linear decay of r-modes
[Kastaun]
- coupling to more stable modes
[Gusakov, Chugunov, Kantor]
-vortex flux-tube interactions
[Haskell, Glampedakis, Andersson]