Holographic models for QCD at high densities - part II

HISS “Dense Matter”, Bogoliubov Laboratory of Theoretical Physics,

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ, Дубна

29.06. - 03.07.2015

Matthias Kaminski
University of Alabama

University of Alabama, Tuscaloosa

University of Alabama, Tuscaloosa

- Overview
- Objectives
- Confirmed Participants
- Schedule (PDF)
- Accommodation
- Directions
- Slides

Supported by

Department of Physics & Astronomy

Office for Academic Affairs

Research Workshop & Summer School: Holography near and far-from equilibrium

Saturday, October 24 until Friday, October 30, 2015

Organizer

Matthias Kaminski (University of Alabama)

Location

University of Alabama, Tuscaloosa
(Take a virtual campus tour)

Overview

During the past few years the holographic principle in the form of the gauge/gravity correspondence has been applied to many strongly coupled systems, such as heavy ion collisions. Starting out near equilibrium, this research community has by now developed methods to study far-from equilibrium dynamics. This meeting aims to share the latest methods and ideas for holography near and far-from equilibrium. Experts on the gauge/gravity correspondence and students will come together in order to share knowledge and define the future goals for this thriving field of research.

http://bama.ua.edu/~mkaminski3/UA_Workshop_2015/Overview.htm

Ask me questions!

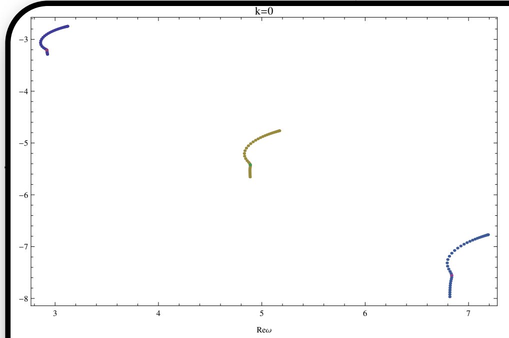
Exercises: The tasks are only suggestions; we can discuss your questions instead!

Recall: heavy ion collisions & neutron stars

see Lecture I by Ohnishi (dense phases of QCD)

Thermalization of charged plasmas near equilibrium

holography



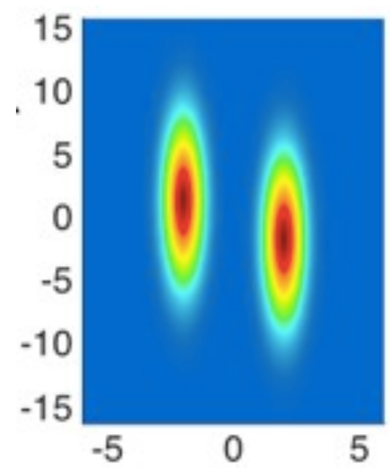
Calculation of quasi-normal modes

- charged plasma
- magnetic field
- non-conformal

[Janiszewski, Kaminski; to appear (2015)]

Thermalization of charged plasmas far-from equilibrium

holography



Calculation of time-dependent metrics / black hole formation

- charged plasma
- magnetic field
- non-conformal
- isotropization
- off-center collision

[Chesler, Yaffe; PRL (2011)]
[Chesler, Yaffe; arXiv (2015)]

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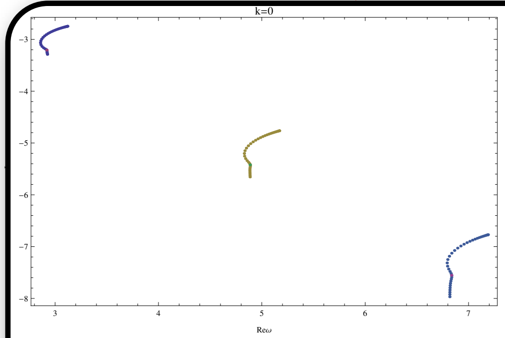

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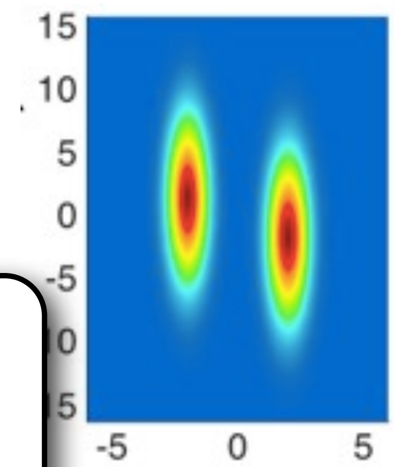

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- charged plasma
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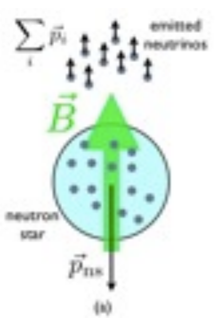
Inspired by holography:



Anomalous hydrodynamics leads to neutron star kicks

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

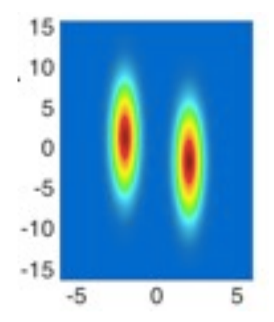
Contents: Lecture II

1. Recall: chiral/ anomalous hydrodynamics

2. Neutron star kicks

3. Quasi-normal modes (QNMs)

4. Holographic thermalization



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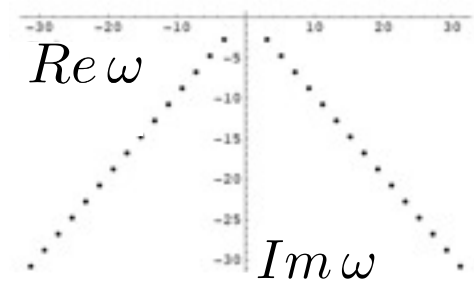

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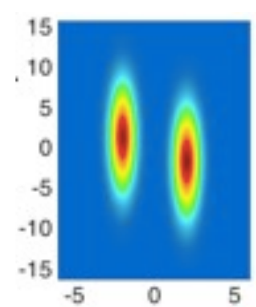

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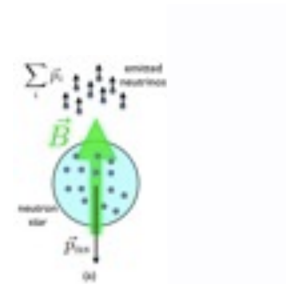

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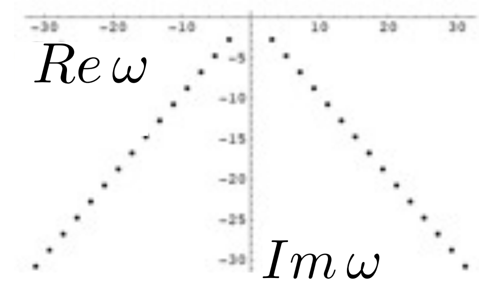

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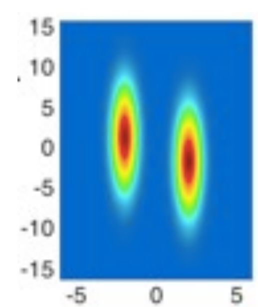

1. Recall: chiral/anomalous hydrodynamics

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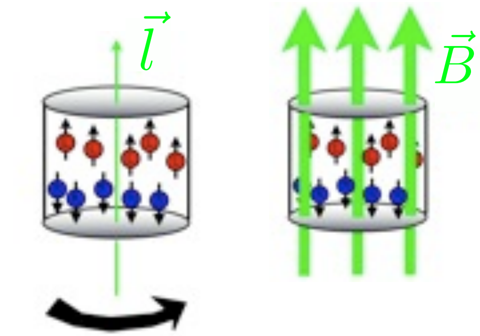

Recall: chiral effects in vector/axial currents

see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral magnetic effect

Axial current (e.g. QCD axial *U(1)*)

$$J_A^\mu = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B_A^\mu$$

chiral vortical effect

chiral separation effect

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Full chiral effects & gravity

[Neiman, Oz; JHEP (2010)]

More than one anomalous current

$$\langle \partial_\mu J^\mu_a \rangle = \frac{1}{8} C_{abc} \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma}$$

Constitutive relation:

$$J^\mu_a = n_a u^\mu + \sigma_a{}^b V^\mu_b + \sigma^V_a \omega^\mu + \sigma^B_{ab} B^{b\,\mu} + \mathcal{O}(\partial^2)$$

Chiral vortical conductivity:

$$\xi_a = C_{abc}\mu^b\mu^c + 2\beta_a T^2 - \frac{2n_a}{\epsilon + p}\left(\frac{1}{3}C_{bcd}\mu^b\mu^c\mu^d + 2\beta_b\mu^b T^2 + \gamma T^3\right)$$

Chiral magnetic conductivity:

$$\xi^{(B)}_{ab} = C_{abc}\mu^c - \frac{n_a}{\epsilon + p}\left(\frac{1}{2}C_{bcd}\mu^c\mu^d + \beta_b T^2\right)$$

Full chiral effects & gravity

[Neiman, Oz; JHEP (2010)]

More than one anomalous current $\langle \partial_\mu J^\mu_a \rangle = \frac{1}{8} C_{abc} \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma}$

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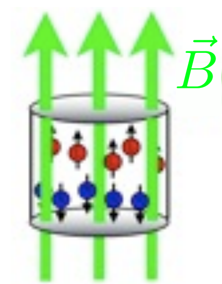
Different frame of reference!

* chiral conductivities look different
* focus on magnetic effect

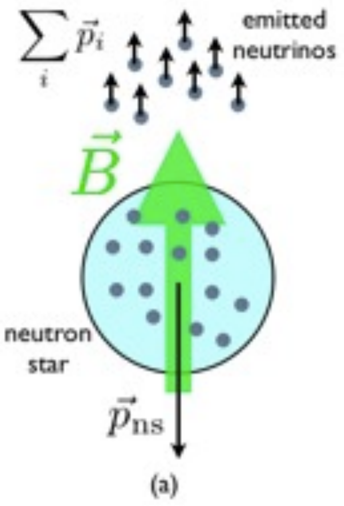
Chiral magnetic conductivity:

$$\sigma^B_{ab} = C_{abc}\, \mu^c$$

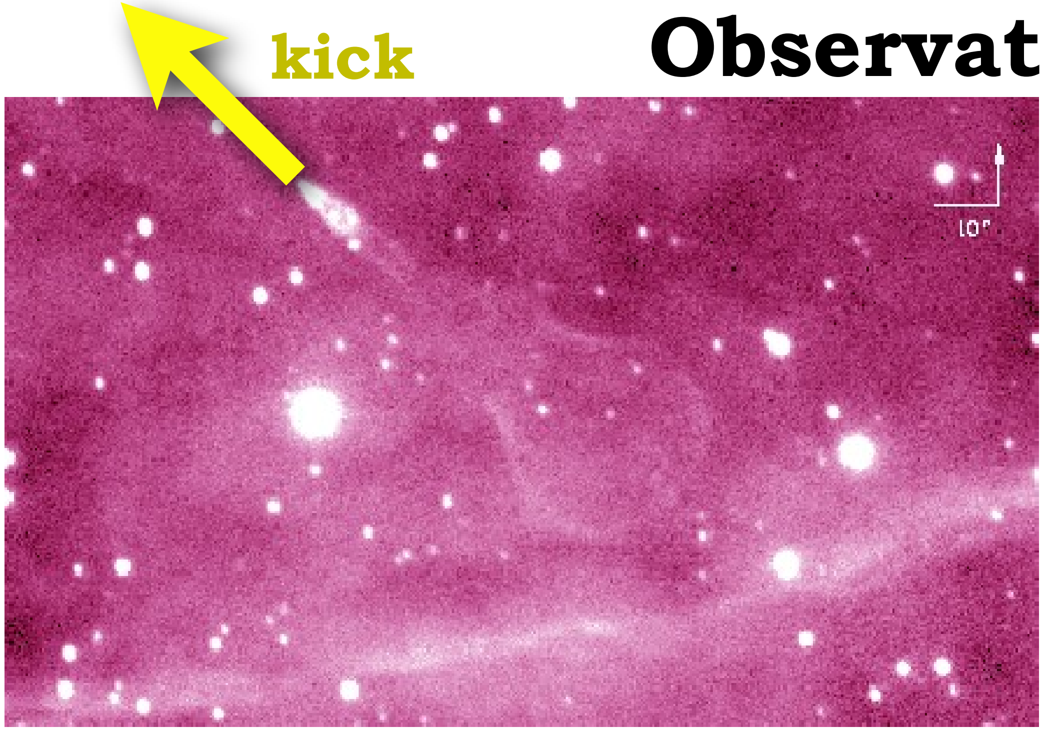


various charges
(e.g. lepton number,
electromagnetic charge, ...)

2. Neutron star kicks



Observations



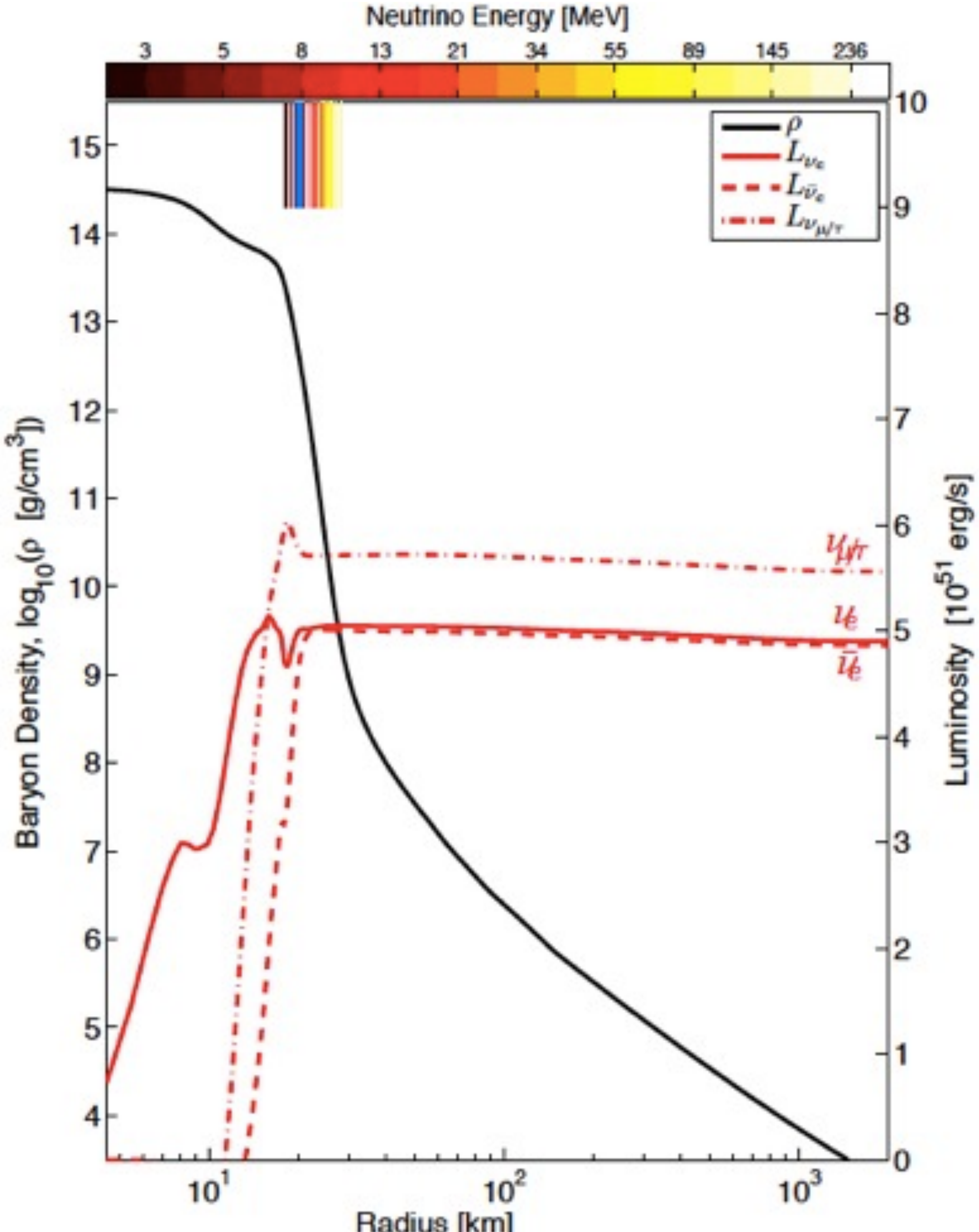
Neutron stars kicked out of their initial position
with velocities ~ 1000 km/s

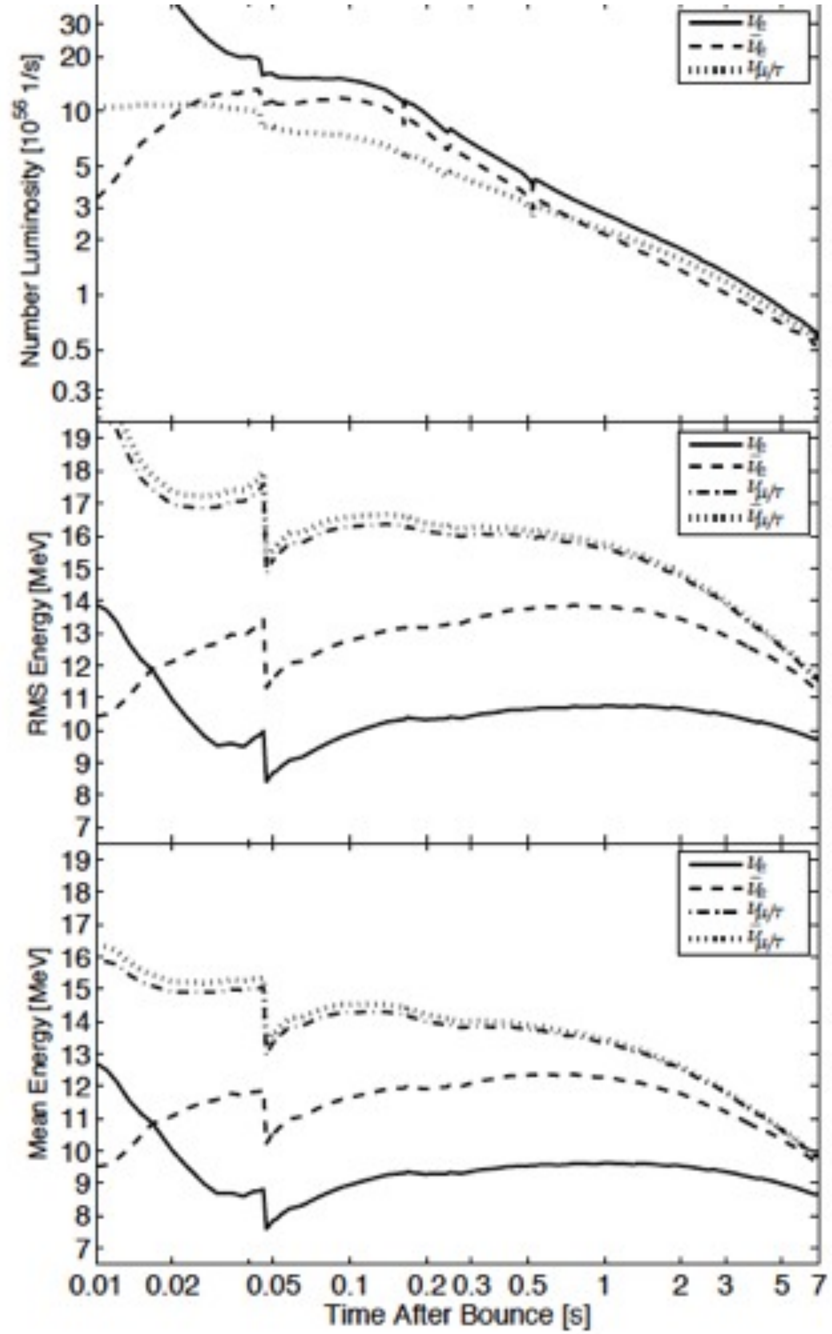
Observations

[Hubble Space Telescope (NASA/ESA), Shami Shatterjee]

Neutron stars kicked out of their initial position with velocities ~ 1000 km/s

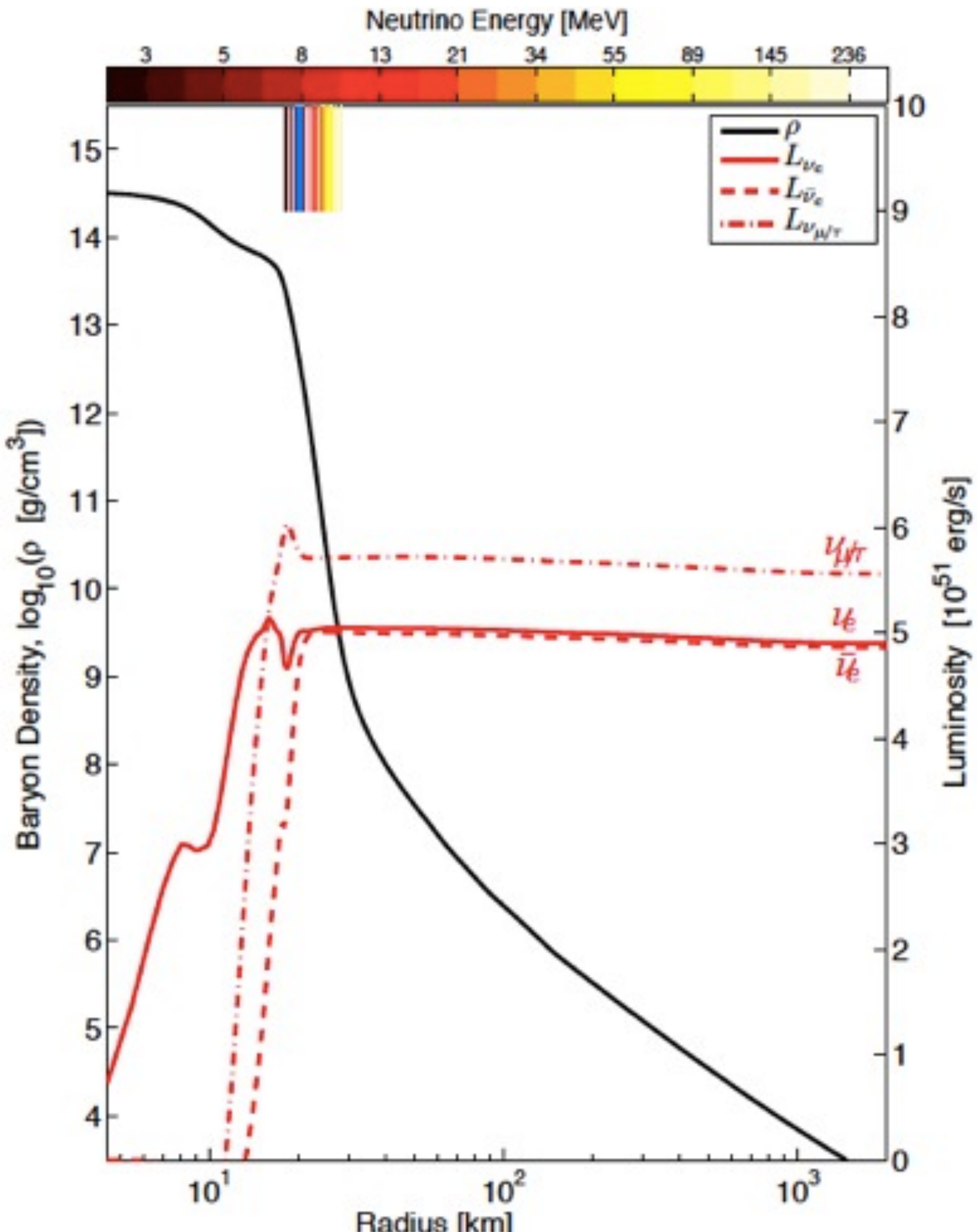
First 10 seconds inside proto-neutron stars

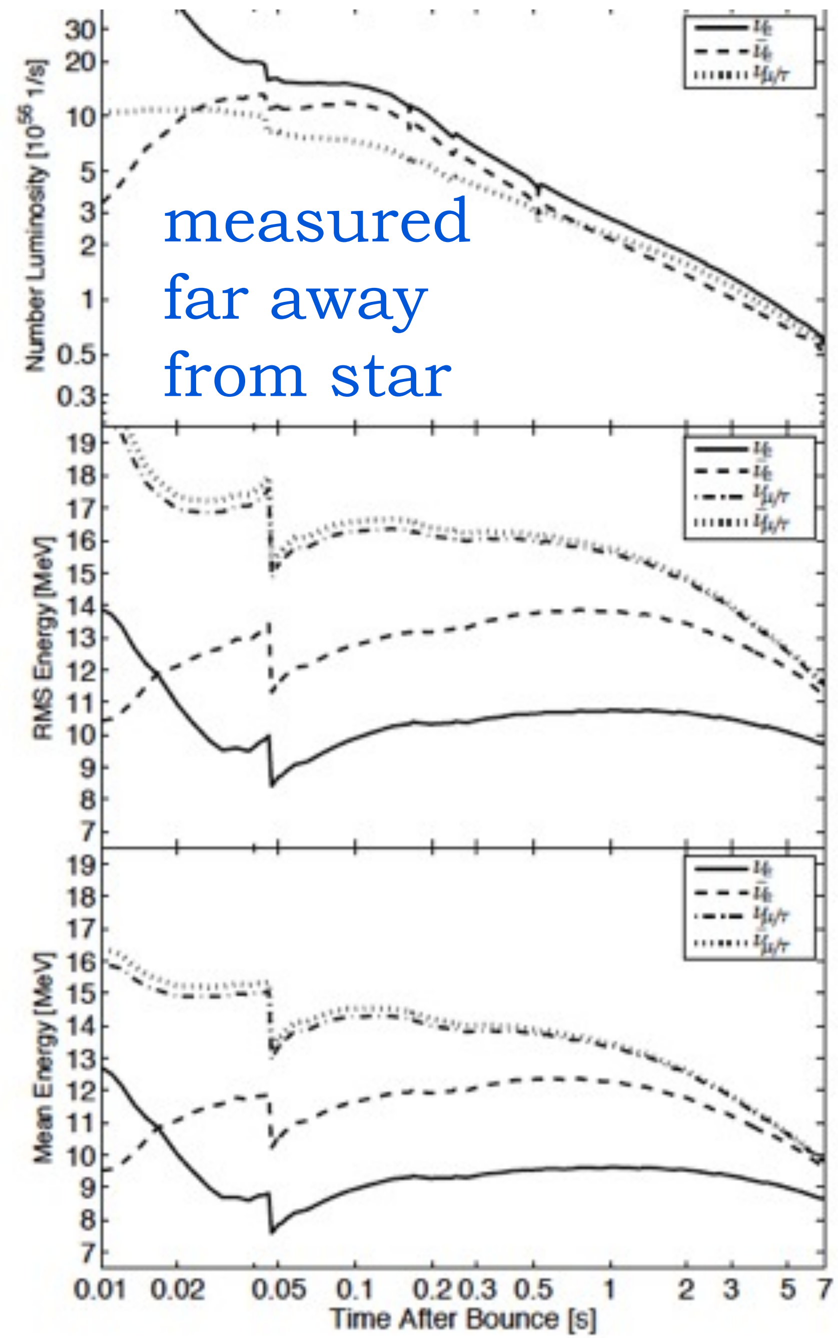




First 10 seconds inside proto-neutron stars

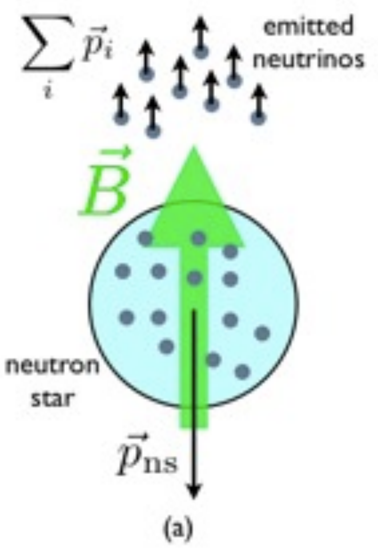
[Fischer et al.; PRD (2011)]



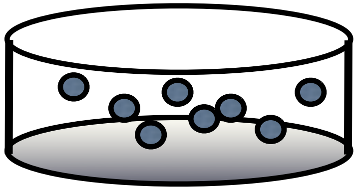


Estimate of the neutron star kick

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]



A bucket full of electrons and electron neutrinos with short mean free path



$$B = 0.1\,\mathrm{MeV}^2$$

$$\mu^\ell \approx 300\,\mathrm{MeV}$$

see lectures by Hempel and Kolomeitsev

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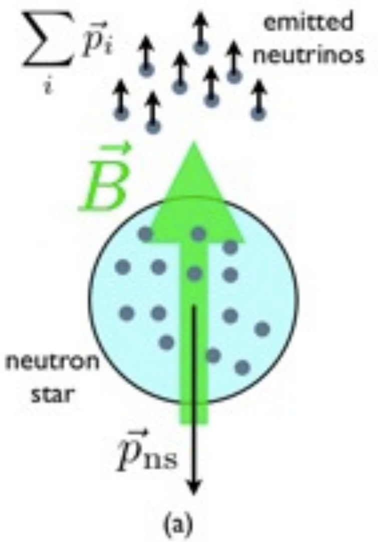

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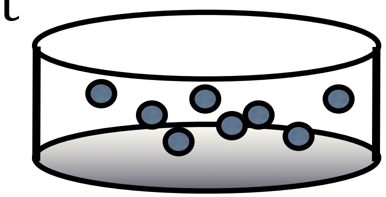

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Microscopic currents: axial/lepton/EM

$$J^\mu_{\ell 5} = \bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R + \bar{\nu}_L \gamma^\mu \nu_L$$

$$J^\mu_\ell = \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R + \bar{\nu}_L \gamma^\mu \nu_L$$

$$J^\mu_{EM}$$

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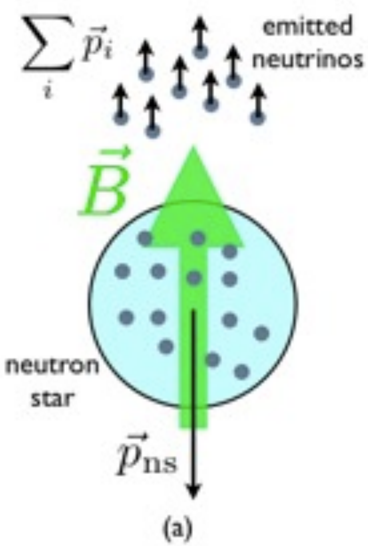

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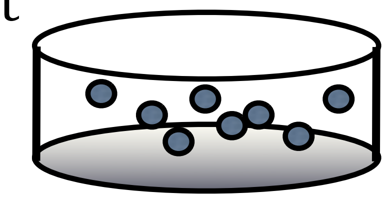

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Macroscopic description:

$$J^\mu_a = n_a u^\mu + \sigma_a{}^b V^\mu_b + \sigma^V_a \omega^\mu + \sigma^B_{ab} B^{b\,\mu} + \mathcal{O}(\partial^2)$$

$$\sigma^B_{ab} = C_{abc}\,\mu^c$$

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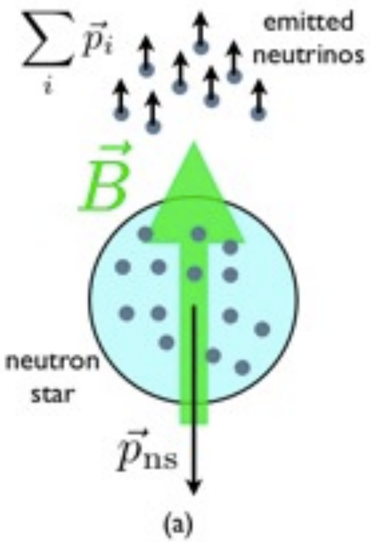

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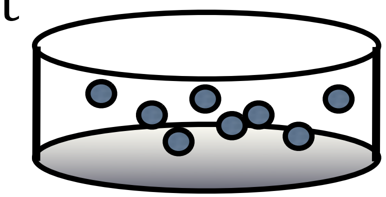

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$$J^\mu_{EM}$$

$$\Rightarrow \quad \sigma^B_{\ell 5, EM} = C_{\ell, \ell 5, EM}\, \mu^\ell$$

$$\vec{J}_\ell \approx 0\,, \qquad \vec{J}_{\ell 5} = C \mu^\ell \vec{B} \approx \vec{e}_B \cdot 1\,\text{MeV}^3$$

Exercise

Kick velocity agrees with observations:

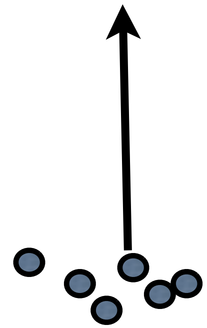
$$\Rightarrow \quad v_{\text{kick}} \approx 1000 \frac{\text{km}}{\text{s}}$$

Macroscopic description:

$$J^\mu_a = n_a u^\mu + \sigma_a{}^b V^\mu_b + \sigma^V_a \omega^\mu + \sigma^B_{ab} B^{b\,\mu} + \mathcal{O}(\partial^2)$$

$$\sigma^B_{ab} = C_{abc}\, \mu^c$$

Observable signal?



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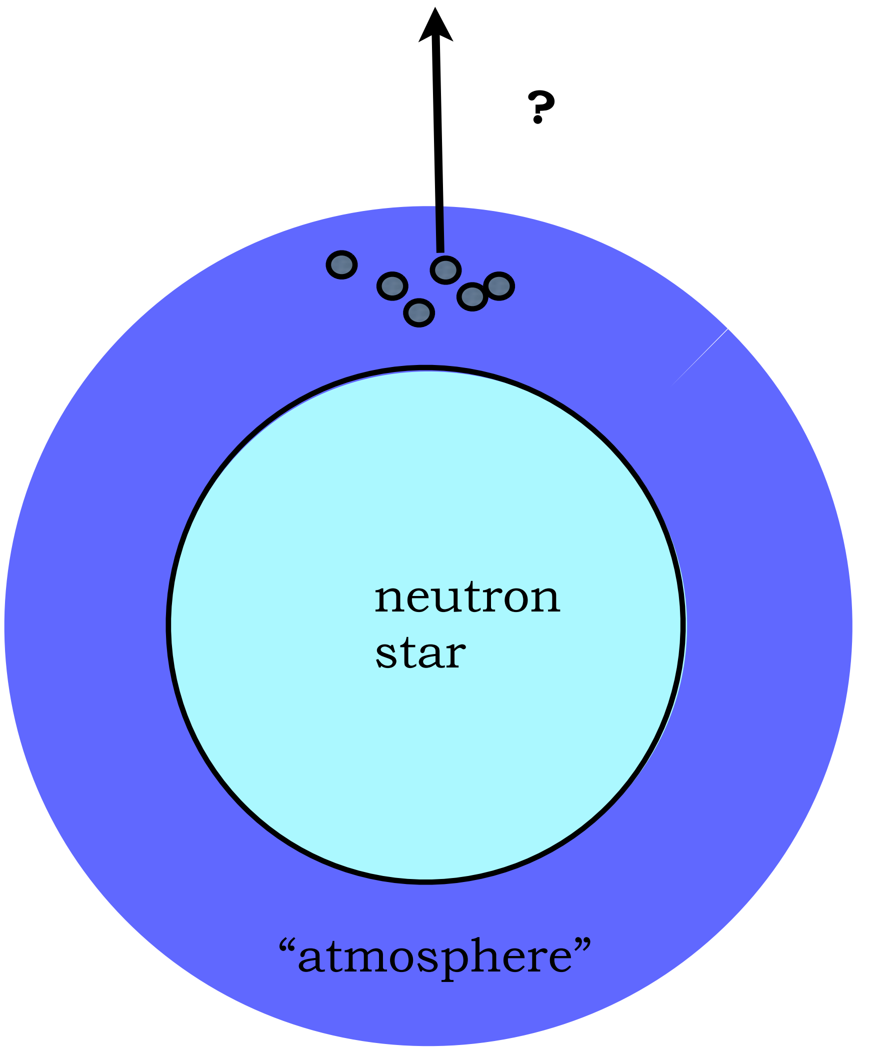
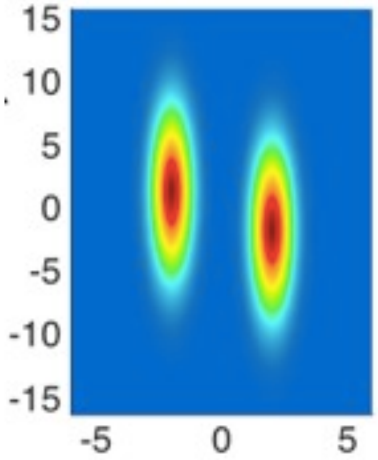
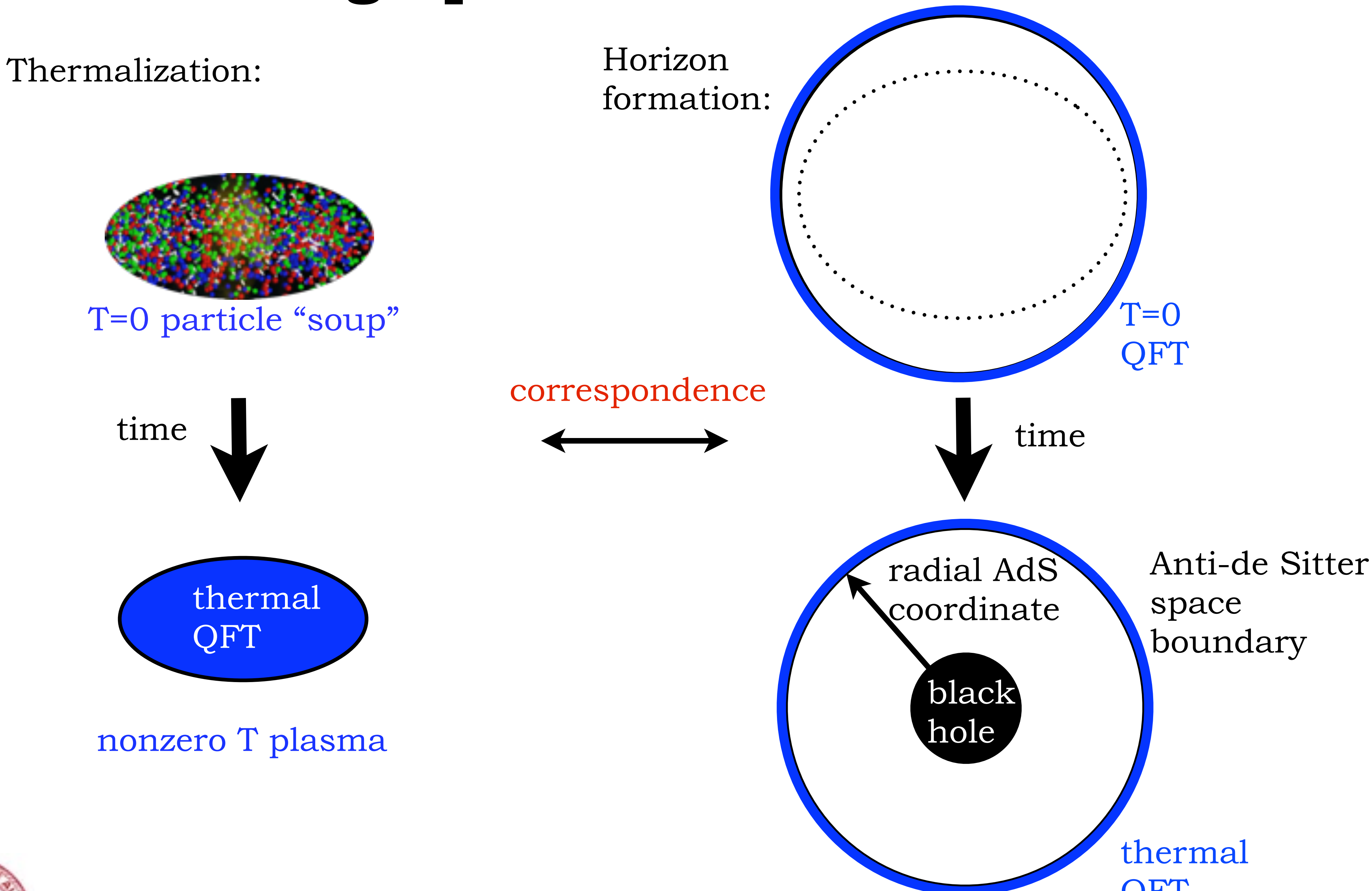


3. Holographic thermalization



Far-from equilibrium states: holographic thermalization



Isotropization in AdS

[Chesler, Yaffe; PRL (2011)]
revolutionary new method for numerical computation in gravity

Ansatz: (all functions of v and r)

$$ds^2 = 2drdv - A\,dv^2 + \Sigma^2 e^B\,(dx^2 + dy^2) + \Sigma^2 e^{-2B} dz^2 + dr^2$$

$$F_{vr} = -F_{rv},\; F_{xy} = -F_{yx}$$

$$\partial_+ = \dot{} = \partial_v + \frac{A}{2}\partial_r$$

Einstein-Maxwell equations [Fuini, Yaffe; (2015)]

$$0 = \frac{B'^2}{2}\Sigma + \Sigma'',$$

$$0 = -\frac{20e^{-2B}F_{xy}^2\ell^2}{3g^2\Sigma^2} + 8\Sigma^2 + 2\Sigma^2 A'' + 6\Sigma^2 B'\partial_+ B - 24\Sigma'\partial_+\Sigma - \frac{28\ell^2 F_{vr}^2}{3g^2},$$

$$0 = 6\Sigma^2\partial_r\partial_+ B + 9\Sigma(\Sigma'\partial_+ B + B'\partial_+\Sigma) + \frac{4e^{-2B}F_{xy}^2\ell^2}{g^2\Sigma^3},$$

$$0 = -6\Sigma^2 + 3\Sigma\partial_r\partial_+\Sigma + 6\Sigma'\partial_+\Sigma + \frac{\ell^2 F_{vr}^2\Sigma^2}{g^2} + \frac{e^{-2B}F_{xy}^2\ell^2}{g^2\Sigma^2},$$

$$0 = 3\Sigma^2(\Sigma(\partial_+ B)^2 - A'\partial_+\Sigma + 2\partial_+\partial_+\Sigma),$$

$$0 = 3F_{vr}\partial_+\Sigma + \Sigma\partial_+ F_{vr} - \frac{1}{2}A(\Sigma F_{vr}' + 3F_{vr}\Sigma'),$$

$$0 = \Sigma F_{vr}' + 3F_{vr}\Sigma'.$$

infalling Eddington-Finkelstein coordinates

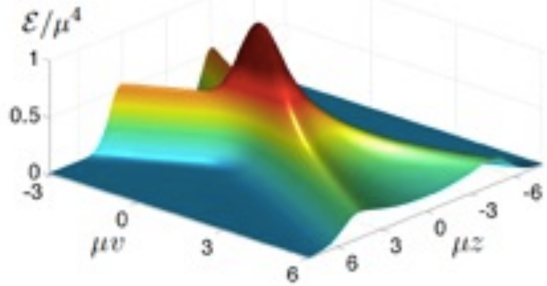
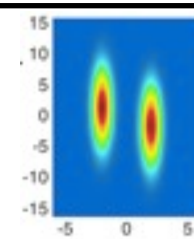
Algorithm:

1. Find boundary expansions.
2. Solve Sigma-equation.
3. Solve Sigma-Dot-equation.
4. Solve B-Dot-equation.
5. Solve A-equation.
6. Find F.
7. Extract new initial data.
8. Repeat.

Boundary conditions on initial time slice:

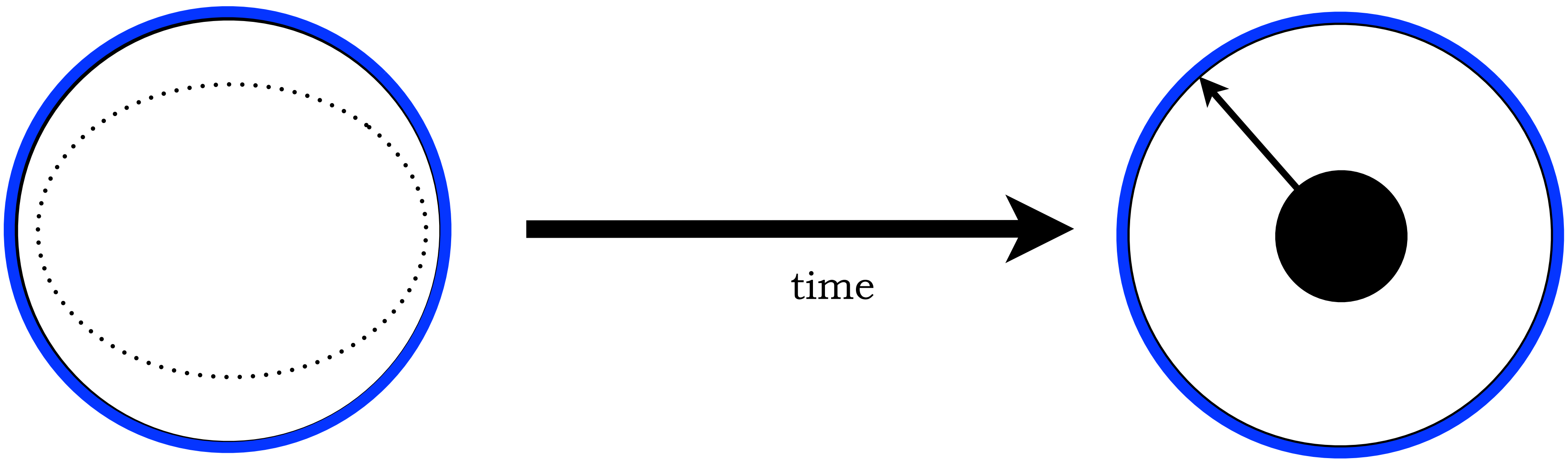
B is Gaussian on initial time slice;
magnetic field fixed;
fix source in Sigma.

Results

Model	Equilibration time
Central collision of two energy lumps in $\mathcal{N}$=4 Super-Yang-Mills. *[Chesler, Yaffe; PRL (2011)]* 	~ 0.35 fm/c
Initial anisotropy in $\mathcal{N}$=4 Super-Yang-Mills, with charges / magnetic field. Confirmed by non-conformal study. *[Fuini, Yaffe; (2015)]* *[Buchel, Heller, Myers; (2015)]*	~ 0.35 fm/c largely unaffected by charges/magnetic field
Off-center collision of two energy lumps in $\mathcal{N}$=4 Super-Yang-Mills. 	~ 0.25 fm/c
1/N corrections *[Schalm; conference talk]*	equilibration time increased

The importance of quasinormal modes

- describing the system at late times
- invaluable consistency check



initial time:
deformed space-time
e.g.: sheared between x- and z-direction

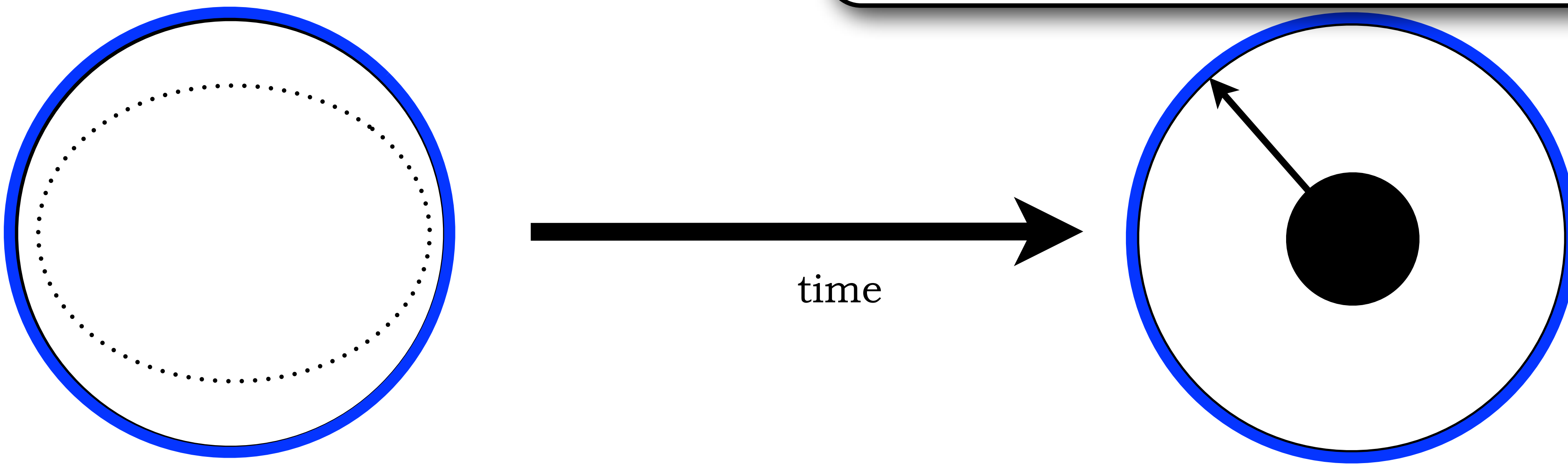
$$ds^2 = 2dr dv - A\, dv^2 + \Sigma^2 e^B \left(dx^2 + dy^2\right) + \Sigma^2 e^{-2B} dz^2 + dr^2$$

final time:
equilibrated space-time
e.g.: AdS5 Schwarzschild black brane

The importance of quasinormal modes

- describing the system at late times
- invaluable consistency check

"Determine late-time evolution of far-from-equilibrium system"
equivalent to
"Relaxation of small perturbations around equilibrium"
aka quasinormal modes of black branes



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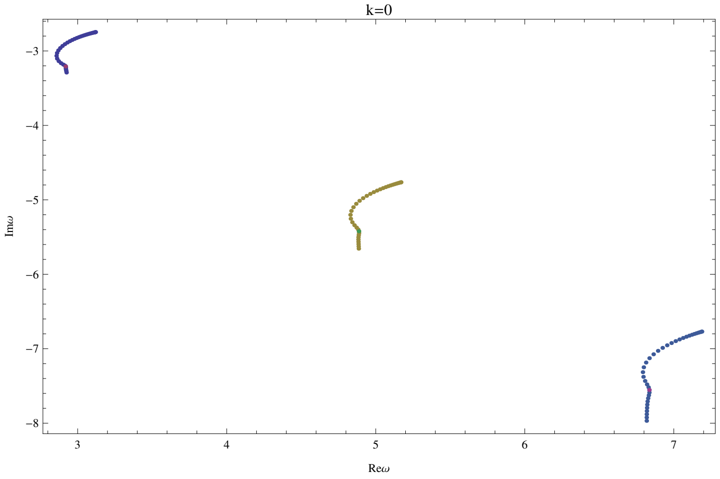
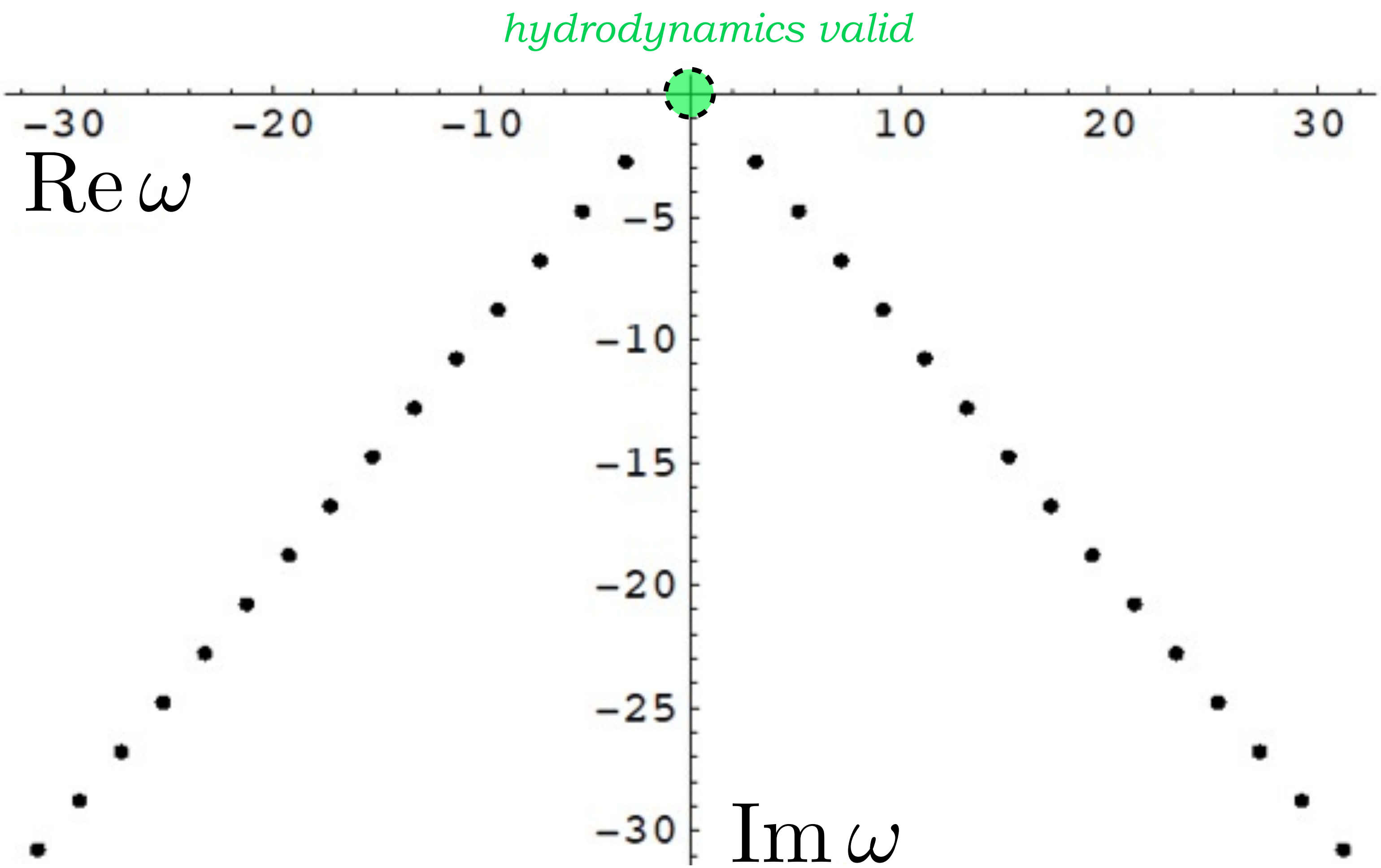


[Janiszewski, Kaminski; to appear (2015)]

Far beyond hydrodynamics

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle (\omega, k) = G^R_{xy,xy}(\omega, k) = -i \int d^4x \; e^{-i\omega t + ikz} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$



Far beyond hydrodynamics : QNMs

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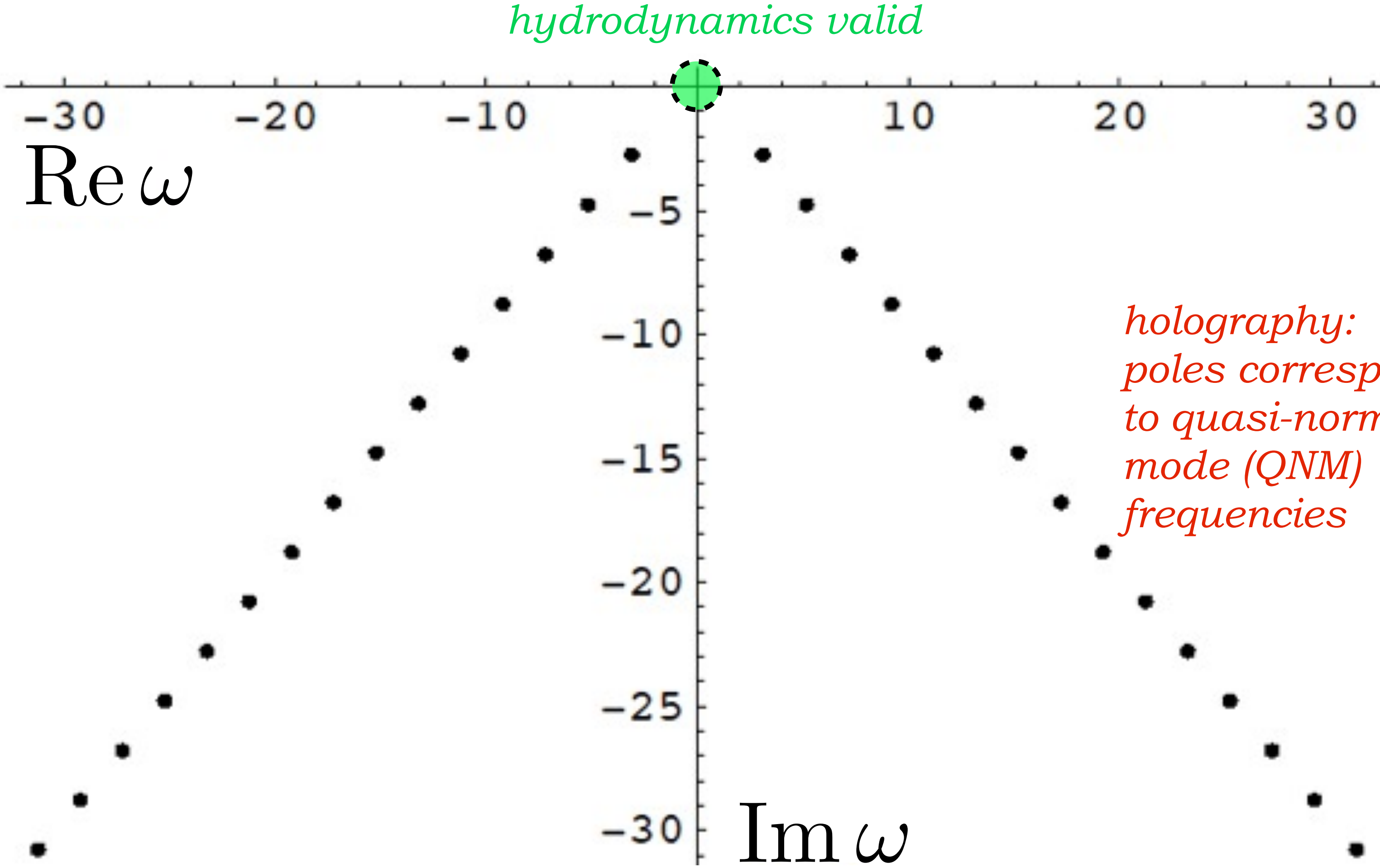


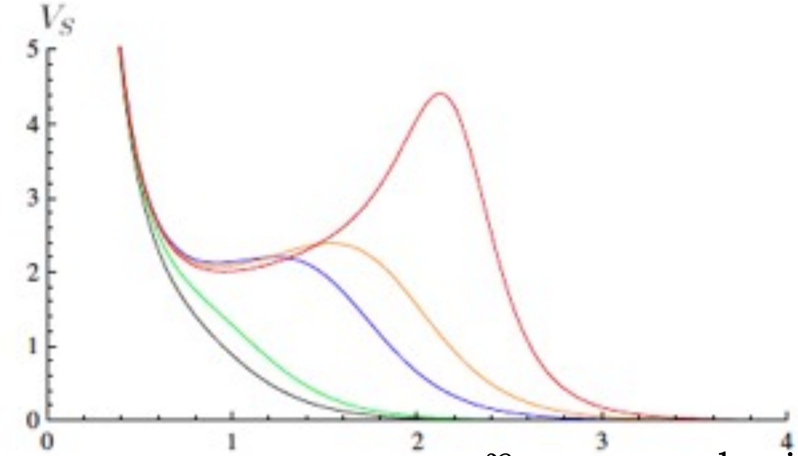
holography: poles correspond to quasi-normal mode (QNM) frequencies

[Starinets; JHEP (2002)]

What are quasi-normal modes?

- heuristically: the eigenmodes of black holes or black branes

$$-\partial_r{}^2\phi + V_S\phi = E\phi$$



- formal definition: (metric) fluctuations that are in-falling at horizon and vanishing at AdS-boundary

- correspond to poles of correlators in dual field theory *[Kovtun, Starinets; 2005]*

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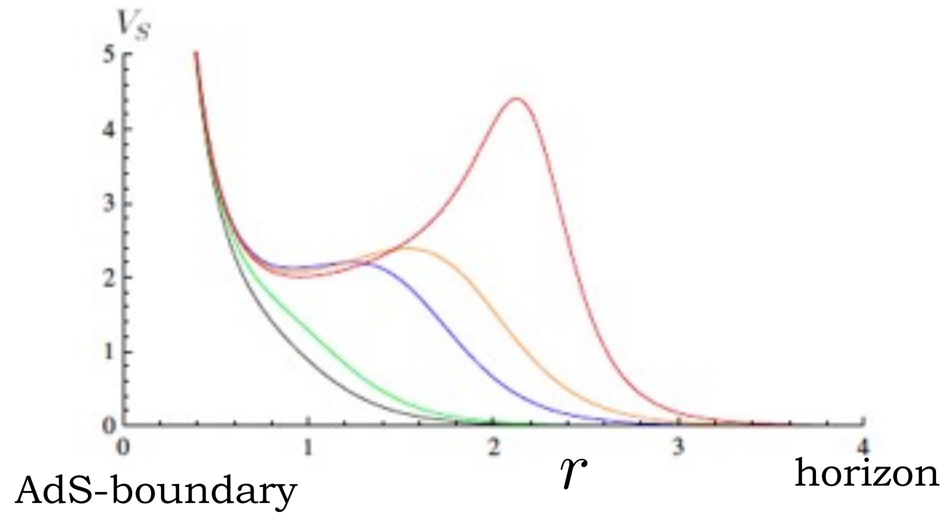


- formal definition: (metric) fluctuations that are in-falling at horizon and vanishing at AdS-boundary

- correspond to poles of correlators in dual field theory [Kovtun, Starinets; 2005]

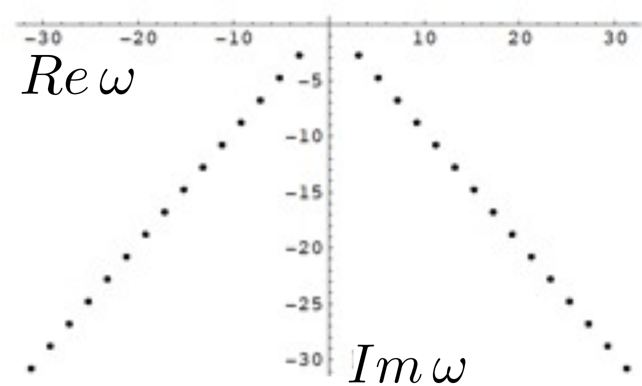
- *example*: tensor fluctuations (known from KSS bound) [Starinets; JHEP (2002)]

  QNMs of $\phi := h_x{}^y$ are poles of $\langle T_{xy} T_{xy}\rangle$

  e.o.m. from linearized Einstein equations:

$$\phi'' - \frac{1+u^2}{uf}\phi' + \frac{\omega^2 - k^2 f}{uf^2}\phi = 0$$

$$f = 1 - u^2$$



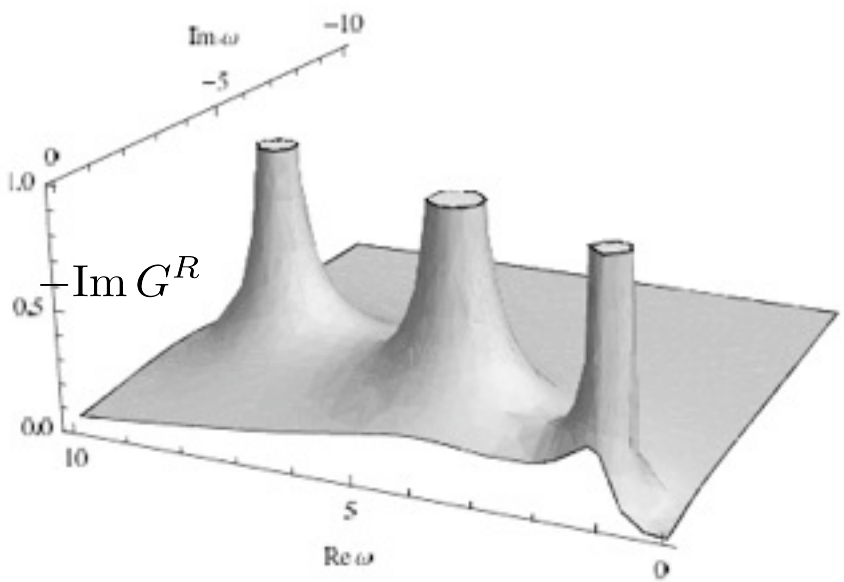
QNMs correspond to poles in retarded Green's

$$G^R(\omega, q) = -\mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}\vec{k}\vec{x}}\, \theta(x^0)\, \langle [J(\vec{x}), J(0)] \rangle \longleftrightarrow \frac{\delta^2}{\delta\phi^{(0)}\delta\phi^{(0)}} S_{gravity}[\phi]$$

correlations between fluctuations around a state

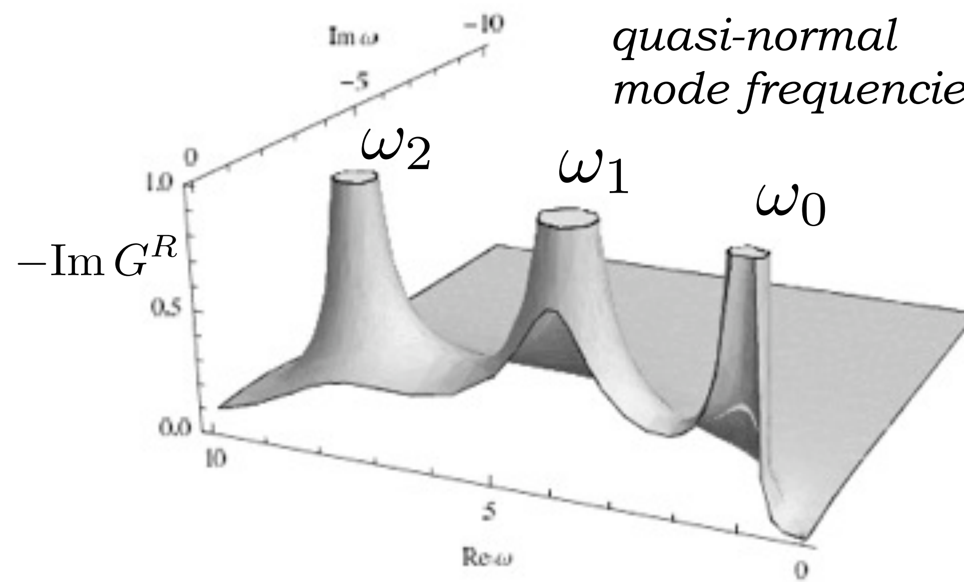
Spectral function (imaginary part of retarded *G*):

see lecture by Kaczmarek



high temperature
no quasiparticles

quasi-normal mode frequencies



low temperature
more stable quasiparticles
(resonances)

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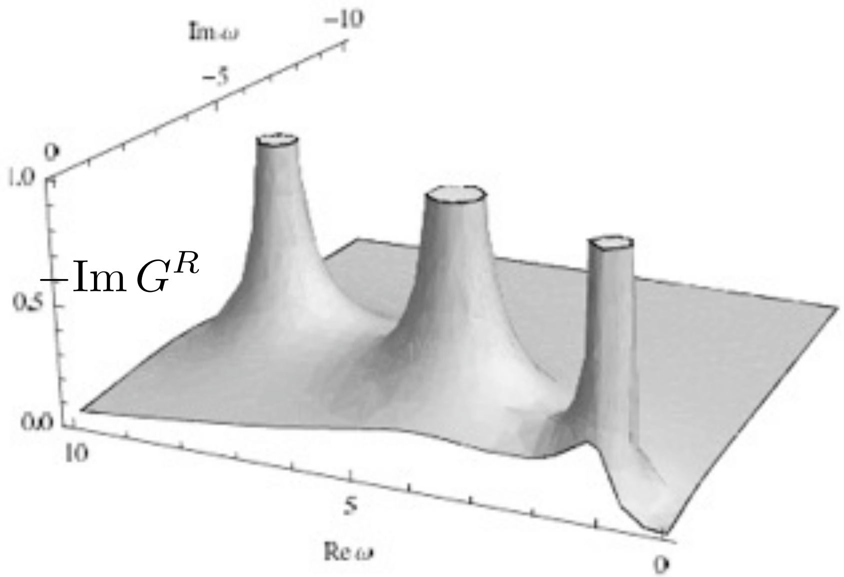
correlations between fluctuations around a state

gravitational fluctuation

BUT: which one ?

Spectral function (imaginary part of retarded G):

see lecture by Kaczmarek



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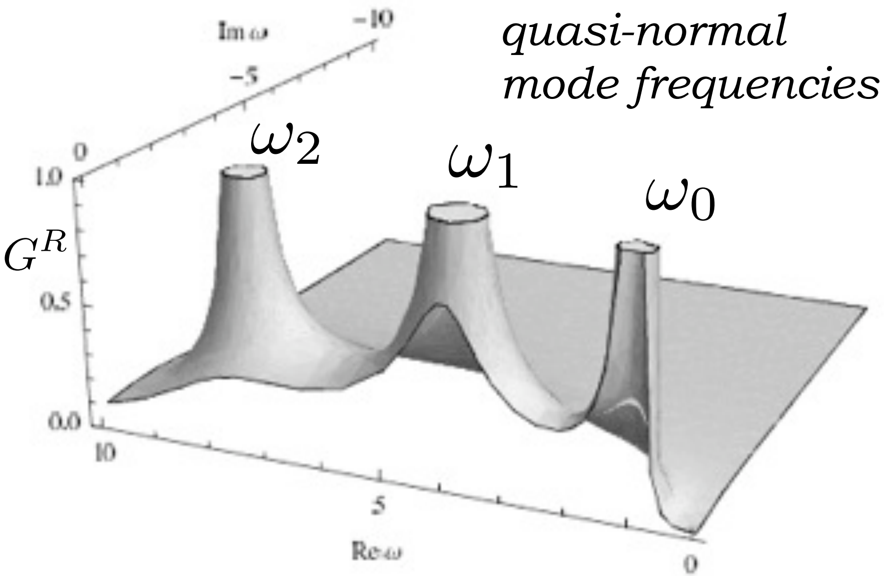

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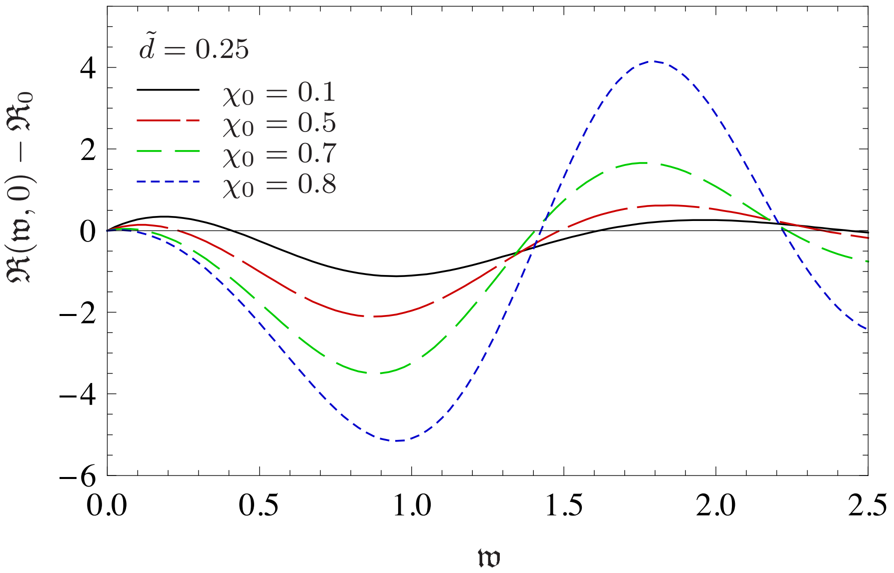
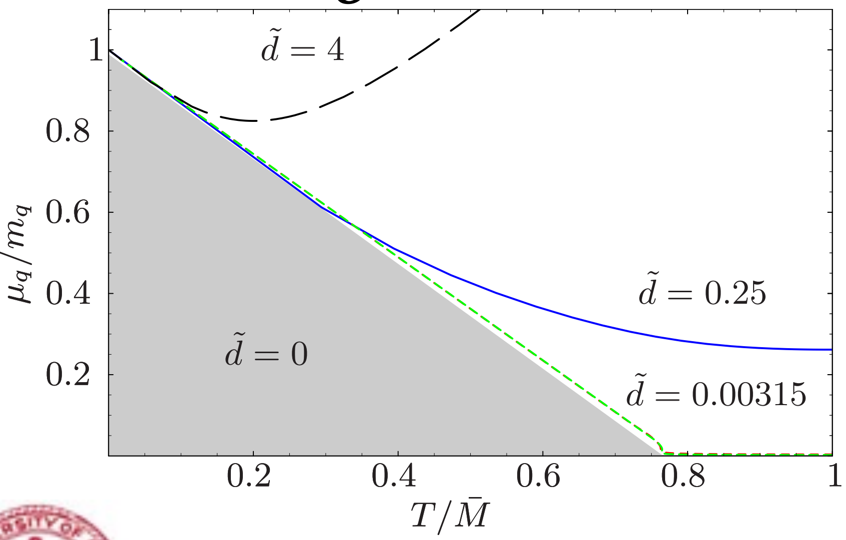
Example: N=2 SYM current correlators

[Erdmenger, Kaminski., Rust 0710.0334]

Nonzero baryon density



Phase diagram



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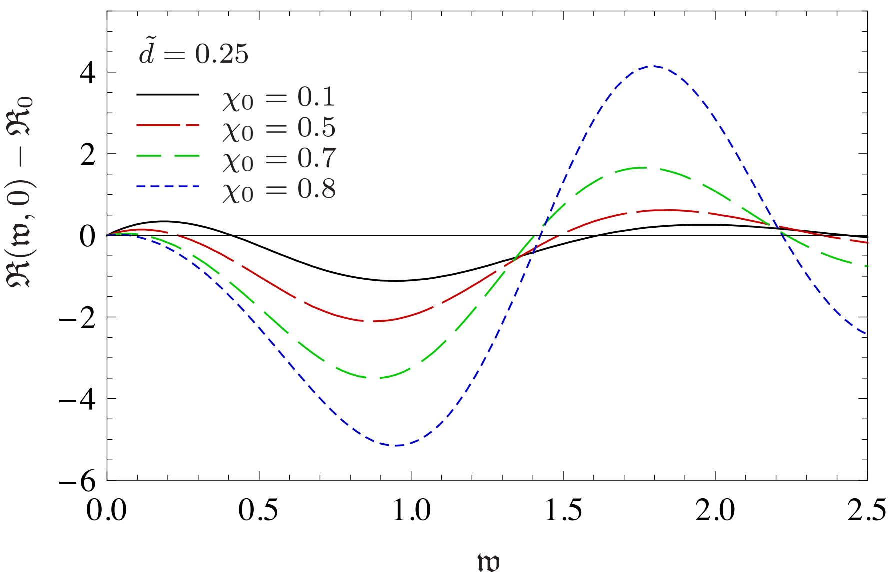

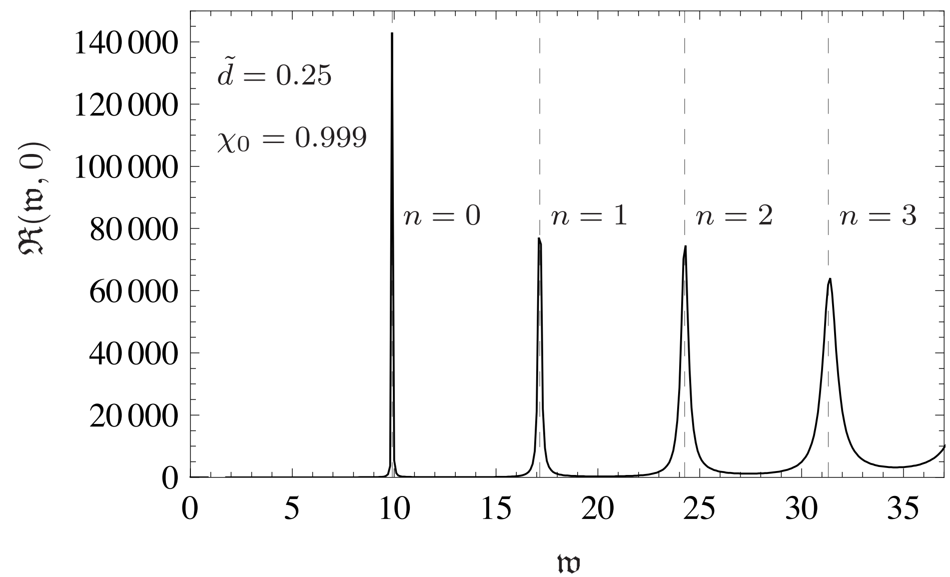

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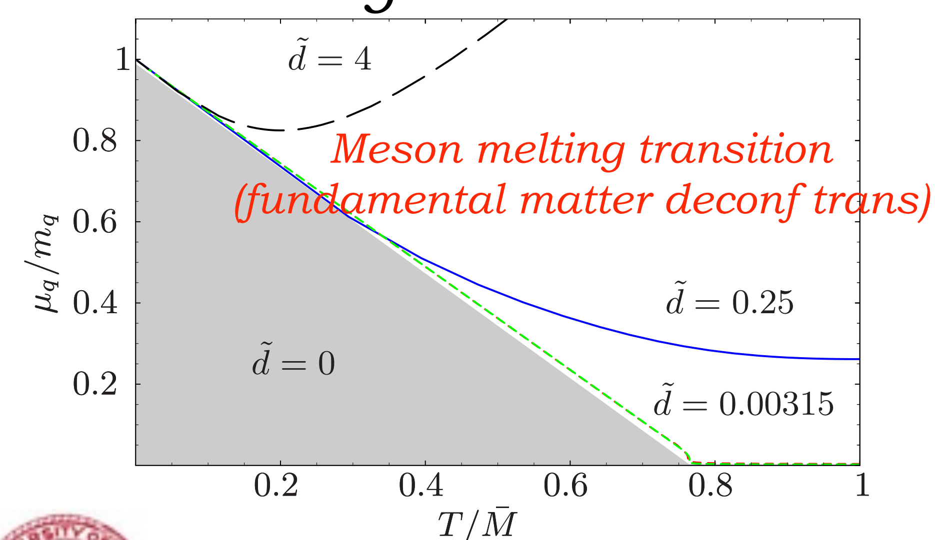

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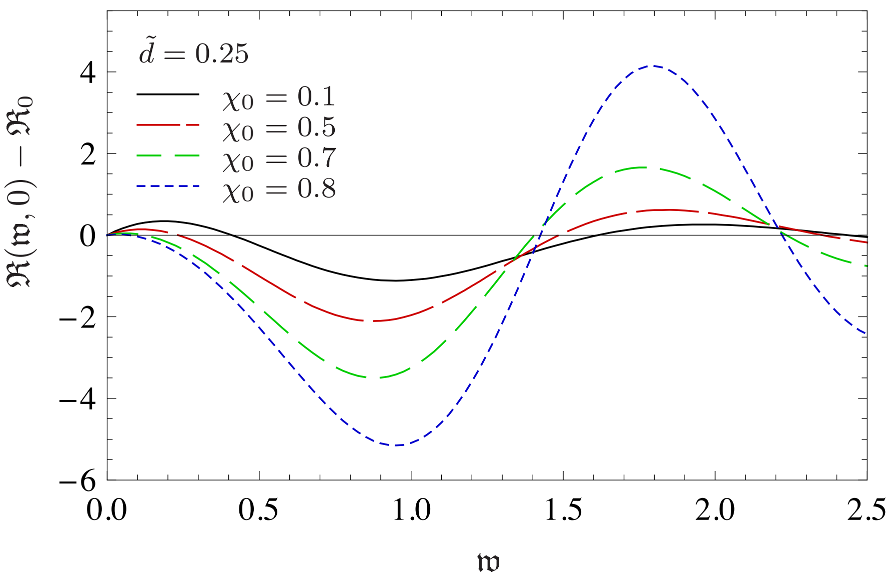

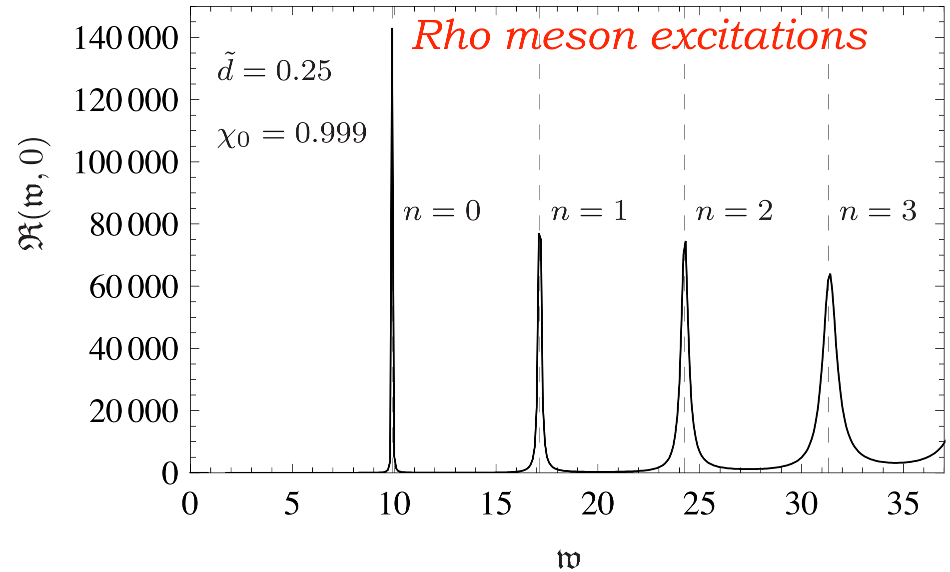

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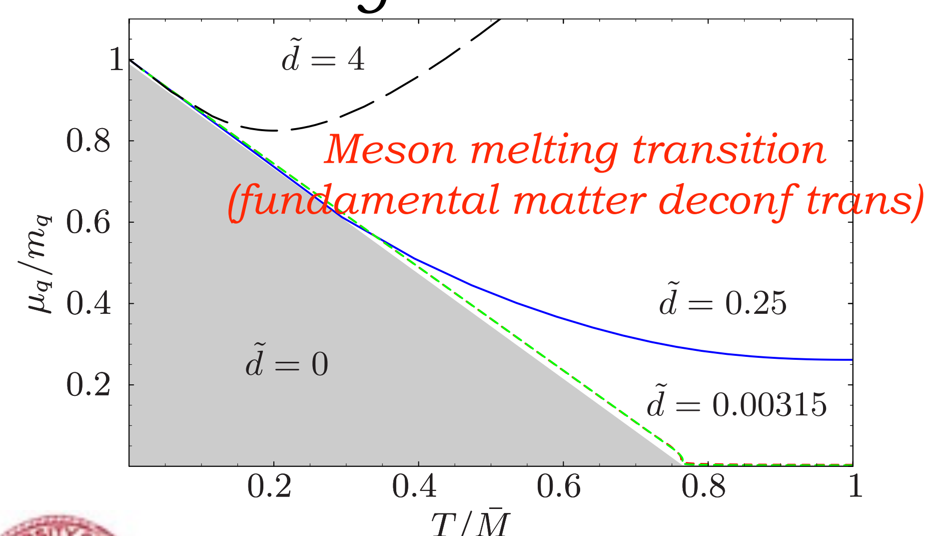
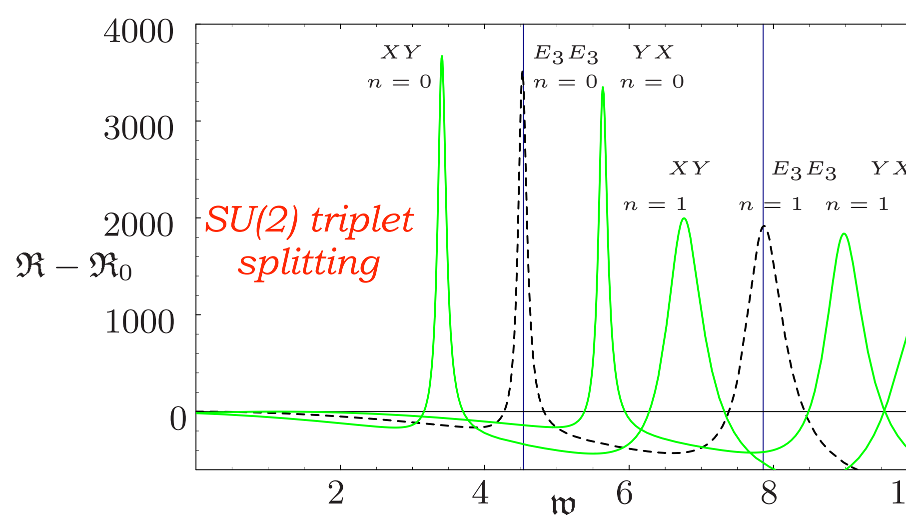
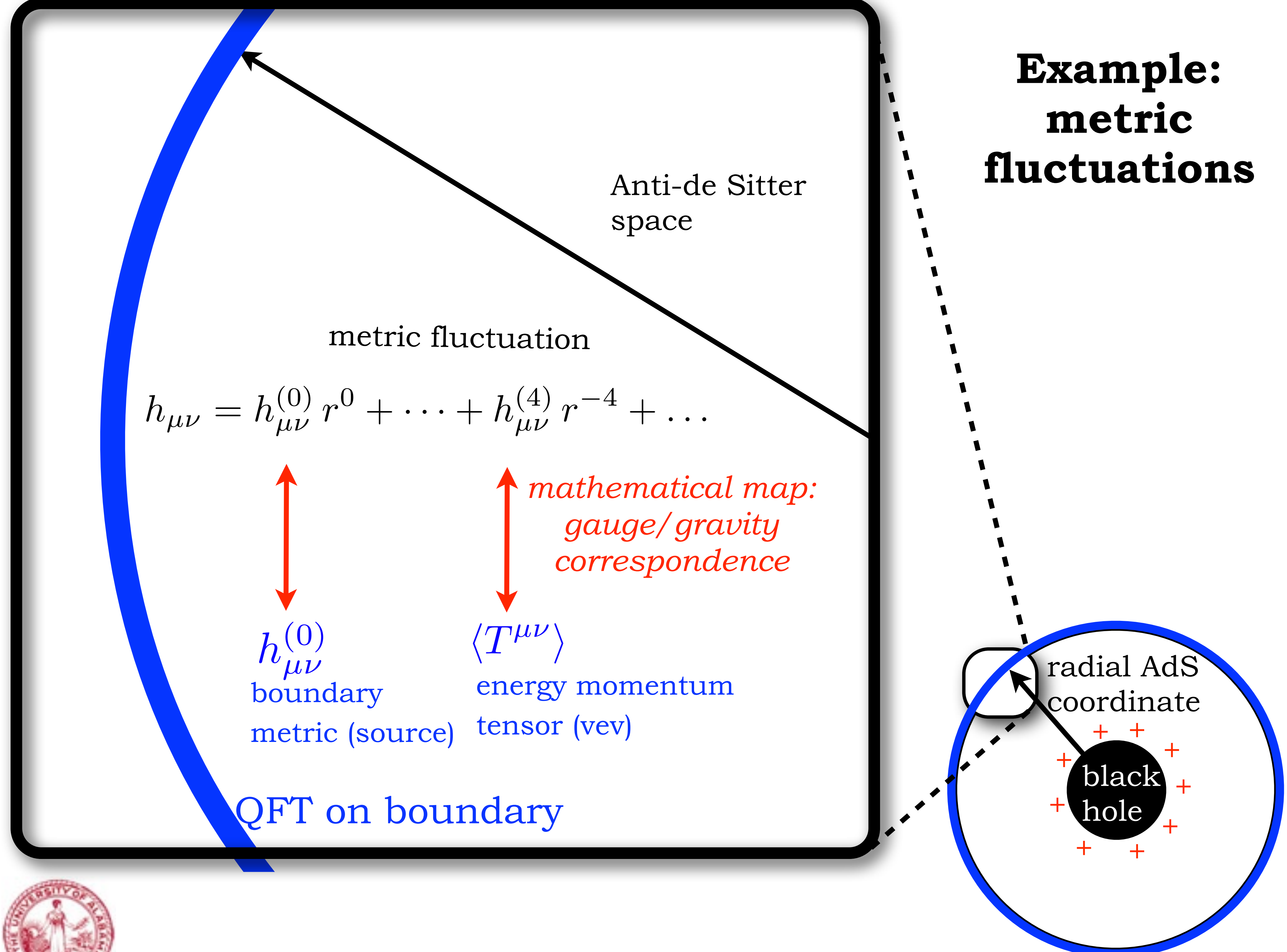


Nonzero isospin density



Analytically: [PhD thesis ’08]

Example: metric fluctuations



How to compute QNMs

- start with any gravitational background (metric, matter content)
  Example: (charged) Reissner-Nordstrom black brane in 5-dim AdS

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2 \qquad f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

$$A_t = \mu - \frac{Q}{Lr^2}$$

- choose one or more fields to fluctuate
  (consistent with the linearized Einstein equations)

  Example: metric tensor fluctuation

$$\phi := {h_x}^y \qquad 0 = \phi'' - \frac{f(u) - u\, f'(u)}{u\, f(u)}\phi' + \frac{\omega^2 - f(u) k^2}{4 r_H^2 u\, f(u)^2}\phi$$

$$u = \left(\frac{r_H}{r}\right)^2$$

- impose boundary conditions that are
  in-falling at horizon:

$$\phi = (1-u)^{\pm \frac{i\tilde{\omega}}{2(2-\tilde{q}^2)}} \left[\phi^{(0)} + \phi^{(1)}(1-u) + \phi^{(2)}(1-u)^2 + \dots\right]$$

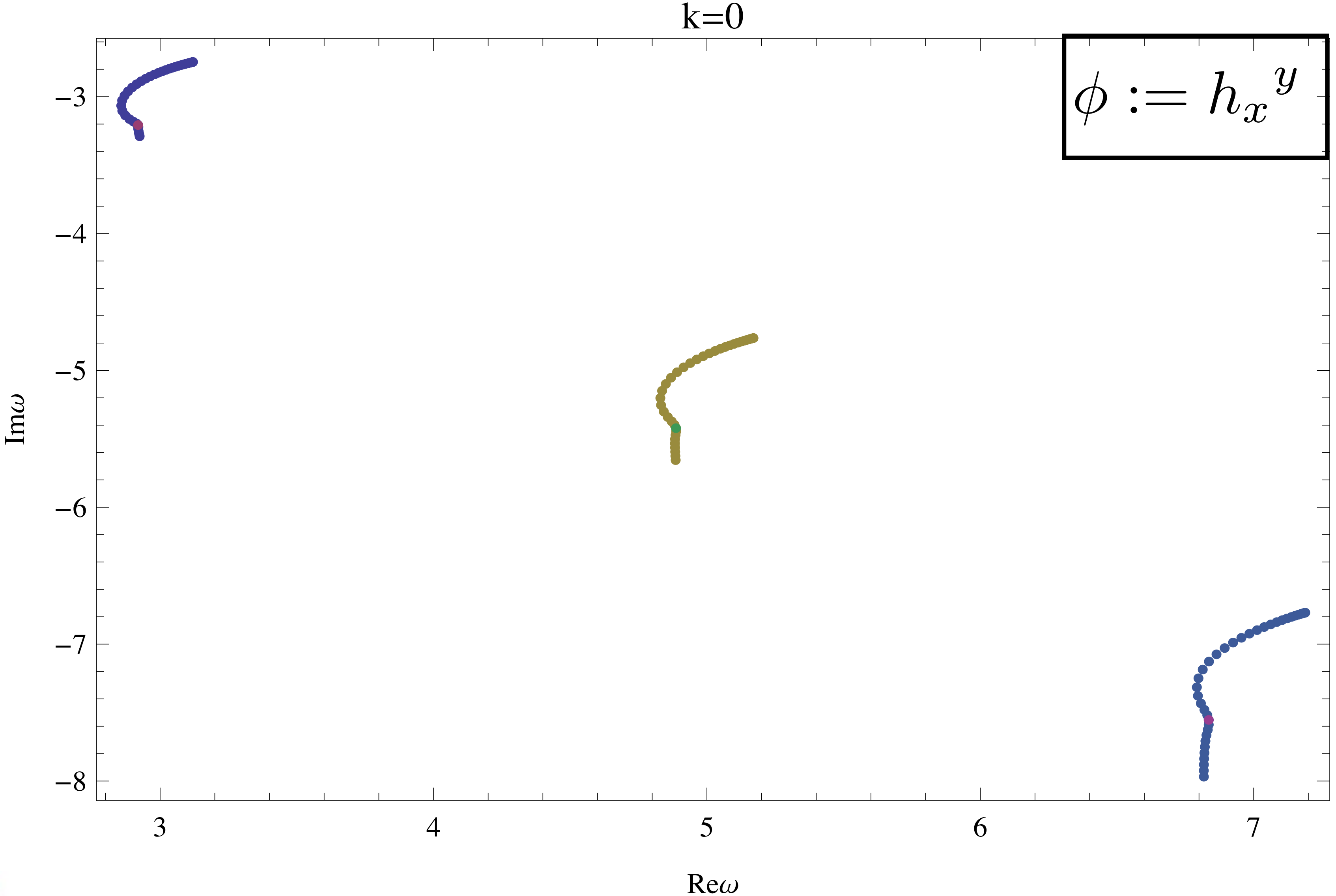
and
  vanishing at AdS-boundary: $\lim_{r \to r_{bdy}} \phi(r) = 0$

QNMs of tensor fluctuation in RN black brane

[Janiszewski, Kaminski; to appear (2015)]

Equilibrium solution

Reissner-Nordstrom (charged) black branes in 5-dim AdS



Less stable resonances at larger charges. Equilibration happens faster.

Near equilibrium results

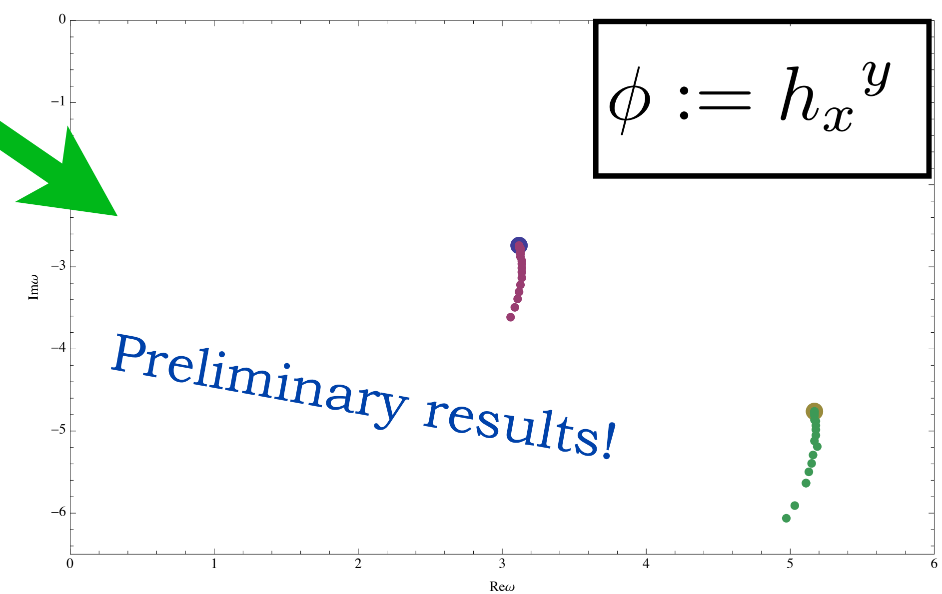
Quasinormal modes
[Janiszewski, Kaminski; ...]

Equilibrium solution

Magnetic black branes
[D'Hoker, Kraus; JHEP (2009)]

- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

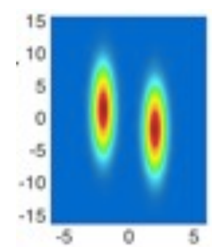
Final state for fluids in magnetic field.

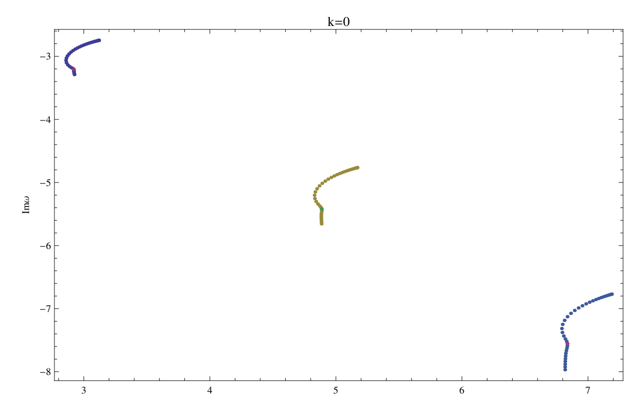


Require agreement with far from equilibrium setup at late times!

Summary

1. Recall: Anomalous/chiral hydro
    - chiral transport coefficients known exactly
    - measure gravitational anomalies?
2. Neutron star kicks
    - kick produced by chiral hydro consistent with observations
3. Holographic thermalization
    - quick thermalization
    - additional scales have little influence





4. Quasi-normal modes
    - black hole/brane “ringing”
    - thermalization code benchmarks

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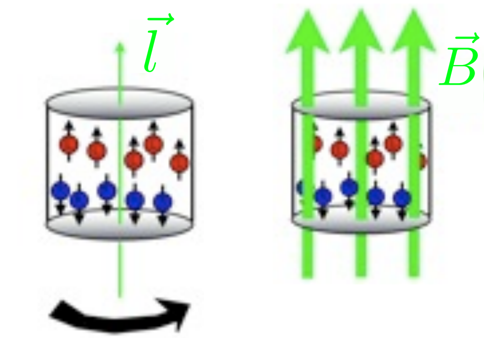

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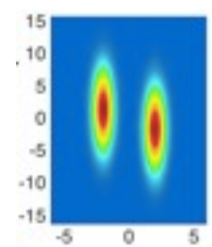

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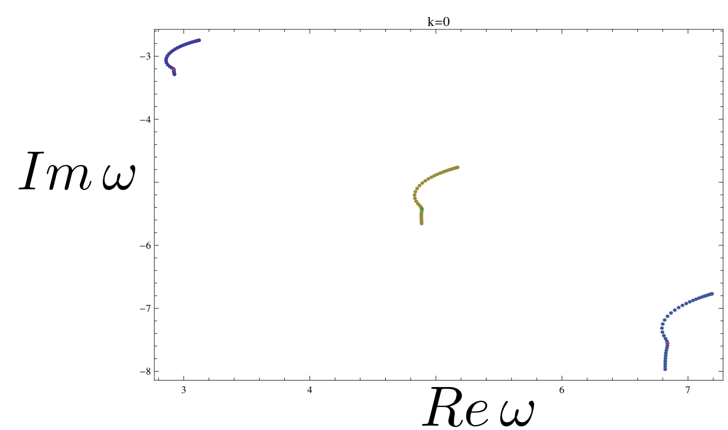

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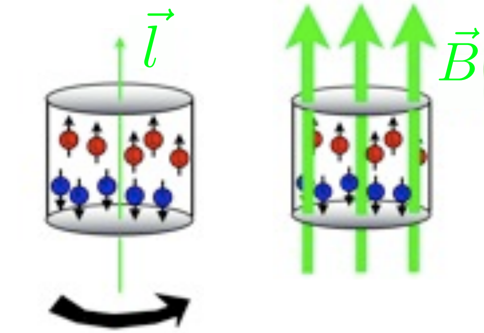

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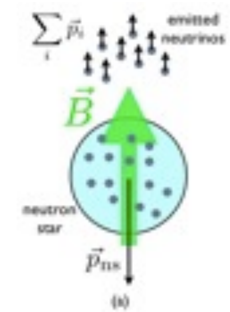

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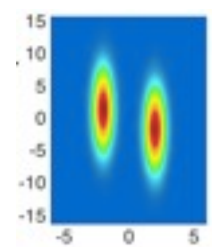

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