Holographic models for QCD at high densities

HISS “Dense Matter”, Bogoliubov Laboratory of Theoretical Physics,

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ, Дубна

29.06. - 03.07.2015

Matthias Kaminski
University of Alabama

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Also, we will talk about “how”, not “why”

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Ask me questions!

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My answer:

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100% of (A), 50% of (B), 20% of (C), 0% of (D).

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Tool with strengths and limitations (like e.g. hydrodynamics).

Basic idea

Assume we have a hard problem that is difficult to solve in a given theory, for example **QCD**

model or effective description

(Hard) problem in "similar" theory

holography (gauge/gravity correspondence)

Simple problem in a particular gravitational theory

Basic idea

Assume we have a hard problem that is difficult to solve in a given theory, for example **QCD**

?

- gravity dual to QCD?
- not known yet

model or effective description

holography (gauge/gravity correspondence)

(Hard) problem in "similar" theory

Simple problem in a particular gravitational theory

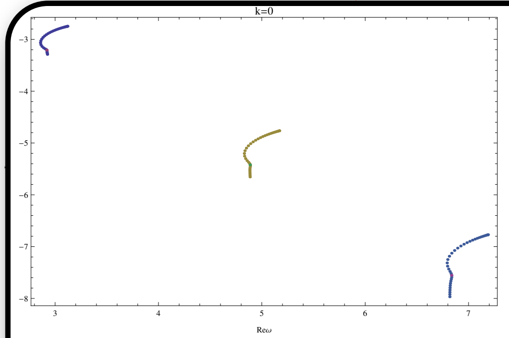
➡ Holography is good at predictions that are qualitative or universal.

Topics: heavy ion collisions & neutron stars

see Lecture I by Ohnishi (dense phases of QCD)

Thermalization of charged plasmas near equilibrium

holography ⟷



Calculation of quasi-normal modes

- charged plasma
- magnetic field
- non-conformal

[Janiszewski, Kaminski; to appear (2015)]

Thermalization of charged plasmas far-from equilibrium

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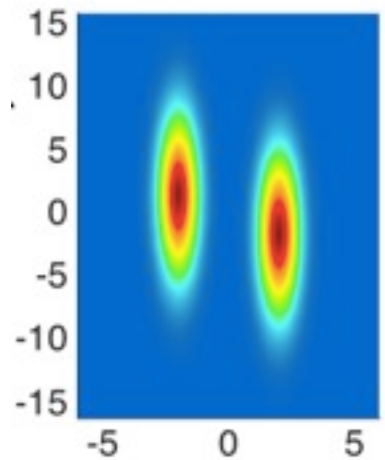

Calculation of time-dependent metrics / black hole formation

- charged plasma
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- isotropization
- off-center collision

[Chesler, Yaffe; PRL (2011)]
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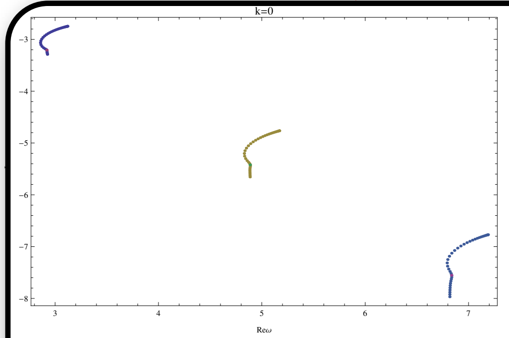

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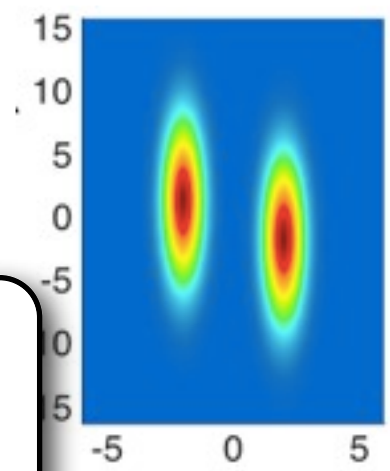

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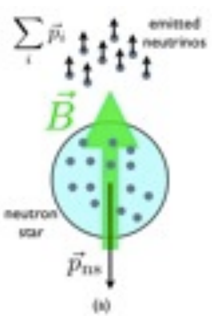
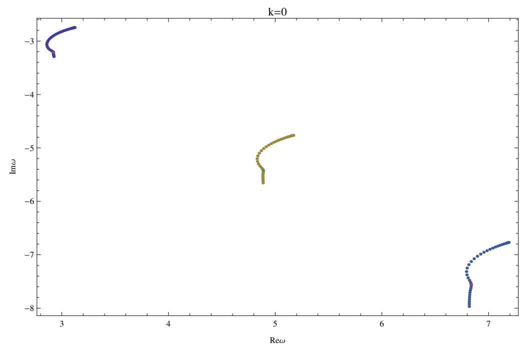
Inspired by holography:

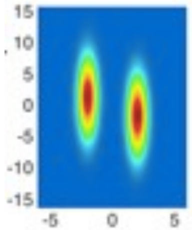


Anomalous hydrodynamics leads to neutron star kicks

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

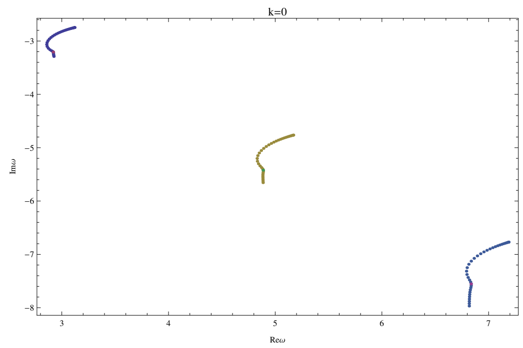
Schedule



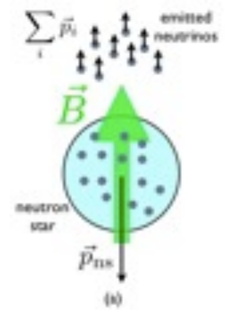


Schedule

Lecture I:
Holographic thermalization of charged plasmas near equilibrium

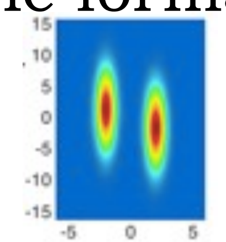


Exercises I:
Calculation of quasi-normal modes



Lecture II:
A. Anomalous hydrodynamics kicks neutron stars
B. Holo thermalization of charged plasmas far-from equilibrium

Exercises II:
A. Details of neutron star kick calculation
B. Calculation of time-dependent metrics / black hole formation

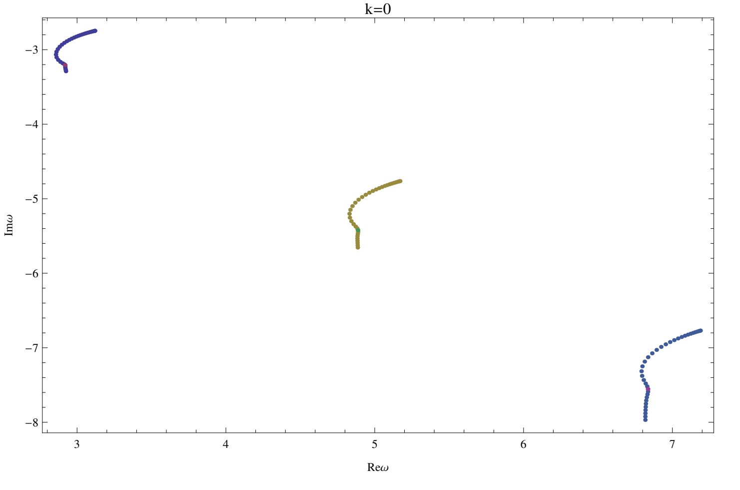


Contents: Lecture I

1. Hydrodynamics 2.0

2. Holography basics

3. Quasi-normal modes (QNMs)



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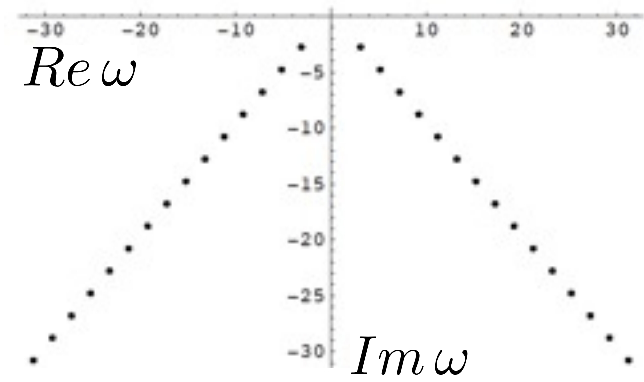
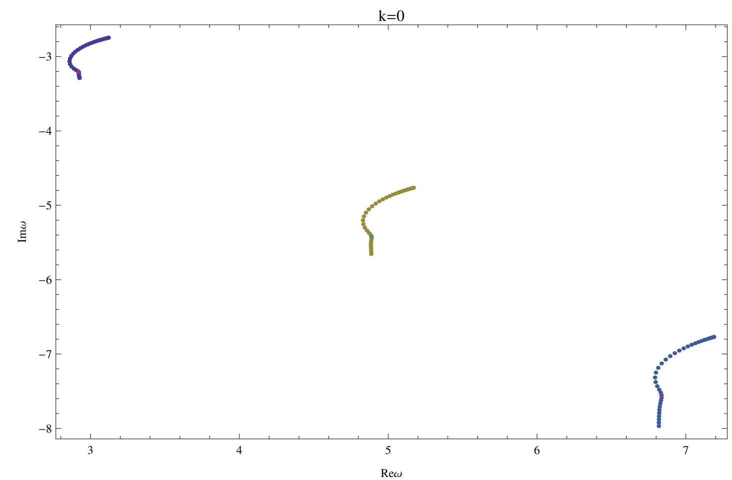


[Starinets; JHEP (2002)]



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1. Hydrodynamics 2.0

Hydrodynamic variables

Thermodynamics

$$T,\ \mu,\ u^\nu$$

Hydrodynamics

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu(t,\vec{x})$$

Hydrodynamic variables

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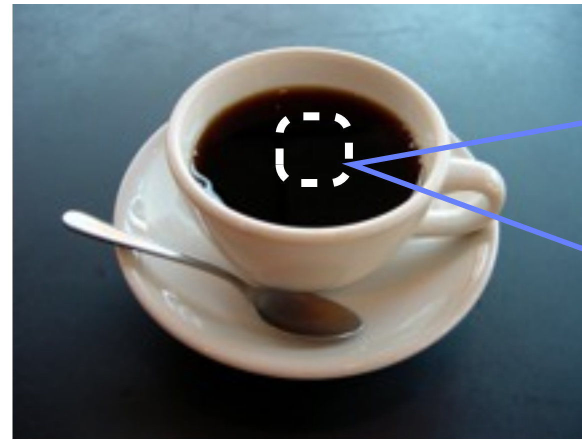

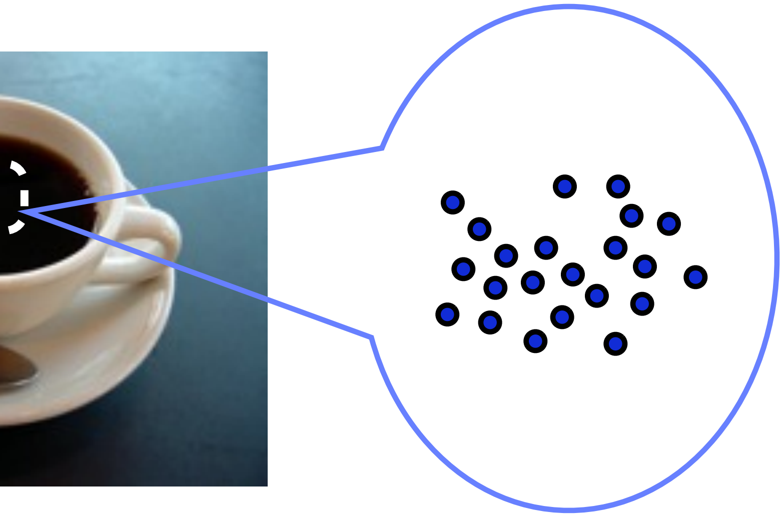

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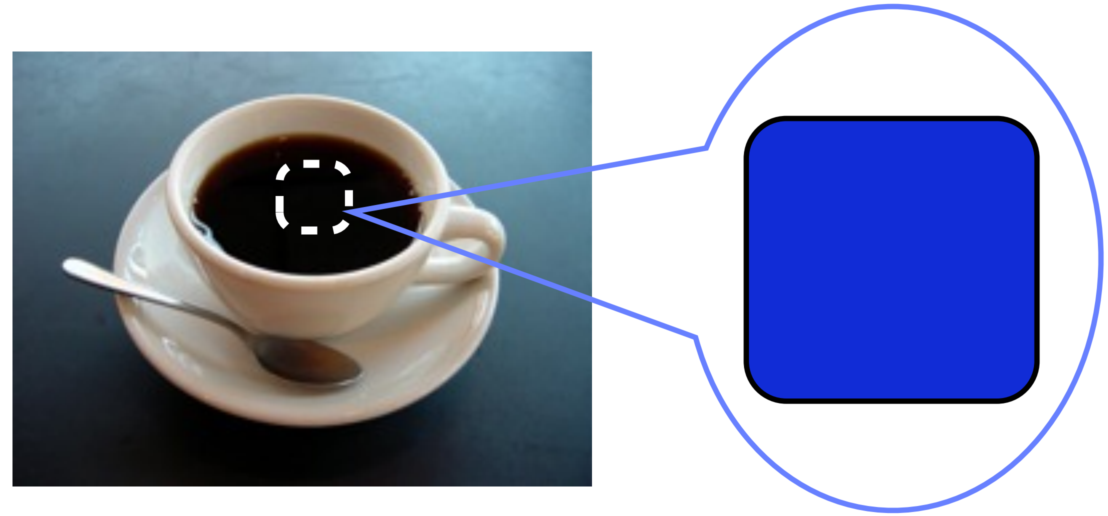

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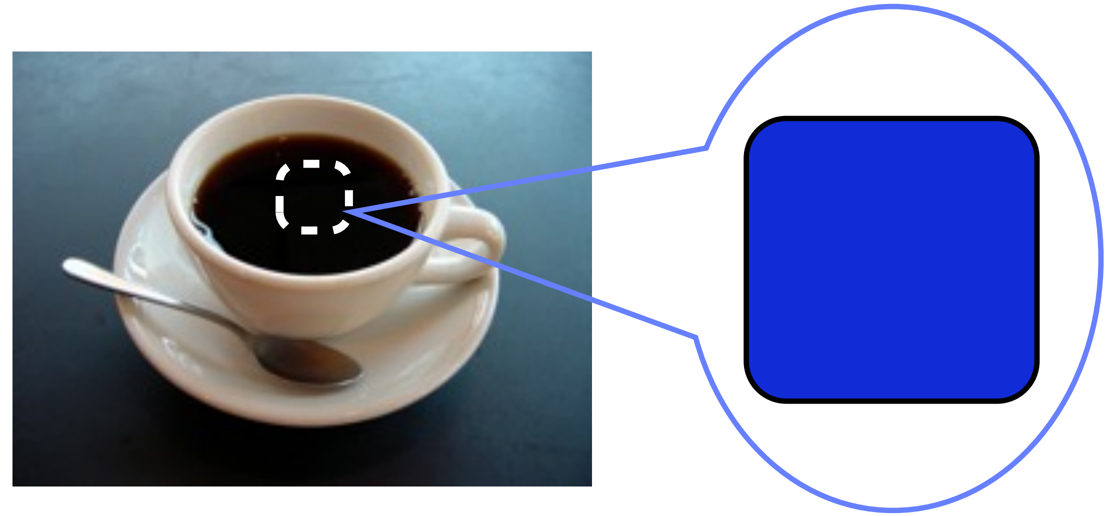

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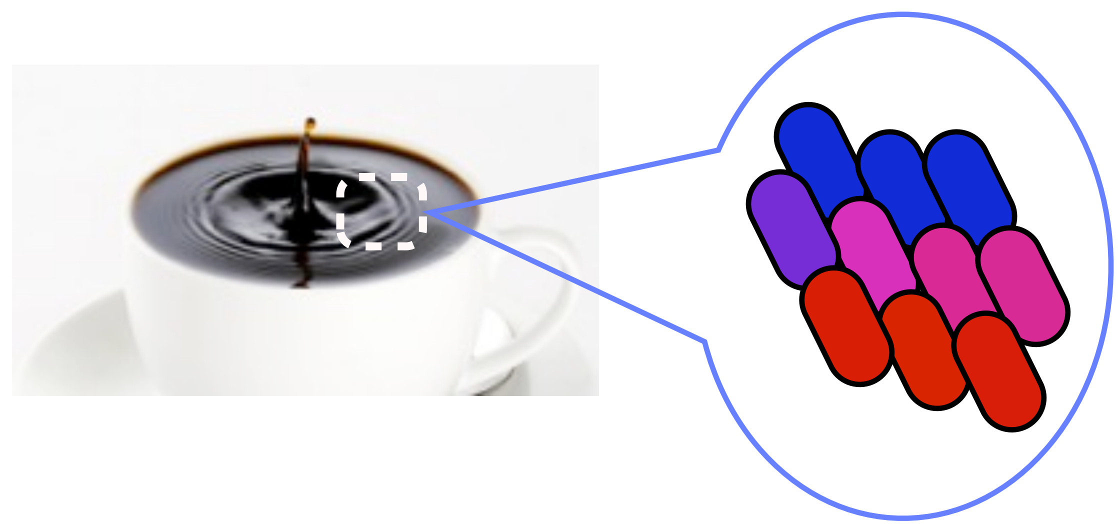

Hydrodynamics

Universal effective field theory for microscopic QFTs, expansion in gradients of temperature, chemical potential and velocity

- fields $T(x),\ \mu(x),\ u^\nu(x)$
- conservation equations
- constitutive equations (Landau frame)

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Energy momentum tensor $$T^{\mu\nu} = \epsilon u^\mu u^\nu + P(g^{\mu\nu} + u^\mu u^\nu) + \tau^{\mu\nu}$$

Conserved current $$j^\mu = n u^\mu + \nu^\mu$$

Chiral hydrodynamics

[Son,Surowka; PRL (2009)]
[Loganayagam; arXiv (2011)]
[Jensen et al.; JHEP (2012)]
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Derived for any QFT with a chiral anomaly (e.g. QCD)

$\nabla_\nu j^\nu = 0$ classical theory

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Def.: $V^\mu = E^\mu - T\Delta^{\mu\nu}\nabla_\nu\left(\frac{\mu}{T}\right)$

Generalized constitutive equation with external fields

$$j^\mu = nu^\mu + \sigma V^\mu + \xi\omega^\mu + \xi_B B^\mu + \ldots$$

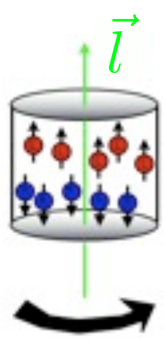
vorticity *magnetic field*

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\nabla_\lambda u_\rho \quad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu F_{\lambda\rho}$$

Agrees with gauge/gravity prediction:

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]

chiral vortical effect



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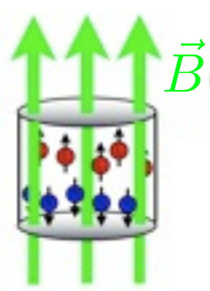

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$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right), \quad \xi_B = C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon+P}\right)$$

anomaly-coefficient C

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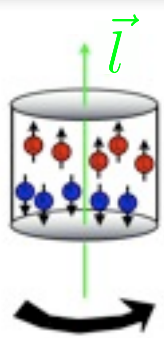

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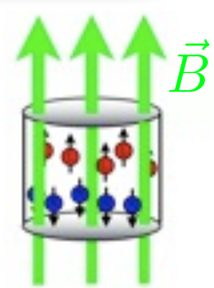

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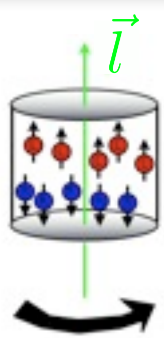

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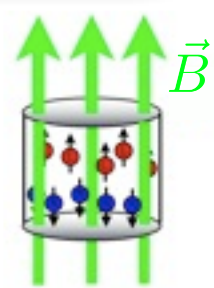

Observable in:
heavy ion collisions?

[Kharzeev, Son.; PRL (2011)]

neutron stars?

[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

see Lecture II

Chiral effects in vector and axial currents

Vector current (e.g. QCD *U(1)*)

$$J_V^\mu = \cdots + \xi_V \omega^\mu + \xi_{VV} B^\mu + \xi_{VA} B_A^\mu$$

chiral
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Axial current (e.g. QCD axial *U(1)*)

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chiral
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formal approach guarantees completeness

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More than one anomalous current

$$\nabla_\nu J^\nu_a = \frac{1}{8} C_{abc}\epsilon^{\nu\rho\sigma\gamma} F^b_{\nu\rho} F^c_{\sigma\gamma}$$

$$\xi_a = C_{abc}\mu^b\mu^c + 2\beta_a T^2 - \frac{2n_a}{\epsilon + p}\left(\frac{1}{3}C_{bcd}\mu^b\mu^c\mu^d + 2\beta_b\mu^b T^2 + \gamma T^3\right)$$

[Neiman, Oz; JHEP (2010)]

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various charges (e.g. axial, vector)

previously neglected

$$\beta = -4\pi^2 c_m$$

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Gravitational anomalies

$$\nabla_\nu T^{\mu\nu}_{cov} = F^\mu{}_\nu J^\nu_{cov} + \frac{c_m}{2}\nabla_\nu\left[\epsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} R^{\mu\nu}{}_{\alpha\beta}\right]$$

full transport coefficient exactly known; first measurement of gravitational anomaly?

Exercise 1.a): hydrodynamic correlators

Simple (non-chiral) example in 2+1:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1, 0, 0)$$

Exercise 1.a): hydrodynamic correlators

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sources $A_t,\, A_x \propto e^{-i\omega t + ikx}$ $\qquad u^\mu = (1, 0, 0)$

fluctuations $n = n(t, x, y) \propto e^{-i\omega t + ikx}$ (fix T and u)

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one point functions $\quad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega,k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^y \rangle = 0$$

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Exercise 1.a): hydrodynamic correlators

Simple (non-chiral) example in 2+1:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right] \qquad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

sources $\quad A_t,\, A_x \propto e^{-i\omega t + ikx} \qquad u^\mu = (1, 0, 0)$

fluctuations $\quad n = n(t, x, y) \propto e^{-i\omega t + ikx}$ (fix T and u)

one point functions $\quad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^y \rangle = 0$$

susceptibility: $\quad \chi = \frac{\partial n}{\partial \mu}$

Einstein relation: $\quad D = \frac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\quad \langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ hydrodynamic poles in spectral function

Exercise 1.a): hydrodynamic correlators

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sources $\quad A_t,\ A_x \propto e^{-i\omega t + ikx} \qquad u^\mu = (1, 0, 0)$

+other sources

fluctuations $\quad n = n(t, x, y) \propto e^{-i\omega t + ikx}$ (fix T and u)

+ fluctuations in T and u

one point functions $\quad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

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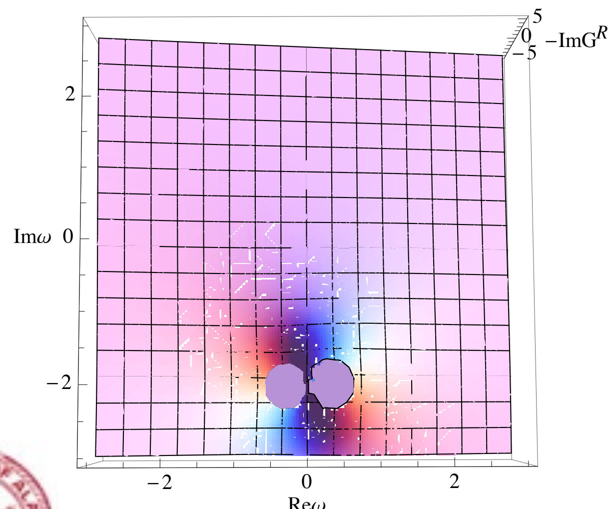
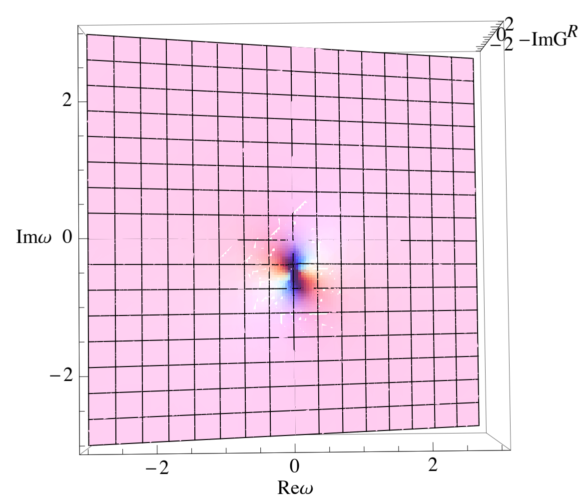
Exercise 1.b): hydrodynamic correlators

Simple (non-chiral) example in 2+1

$$-\mathrm{Im}\, G^R = -\mathrm{Im}\,\langle j_x j_x \rangle = -\sigma\,\omega_R \frac{2Dk^2\omega_I + \omega_R^2 + \omega_I^2}{\omega_R^2 + (\omega_I + Dk^2)^2}$$





- pole goes to zero
- spectral function vanishes with k

Exercise 1.b): hydrodynamic correlators

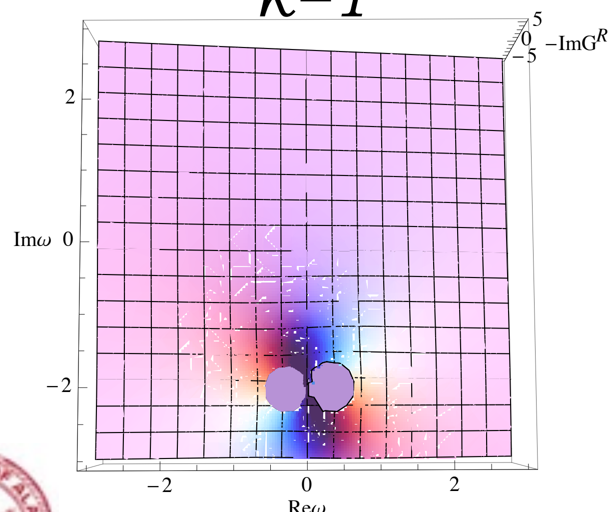
Simple (non-chiral) example in 2+1

two point function: $\langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

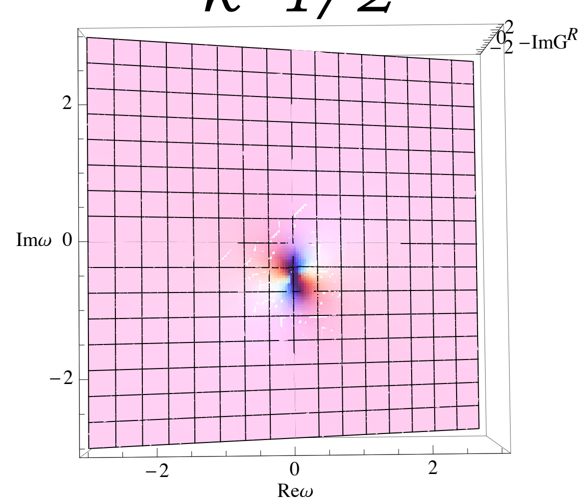
spectral function: $-\mathrm{Im}\, G^R = -\mathrm{Im}\, \langle j_x j_x \rangle = -\sigma\, \omega_R \frac{2Dk^2\omega_I + \omega_R^2 + \omega_I^2}{\omega_R^2 + (\omega_I + Dk^2)^2}$

hydrodynamic pole (diffusion pole) in spectral function
at decreasing momentum *k*:

k=1



k=1/2

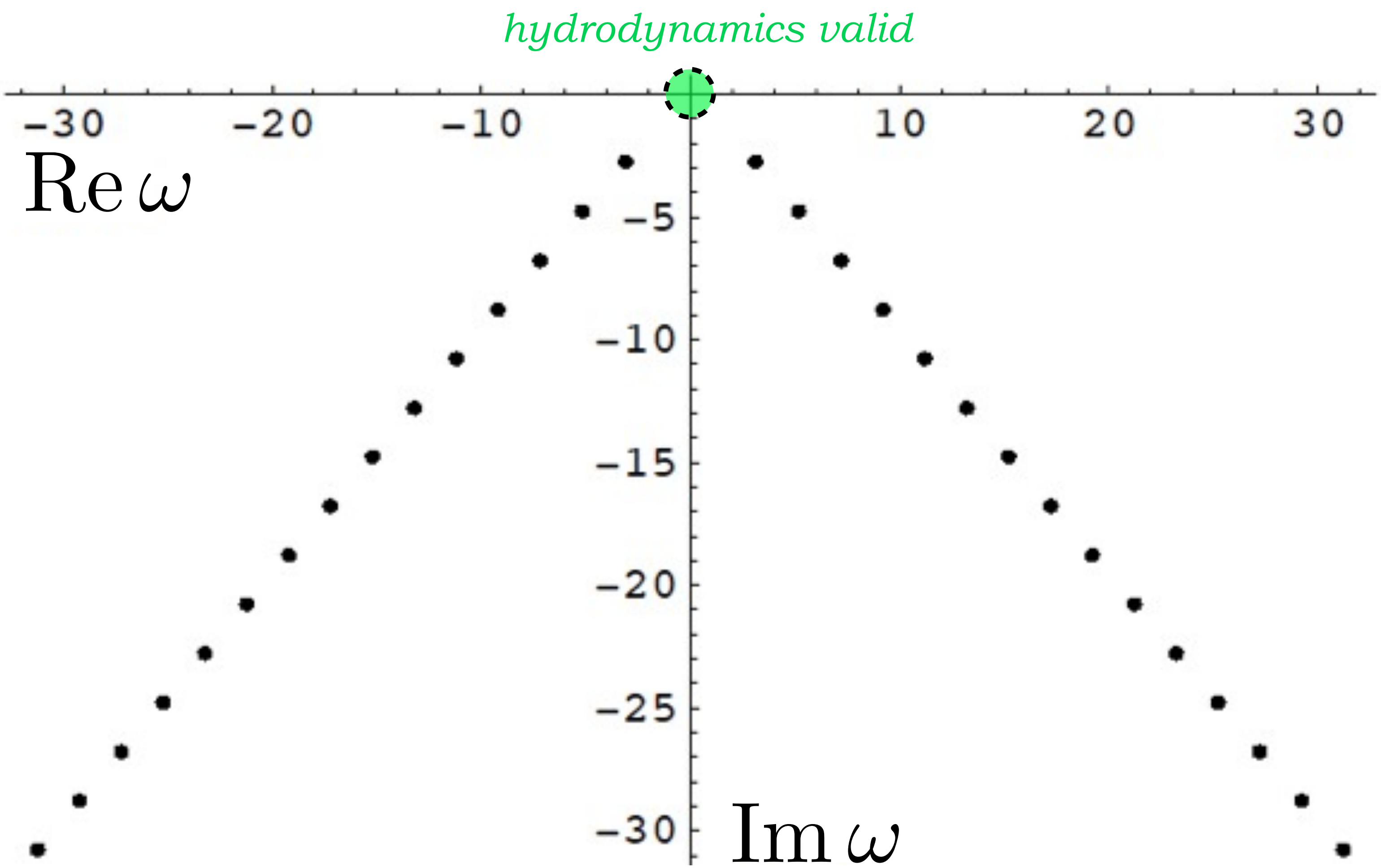


- pole goes to zero
- spectral function vanishes with *k*

Far beyond hydrodynamics

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle(\omega, k) = G^R_{xy,xy}(\omega, k) = -i \int d^4x \; e^{-i\omega t + ikz} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$



Far beyond hydrodynamics : QNMs

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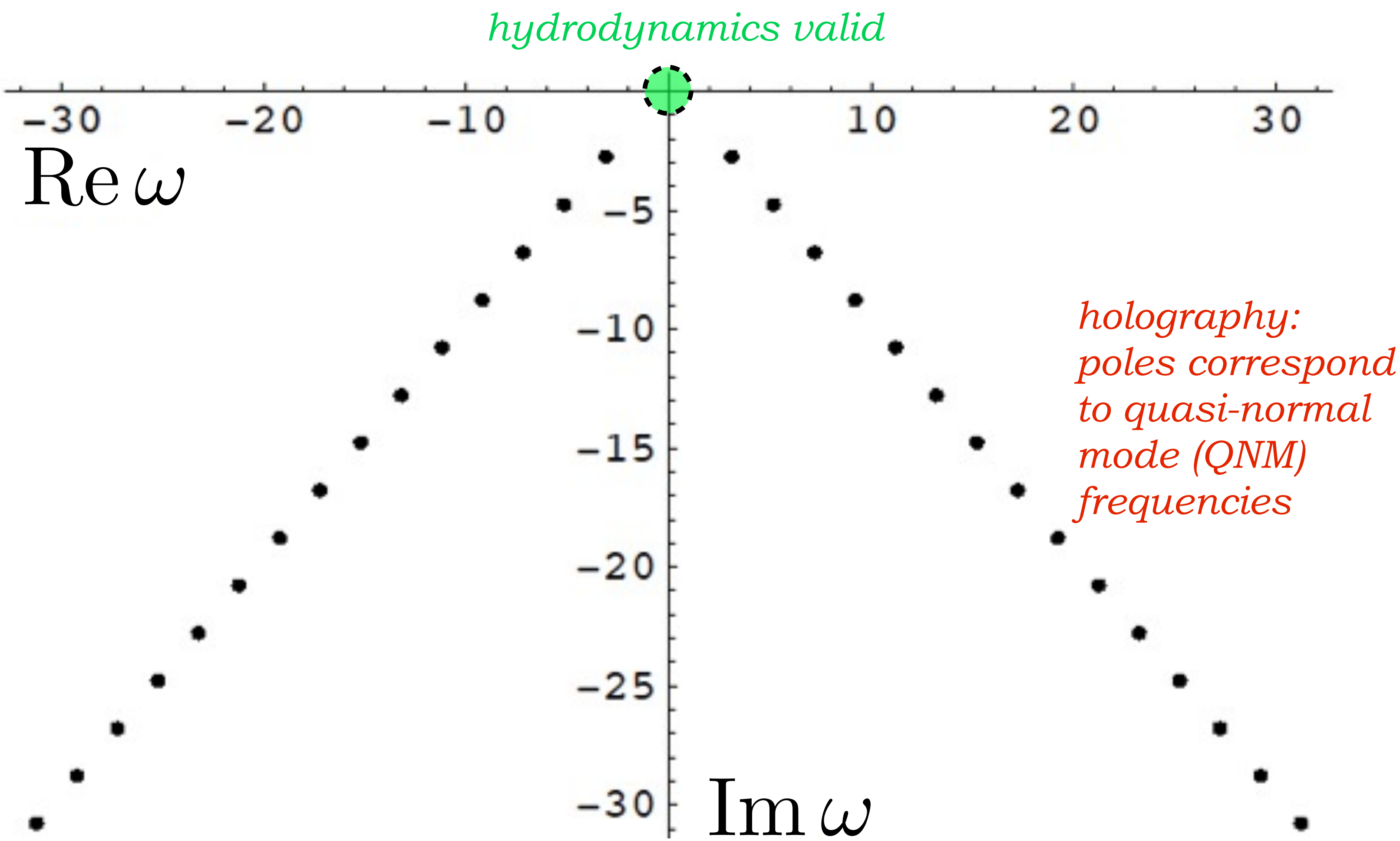


[Starinets; JHEP (2002)]

2. Holography basics

Gauge/Gravity concepts

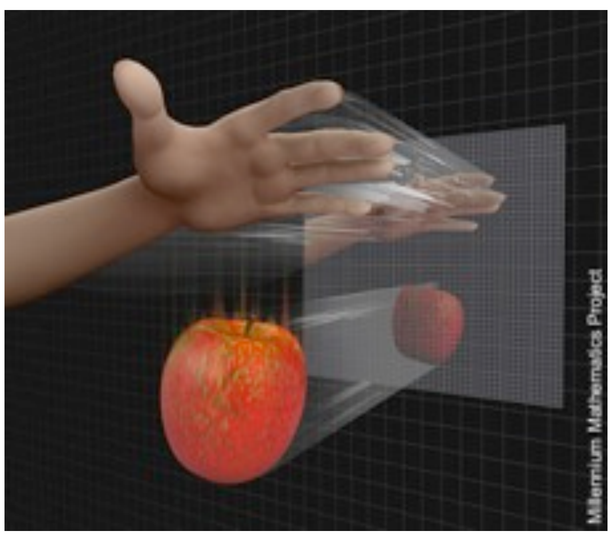
The Gauge/Gravity correspondence is based on the holographic principle. *['t Hooft (1993)]*

$$S_{max}(\text{volume}) \propto \text{surface area}$$

String theory gives one example (AdS/CFT).

N=4 Super-Yang-Mills in 3+1 dimensions (CFT) $\Longleftrightarrow$ Typ II B Supergravity in (4+1)-dimensional Anti de Sitter space (AdS)

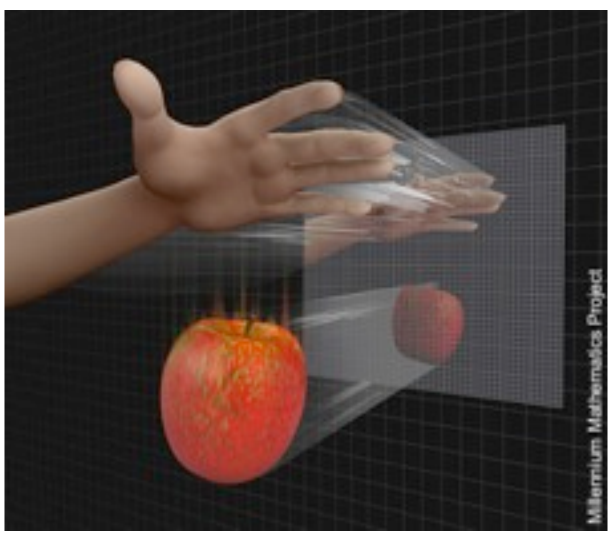
[Susskind (1995)]
[Maldacena (1997)]

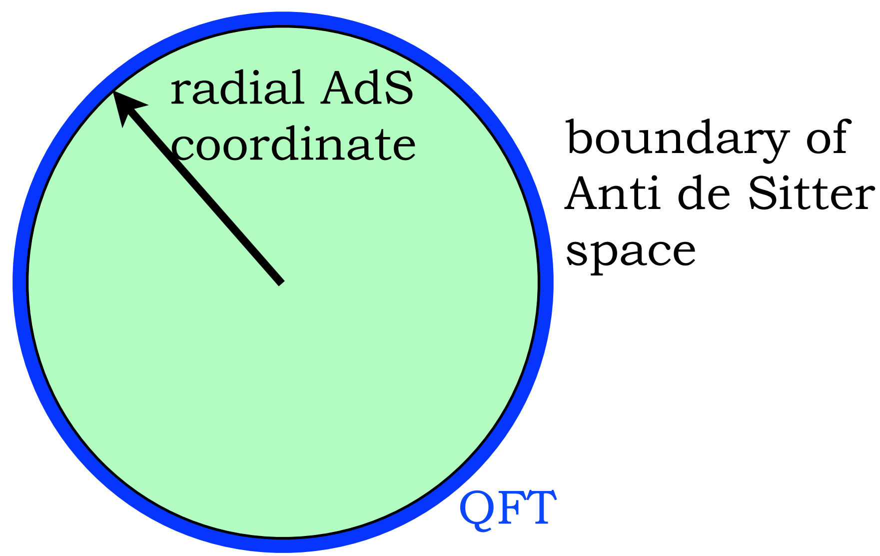


Equilibrium states

strongly coupled quantum field theory (QFT) ⟷ *correspondence* ⟷ weakly curved gravitational theory

renormalization scale ⟷ radial AdS coordinate

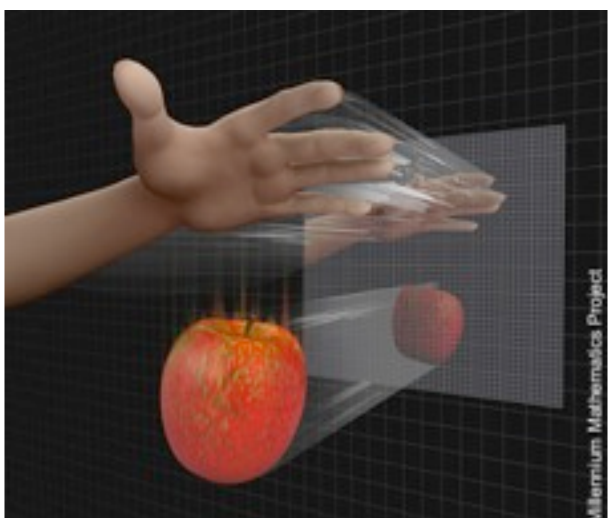


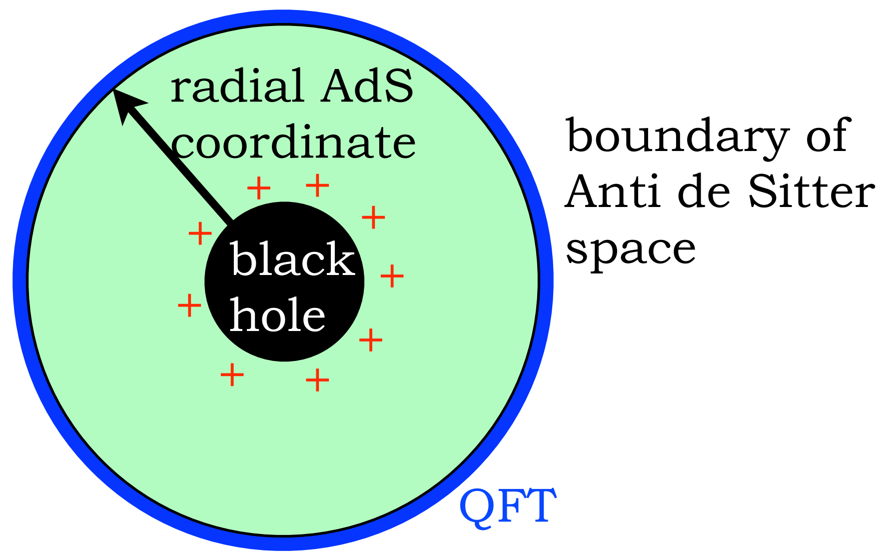


Equilibrium states

strongly coupled quantum field theory (QFT) ⟷ *correspondence* ⟷ weakly curved gravitational theory

renormalization scale	⟷	radial AdS coordinate
QFT temperature	⟷	Hawking temperature
conserved charge	⟷	charged black hole/brane





Example: Reissner-Nordström black brane

$\mathcal{N}$=4 Super-Yang-Mills theory at nonzero temperature & charge

correspondence ⟷

metric & gauge field defining a RN black brane (solve Einstein-Maxwell eq's)

metric: $ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2$

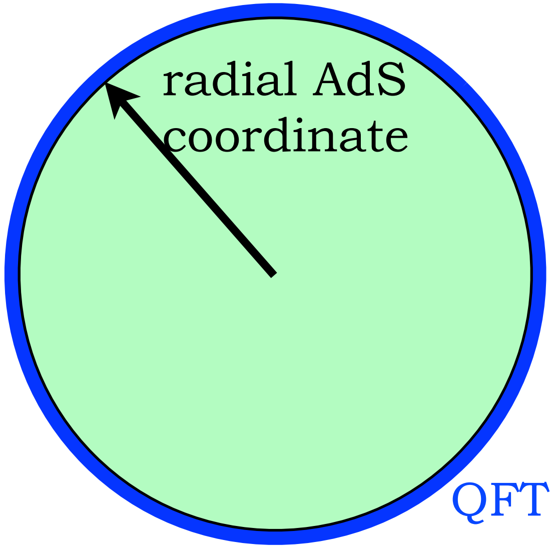
$$T = r_H^2 \frac{|f'(r_H)|}{4\pi}$$

$$f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

gauge field:

$$A_t = \mu - \frac{Q}{Lr^2}$$

$$\mu = \frac{\sqrt{3}q}{2r_H^2}$$



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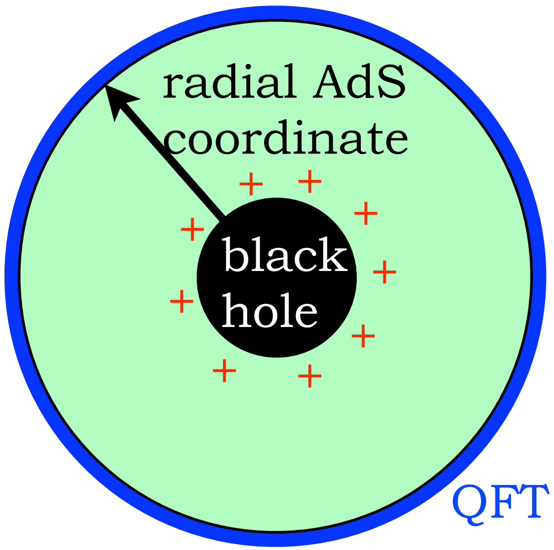
$$T = r_H^2 \frac{|f'(r_H)|}{4\pi}$$

$$f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

conserved charge Q, thermodynamically dual to chemical potential: ⟷ gauge field:

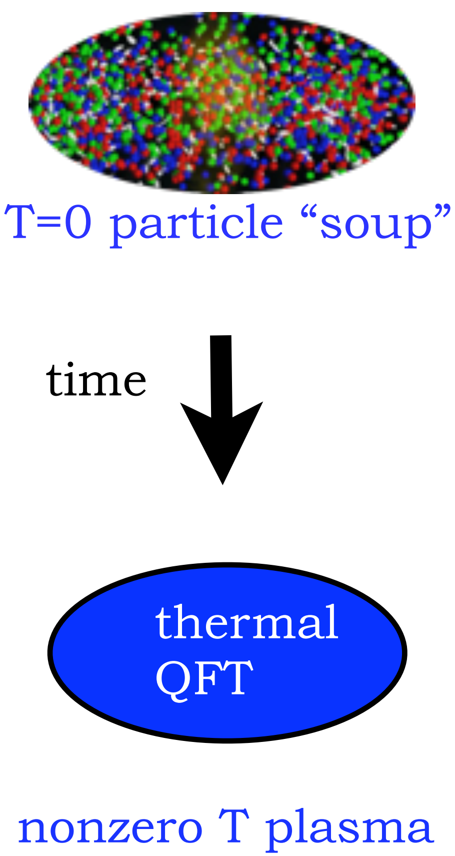
$$\mu = \frac{\sqrt{3}q}{2r_H^2}$$

$$A_t = \mu - \frac{Q}{Lr^2}$$



Far-from equilibrium states: holographic thermalization

Thermalization:



Far-from equilibrium states: holographic thermalization

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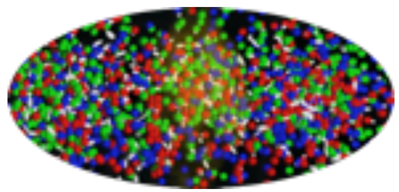


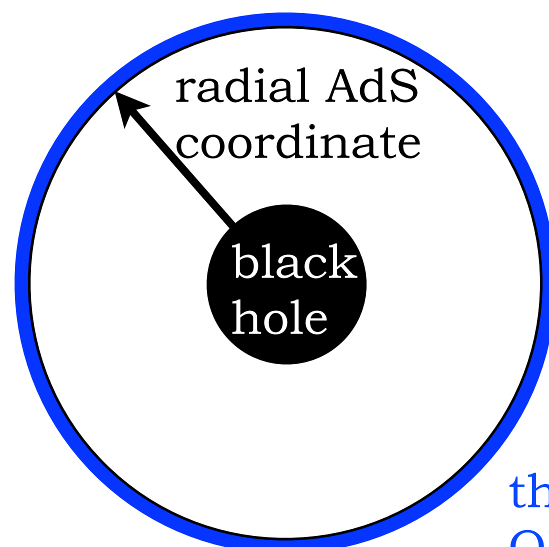
T=0 particle "soup"

time

correspondence

thermal QFT

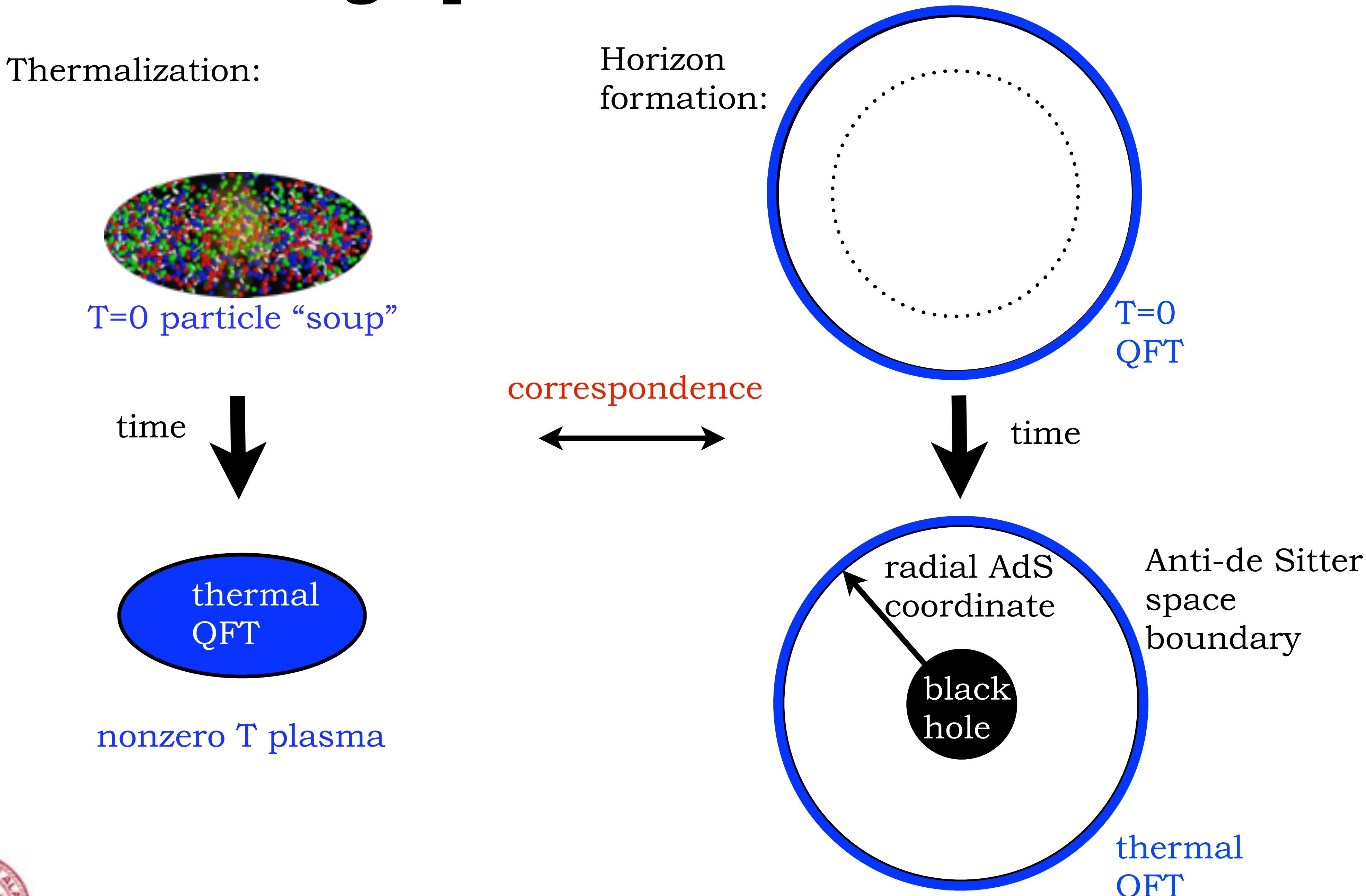
nonzero T plasma



Anti-de Sitter space boundary

thermal QFT

Far-from equilibrium states: holographic thermalization



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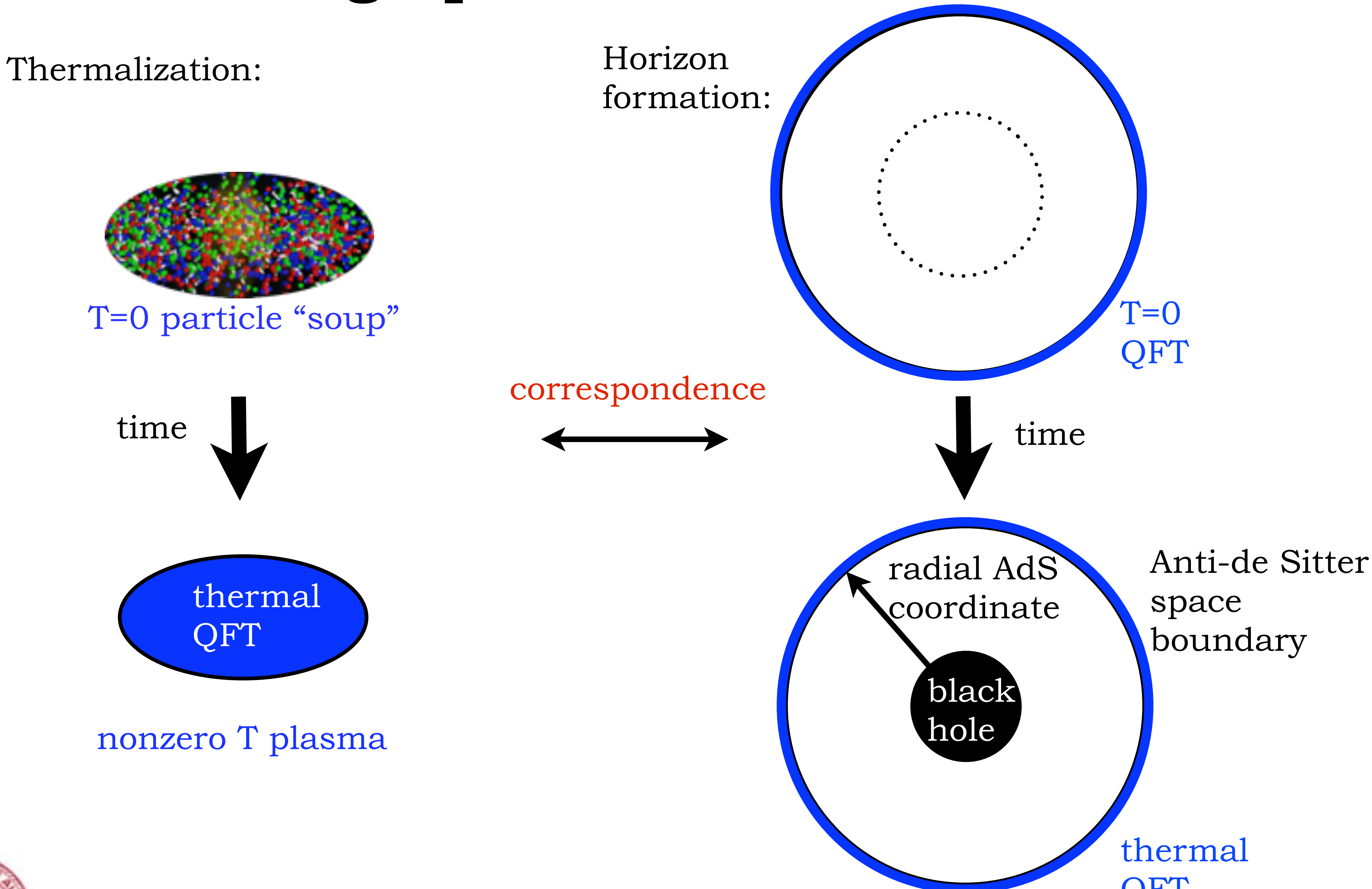

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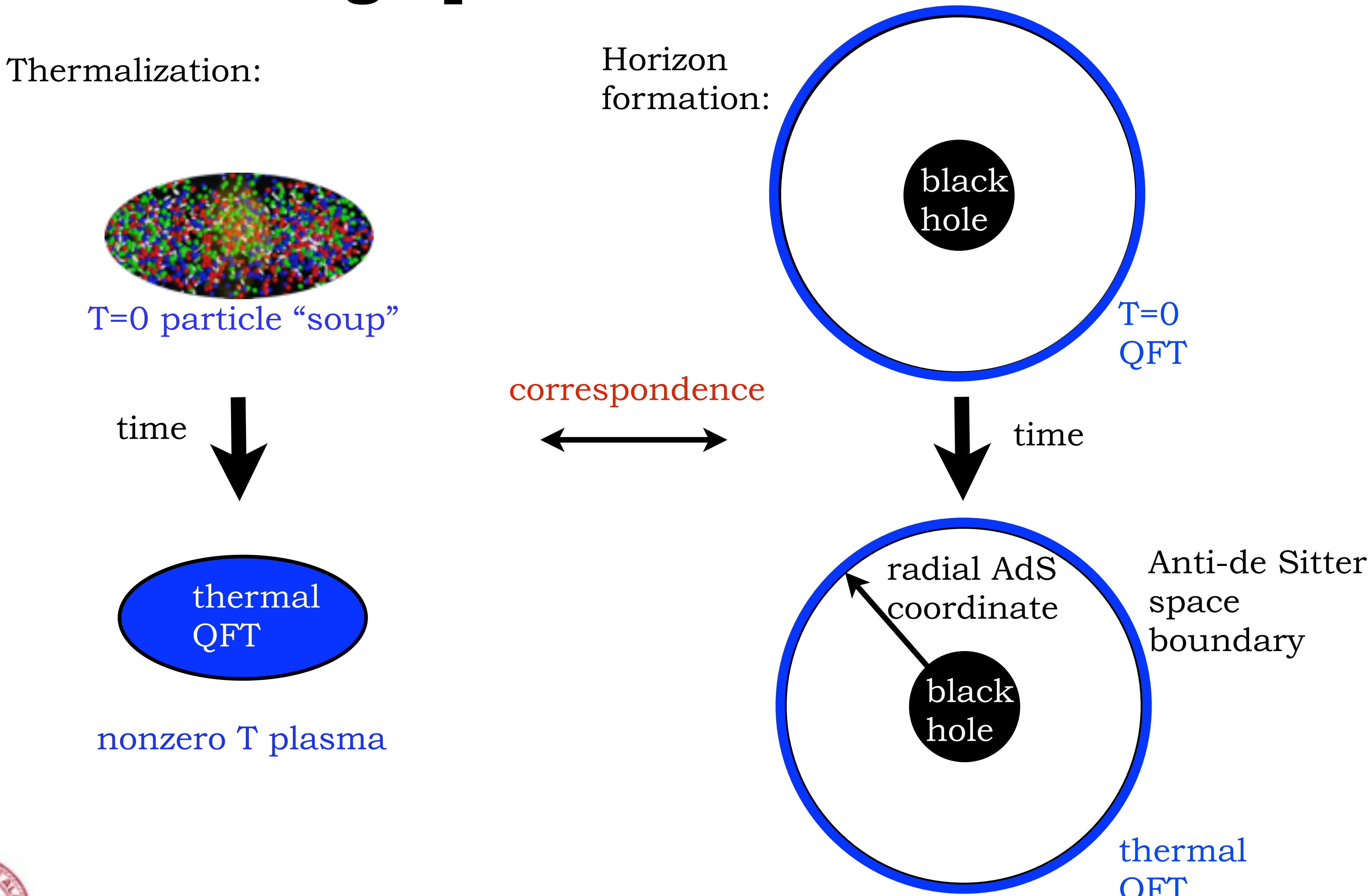
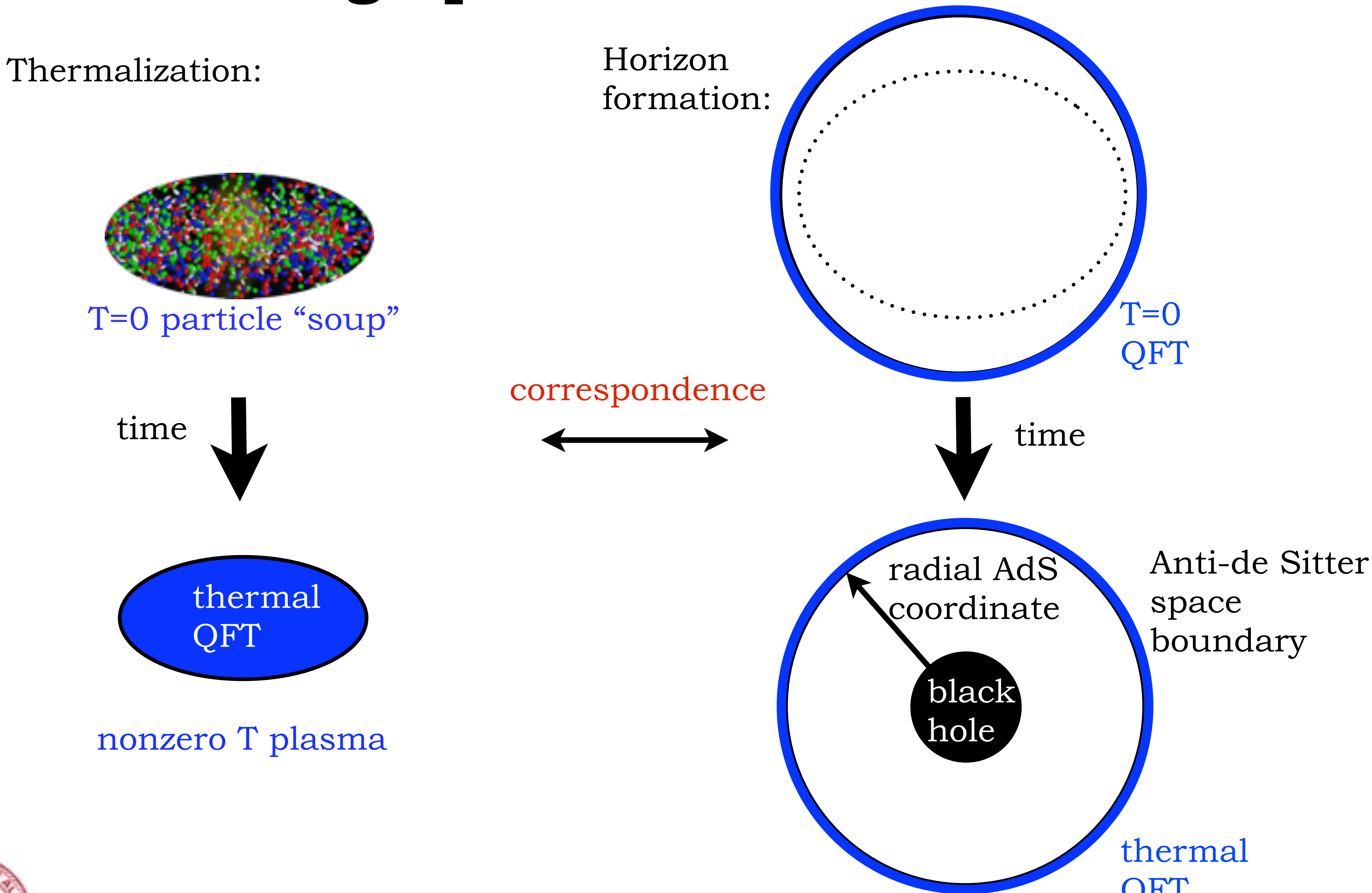


Far-from equilibrium states: holographic thermalization

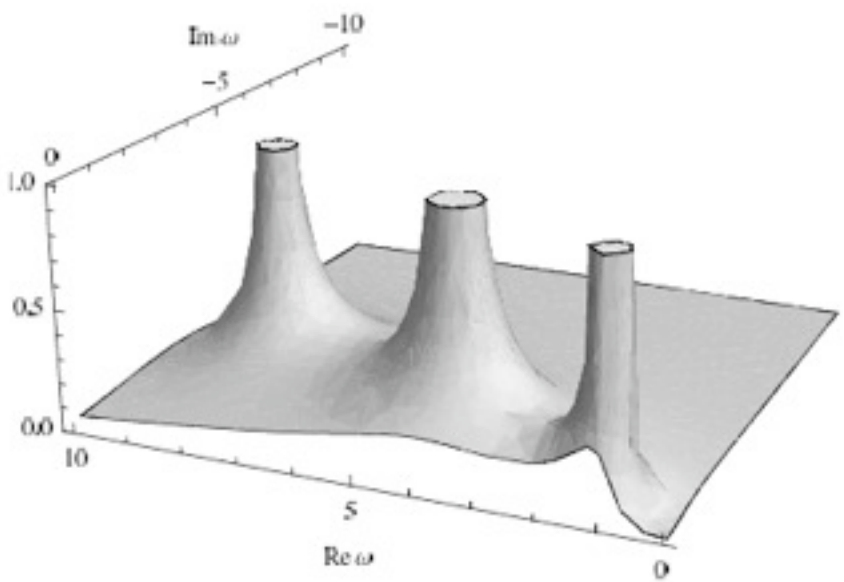


Retarded Green's function from holography

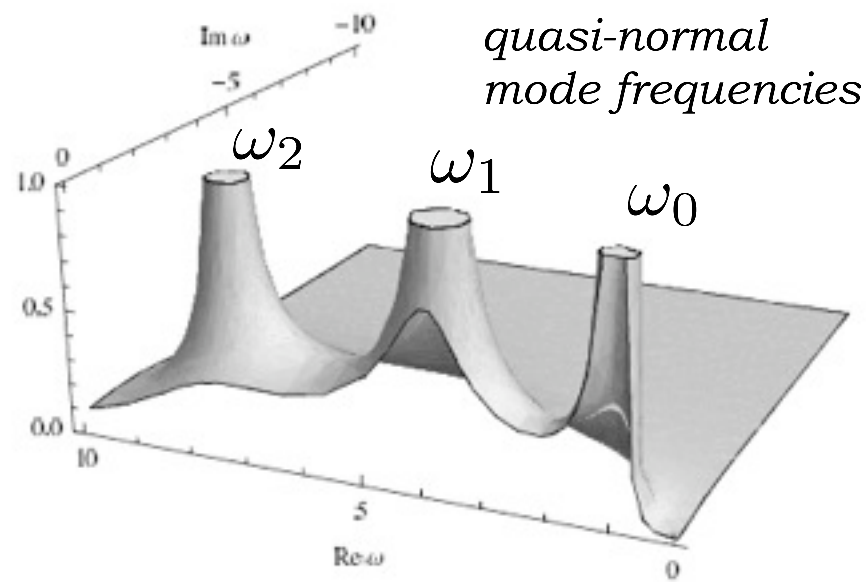
$$G^R(\omega, \boldsymbol{q}) = -\mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}\vec{k}\vec{x}}\,\theta(x^0)\,\langle[J(\vec{x}), J(0)]\rangle \longleftrightarrow \frac{\delta^2}{\delta\phi^{(0)}\delta\phi^{(0)}} S_{gravity}[\phi]$$

correlations between fluctuations around a state

Spectral function (imaginary part of retarded *G*):



high temperature
no quasiparticles

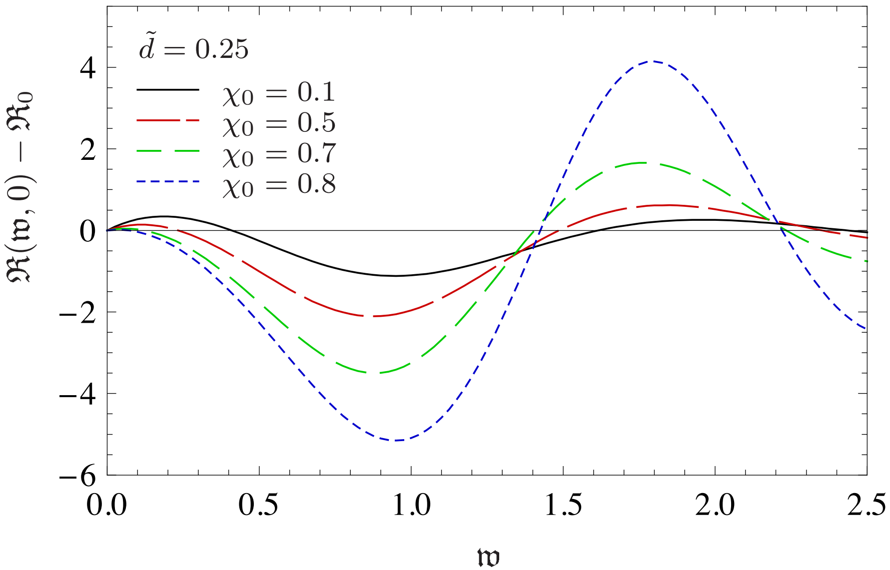


low temperature
more stable quasiparticles
(resonances)

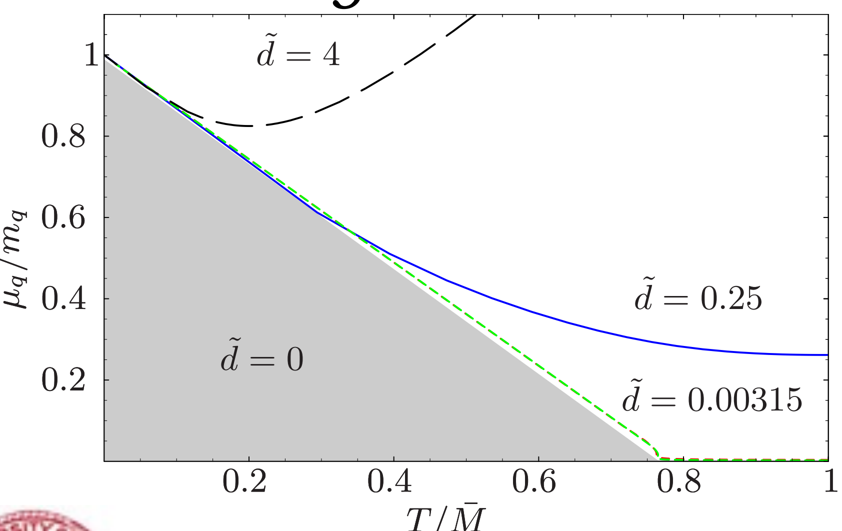
Example: N=2 SYM current correlators

[Erdmenger, Kaminski., Rust 0710.0334]

Nonzero baryon density



Phase diagram



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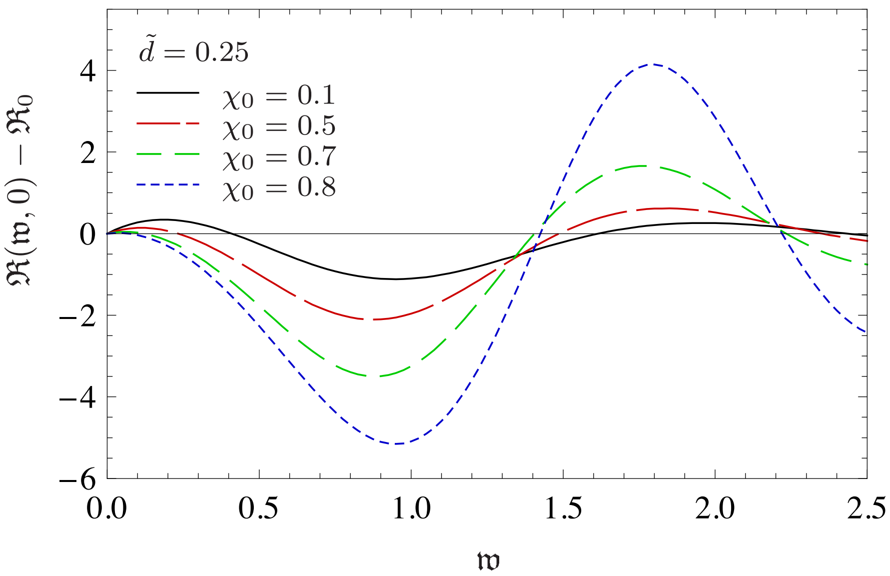

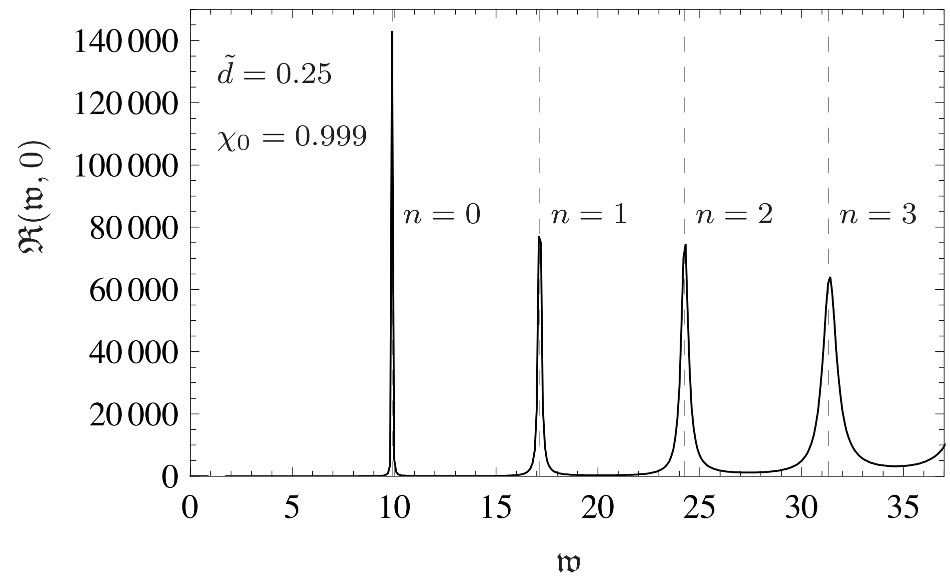

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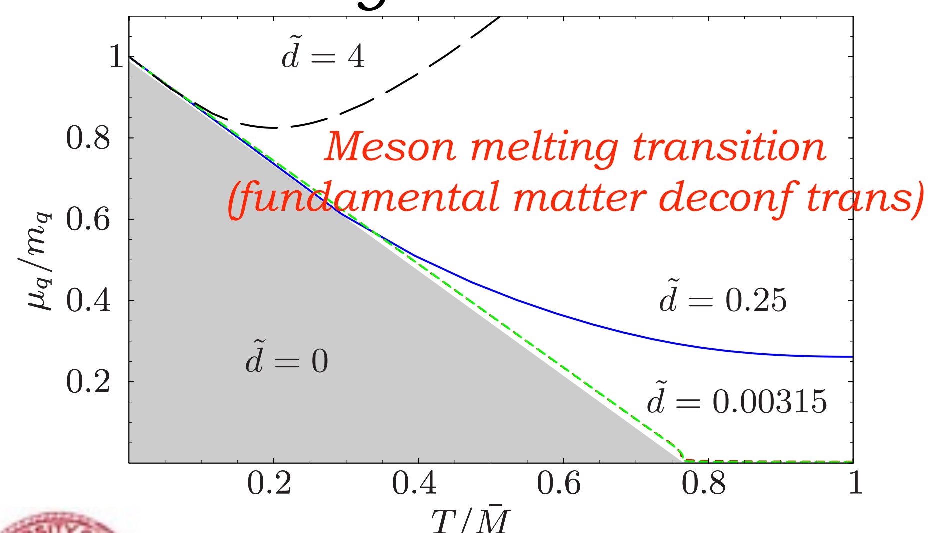

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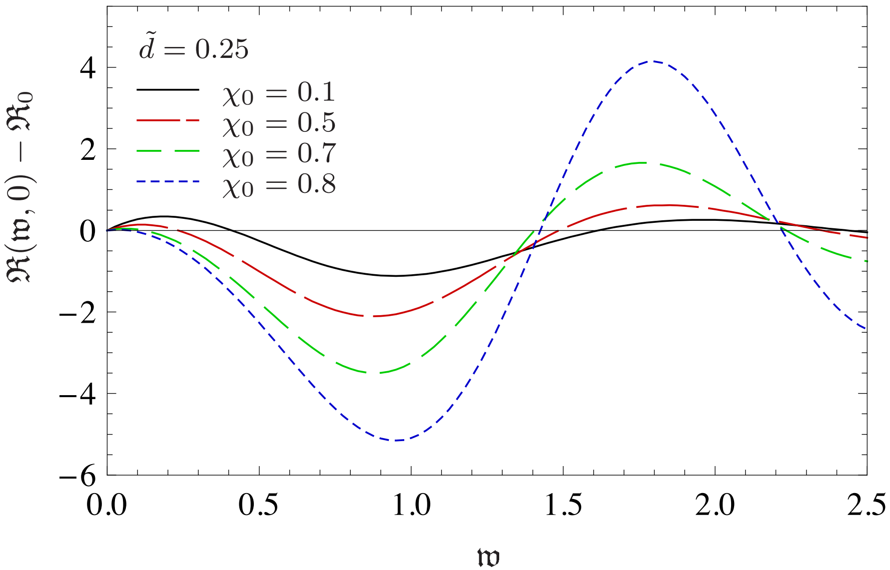

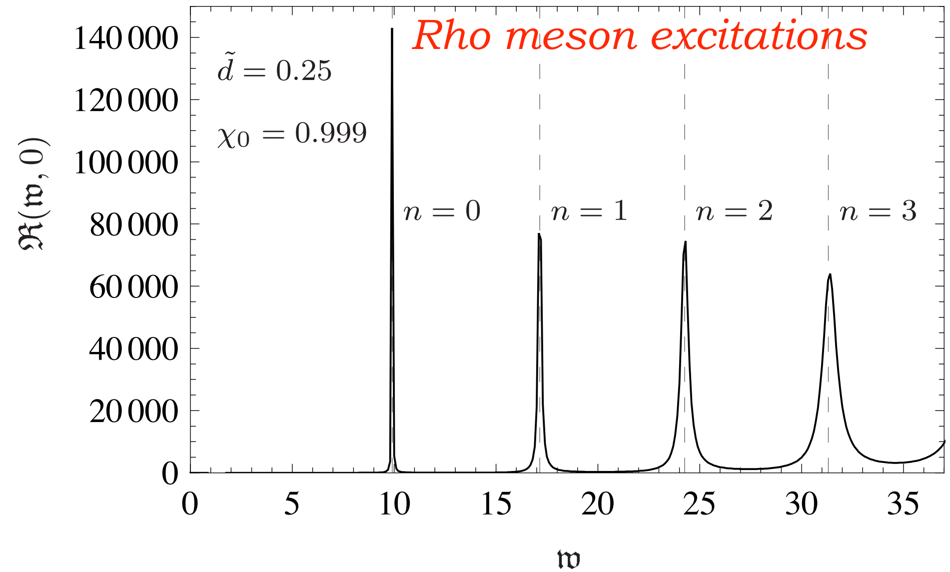

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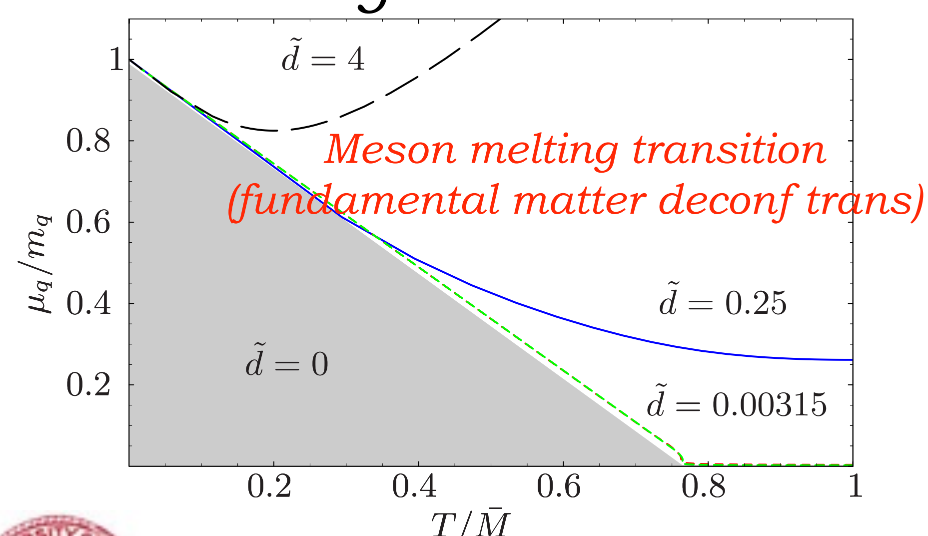
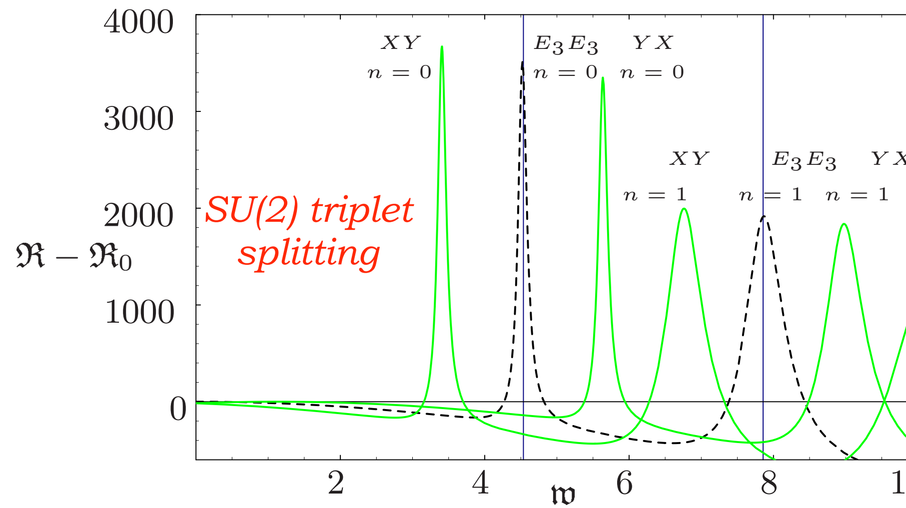


Nonzero isospin density



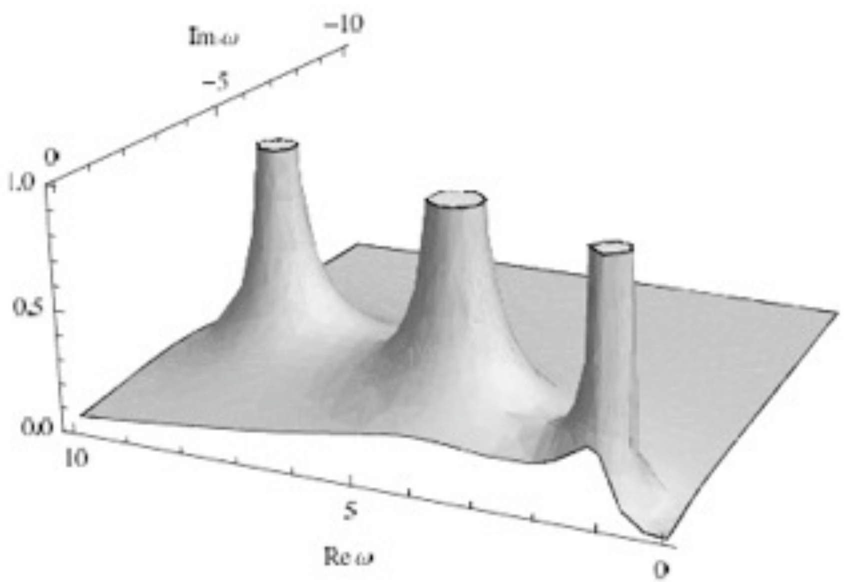
Analytically: [PhD thesis '08]

Retarded Green's function from holography

$$G^R(\omega, \boldsymbol{q}) = -\mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}\vec{k}\vec{x}}\, \theta(x^0)\, \langle [J(\vec{x}), J(0)] \rangle \longleftrightarrow \frac{\delta^2}{\delta\phi^{(0)}\delta\phi^{(0)}} S_{gravity}[\phi]$$

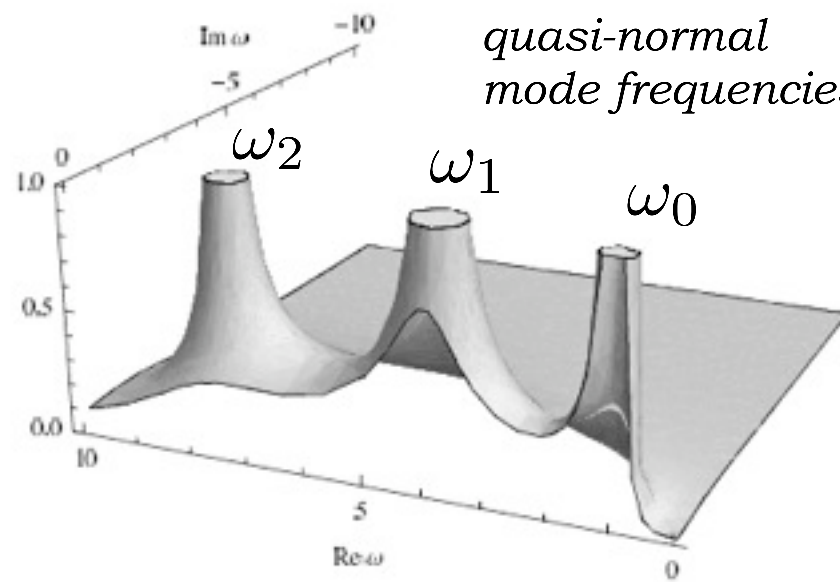
correlations between fluctuations around a state

Spectral function (imaginary part of retarded *G*):



high temperature
no quasiparticles

quasi-normal mode frequencies



low temperature
more stable quasiparticles
(resonances)

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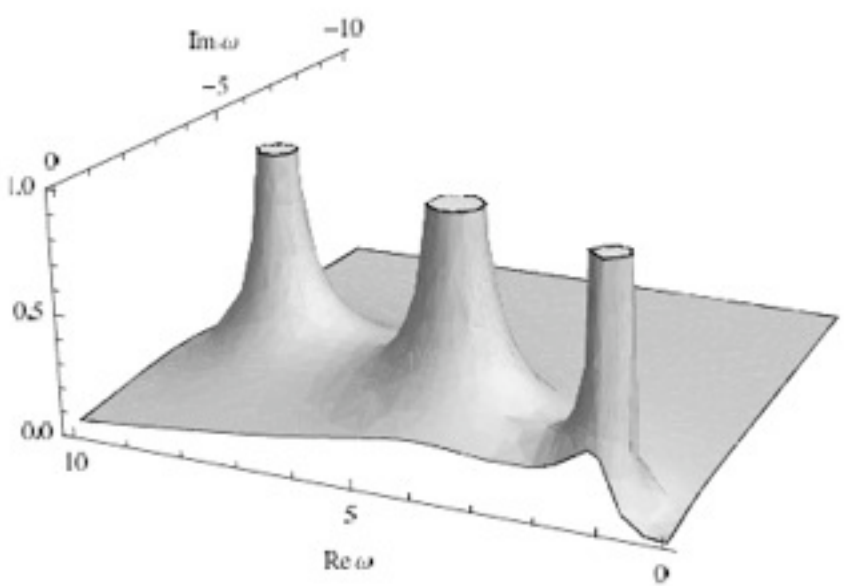
correlations between fluctuations around a state

gravitational fluctuation

BUT: which one ?

Spectral function (imaginary part of retarded G):



high temperature
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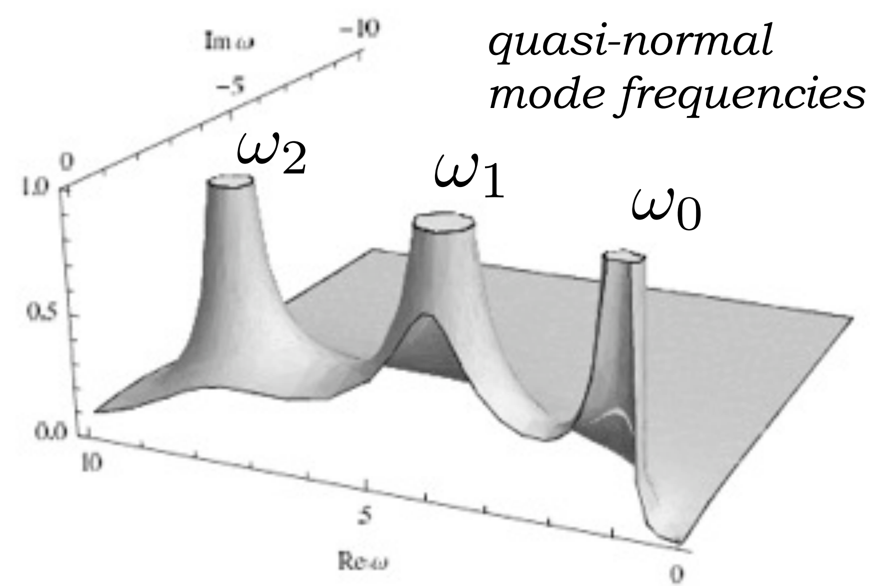
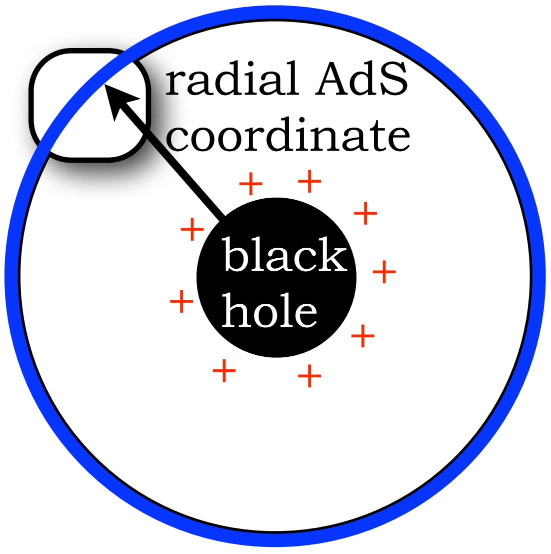


low temperature
more stable quasiparticles
(resonances)

Correspondence by zooming in on boundary



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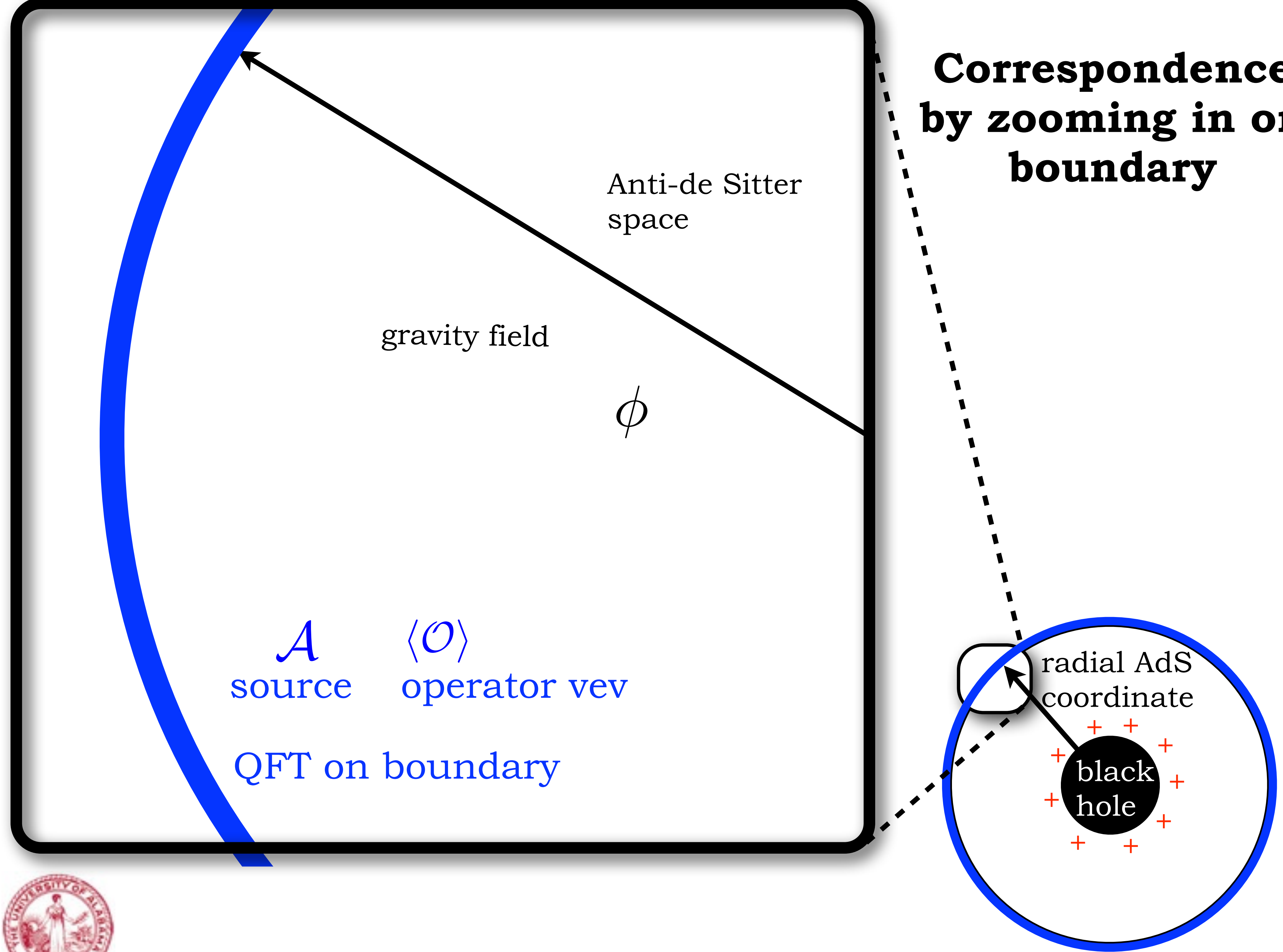

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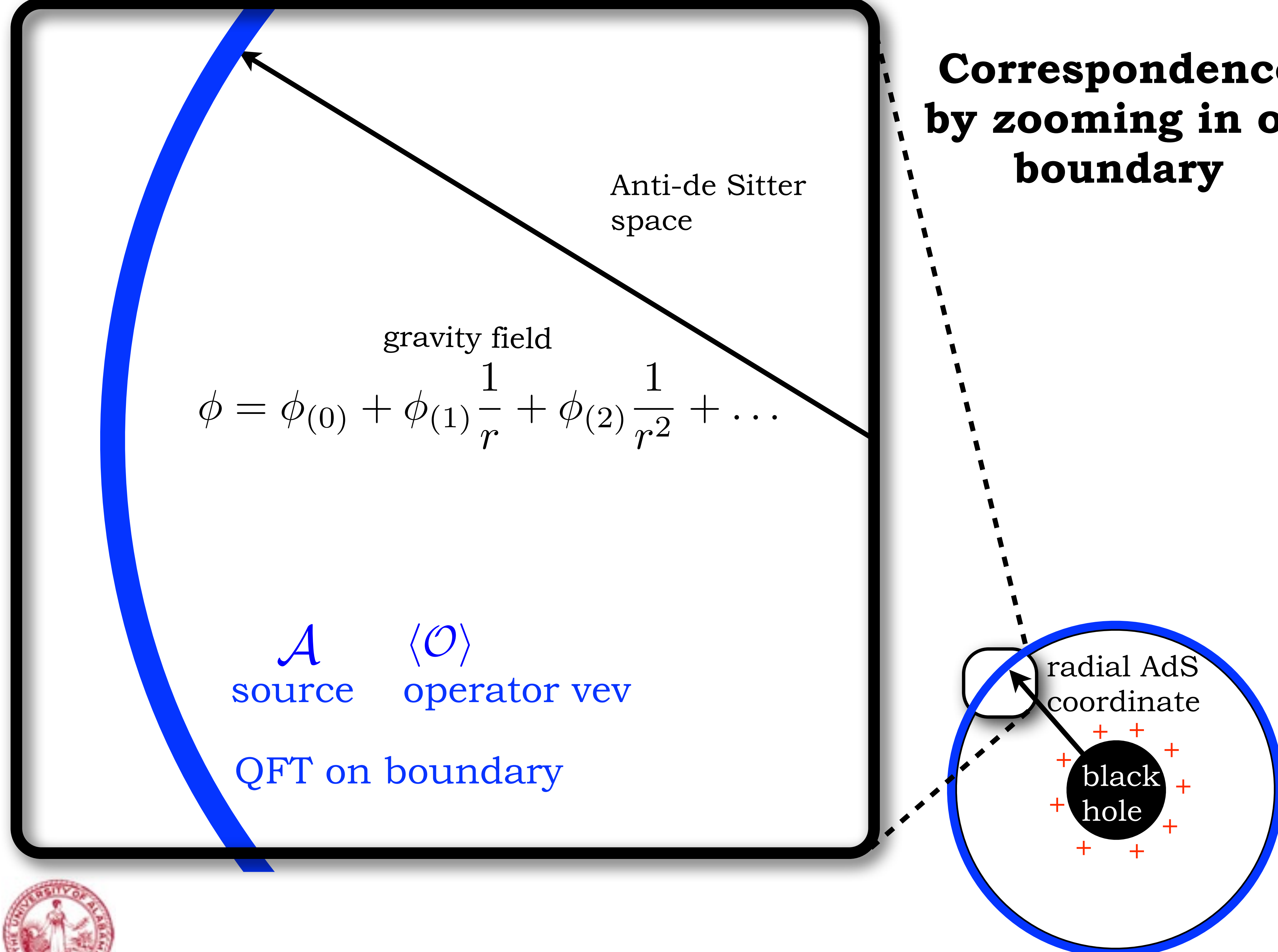

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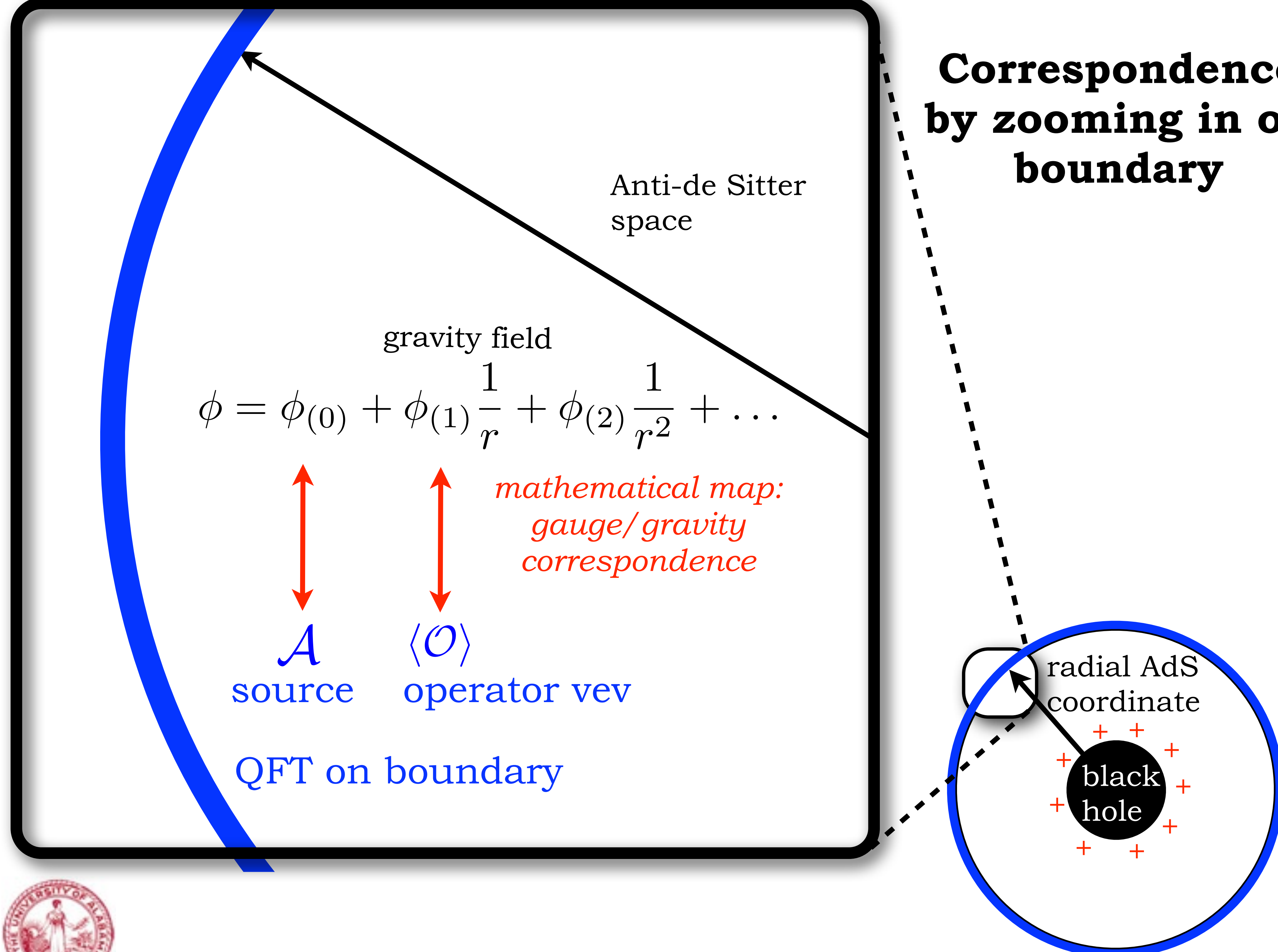
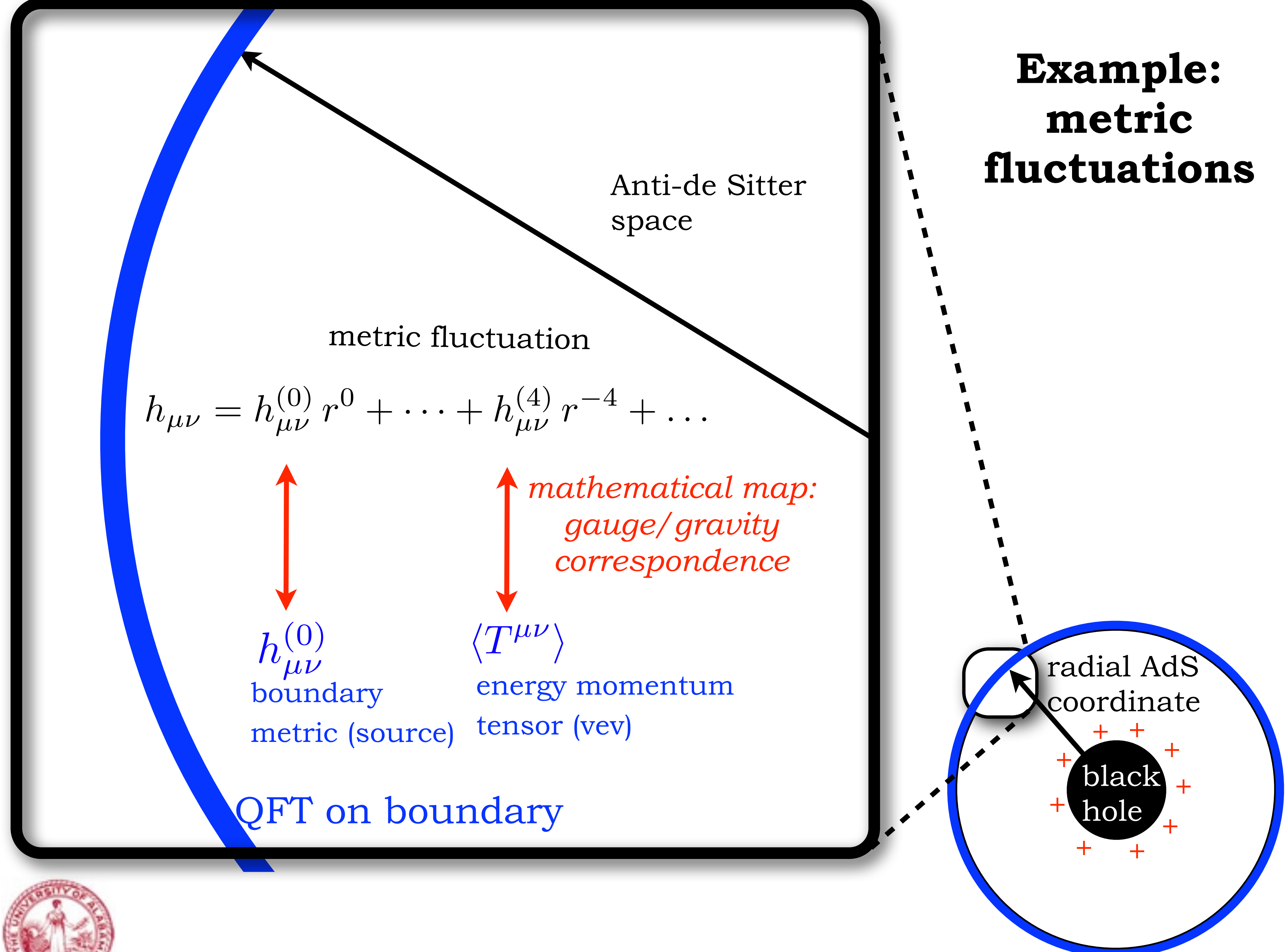
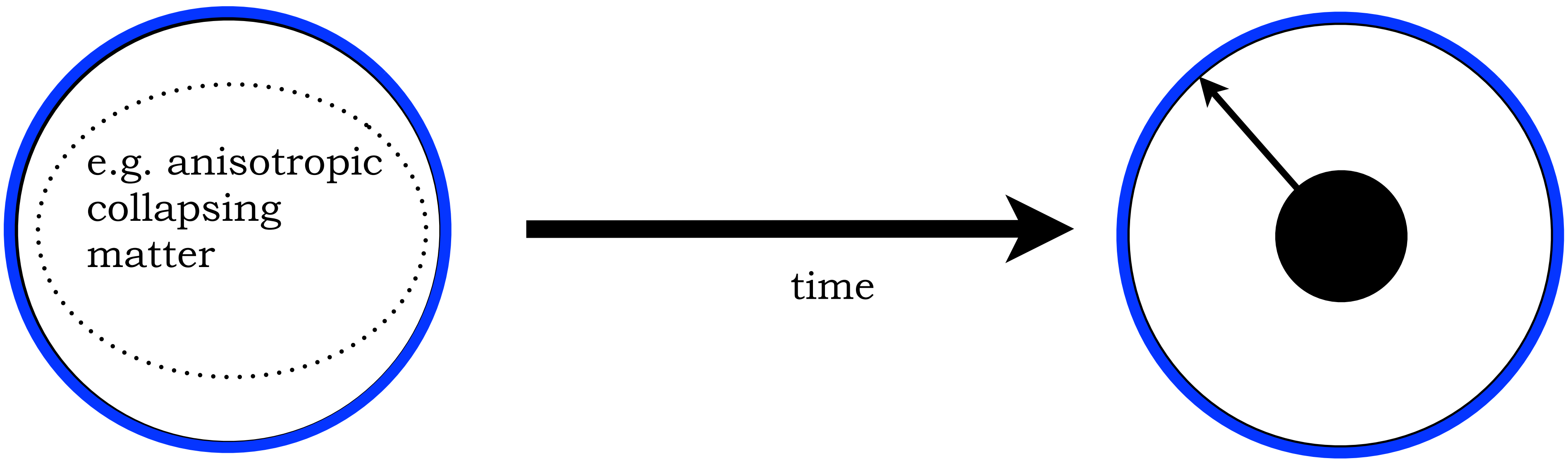


Example: metric fluctuations



The importance of quasinormal modes

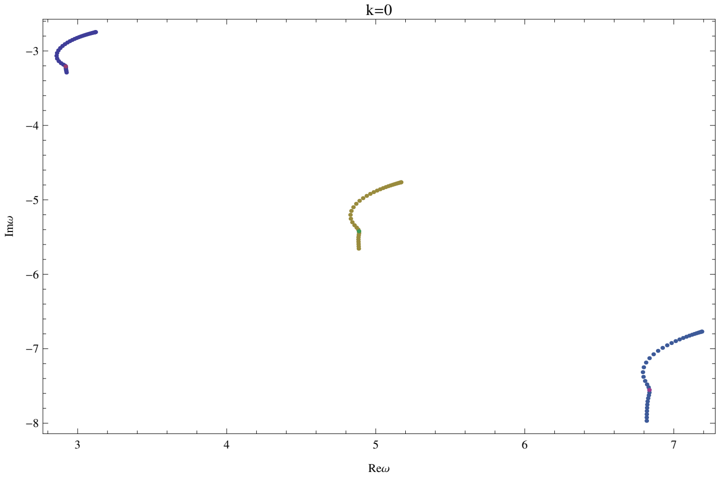
- describing thermalizing system at late times
- invaluable consistency check for holographic thermalization codes



initial time:
deformed space-time
e.g.: sheared between *x*- and *z*-direction

final time:
equilibrated space-time
e.g.: AdS5 Schwarzschild black brane

3. Quasi-normal modes (QNMs)

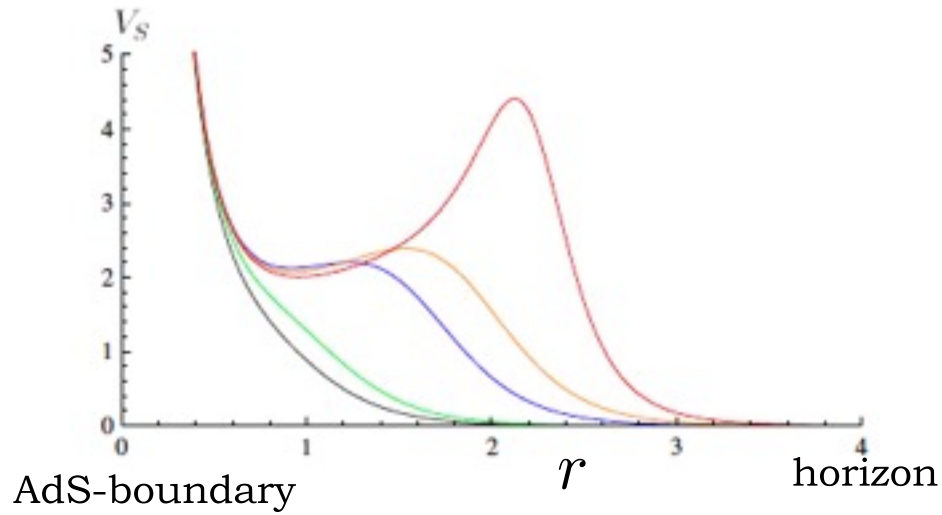


[Janiszewski, Kaminski; to appear (2015)]

What are quasi-normal modes?

- heuristically: the eigenmodes of black holes or black branes

$$-\partial_r{}^2\phi + V_S\phi = E\phi$$



- formal definition: (metric) fluctuations that are in-falling at horizon and vanishing at AdS-boundary

- correspond to poles of correlators in dual field theory

[Kovtun, Starinets; 2005]

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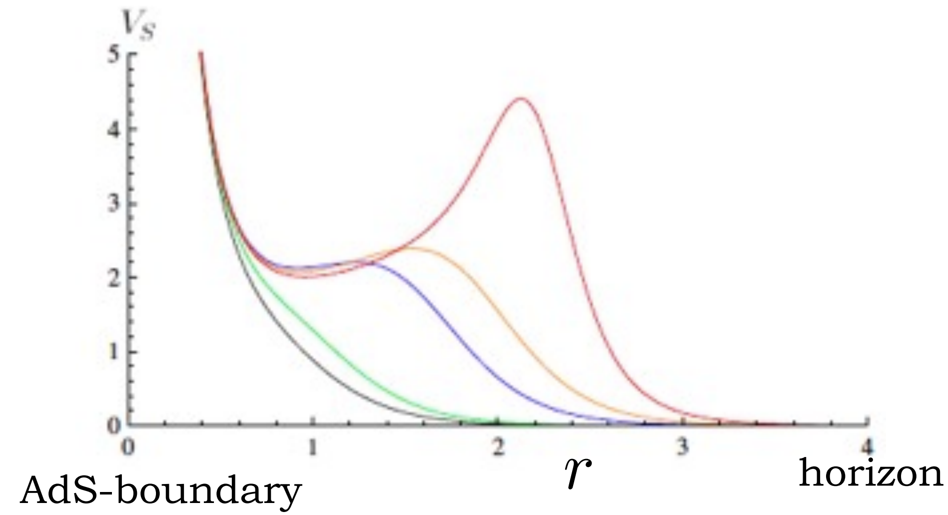


- formal definition: (metric) fluctuations that are in-falling at horizon and vanishing at AdS-boundary

- correspond to poles of correlators in dual field theory *[Kovtun, Starinets; 2005]*

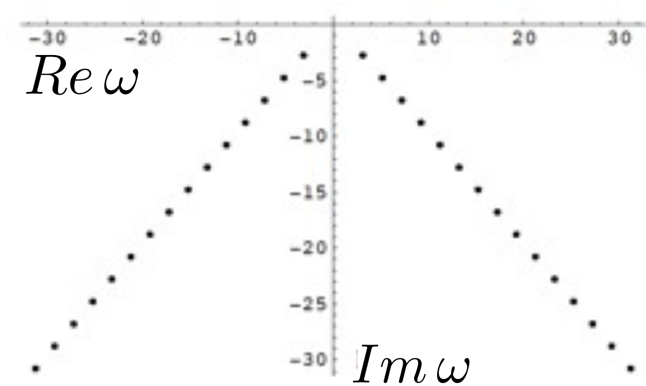
- *example*: tensor fluctuations (known from KSS bound) *[Starinets; JHEP (2002)]*

  $\phi := h_x{}^y$

  e.o.m. from linearized Einstein equations:

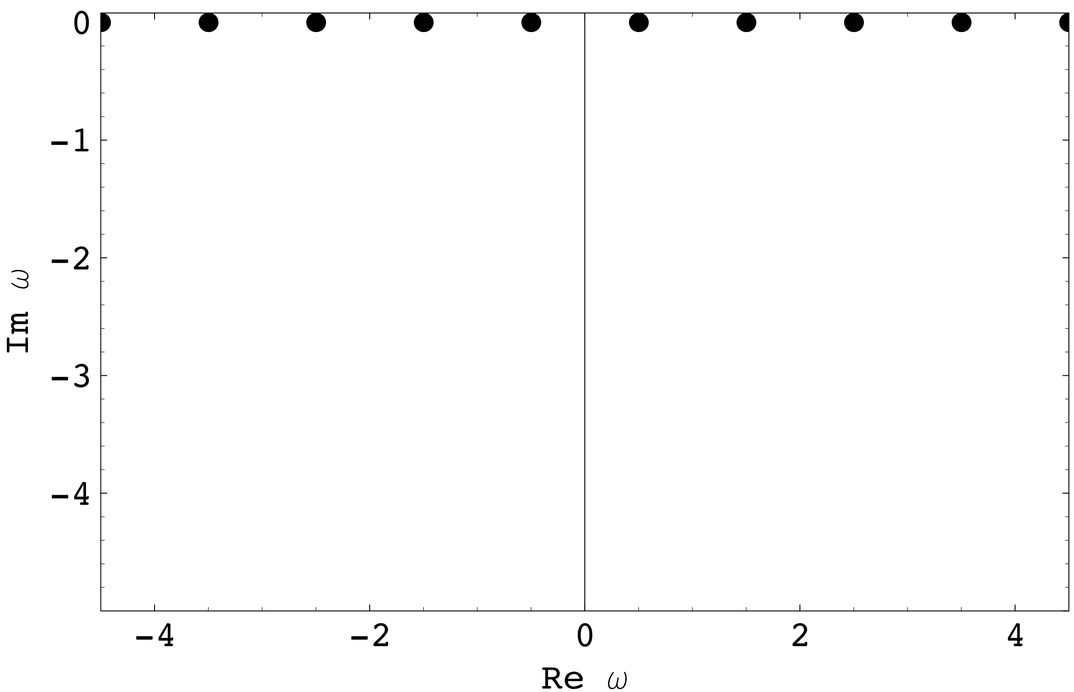
$$\phi'' - \frac{1+u^2}{uf}\phi' + \frac{\omega^2 - k^2 f}{uf^2}\phi = 0$$

$$f = 1 - u^2$$



Contrast: Normal modes

normal frequencies



Simple example:
Eigenfrequencies / normal frequencies
of the quantum mechanical
harmonic oscillator
(no damping)

$$\omega_n = \frac{1}{2} + n$$

How to compute QNMs

- start with any gravitational background (metric, matter content)
  Example: (charged) Reissner-Nordstrom black brane in 5-dim AdS

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2 \qquad f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

$$A_t = \mu - \frac{Q}{L r^2}$$

- choose one or more fields to fluctuate
  (consistent with the linearized Einstein equations)

  Example: metric tensor fluctuation

$$\phi := {h_x}^y \qquad 0 = \phi'' - \frac{f(u) - u f'(u)}{u f(u)}\phi' + \frac{\omega^2 - f(u)k^2}{4 r_H^2 u f(u)^2}\phi$$

$$u = \left(\frac{r_H}{r}\right)^2$$

- impose boundary conditions that are
  in-falling at horizon:

$$\phi = (1-u)^{\pm\frac{i\tilde{\omega}}{2(2-\tilde{q}^2)}}\left[\phi^{(0)} + \phi^{(1)}(1-u) + \phi^{(2)}(1-u)^2 + \dots\right]$$

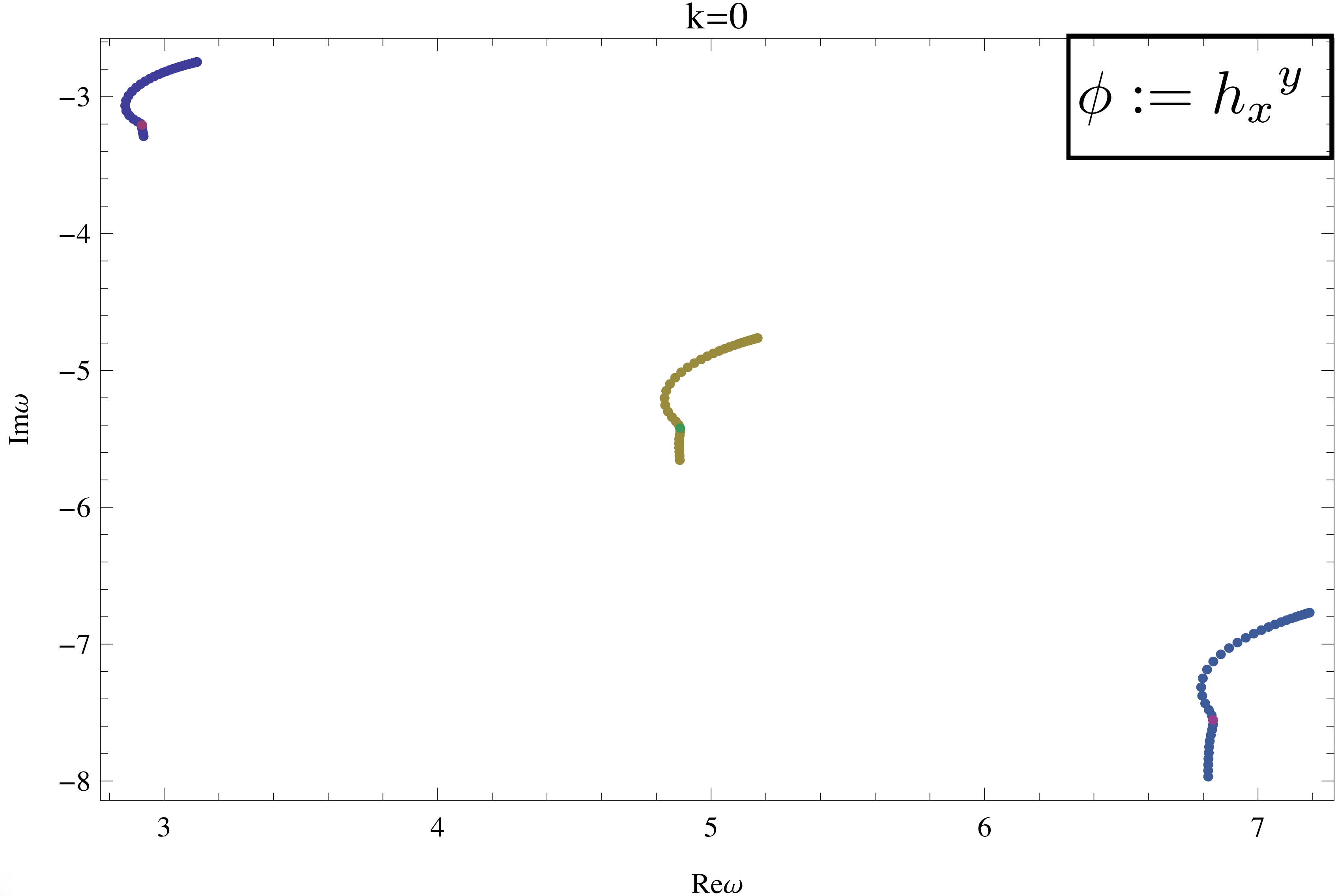
and
  vanishing at AdS-boundary:
$$\lim_{r \to r_{bdy}} \phi(r) = 0$$

QNMs of tensor fluctuation in RN black brane

[Janiszewski, Kaminski; to appear (2015)]

Equilibrium solution

Reissner-Nordstrom (charged) black branes in 5-dim AdS



Less stable resonances at larger charges.

Compare to far-from equilibrium results

Equilibrium solution

Reissner-Nordstrom black branes

- charged
- inner/outer horizon
- extremal case: maximum charge

Final state for charged fluids.

Magnetic black branes

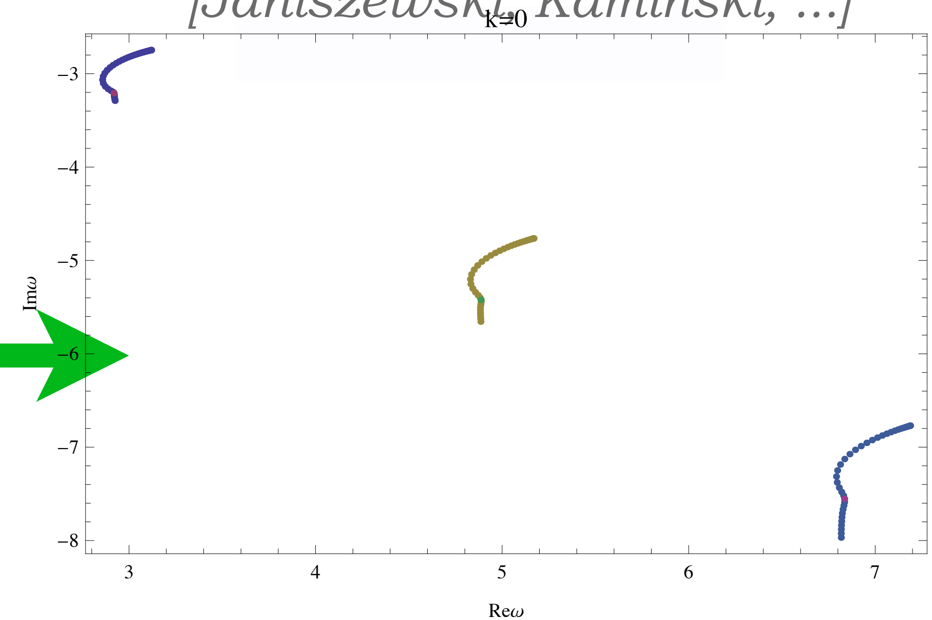
[D'Hoker, Kraus; JHEP (2009)]

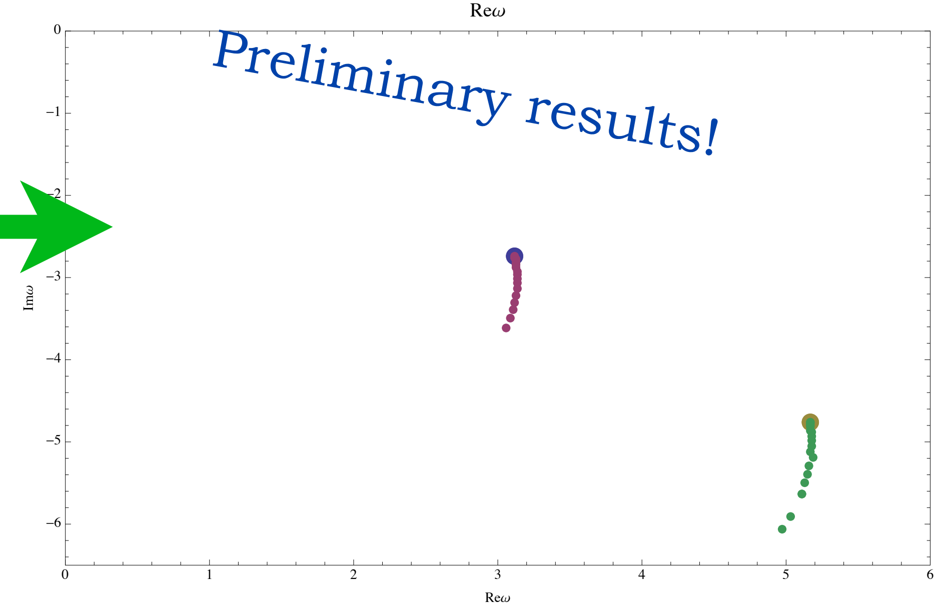
- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

Final state for fluids in magnetic field.

Quasinormal modes

[Janiszewski, Kaminski; ...]





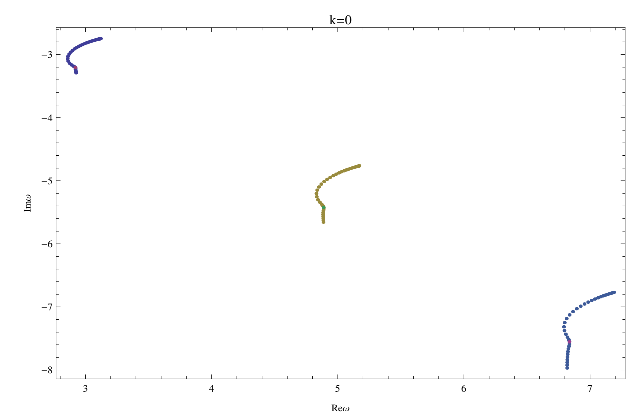
Require agreement with far from equilibrium setup at late times.

Summary

1. Hydrodynamics 2.0
    - chiral hydro
    - chiral transport coefficient known exactly
    - measure gravitational anomalies

2. Holography basics
    - charged equilibrium states = charged black branes/holes
    - QNMs = correlator poles
    - thermalization = black brane/hole formation

3. Quasi-normal modes
    - black hole/brane "ringing"
    - thermalization code benchmarks



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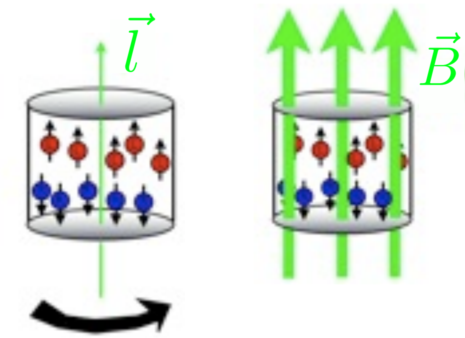

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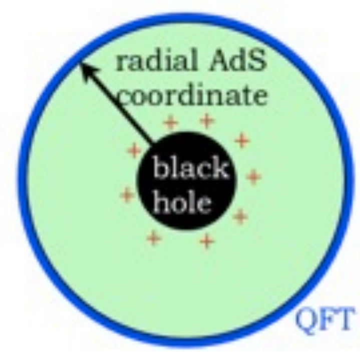

3. Quasi-normal modes
    - black hole/brane “ringing”
    - thermalization code benchmarks (see Lecture II)