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Correlation functions for heavy quarks in the QGP from lattice QCD calculations

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Lattice calculations of hadronic correlation functions

... and how we try to

extract transport properties and spectral properties from them

1) Vector meson correlation functions for light quarks

continuum extrapolation — with H-T.Ding, F.Meyer, et al.

comparison to perturbation theory — with J.Ghiglieri, M.Laine, F.Meyer

→ Electrical conductivity

→ Thermal dilepton rates and thermal photon rates

2) Color electric field correlation function — with A.Francis, M. Laine, T.Neuhaus, H.Ohno

Heavy quark momentum diffusion coefficient κ

3) Vector meson correlation functions for heavy quarks — with H-T.Ding, H.Ohno et al.

Heavy quark diffusion coefficients

Charmonium and Bottomonium dissociation patterns

Motivation - Transport Coefficients

Transport Coefficients are important ingredients into **hydro/transport models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

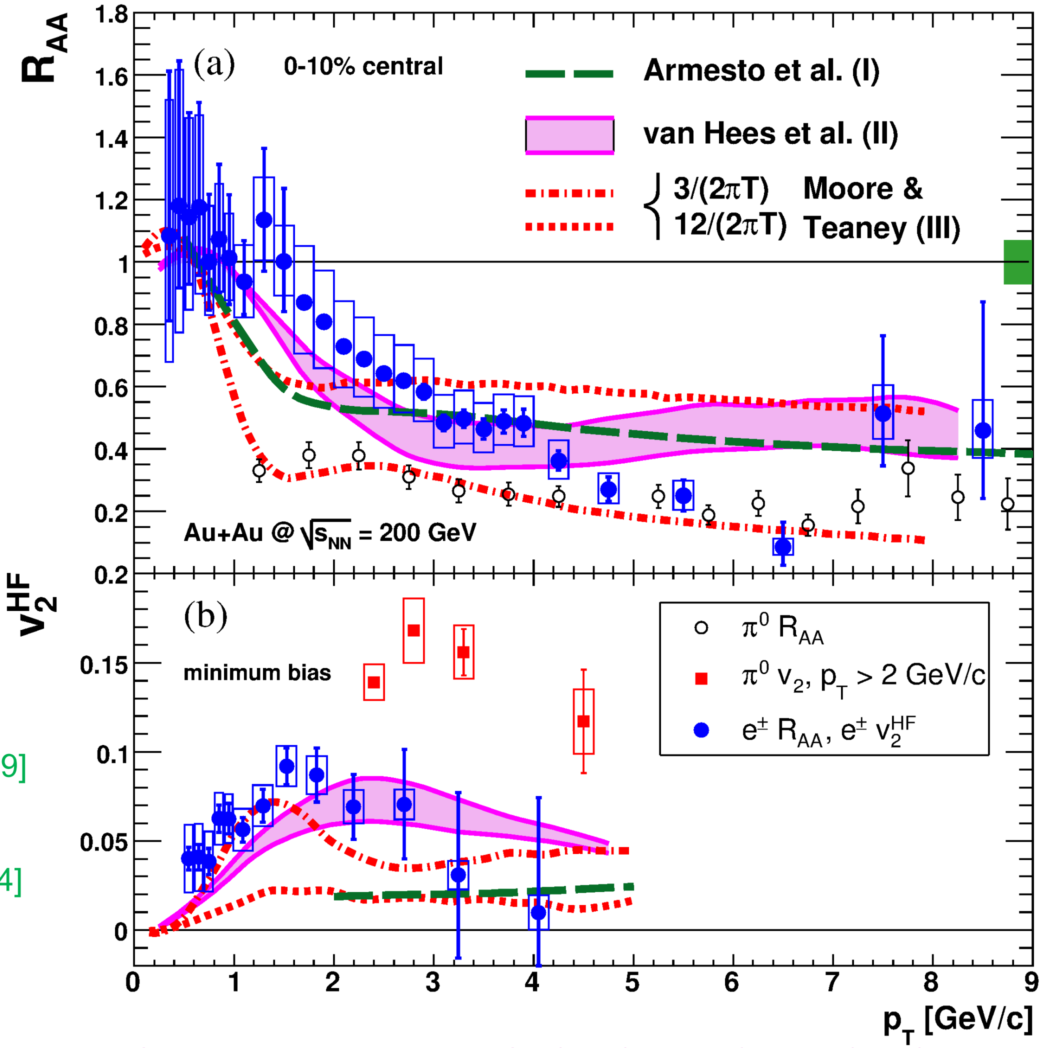
Need to be determined from QCD using first principle lattice calculations!

here heavy flavour:

- Heavy Quark Diffusion Constant D
  [H.T.Ding, OK et al., PRD86(2012)014509]
- Heavy Quark Momentum Diffusion κ
  [OK, arXiv:1409.3724]

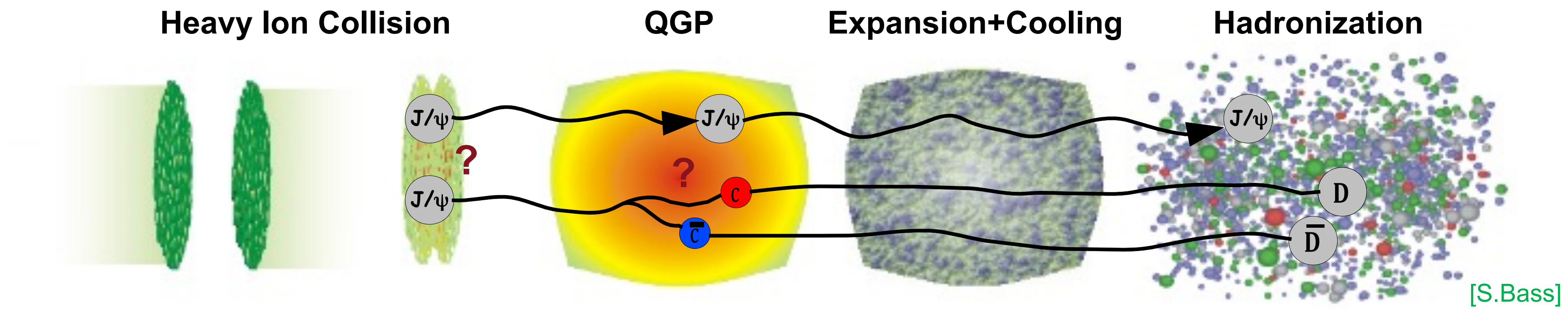
or for light quarks:

- Light quark flavour diffusion
- Electrical conductivity
  [A.Francis, OK et al., PRD83(2011)034504]



[PHENIX Collaboration, Adare et al.,PRC84(2011)044905 & PRL98(2007)172301]

Motivation - Quarkonium in Heavy Ion Collisions

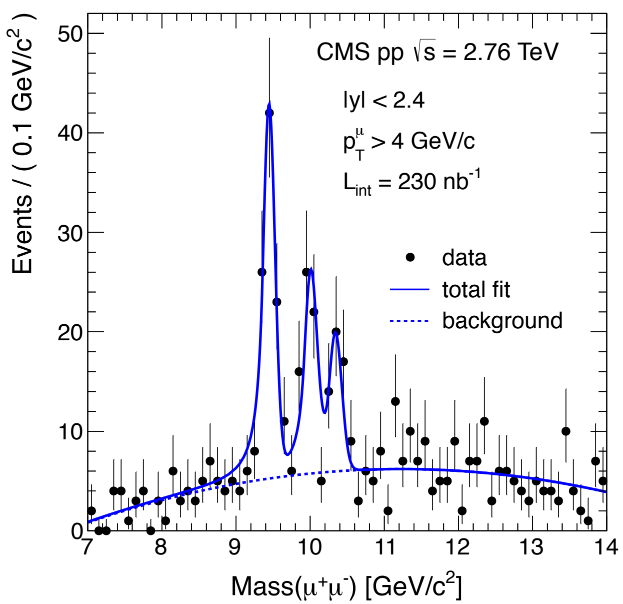


Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

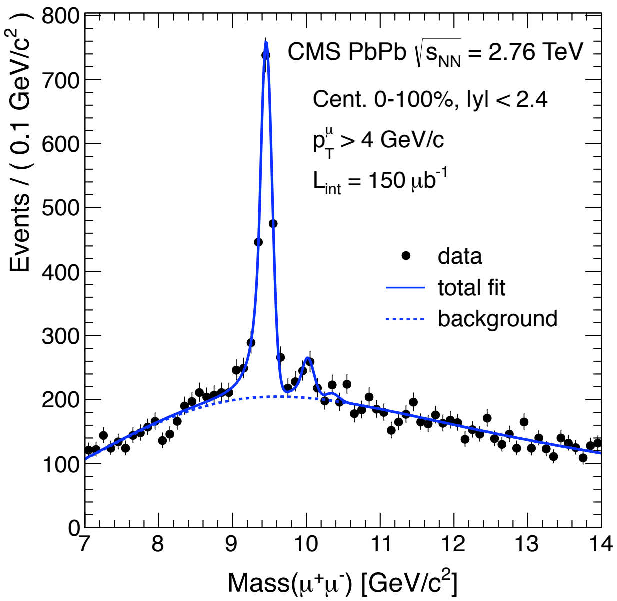
Depending on the Dissociation Temperature

- remain as bound states in the whole evolution

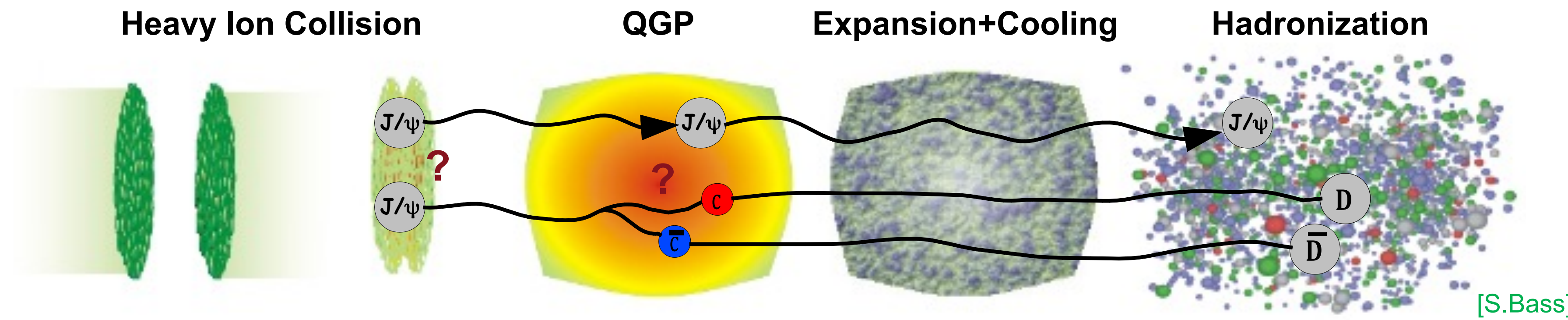
- release their constituents in the plasma



Sequential suppression for bottomonium observed at CMS



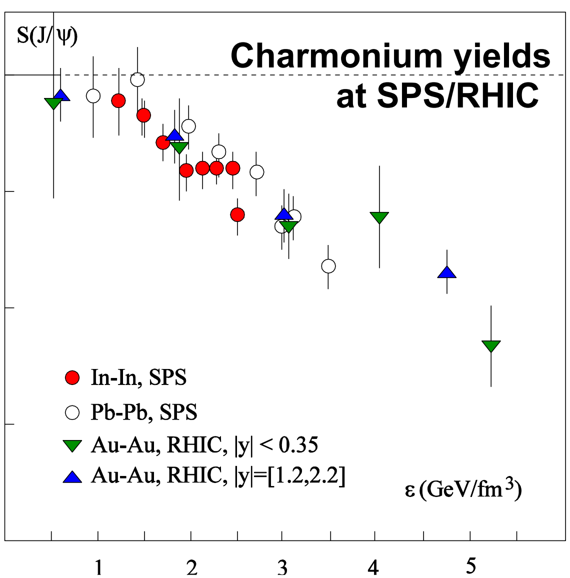
Motivation - Quarkonium in Heavy Ion Collisions



Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

Depending on the Dissociation Temperature

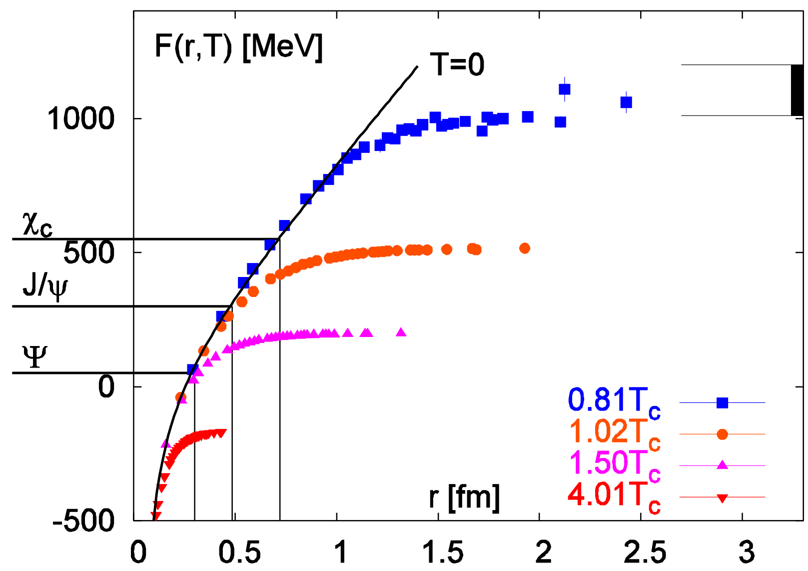
- **remain as bound states in the whole evolution**
- **release their constituents in the plasma**



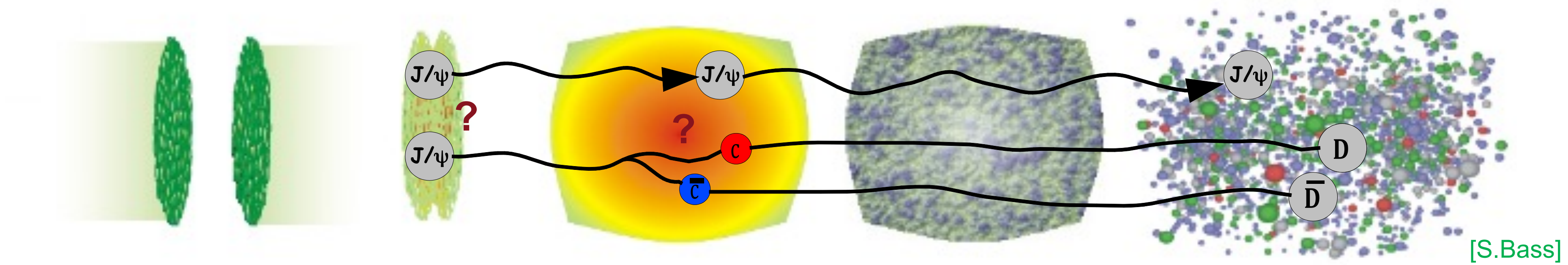
First estimates on Dissociation Temperatures

from detailed knowledge of Heavy Quark Free Energies and Potential Models

[Kaczmarek, Zantow, 2005]



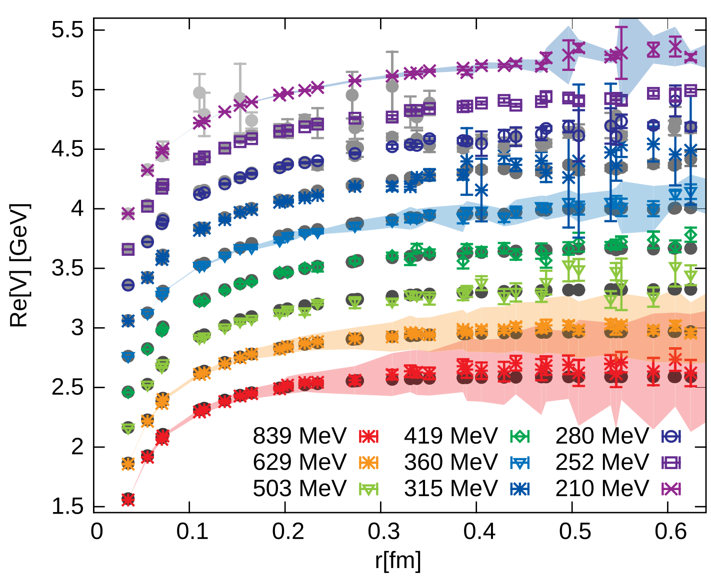
Motivation - Quarkonium in Heavy Ion Collisions

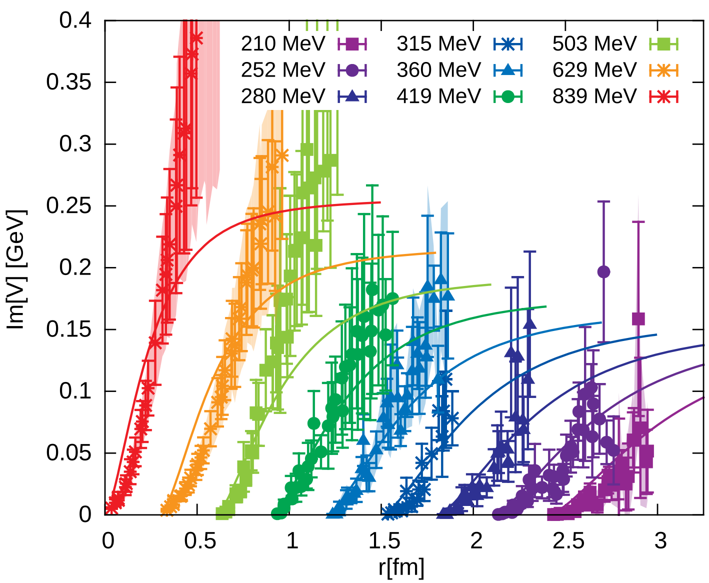


Heavy quark potential complex valued at finite temperature

[Y.Burnier, OK, A.Rothkopf, PRL114(2015)082001]

$$V(r) = \lim_{t\to\infty} \frac{i\partial_t W(t,r)}{W(t,r)} \leftrightarrow V(r) = \lim_{t\to\infty} \int d\omega \omega e^{-i\omega t}\rho(\omega,r) / \int d\omega e^{-i\omega t}\rho(\omega,r)$$





Transport coefficients from Lattice QCD – Flavour Diffusion

Transport coefficients usually calculated using correlation function of conserved currents

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\mathbf{p},T) K(\tau,\omega,T)$$

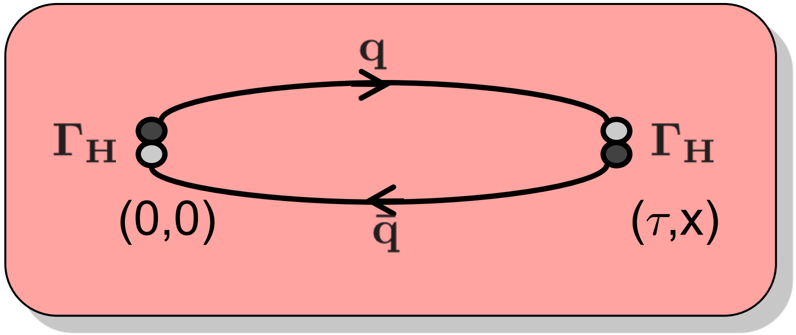
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J^\dagger_\nu(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x})$$

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x}) e^{i\vec{p}\vec{x}}$$



related to a conserved current

only correlation functions calculable on lattice but

Transport coefficient determined by slope of spectral function at ω=0 (Kubo formula)

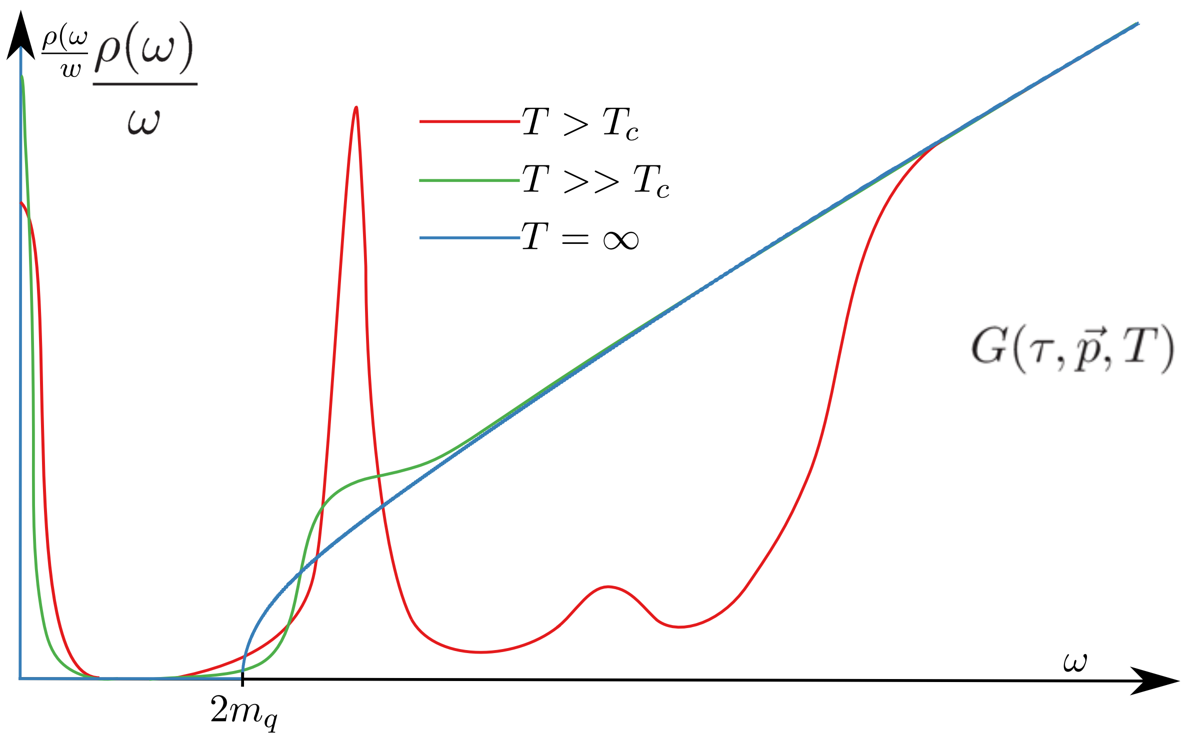
$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega\to 0} \frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

Vector spectral function – hard to separate different scales

Different contributions and scales enter in the spectral function

- continuum at large frequencies
- possible bound states at intermediate frequencies
- transport contributions at small frequencies
- in addition cut-off effects on the lattice

notoriously difficult to extract from correlation functions



$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

(narrow) transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2+\eta^2}$, $\eta = \frac{T}{MD}$

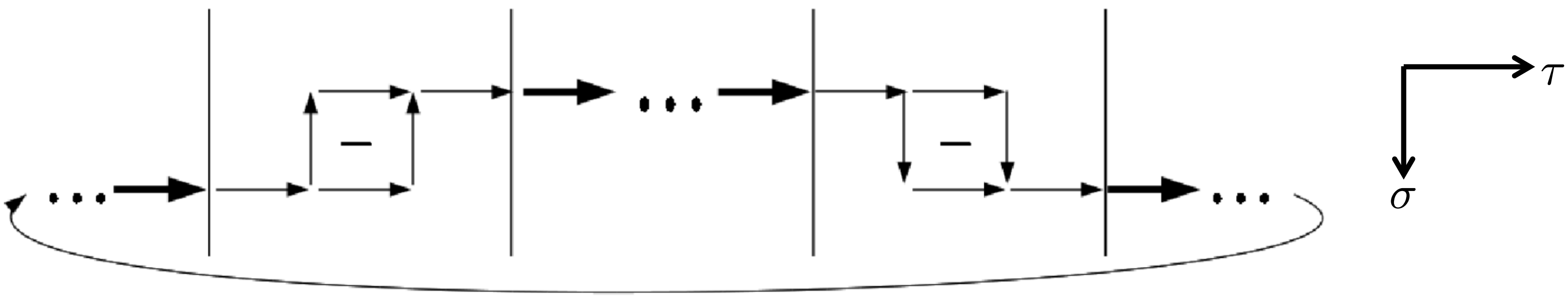
Heavy Quark Momentum Diffusion Constant – Single Quark in the Medium

Heavy Quark Effective Theory (HQET) in the large quark mass limit

for a single quark in medium

leads to a (pure gluonic) "color-electric correlator"

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re}\,\mathrm{Tr}\left[U(\tfrac{1}{T};\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re}\,\mathrm{Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

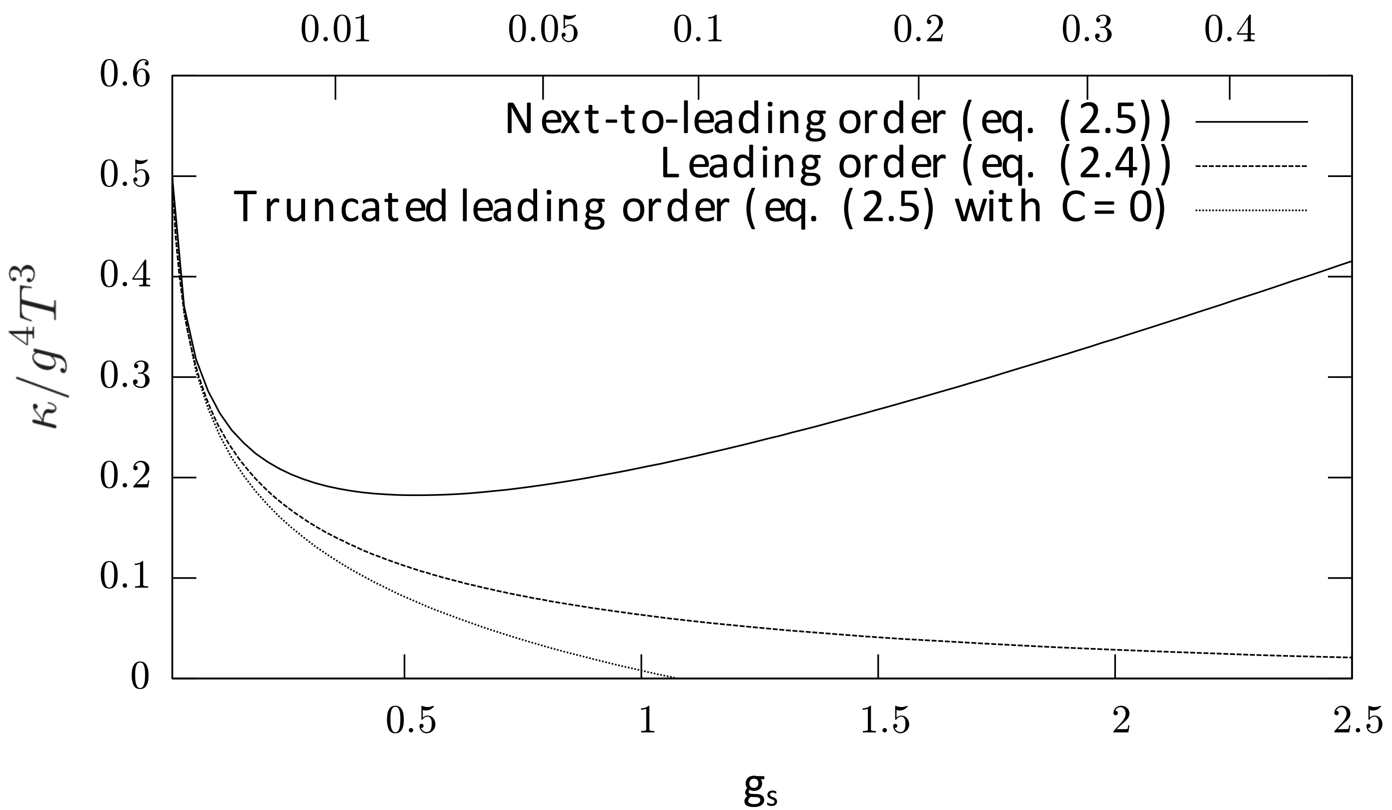
Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \qquad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

can be related to the thermalization rate: $$\eta_D = \frac{\kappa}{2M_{kin}T}\left(1 + O\left(\frac{\alpha_s^{3/2}T}{M_{kin}}\right)\right)$$

NLO in perturbation theory: [Caron-Huot, G.Moore, JHEP 0802 (2008) 081]

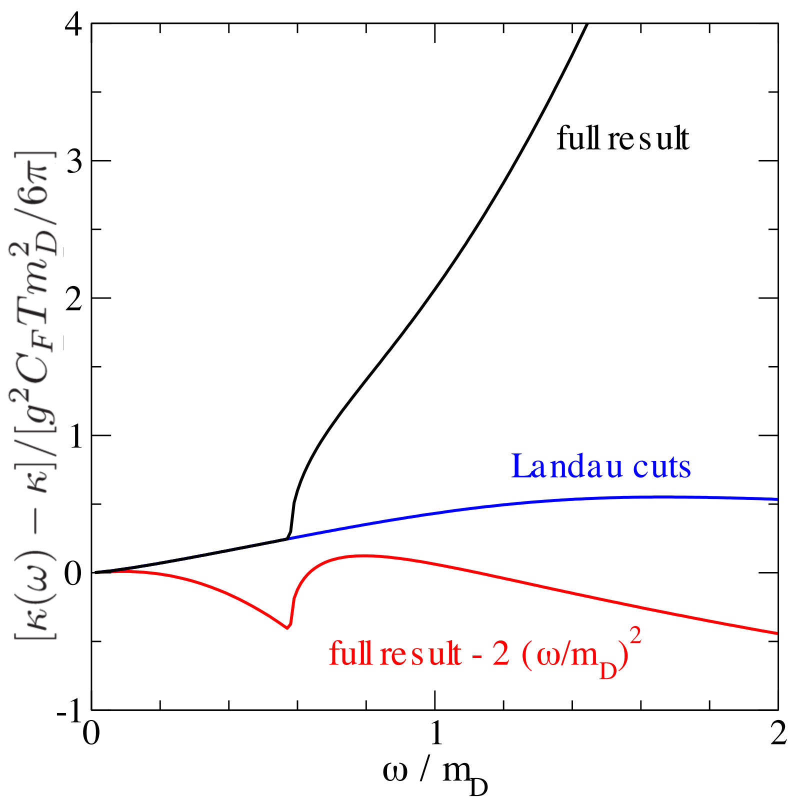


very poor convergence

→ **Lattice QCD study required in the relevant temperature region**

Heavy Quark Momentum Diffusion Constant – Perturbation Theory

NLO spectral function in perturbation theory: [Caron-Huot, M.Laine, G.Moore, JHEP 0904 (2009) 053]



in contrast to a narrow transport peak, from this a smooth limit

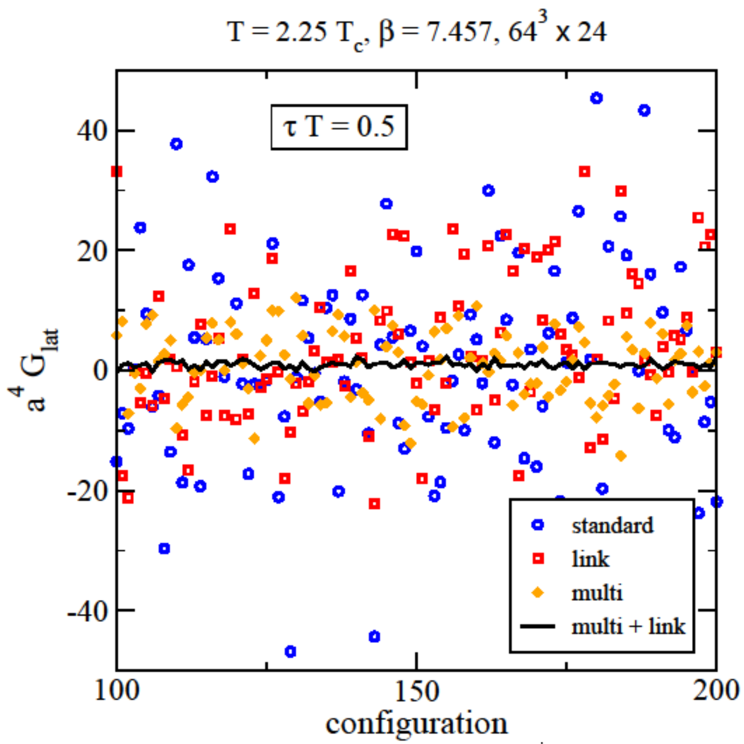
$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\rm E}(\omega)}{\omega}$$

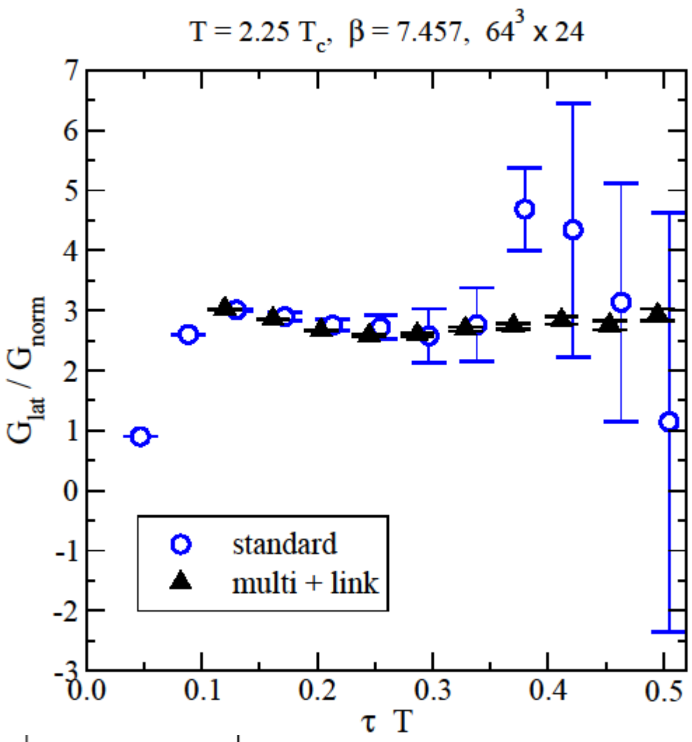
is expected

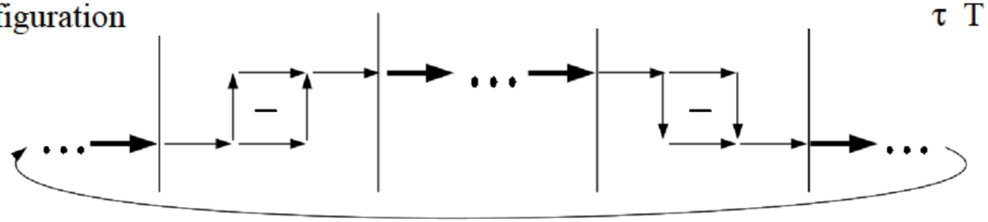
qualitatively similar behavior also found in AdS/CFT [S.Gubser, Nucl.Phys.B790 (2008)175]

Heavy Quark Momentum Diffusion Constant – Lattice algorithms

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]







due to the gluonic nature of the operator, signal is extremely noisy

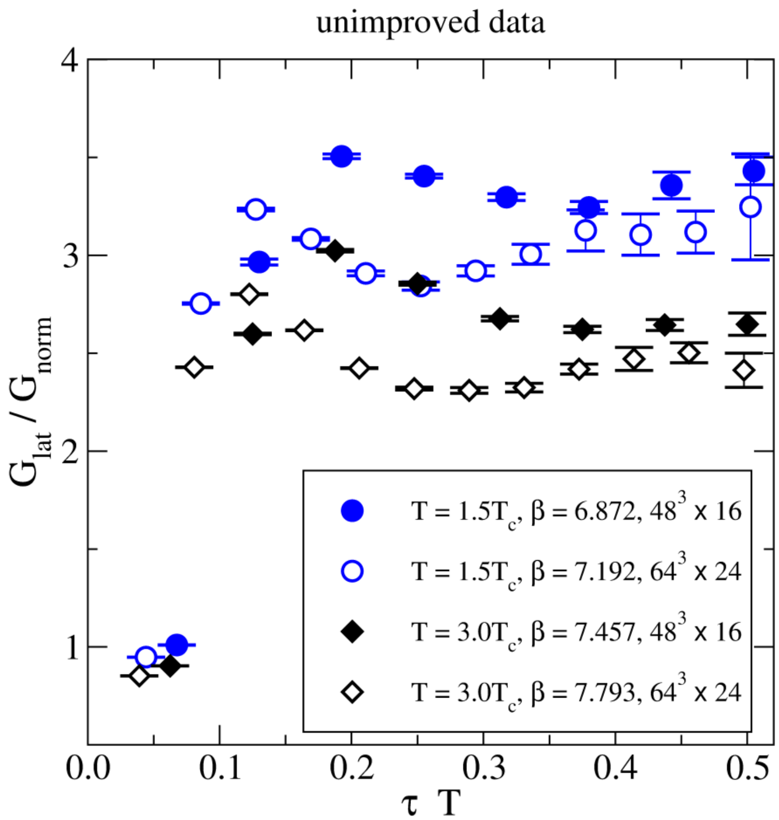
→ multilevel combined with link-integration techniques to improve the signal

[Lüscher,Weisz JHEP 0109 (2001)010
and H.B.Meyer PRD (2007) 101701]

[Parisi,Petronzio,Rapuano PLB 128 (1983) 418,
and de Forcrand PLB 151 (1985) 77]

Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



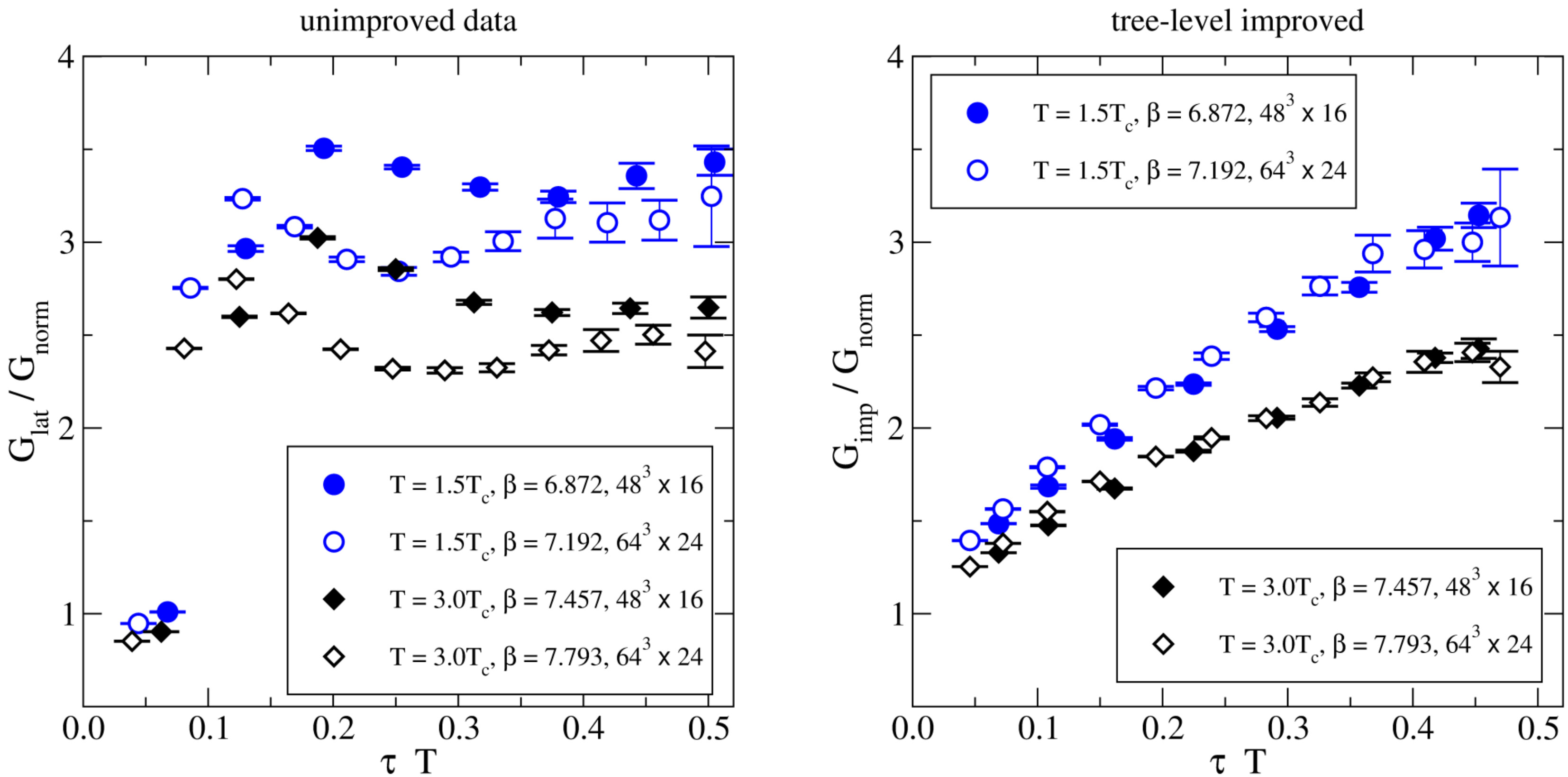
normalized by the LO-perturbative correlation function:

$$G_{\text{norm}}(\tau T) \equiv \frac{G^{\text{LO}}_{\text{cont}}(\tau T)}{g^2 C_F} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right] \qquad C_F \equiv \frac{N_c^2 - 1}{2N_c}$$

and renormalized using NLO renormalization constants $Z(g^2)$

Heavy Quark Momentum Diffusion Constant – Tree-Level Improvement

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



lattice cut-off effects visible at small separations (left figure)

→ **tree-level improvement** (right figure) to reduce discretization effects

$$G^{\rm LO}_{\rm cont}(\overline{\tau T}) = G^{\rm LO}_{\rm lat}(\tau T)$$

leads to an effective reduction of cut-off effect for all τT

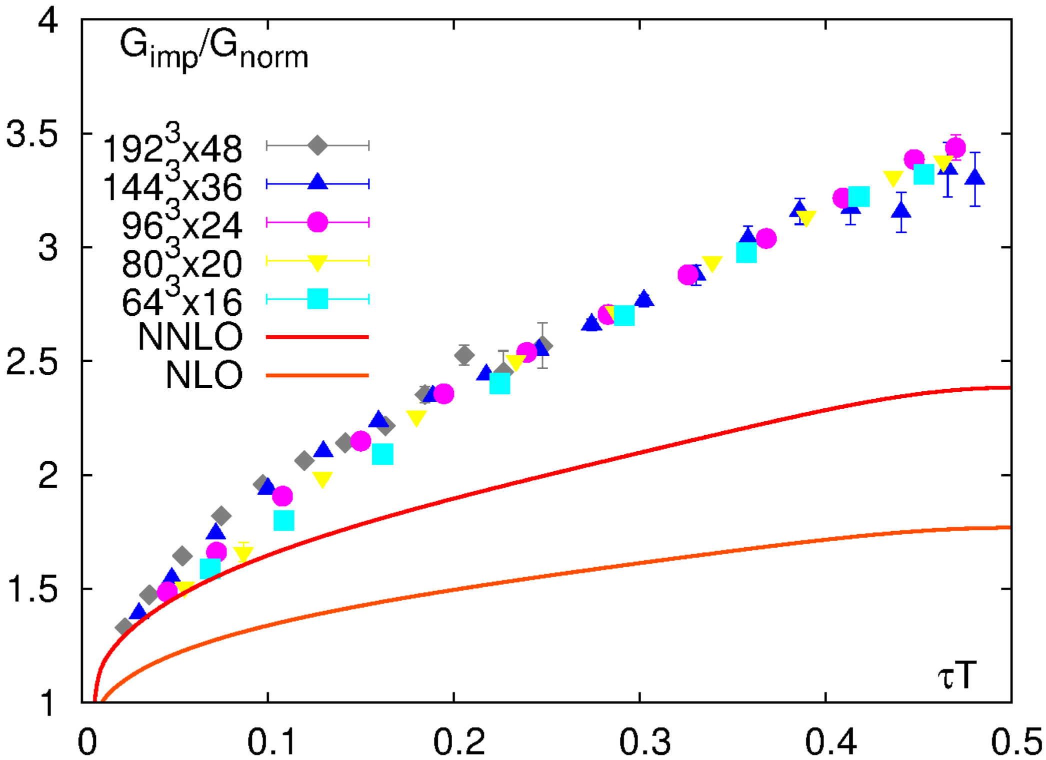
Heavy Quark Momentum Diffusion Constant – Lattice results

Quenched Lattice QCD on large and fine isotropic lattices at $T \simeq 1.4\, T_c$

- standard Wilson gauge action
- algorithmic improvements to enhance signal/noise ratio
- fixed aspect ration $N_s/N_t = 4$, i.e. fixed physical volume $(2\text{fm})^3$
- perform the continuum limit, $a \to 0 \;\leftrightarrow\; N_t \to \infty$
- determine κ in the continuum using an Ansatz for the spectral fct. $\rho(\omega)$

N_σ	N_τ	β	$1/a$[GeV]	a[fm]	#Confs
64	16	6.872	7.16	0.03	172
80	20	7.035	8.74	0.023	180
96	24	7.192	10.4	0.019	160
144	36	7.544	15.5	0.013	693
192	48	7.793	20.4	0.010	223

Heavy Quark Momentum Diffusion Constant – Lattice results



finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit $\tau T \to 0$

N_σ	N_τ	β	$1/a$[GeV]	a[fm]	#Confs
64	16	6.872	7.16	0.03	172
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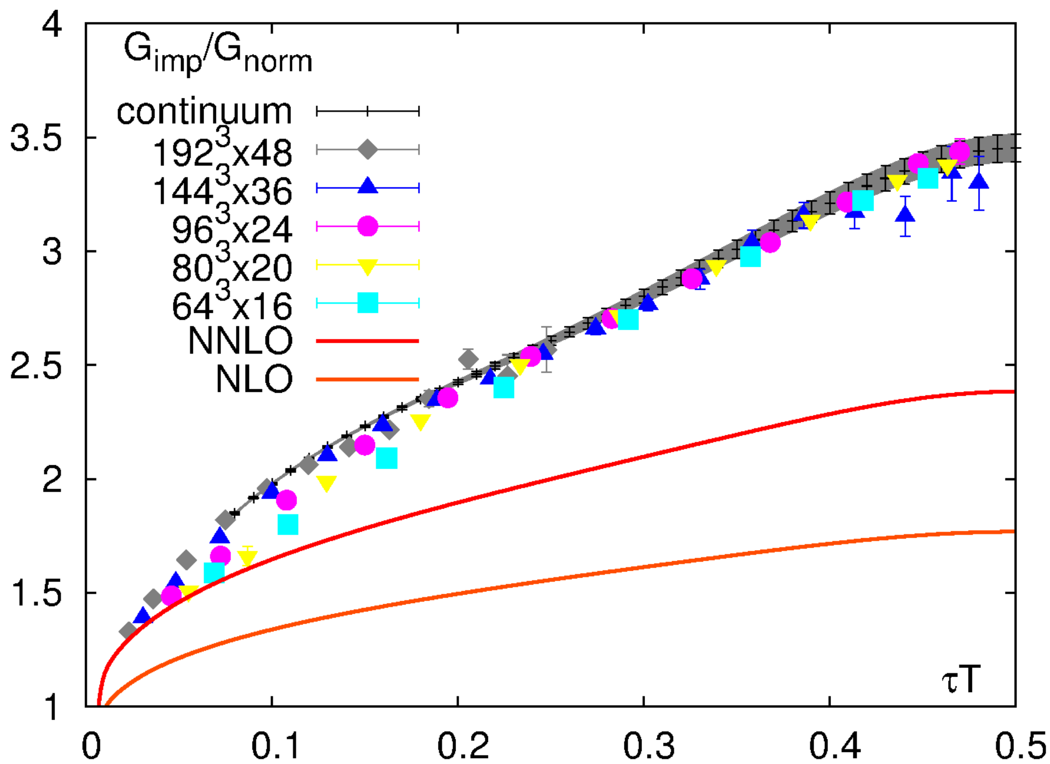
96	24	7.192	10.4	0.019	160
144	36	7.544	15.5	0.013	693
192	48	7.793	20.4	0.010	223

allows to perform continuum extrapolation, $a \to 0 \leftrightarrow N_t \to \infty$ **, at fixed** $T=1/a N_t$

Heavy Quark Momentum Diffusion Constant – Continuum extrapolation



finest lattices still quite noisy at large τT

but only

small cut-off effects at intermediate τT

cut-off effects become visible at small τT

need to extrapolate to the continuum

perturbative behavior in the limit $\tau T \rightarrow 0$

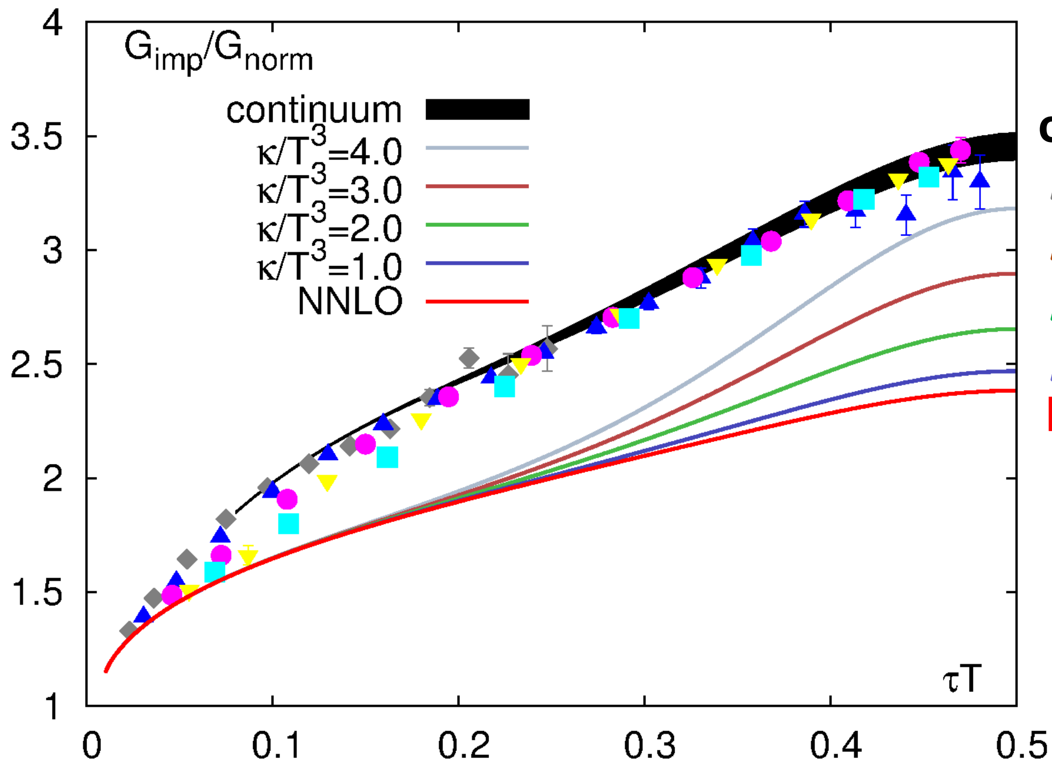
well behaved continuum extrapolation for $0.05 \leq \tau T \leq 0.5$

finest lattice already close to the continuum

coarser lattices at larger τT close to the continuum

how to extract the spectral function from the correlator?

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



continuum extrapolation

$\kappa/T^3 = 4.0$

$\kappa/T^3 = 3.0$

$\kappa/T^3 = 2.0$

$\kappa/T^3 = 1.0$

NNLO (vacuum) perturbation theory

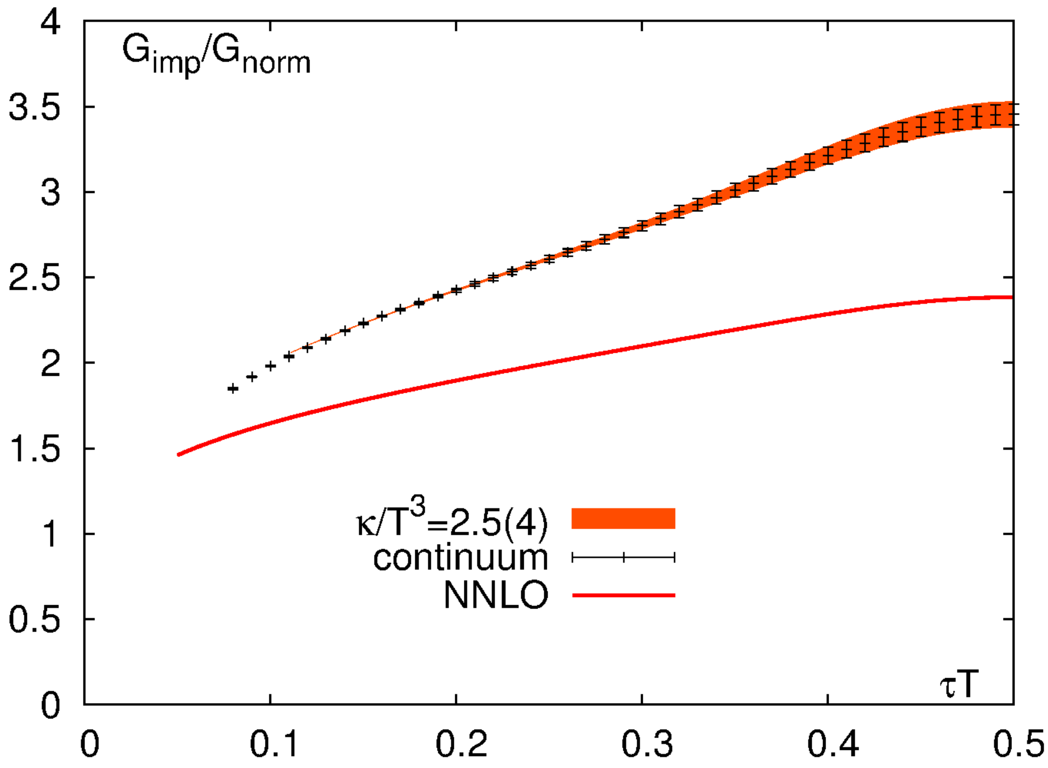
Model spectral function: transport contribution + NNLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\mathrm{model}}(\omega) \equiv \max\left\{\rho_{\mathrm{NNLO}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\mathrm{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{\mathrm{model}}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

some contribution at intermediate distance/frequency seems to be missing

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



result of the fit to $\rho_{model}(\omega)$

with three parameters: κ,A,B

NNLO (vacuum) perturbation theory

Model spectral function: transport contribution + NNLO + correction

$$\rho_{\text{model}}(\omega) \equiv \max\left\{ A\rho_{\text{NNLO}}(\omega) + B\omega^3, \frac{\omega\kappa}{2T} \right\}$$

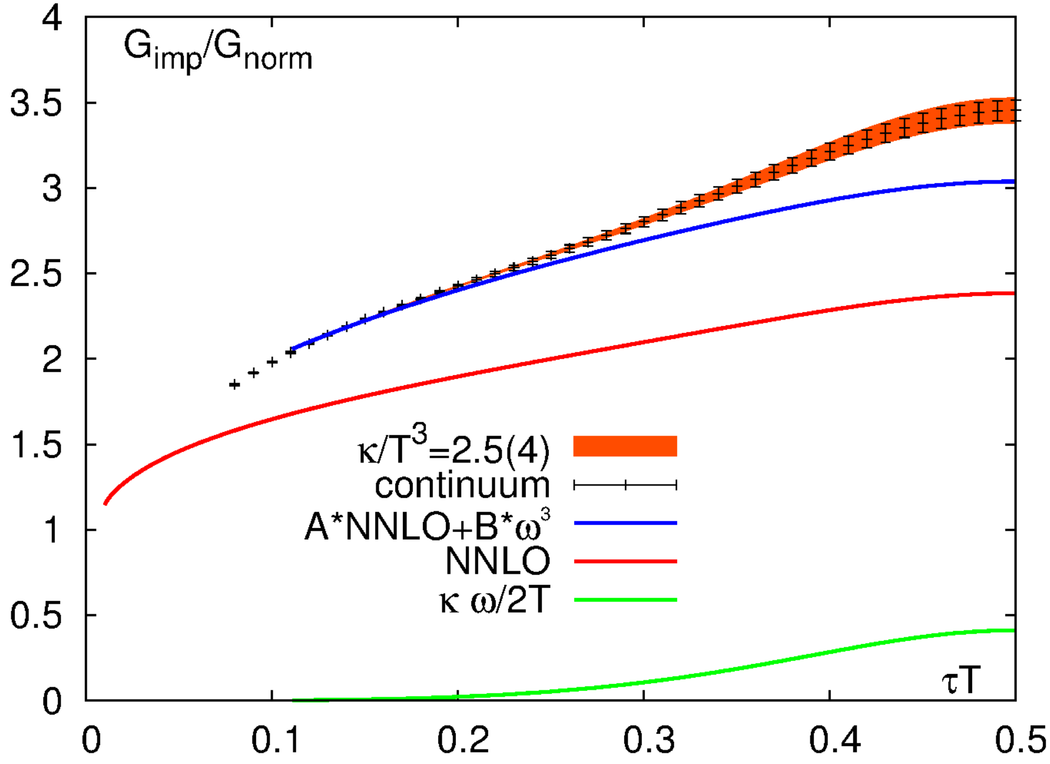
$$G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\text{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

used to fit the continuum extrapolated data

→ **first continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\text{E}}(\omega)}{\omega} \simeq 2.5(4)$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



result of the fit to $\rho_{model}(\omega)$

$A\,\rho_{NNLO}(\omega)$ + B w^3

NNLO (vacuum) perturbation theory

$\frac{\omega\kappa}{2T}$ small but relevant contribution at τT > 0.2 !

Model spectral function: transport contribution + NNLO + correction

$$\rho_{\rm model}(\omega) \equiv \max\left\{A\rho_{\rm NNLO}(\omega) + B\omega^3, \frac{\omega\kappa}{2T}\right\}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{{\rm d}\omega}{\pi}\rho_{\rm model}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

used to fit the continuum extrapolated data

→ **first continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega} \simeq 2.5(4)$$

Heavy Quark Momentum Diffusion Constant – IR and UV asymptotics

$\omega \ll$T: linear behavior motivated at small frequencies

$$\rho_{\rm IR}(\omega) = \frac{\kappa\omega}{2T}$$

$\omega \gg$T: vacuum perturbative results and leading order thermal correction:

$$\rho_{\rm UV}(\omega) = \left[\rho_{\rm UV}(\omega)\right]_{T=0} + \mathcal{O}\left(\frac{g^4 T^4}{\omega}\right)$$

using a renormalization scale $\bar{\mu}_\omega = \omega$ for $\omega \gg \Lambda_{\overline{MS}}$ leading order becomes

$$\rho_{\rm UV}(\omega) = \Phi_{UV}(\omega)\left[1 + \mathcal{O}\left(\frac{1}{\ln(\omega/\Lambda_{MS})}\right)\right]$$

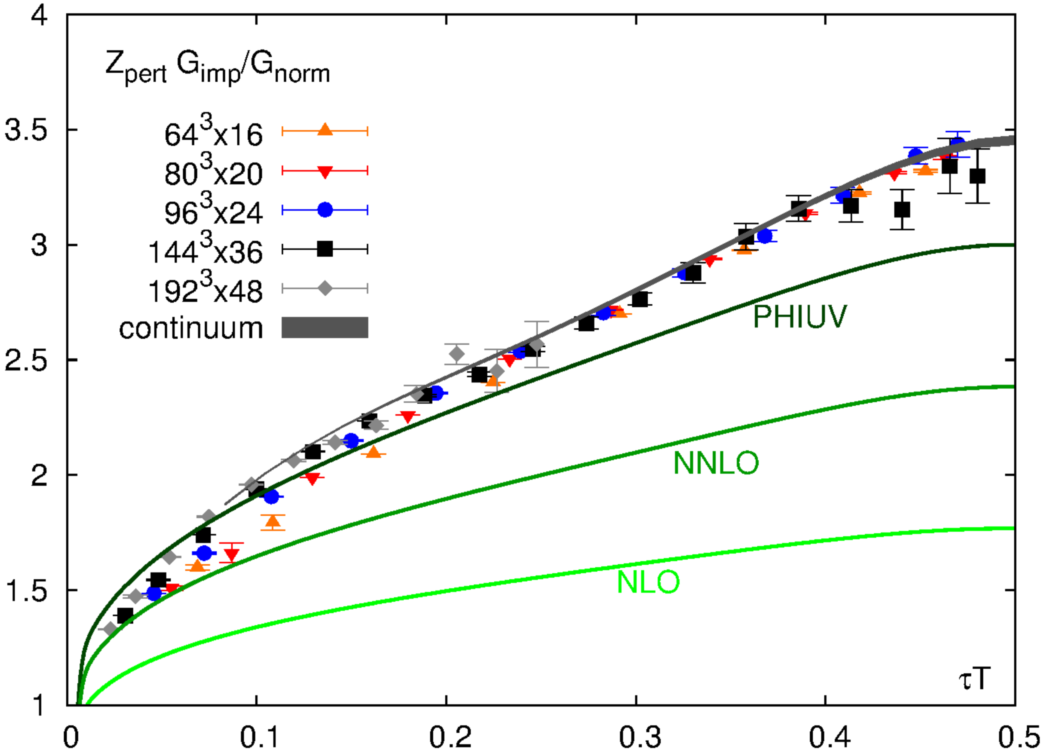
$$\Phi_{\rm UV}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi} \quad , \quad \bar{\mu}_\omega \equiv \max(\omega, \pi T)$$

here we used 4-loop running of the coupling

model the spectral function using these asymptotics with two free parameters

$$\rho_{\rm model}(\omega) \equiv \max\left\{A\Phi_{\rm UV}(\omega), \frac{\omega\kappa}{2T}\right\}$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



$$\rho_{\text{UV}}(\omega) = \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}$$

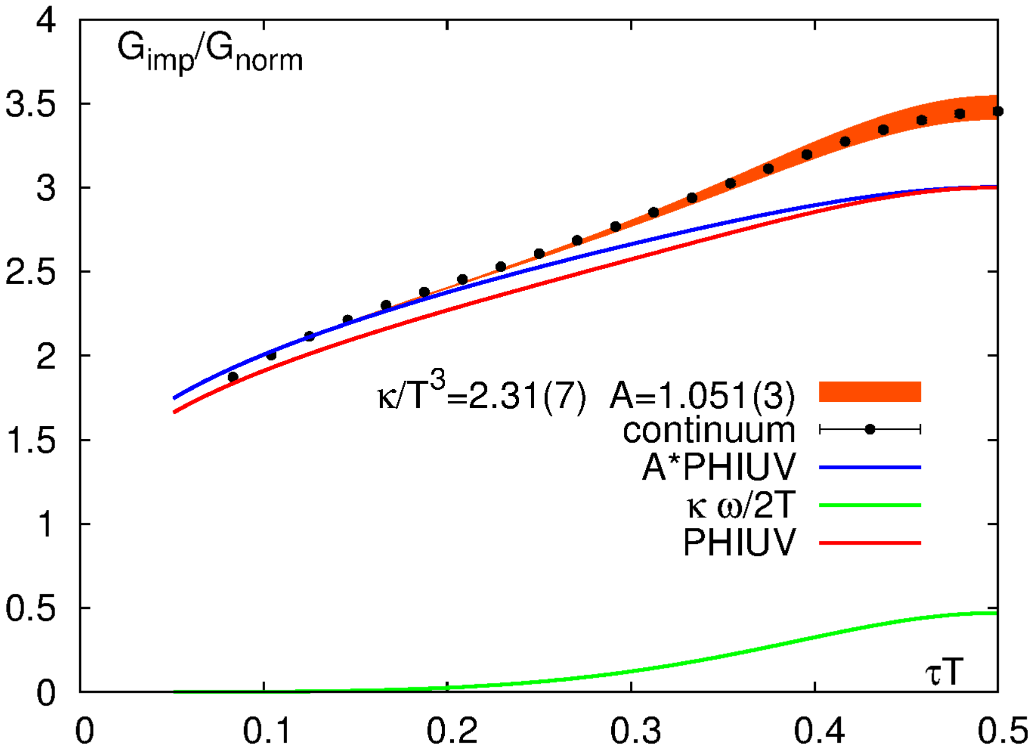
already closer to the data

Model spectral function: transport contribution + UV-asymtotics

$$\rho_{\text{model}}(\omega) \equiv \max\left\{ A\rho_{\text{UV}}(\omega), \frac{\omega\kappa}{2T} \right\}$$

$$G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\text{model}}(\omega) \frac{\cosh\left(\frac{1}{2} - \tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

Heavy Quark Momentum Diffusion Constant – Model Spectral Function



result of the fit to $\rho_{model}(\omega)$

A $\rho_{UV}(\omega)$

$\frac{\omega\kappa}{2T}$ small but relevant contribution at τT > 0.2 !

Model spectral function: transport contribution + UV-asymtotics

$$\rho_{\rm model}(\omega) \equiv \max\left\{ A\rho_{\rm UV}(\omega), \frac{\omega\kappa}{2T} \right\}$$

$$G_{\rm model}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi}\rho_{\rm model}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

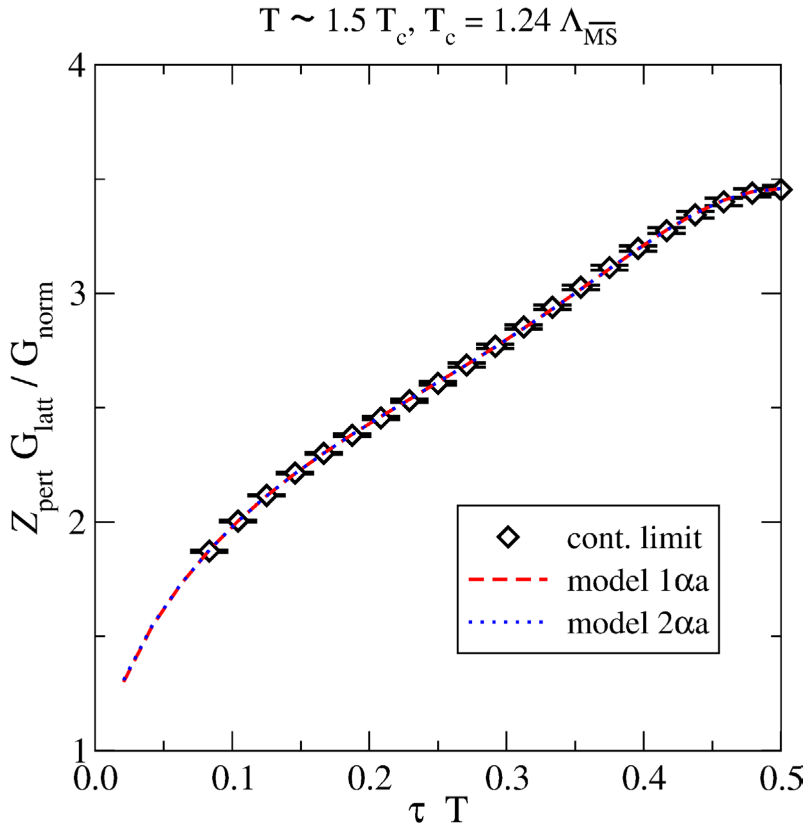
used to fit the continuum extrapolated data

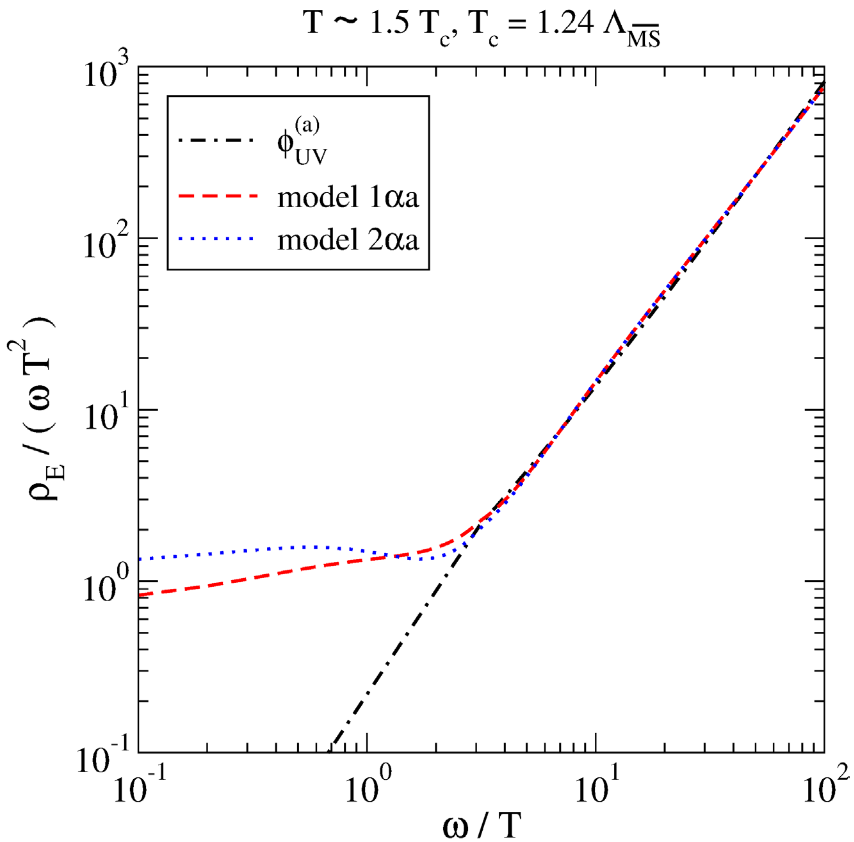
→ **second continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega} \simeq 2.31(7)$$

Heavy Quark Momentum Diffusion Constant – systematic uncertainties

model corrections to ρ_{IR} by a power series in ω





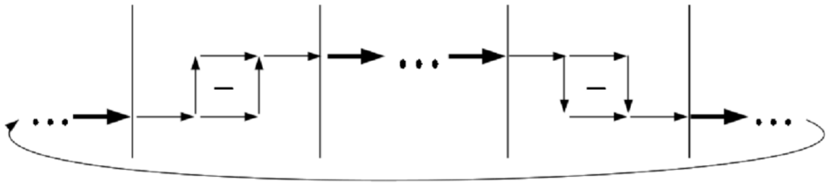
analysis of the systematic uncertainties

→ **continuum estimate of κ :**

$$\kappa/T^3 = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} = 1.8 \ldots 3.6$$

Conclusions and Outlook – Heavy Quark Momentum Diffusion

$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \mathrm{Re}\,\mathrm{Tr}\left[U(\tfrac{1}{T};\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re}\,\mathrm{Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$



→ continuum extrapolation for the color electric correlation function

extracted from quenched Lattice QCD

- using noise reduction techniques to improve signal
- and an Ansatz for the spectral function

→ first continuum estimate for the Heavy Quark Momentum Diffusion Coefficient κ

- still based on a simple Ansatz for the spectral function

→ detailed analysis of the systematic uncertainties

- different Ansätze for the spectral function
- using contributions from thermal perturbation theory
- other techniques to extract the spectral function

other Transport coefficients from Effective Field Theories?

Bottomonium - Lattice NRQCD

[G.Aarts et al., JHEP11(2011)103]

In non-relativistic QCD the Lagrangian is expanded in terms of v=|**p**|/M

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}$$

with

$$\mathcal{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi$$

and

$$\begin{aligned}
\delta\mathcal{L} = & -\frac{c_1}{8M^3} \left[\psi^\dagger (\mathbf{D}^2)^2 \psi - \chi^\dagger (\mathbf{D}^2)^2 \chi \right] \\
& + c_2 \frac{ig}{8M^2} \left[\psi^\dagger \left(\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} \right) \psi + \chi^\dagger \left(\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} \right) \chi \right] \\
& - c_3 \frac{g}{8M^2} \left[\psi^\dagger \boldsymbol{\sigma} \cdot \left(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D} \right) \psi + \chi^\dagger \boldsymbol{\sigma} \cdot \left(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D} \right) \chi \right] \\
& - c_4 \frac{g}{2M} \left[\psi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \psi - \chi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \chi \right]
\end{aligned}$$

which is correct up to order O(v^4) [G.T.Bodwin,E.Braaten,G.P.Lepage, PRD 51 (1995) 1125]

Bottomonium - Lattice NRQCD

[G.Aarts et al., JHEP1407(2014)097]

NRQCD is more sensitive to the bound state region

Kernel is T-independent

→ contributions at ω < 2M absent

→ no small-ω contribution

→ no information on transport properties

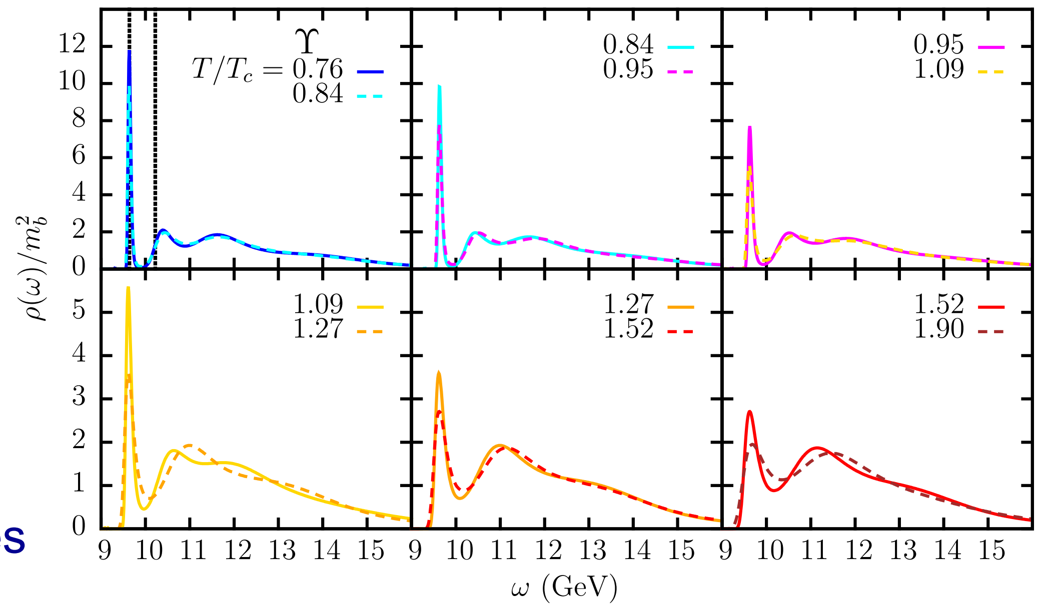
$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega')$$

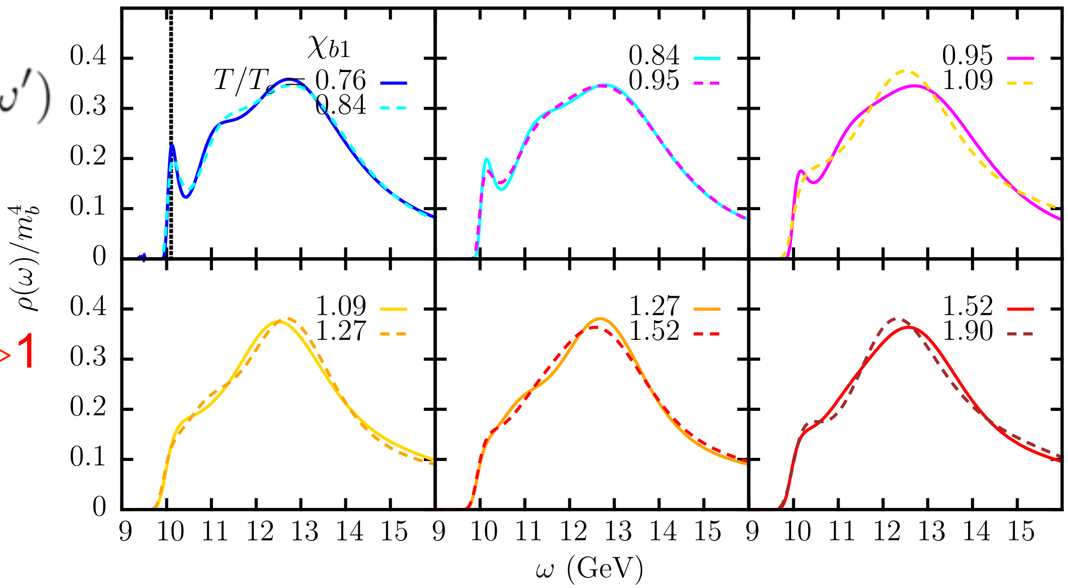
$$\omega' = \omega - 2M$$

requires anisotropic lattices with $a_s M \gg 1$

no continuum limit in NRQCD

only small energy region accessible





Lattice cut-off effects – free spectral functions

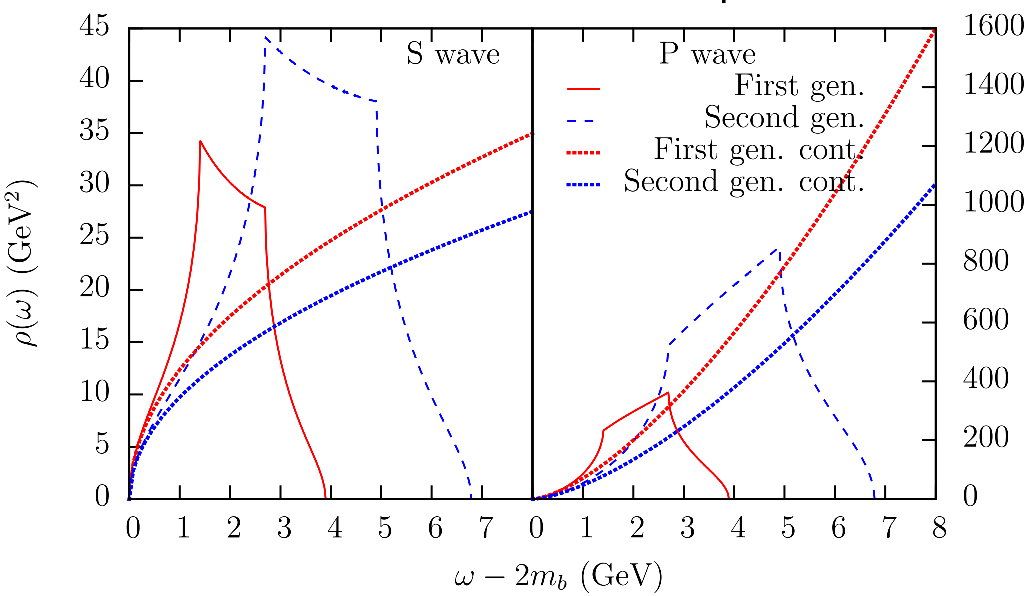
[G.Aarts et al., JHEP1407(2014)097]

gauge configurations from n_f=2+1 dynamical Wilson fermion action

$a_s \simeq 0.16$ fm $\quad a_s \simeq 0.13$ fm

$1/a_t \simeq 7.35$ GeV $\quad 1/a_t \simeq 5.63$ GeV

anisotropic NRQCD



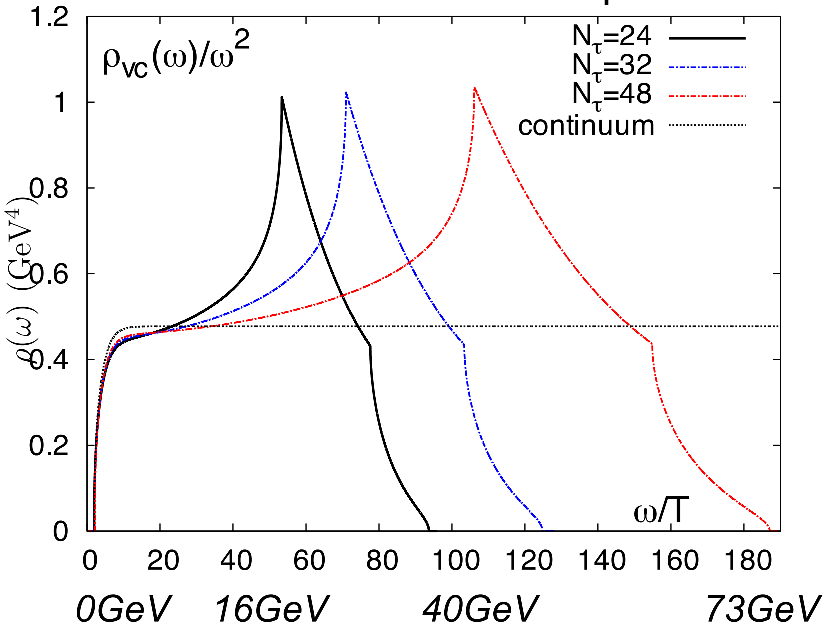
[H.T.Ding, OK et al., arXiv:1204.4945]

gauge configurations from quenched action

$a \simeq 0.01$ fm

$1/a \simeq 19$ GeV

isotropic Wilson



cut-off effects and energy resolution determined by spatial lattice spacing

no continuum limit in NRQCD, $a_s M \gg 1$

only small energy region accessible

continuum limit straight forward, but expensive

transport properties accessible

[see also F.Karsch et al., PRD68 (2003) 014504]

In the following: Meson Correlation Functions

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\mathbf{p},T) K(\tau,\omega,T)$$

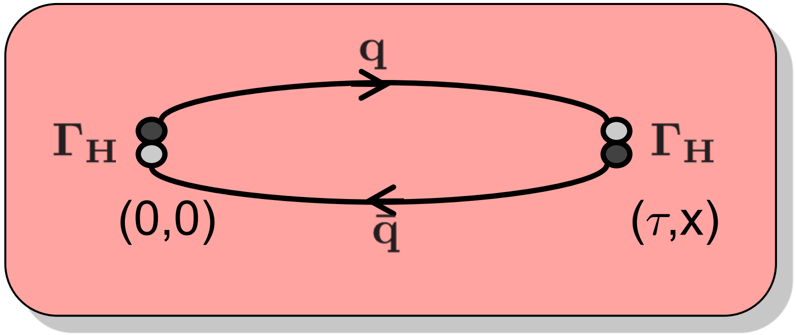
$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x})$$

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x}) e^{i\vec{p}\vec{x}}$$



related to a conserved current in the vector channel

Channel	Γ_H	$^{2S+1}L_J$	J^{PC}	Quarkonia
Pseudoscalar (PS)	γ_5	$^1S_0$	0^{-+}	$\eta_c,\ \eta_b$
Vector (V)	γ_i	$^3S_1$	1^{--}	$J/\psi,\ \Upsilon$
Scalar (S)	$\mathbf{1}$	$^1P_0$	0^{++}	$\chi_{c0},\ \chi_{b0}$
Axialvector (AV)	$\gamma_i\gamma_5$	$^3P_1$	1^{++}	$\chi_{c1},\ \chi_{b1}$

Quarkonium Spectral Functions – HQ Diffusion and Dissociation

In the following: Meson Correlation Functions

$$G(\tau,\mathbf{p},T) = \int_0^\infty \frac{d\omega}{2\pi}\rho(\omega,\mathbf{p},T)K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

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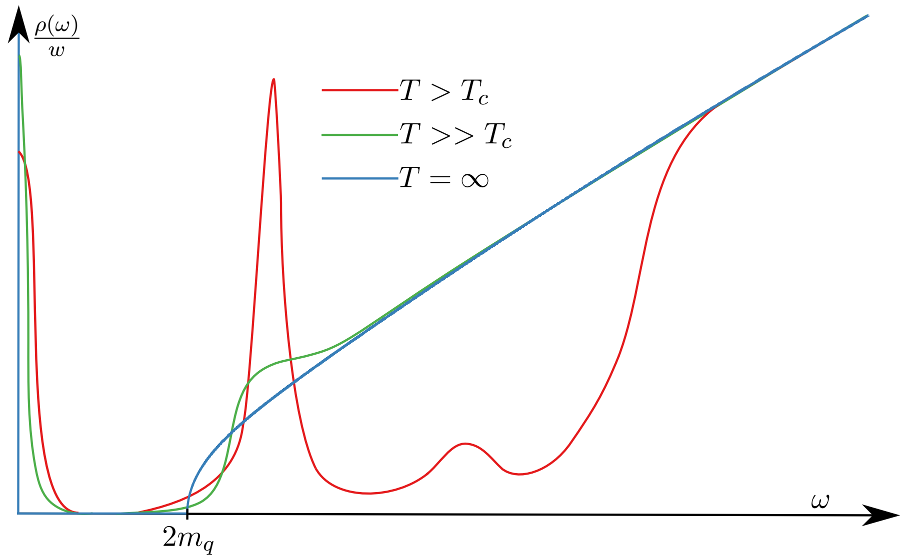


only correlation functions calculable on lattice but

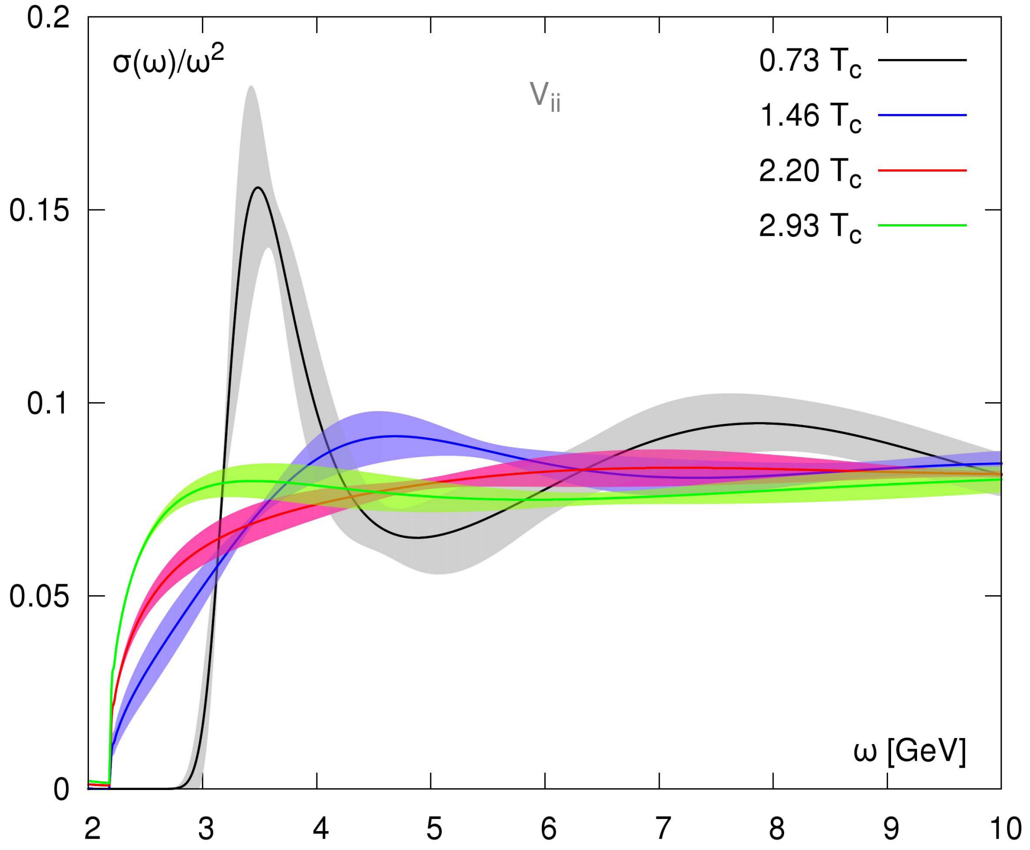
Transport coefficient determined by slope of spectral function at ω=0 (Kubo formula)

$$D = \frac{\pi}{3\chi_{00}}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

Charmonium Spectral function

[H.T.Ding, OK et al., PRD86(2012)014509]

from Maximum Entropy Method analysis on a fine but finite lattice:



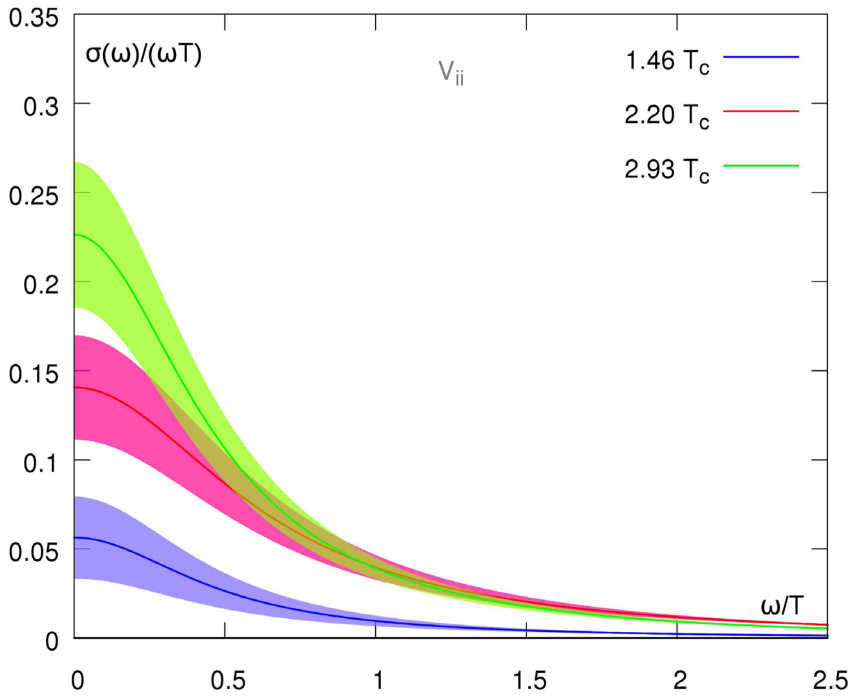
statistical error band from Jackknife analysis

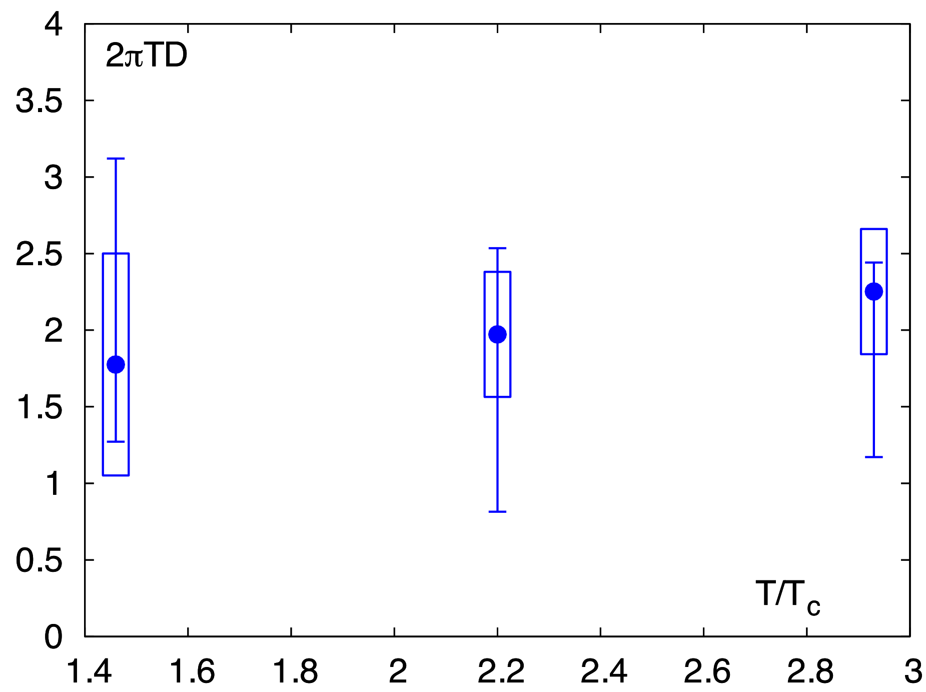
no clear signal for bound states at and above 1.46 T_c

study of the continuum limit and quark mass dependence required!

Charmonium Spectral function – Transport Peak

[H.T.Ding, OK et al., PRD86(2012)014509]





$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

Perturbative estimate (α_s~0.2, g~1.6):

LO: $2\pi TD \simeq 71.2$
NLO: $2\pi TD \simeq 8.4$

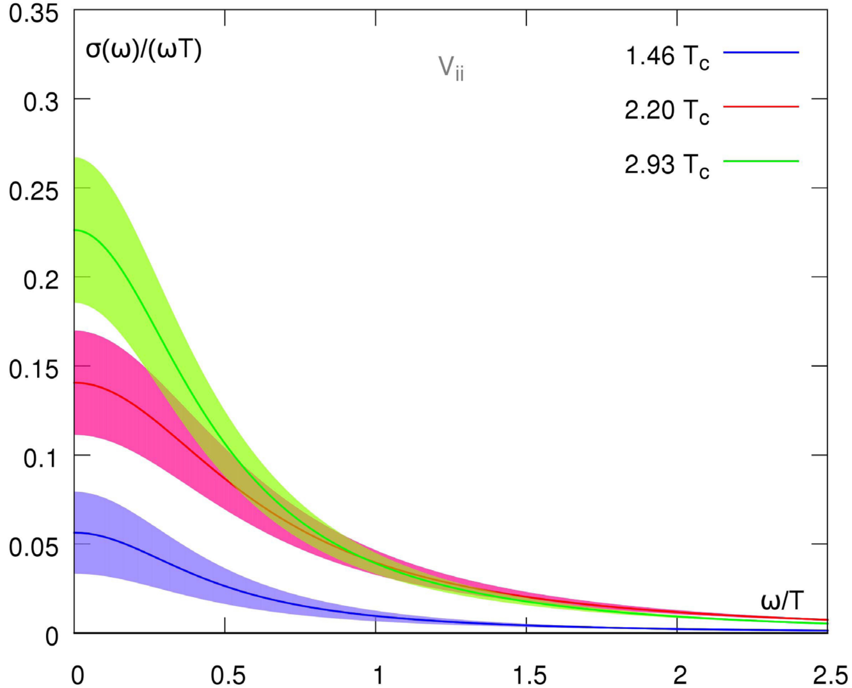
[Moore&Teaney, PRD71(2005)064904,
Caron-Huot&Moore, PRL100(2008)052301]

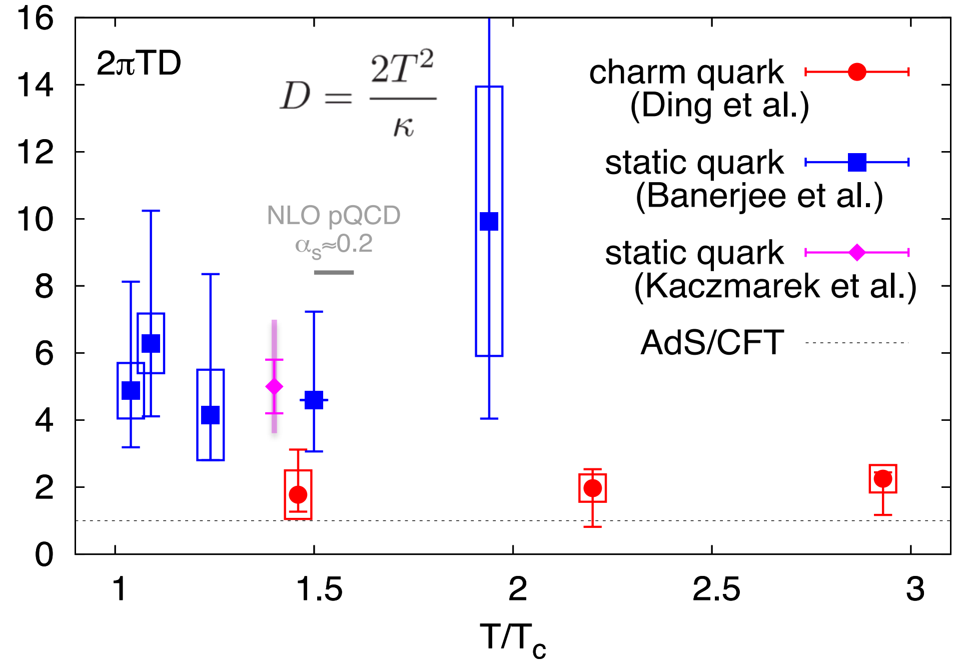
Strong coupling limit:

$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

Charmonium Spectral function – Transport Peak





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Strong coupling limit:

$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

Charmonium and Bottomonium correlators

[H.T.Ding, H.Ohno, OK, M.Laine, T.Neuhaus, work in progress]

- standard plaquette gauge & O(a)-improved Wilson quarks
- quenched gauge field configurations
- on fine and large isotropic lattices
- $T = 0.7 - 1.4\, T_c$
- 2 different lattice spacing
  - analysis of cut-off effects
  - continuum limit (in the future)
- both charm & bottom
  - tuned close to their physical masses

β	N_σ	N_τ	T/T_c	# confs.
7.192	96	48	0.7	259
		32	1.1	476
		28	1.2	336
		24	1.4	336
7.793	192	96	0.7	66
		56	1.2	66
		48	1.4	217

β	a [fm]	a^{-1} [GeV]	κ_{charm}	κ_{bottom}	$m_{J/\Psi}$ [GeV]	m_Υ [GeV]
7.192	0.0190	10.4	0.13194	0.12257	3.105(3)	9.468(3)
7.793	0.00968	20.4	0.13221	0.12798	3.092(5)	9.431(5)

Experimental values: $m_{J/\psi}$ = 3.096.916(11) GeV, m_Υ = 9.46030(26) GeV

Reconstructed correlation function

$$G_{\rm rec}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$

$$\frac{G(\tau, T)}{G_{\rm rec}(\tau, T; T')}$$ equals to unity at all τ

if the spectral function doesn't vary with temperature

S. Datta *et al.*, PRD 69 (2004) 094507

can be calculated directly from correlation function
for suitable ratios of N'_τ / N_τ without knowledge of spectral function:

$$\frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh[\omega N_\tau/2]} = \sum_{\tau'=\tau;\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh[\omega N'_\tau/2]}$$

$$T = 1/(N_\tau a) \qquad N'_\tau = m N_\tau \qquad m = 1, 2, 3, \cdots$$

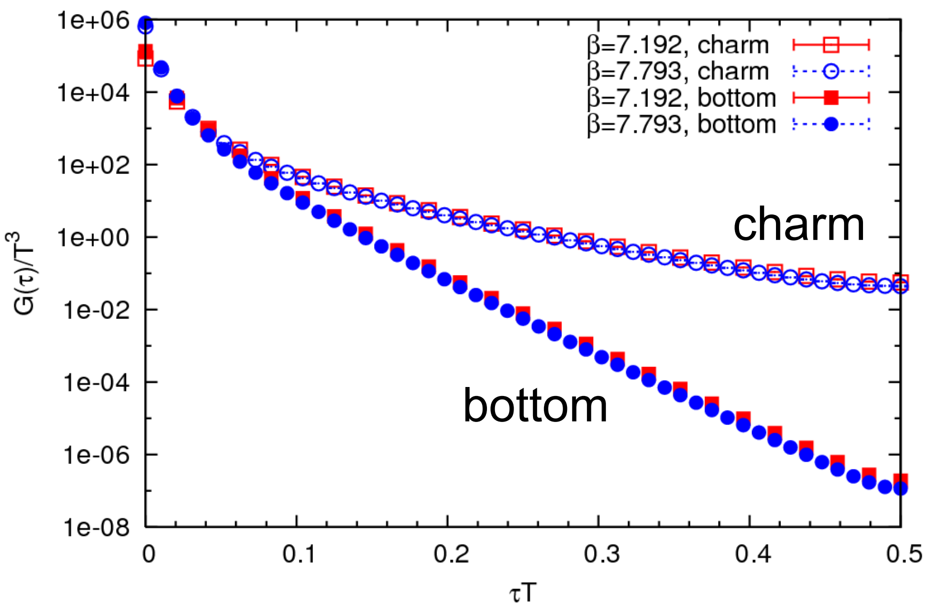
$$G_{\rm rec}(\tau, T; T') = \sum_{\tau'=\tilde{\tau};\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau', T')$$

H.-T. Ding *et al.*, PRD 86 (2012) 014509

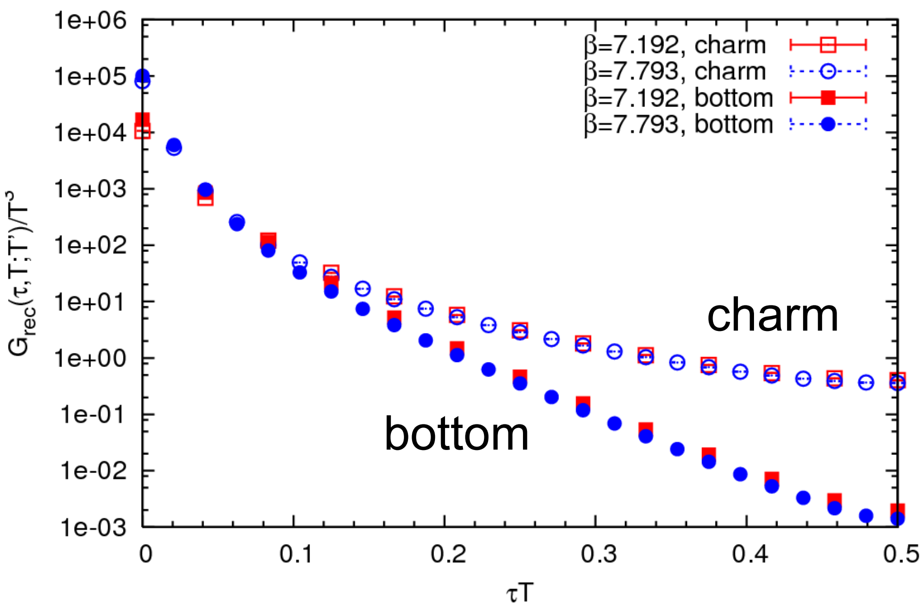
Charmonium and Bottomonium correlators

$$G_{\text{rec}}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$

$G(\tau, 0.7T_c)/T^3$

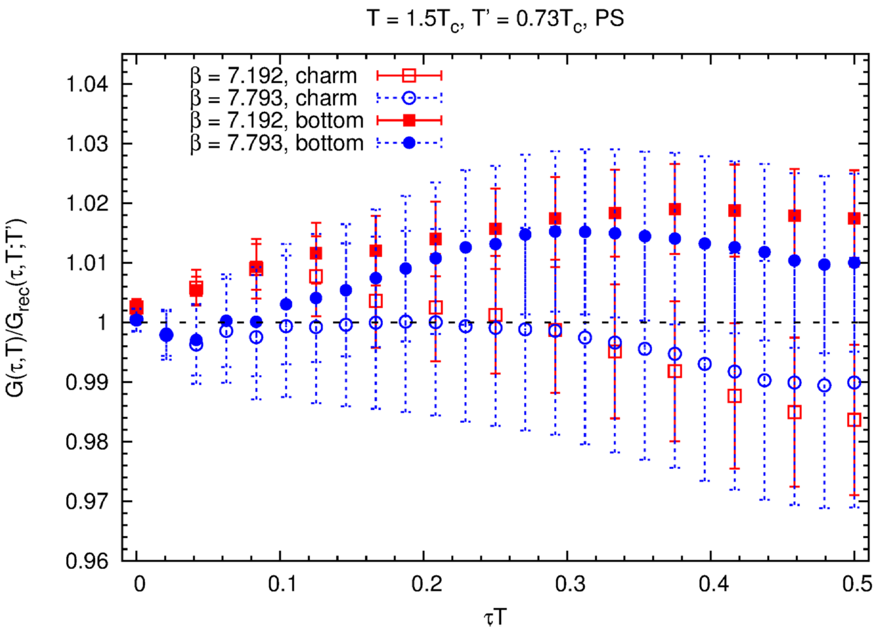


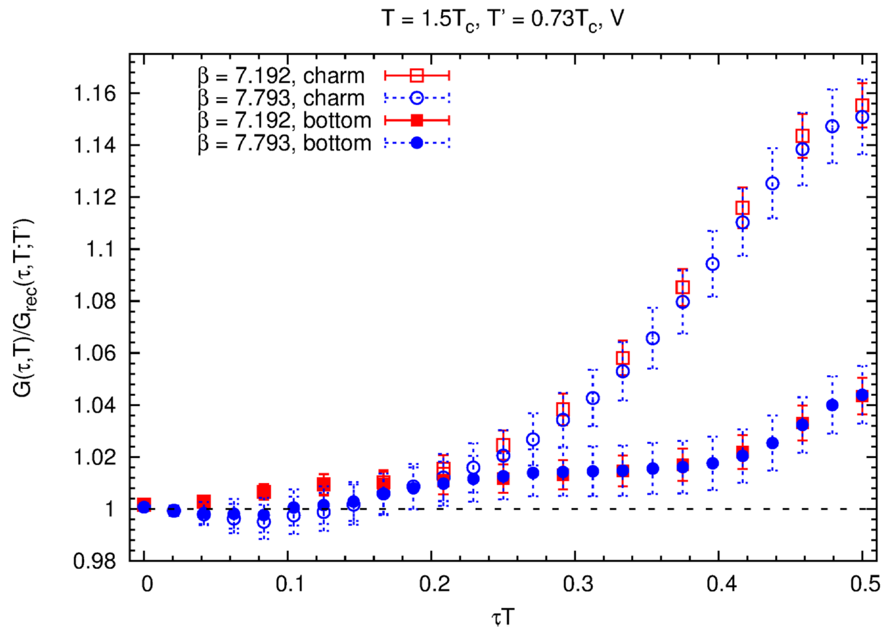
$G_{rec}(\tau, 1.4T_c; 0.7T_c)/T^3$



Charmonium and Bottomonium correlators – S-wave channels

$$G_{\rm rec}(\tau,T;T') \equiv \int_0^\infty d\omega\rho(\omega,T')K(\omega,\tau,T)$$





different behavior in pseudo-scalar (left) and vector (right) channel

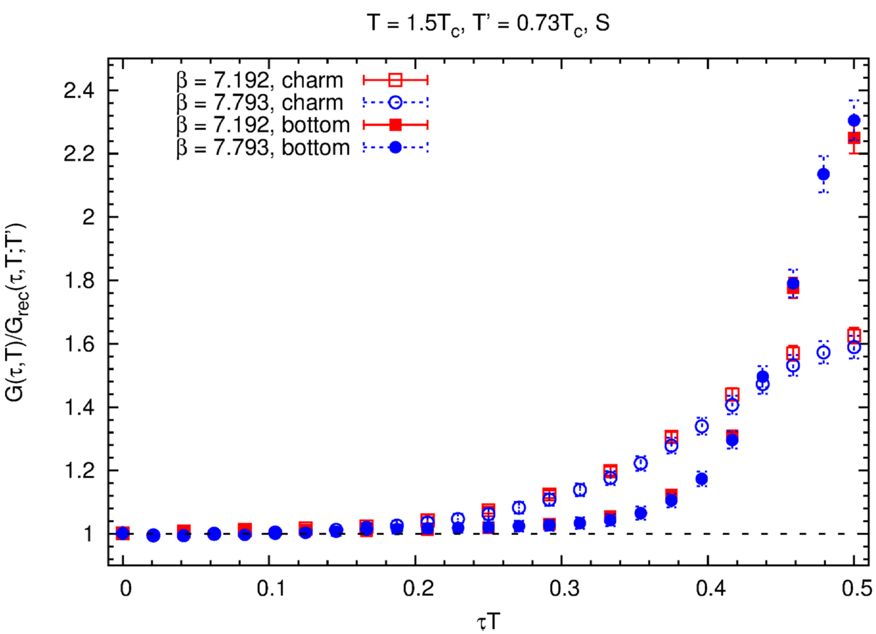
strong modification at large τ in vector channel

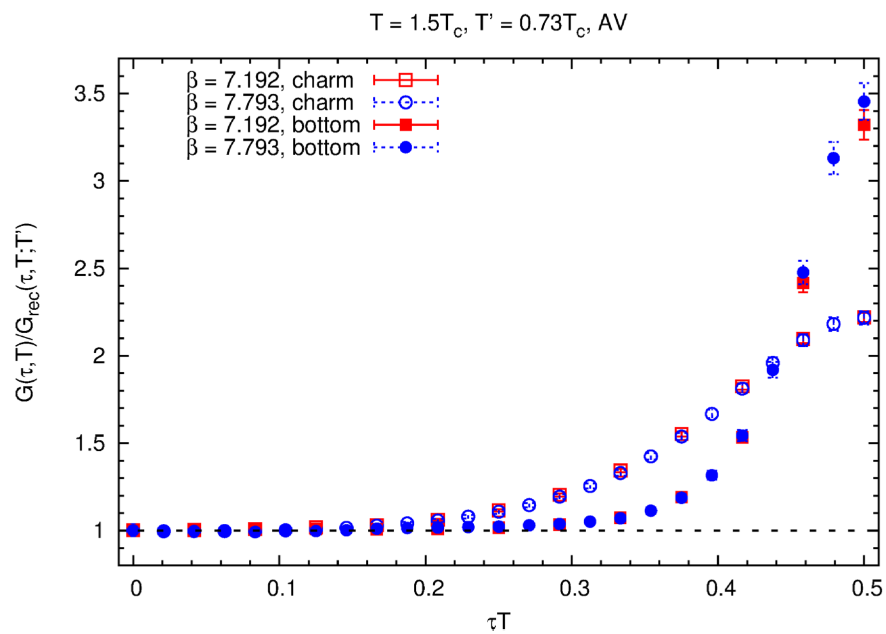
stronger for charm compared to bottom

related to transport contribution

Charmonium and Bottomonium correlators – P-wave channels

$$G_{\text{rec}}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$





comparable behavior in scalar (left) and axial-vector (right) channel

strong modification at large τ in both channels

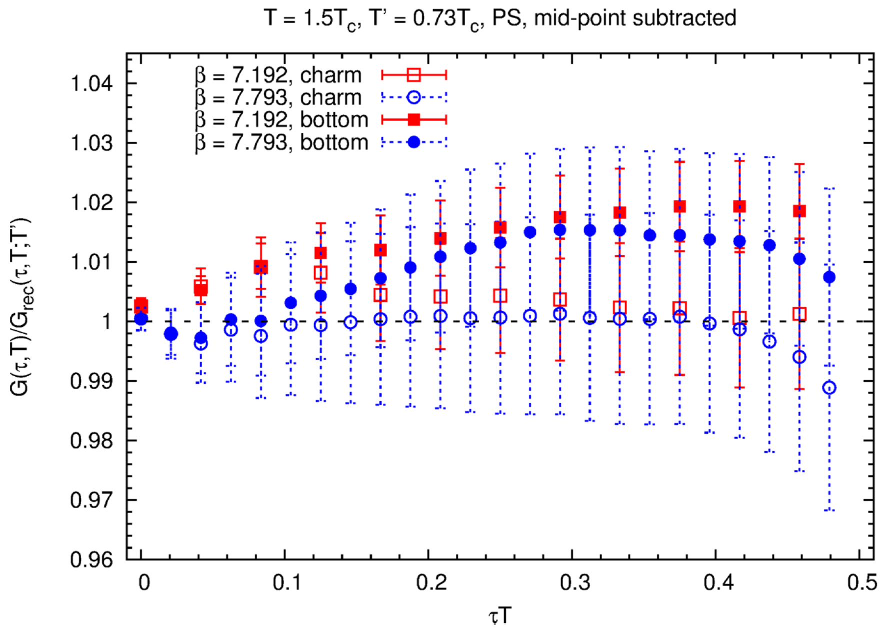
stronger for bottom compared to charm

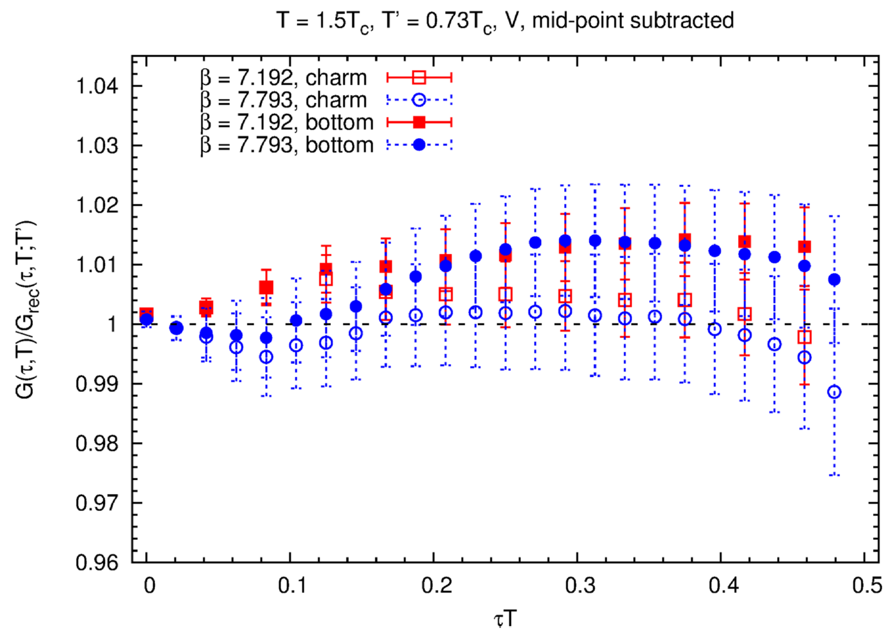
related to transport/constant contribution

Mid-point subtracted correlators – S-wave channels

$$\bar{G}(\tau) \equiv G(\tau) - G(1/2T)$$

mid-point subtracted correlator





small ω region gives (almost) constant contribution to correlators

effectively removed by mid-point subtraction

pseudo-scalar (left) and vector (right) very comparable

need to understand cut-off effects and quark-mass effects

Outlook

work is still in progress

continuum extrapolation for the quarkonium correlators still needed

detailed analysis of the systematic uncertainties

extract spectral properties (on continuum extrapolated correlators) by

- comparing to perturbation theory
- Fits using Ansätze for the spectral function
- Bayesian techniques to extract the spectral function

final goal:

understand the temperature and quark mass dependence of

heavy quark diffusion coefficient

dissociation temperatures for different states

Spatial correlation function and screening masses

[“Signatures of charmonium modification in spatial correlation functions“, F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky, (2012) arXiv:1203.3770]

Correlation functions along the **spatial direction**

$$G(z,T) = \int dxdy \int_0^{1/T} d\tau \langle J(x,y,z,\tau) J(0,0,0,0) \rangle$$

are related to the meson spectral function at **non-zero spatial momentum**

$$G(z,T) = \int_{-\infty}^{\infty} dp_z e^{ip_z z} \int_0^{\infty} d\omega \frac{\sigma(\omega, p_z, T)}{\omega}$$

exponential decay defines **screening mass M_{scr} :** $\mathbf{G(z,T)} \underset{z \gg 1/T}{\longrightarrow} \mathbf{e^{-M_{scr} z}}$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

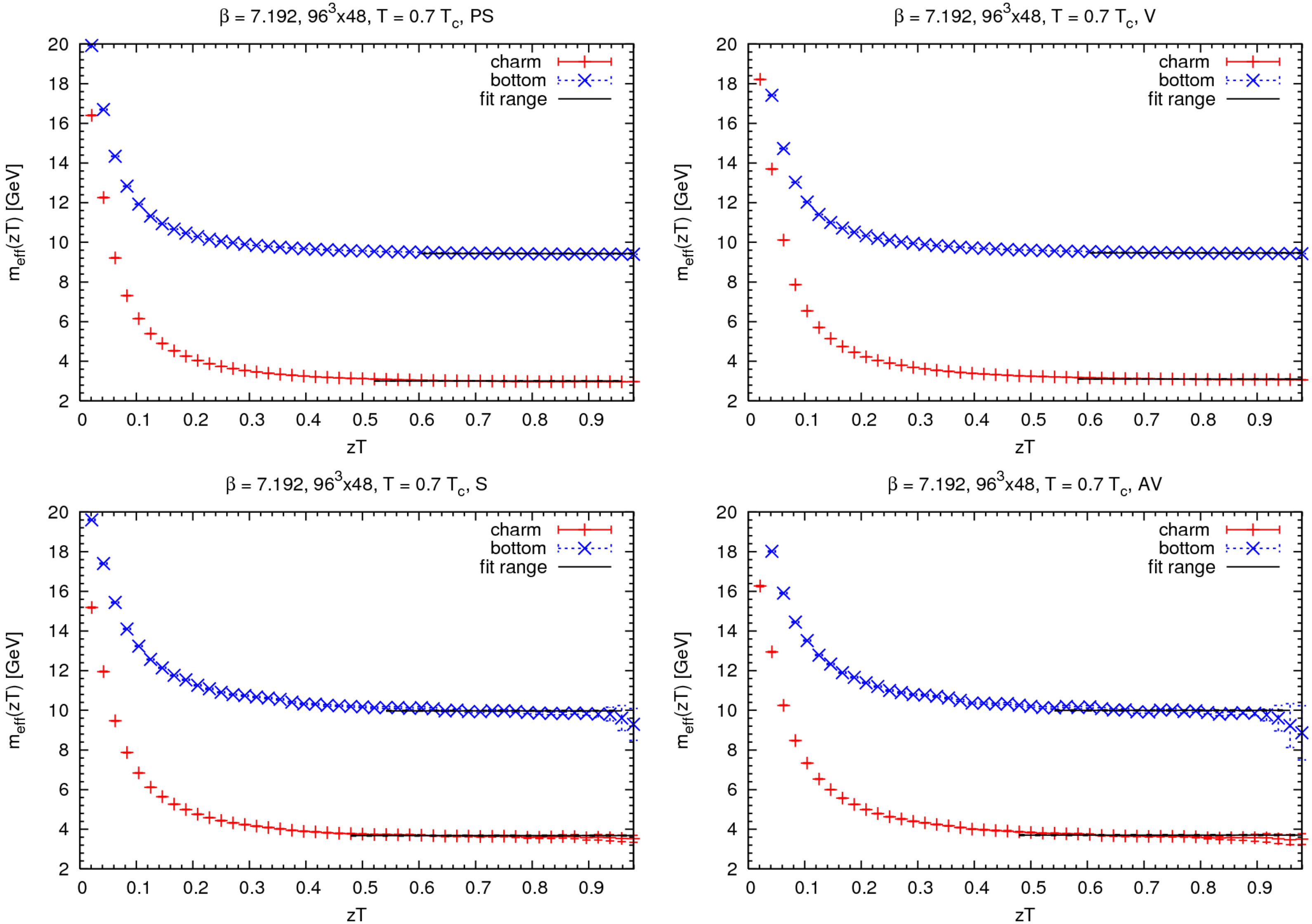
$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

$M_{scr} = M$ ⟶ indications for medium modifications/dissociation ⟵ $M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}$

Spatial Correlation Function and Screening Masses

[H.T.Ding, H.Ohno, OK, M.Laine, T.Neuhaus, work in progress]

$$m_{eff}(zT) = \log\left(\frac{G(zT)}{G((z+1)T)}\right)$$



Spatial Correlation Function and Screening Masses

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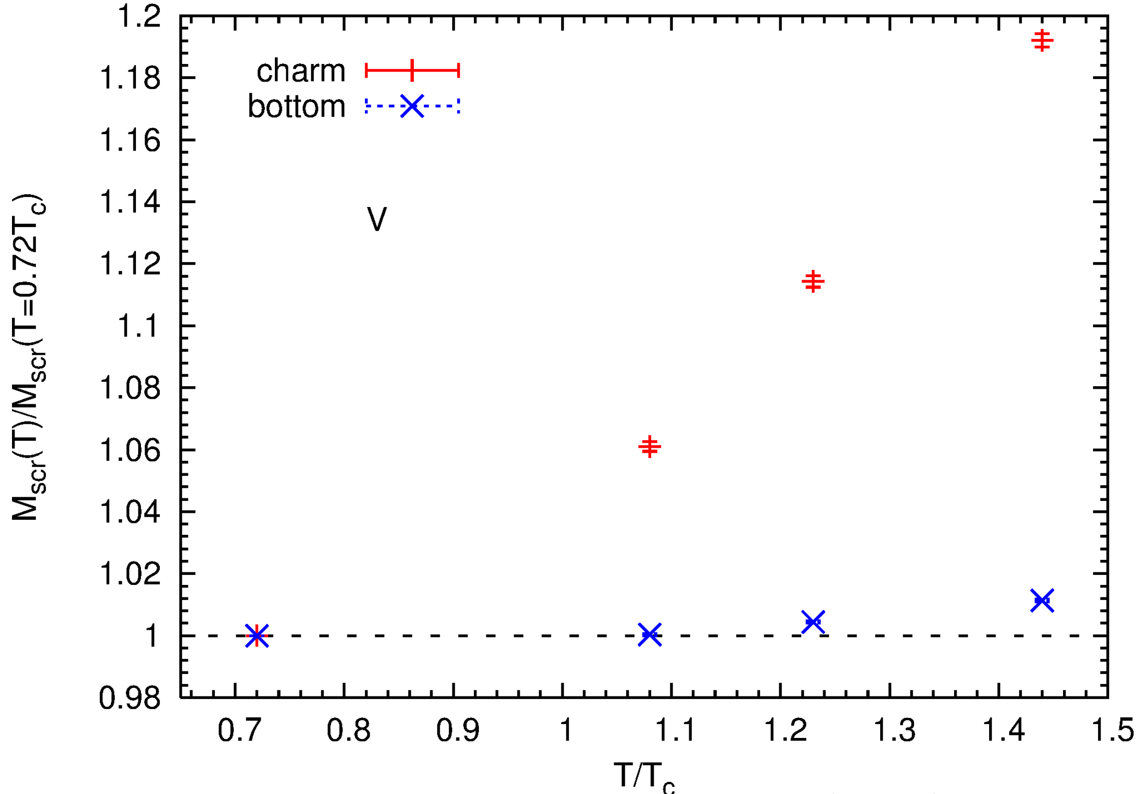


exponential decay defines **screening mass M_{scr}** : $\mathbf{G(z, T)} \underset{z \gg 1/T}{\longrightarrow} \mathbf{e^{-M_{scr} z}}$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

$M_{scr} = M$ → indications for medium modifications/dissociation ← $M_{scr} = 2\sqrt{(\pi T)^2 + m_c{}^2}$

Spatial correlation function and screening masses

[A.Bazavov, F.Karsch, Y.Maezawa et al., PRD91 (2015) 054503]

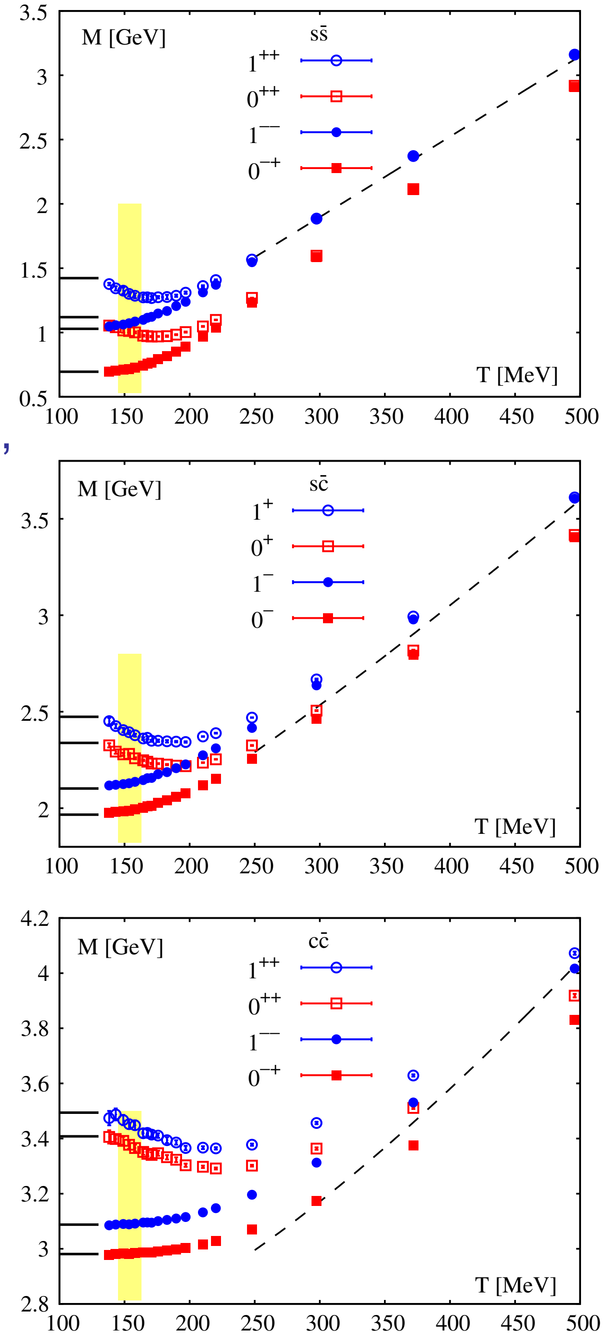
2+1 flavor HISQ with almost physical quark masses

$48^3 \times 12$ lattices with $m_l = m_s/20$ and physical m_s

"$s\bar{s}$ and $s\bar{c}$ possibly dissolve close to crossover temperature"

"$c\bar{c}$ in line with the sequential melting of charmonia states"

	$-\tilde{\phi}(x)$	Γ	J^{PC}	$s\bar{s}$	$s\bar{c}$	$c\bar{c}$
M_-^S	1	$\gamma_4\gamma_5$	0^{-+}	$\eta_{s\bar{s}}$	D_s	η_c
M_+^S		1	0^{++}		D_{s0}^*	χ_{c0}
M_-^{PS}	$(-1)^{x+y+z}$	γ_5	0^{-+}	$\eta_{s\bar{s}}$	D_s	η_c
M_+^{PS}		γ_4	0^{+-}	–		–
M_-^{AV}	$(-1)^x, (-1)^y$	$\gamma_i\gamma_4$	1^{--}	ϕ	D_s^*	J/ψ
M_+^{AV}		$\gamma_i\gamma_5$	1^{++}	$f_1(1420)$	D_{s1}	χ_{c1}
M_-^V	$(-1)^{x+z}, (-1)^{y+z}$	γ_i	1^{--}	ϕ	D_s^*	J/ψ
M_+^V		$\gamma_j\gamma_k$	1^{+-}			h_c



Correlations of conserved charges – open charm sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klmn}^{BQSC}(T) \left(\frac{\mu_B}{T}\right)^k \left(\frac{\mu_Q}{T}\right)^l \left(\frac{\mu_S}{T}\right)^m \left(\frac{\mu_C}{T}\right)^n$$

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n}\right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

$$P^C = P_M^C \cosh(\hat{\mu}_C) + \sum_{k=1,2,3} P_B^{C=k} \cosh(B\hat{\mu}_B + k\hat{\mu}_C)$$

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

$$\chi_{mn}^{BC} = B^m P_B^{C=1} + B^m 2^n P_B^{C=2} + B^m 3^n P_B^{C=3} \simeq B^m P_B^{C=1}$$

relative contribution of C=2 and C=3 baryons negligible

ratios independent of the detailed spectrum and sensitive to special sectors:

charmed baryon sector

$$\frac{\chi_{mn}^{BC}}{\chi_{m+1,n-1}^{BC}} = B^{-1}$$

=1 when DoF are hadronic

=3 when DoF are quarks

$$\frac{\chi_{mn}^{BC}}{\chi_{m,n+2}^{BC}} = 1$$ always

Correlations of conserved charges – open charm sector

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

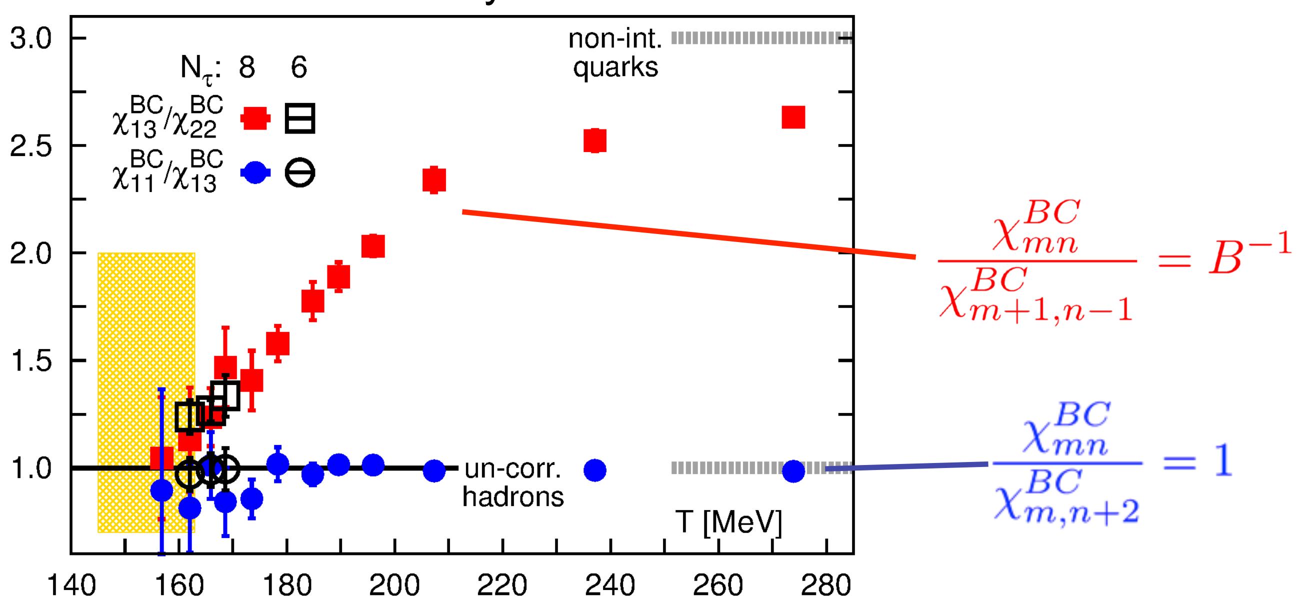
$32^3\times 8$ and $24^3\times 6$ lattices with $m_l = m_s/20$ and physical m_s and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial \hat{\mu}_B^k \partial \hat{\mu}_Q^l \hat{\mu}_S^m \partial \hat{\mu}_C^n}\bigg|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

charmed baryon sector



→ indications that open charm hadrons start to dissolve already close to the chiral crossover

Correlations of conserved charges – open charm sector

Taylor expansion of pressure in terms of chemical potentials related to conserved charges

$$\frac{P}{T^4} = \sum_{k,l,m,n=0}^{\infty} \frac{1}{k!l!m!n!} \chi_{klmn}^{BQSC}(T) \left(\frac{\mu_B}{T}\right)^k \left(\frac{\mu_Q}{T}\right)^l \left(\frac{\mu_S}{T}\right)^m \left(\frac{\mu_C}{T}\right)^n$$

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left.\frac{\partial^{(k+l+m+n)}[P(\hat\mu_B, \hat\mu_Q, \hat\mu_S, \hat\mu_C)/T^4]}{\partial \hat\mu_B^k \partial \hat\mu_Q^l \hat\mu_S^m \partial \hat\mu_C^n}\right|_{\vec\mu=0}$$

are sensitive to the underlying degrees of freedom

charm contributions to pressure in a hadron gas:

$$P^C = P_M^C \cosh(\hat\mu_C) + \sum_{k=1,2,3} P_B^{C=k} \cosh(B\hat\mu_B + k\hat\mu_C)$$

partial pressure of open-charm mesons and charmed baryons depends on hadron spectra

$$\chi_{mn}^{B=1,C} = P_B^{C=1} + 2^n P_B^{C=2} + 3^n P_B^{C=3} \simeq P_B^{C=1}$$

$$\chi_k^C = P_M^C + 2^n P_B^{C=2} + 3^n P_B^{C=3} \simeq P_M^C + P_B^{C=1}$$

ratios independent of the detailed spectrum and sensitive to special sectors:

partial pressure of open-charm mesons:

open charm meson sector

$$P_M^C = \chi_2^C - \chi_{22}^{BC} = \chi_4^C - \chi_{13}^{BC}$$

$$\frac{\chi_4^C}{\chi_2^C} = 1$$

Correlations of conserved charges – open charm sector

[A.Bazavov, H.T.Ding, P.Hegde, OK et al., PLB737 (2014) 210]

2+1 flavor HISQ with almost physical quark masses

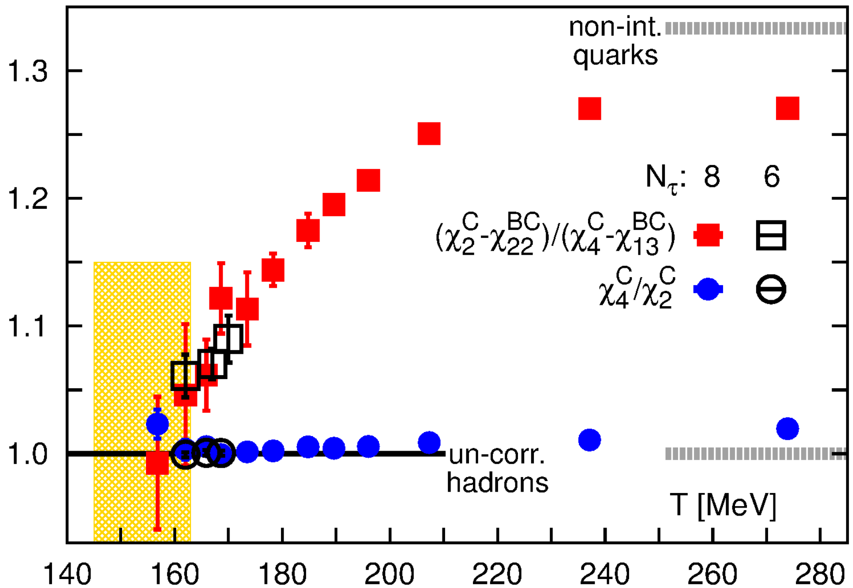
$32^3\times 8$ and $24^3\times 6$ lattices with $m_l = m_s/20$ and physical m_s and quenched charm quarks

generalized susceptibilities of conserved charges

$$\chi_{klmn}^{BQSC} = \left. \frac{\partial^{(k+l+m+n)}[P(\hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S, \hat{\mu}_C)/T^4]}{\partial\hat{\mu}_B^k \partial\hat{\mu}_Q^l \hat{\mu}_S^m \partial\hat{\mu}_C^n} \right|_{\vec{\mu}=0}$$

are sensitive to the underlying degrees of freedom

open charm meson sector



partial pressure of open-charm mesons:

$$P_M^C = \chi_2^C - \chi_{22}^{BC} = \chi_4^C - \chi_{13}^{BC}$$

$$\frac{\chi_4^C}{\chi_2^C} = 1$$

→ **indications that open charm hadrons start to dissolve already close to the chiral crossover**