Hadron production as Hawking-Unruh effect.1

Unruh effect and tunneling-Event horizon in QCD

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DUBNA
basic observation in all high energy multihadron production

thermal production pattern

Fermi, Landau, Pomeranchuk, Hagedorn

• species abundances $\sim$ ideal resonance gas at $T_H$
• universal $T_H \sim 165 \pm 15 \text{MeV}$ for all (large) $\sqrt{s}$

caveats

• strangeness suppression in elementary collisions
• strangeness suppression weakened/removed in nuclear collisions

Statistical model
First Question

1) Why do elementary high energy collisions show a statistical behavior?

Is there another non-kinetic mechanism providing a common origin of the statistical features?
The Unruh effect

a conceptually subtle quantum field theory result

In short: the Unruh effect expresses the fact that uniformly accelerated observers in Minkowski spacetime, i.e. linearly accelerated observers with constant proper acceleration (also called Rindler observers), associate a thermal bath to the no-particle state of inertial observers (Minkowski vacuum).

\[ T = \frac{\hbar a}{2\pi k c}, \]

arXiv:0710.5373

The Unruh effect and its applications
Luis C. B. Crispino, Atsushi Higuchi, George E. A. Matsa
Prologo 1

\begin{align*}
\Sigma' (X, Y, Z, T) & \quad \Sigma (x, y, z, t) \\
\text{Proper instantaneous r.f.} & \quad \text{Inertial frame - } v(t)
\end{align*}

\[
\begin{cases}
x = \gamma [X - vT] \\
t = \gamma [T - vX] \\
X = \gamma [x + vt] \\
T = \gamma [t + vx]
\end{cases}
\]

\[
\gamma = \frac{1}{\sqrt{1 - v^2}}
\]

\[
ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \\
st = \gamma dt
\]

\[
ds^2 = -d\tau^2
\]

\[
u^\mu \equiv \frac{dx^\mu}{d\tau} = \left( \frac{dx}{d\tau}, \frac{dt}{d\tau} \right) = \left( \gamma \frac{dx}{dt}, \gamma \frac{dt}{dt} \right) = (\gamma \mathbf{u}, \gamma)
\]
\[ a^\mu \equiv \frac{d u^\mu}{d\tau} = \gamma \frac{d u^\mu}{dt} = \gamma \left( \frac{d (\gamma \ u)}{dt}, \frac{d \gamma}{dt} \right) \]

\[ 0 = \frac{d (u^\mu u_\mu)}{d\tau} = u^\mu \frac{d u_\mu}{d\tau} + u_\mu \frac{d u^\mu}{d\tau} = 2 u^\mu a_\mu \]

In the instantaneous proper frame \( v = 0, \ \gamma = 1 \) & \[ \frac{d \gamma}{dT} = \gamma^3 \left( \mathbf{v} \cdot \frac{d \mathbf{v}}{dT} \right) = 0 \]

\[ A^\mu = \left( \frac{d u}{dT}, 0 \right) = \left( \frac{d^2 \mathbf{x}}{dT^2}, 0 \right) \]

\[ A^\mu A_\mu = \alpha^2 > 0 \]

proper acceleration
\[
\frac{d\gamma}{dt} = \gamma^3 \left( \mathbf{v} \cdot \frac{d\mathbf{v}}{dt} \right) = \gamma^3 v \frac{dv}{dt}
\]

Relative motion in the x-axis

\[
\frac{d (\gamma v)}{dt} = \gamma \frac{dv}{dt} + v \frac{d\gamma}{dt} = \gamma \frac{1}{\gamma^3 v} \frac{d\gamma}{dt} + v \frac{d\gamma}{dt} = \\
= \left( \frac{1}{\gamma^2 v} + v \right) \frac{d\gamma}{dt} = \frac{1}{v} \frac{d\gamma}{dt}
\]

\[
\alpha^2 = a^\mu a_\mu = \gamma^2 \left[ \left( \frac{d (\gamma u)}{dt} \right)^2 - \left( \frac{d \gamma}{dt} \right)^2 \right]
\]

\[
\alpha^2 = \gamma^2 (1 - v^2) \left( \frac{d (\gamma u)}{dt} \right)^2 = \left( \frac{d (\gamma u)}{dt} \right)^2
\]
\[
\frac{d (\gamma u)}{dt} = \alpha \\
\gamma u = \alpha t
\]

\[
u (t) = \frac{dx}{dt} = \frac{\alpha t}{\sqrt{1 + \alpha^2 t^2}}
\]

\[x (t_0) = \alpha^{-1}\]

\[
x (t) = \frac{1}{\alpha} \sqrt{1 + \alpha^2 t^2} \quad \Rightarrow \quad x^2 - t^2 = \frac{1}{\alpha^2}\]
The diagram shows the light-cone in spacetime with the following coordinates:

- The light-cone is defined by the equation $x = \pm ct$.
- The coordinates $-c^2/\alpha$ and $c^2/\alpha$ are marked at the intersections of the light-cone with the x-axis.

The figure illustrates the relationship between space and time in the context of special relativity.
Rindler coordinates

\[ x^2 - c^2 t^2 = \left( \frac{c^2}{\alpha} \right)^2 \]

\[
\begin{cases}
  c t = \frac{c^2}{a} e^{\frac{a \xi}{c^2}} \sinh \left( \frac{a \lambda}{c} \right) \\
  x = \frac{c^2}{a} e^{\frac{a \xi}{c^2}} \cosh \left( \frac{a \lambda}{c} \right)
\end{cases}
\]

\[ \alpha = a e^{-\frac{a \xi}{c^2}} \]

\[ J = c e^{\frac{2a \xi}{c^2}} \in ]0, \infty[ \]
\[
\begin{align*}
\lambda &= \frac{c}{a} \tanh^{-1} \left( \frac{ct}{x} \right) \\
\xi &= \frac{c^2}{2a} \ln \left[ \left( \frac{a}{c^2} \right)^2 (x^2 - c^2 t^2) \right]
\end{align*}
\]

\[
\begin{align*}
x = ct &\implies \lambda = \frac{c}{a} \tanh^{-1} (1) = +\infty \\
x = -ct &\implies \lambda = \frac{c}{a} \tanh^{-1} (-1) = -\infty \\
\xi &= -\infty
\end{align*}
\]

L.C.
\[
\lambda = \pm \infty \quad \& \quad \xi = -\infty
\]

\[
\begin{align*}
\lambda &= \text{cost.} \quad \frac{x}{ct} = \coth \left( \frac{a \lambda}{c} \right) = \text{cost} \\
\xi &= \text{cost} \quad x^2 - c^2 t^2 = \left( \frac{c^2}{a} \right)^2 e^{2a\xi} \frac{2a\xi}{c^2} = \text{cost}
\end{align*}
\]
\[ |t| < -x \]

\[ |t| < x \]

**FIG. 1:** Trajectory of the Rindler observer as seen by the observer at rest.
2D Rindler metric

\[ ds^2 = dx^2 - c^2 \, dt^2 \]

\[ dt = (\partial_\lambda t) \, d\lambda + (\partial_\xi t) \, d\xi \]

\[ dx = (\partial_\lambda x) \, d\lambda + (\partial_\xi x) \, d\xi \]

\[ ds^2 = e^{\frac{2a_\xi}{c^2}} \left( d\xi^2 - c^2 \, d\lambda^2 \right) \]

conformal to Minkowski metric with \[ \Omega = e^{\frac{a_\xi}{c^2}} \]

\[ g_{\alpha\beta} = \begin{pmatrix} g_{\xi\xi} & g_{\xi\lambda} \\ g_{\lambda\xi} & g_{\lambda\lambda} \end{pmatrix} = \begin{pmatrix} 2a_\xi e^{\frac{a_\xi}{c^2}} & 0 \\ 0 & -e^{\frac{2a_\xi}{c^2}} \end{pmatrix} \]

\[ \begin{align*}
ct &= \frac{c^2}{a} \, e^{\frac{a_\xi}{c^2}} \sinh \left( \frac{a_\lambda}{c} \right) \\
x &= \frac{c^2}{a} \, e^{\frac{a_\xi}{c^2}} \cosh \left( \frac{a_\lambda}{c} \right)
\end{align*} \]
Eddington Finkelstein

\[
\begin{align*}
 v &= c \lambda + \xi \\
 u &= c \lambda - \xi
\end{align*}
\implies
\begin{align*}
 \lambda &= \frac{v + u}{2} \\
 \xi &= \frac{2c}{v - u}
\end{align*}
\]

\[ds^2 = -e^{\frac{a(v-u)}{c^2}} dv \, du\]

Kruskal

\[
\begin{align*}
 U &= -\frac{c^2}{a} e^{-\frac{a}{c^2} u} \\
 V &= \frac{c^2}{a} e^{\frac{a}{c^2} v}
\end{align*}
\]

\[ds^2 = -dU \, dV\]
(U, V)_+ = \left(0, e^{\frac{a}{c^2} v} < \infty \right)
(U, V)_- = \left(e^{-\frac{a}{c^2} u} < \infty, 0 \right)

Regular at the horizon

- Uniform acceleration
- hyperbolic motion in Minkowski s-t
- Rindler metric and coordinates for the accelerated observers

\[ ds^2 = e^{\frac{2 a \xi}{c^2}} (d\xi^2 - c^2 d\lambda^2) \]

- Rindler metric does not cover the entire Minkowski (t-x) plane but…

\[ \text{FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.} \]
Extension of the Rindler metric in the whole space-time

\[(U, V)_{R.W} = \left(-\frac{c^2}{a} e^{-\frac{a}{c^2} u}, \frac{c^2}{a} e^{\frac{a}{c^2} v}\right)\]

is regular on the L-C

\[(U, V)_{L.W} = \left(\frac{c^2}{a} e^{\frac{a}{c^2} u}, -\frac{c^2}{a} e^{-\frac{a}{c^2} v}\right)\]

\[(U, V)_{F} = \left(\frac{c^2}{a} e^{-\frac{a}{c^2} u}, \frac{c^2}{a} e^{\frac{a}{c^2} v}\right)\]

\[(U, V)_{P} = \left(-\frac{c^2}{a} e^{-\frac{a}{c^2} u}, -\frac{c^2}{a} e^{\frac{a}{c^2} v}\right)\]
Event horizon is a space-time membrane
\[ (\nabla_\mu \nabla^\mu - m^2) \phi = 0 \]

\[
\begin{align*}
\left[ \hat{\phi}(x, t), \hat{\phi}(x', t) \right] &= \left[ \hat{\pi}(x, t), \hat{\pi}(x', t) \right] = 0 \\
\left[ \hat{\phi}(x, t), \hat{\pi}(x', t) \right] &= i \delta^3(x, x')
\end{align*}
\]

\[
\hat{\pi}(x, t) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\hat{\phi}}(x, t)
\]

\[
\hat{\phi}(x) = \sum_i \left[ \hat{a}_i f_i(x) + \hat{a}_i^\dagger f_i^*(x) \right]
\]

\(\{f_i\}\)

- annihilation operator
- positive frequency normal modes
- negative frequency normal modes
- creation operator

complete set of wave functions
\[ \hat{a}_i = (f_i, \phi)_{KG} \]
\[ \hat{a}^\dagger_i = (\phi, f_i)_{KG} \]

**Definition**

\[
(f_A, f_B)_{KG} = i \int d^3x \left( f_A^* \partial_t f_B - f_B \partial_t f_A^* \right)
= i \int d^3x \left( f_A^* \pi_B - f_B \pi_A^* \right)
\]

\[
[\hat{a}_i, \hat{a}_j] = -(f_i, f_j^*) = 0
\]
\[
[\hat{a}_i^\dagger, \hat{a}_j^\dagger] = -(f_i^*, f_j) = 0
\]
\[
[\hat{a}_i, \hat{a}_j^\dagger] = (f_i, f_j) = \delta_{ij}
\]

\[ \hat{a}_i |0\rangle = 0 \quad \forall i. \]
Bogoliubov transformations

\[ \left\{ f^{(A)}_i \right\} \quad \text{and} \quad \left\{ f^{(B)}_I \right\} \]

two complete sets of positive energy eigenfunctions

\[
\begin{align*}
f^{(B)}_I &= \sum_i \left[ \alpha_{II} f^{(A)}_i + \beta_{II} f^{(A)*}_i \right] \\
(f^{(B)*}_I &= \sum_i \left[ \alpha_{II} f^{(A)*}_i + \beta^{*}_{II} f^{(A)}_i \right] \\
\end{align*}
\]

\[
\begin{align*}
f^{(A)}_i &= \sum_I \left[ \alpha^{*}_{II} f^{(B)}_I - \beta_{II} f^{(B)*}_I \right] \\
(f^{(A)*}_i &= \sum_I \left[ \alpha_{II} f^{(B)*}_I - \beta^{*}_{II} f^{(B)}_I \right] \\
\end{align*}
\]
The coefficients of the previous expansions can be written as

\[
\alpha_{fi} = \left( f_i^{(A)}, f_i^{(B)} \right)_{KG} = \left( f_i^{(B)}, f_i^{(A)} \right)^*_{KG} \quad \quad \quad \quad (f_A, f_B)_{KG} = i \int d^3x \left( f_A^* \partial_t f_B - f_B \partial_t f_A^* \right)
\]

\[
\beta_{fi} = - \left( f_i^{(A)*}, f_i^{(B)} \right)_{KG} = \left( f_i^{(B)*}, f_i^{(A)} \right)_{KG}
\]

\[
\hat{\phi}(x) = \sum_i \left[ \hat{\alpha}_i^A f_i^A (x) + \hat{\alpha}_i^{A*} f_i^{*A} (x) \right]
\]

\[
\hat{\phi}(x) = \sum_i \left[ \hat{\alpha}_i^B f_i^B (x) + \hat{\alpha}_i^{B*} f_i^{*B} (x) \right]
\]

\[
f_i^{(B)} = \sum_i \left[ \alpha_{fi} f_i^{(A)} + \beta_{fi} f_i^{(A)*} \right]
\]

\[
f_i^{(B)*} = \sum_i \left[ \alpha_{fi}^* f_i^{(A)*} + \beta_{fi}^* f_i^{(A)} \right]
\]
\[ \hat{a}_i^{(A)} = \sum_I \left[ \alpha_{Ii} \hat{a}_I^{(B)} + \beta_{Ii}^* \hat{a}_I^{(B)\dagger} \right] \]

\[ \hat{a}_I^{(B)} = \sum_i \left[ \alpha_{Ii}^* \hat{a}_i^{(A)} - \beta_{Ii}^* \hat{a}_i^{(A)\dagger} \right] \]

Bogoliubov transformations
\begin{align*}
\hat{a}_i^{(A)} |0_{(A)}\rangle &= 0 \\
\hat{a}_I^{(B)} |0_{(A)}\rangle \\
= \sum_i \left[ \alpha_{Ii}^* \hat{a}_i^{(A)} - \beta_{Ii}^* \hat{a}_i^{(A)\dagger} \right] |0_{(A)}\rangle = \\
- \sum_i \beta_{Ii}^* \hat{a}_i^{(A)\dagger} |0_{(A)}\rangle &\neq 0
\end{align*}

The ground state associated with the complete basis \( \{ f_i^{(A)} \} \) can be different from the ground state associated with the complete basis \( \{ f_I^{(B)} \} \).
\[ \hat{N}^{(B)}_I \equiv \hat{a}^{(B)\dagger}_I \hat{a}^{(B)}_I \]

Therefore if one the coefficient \( \beta_{Ii} \) is different from zero

The two sets «see» a different particle content
Unruh effect

\[ \hat{\phi}(x, t) \]

\[ ds^2 = dx^2 - dt^2 \quad \text{Minkowski} \]

\[ ds^2 = e^{2a\xi} (d\xi^2 - d\lambda^2) \quad \text{Rindler} \]
Minkowski

\[
\left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2} \right) \hat{\phi} = 0
\]

\[
\hat{\phi}(x, t) = \int_0^\infty \frac{dk}{\sqrt{4\pi k}} \left( \hat{b}_{-k} e^{-ik(t-x)} + \hat{b}_{+k} e^{-ik(x+t)} + \hat{b}^\dagger_{-k} e^{ik(t-x)} + \hat{b}^\dagger_{+k} e^{ik(x+t)} \right)
\]

Creation and annihilation operators for + and - frequencies
Minkowski

\[ U = t - x \]
\[ V = t + x \]

\[ \hat{\phi}(x, t) = \hat{\phi}_- (U) + \hat{\phi}_+ (V) \]

\[ \hat{\phi}_+ (V) = \int_0^\infty dk \left[ \hat{b}_{+k} f_k (V) + \hat{b}_{+k}^\dagger f_k^* (V) \right] \]

\[ f_k (V) \equiv \frac{1}{\sqrt{4 \pi k}} e^{-i k V} \]
Rindler\textsuperscript{\textregistered} RW

\begin{align*}
u &= \lambda - \xi \\
v &= \lambda + \xi \\
U &= -\frac{1}{a} e^{-au} \\
V &= \frac{1}{a} e^{av}
\end{align*}

RW \quad V > 0

FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.

\begin{align*}
\begin{cases}
v &= c\lambda + \xi \\
u &= c\lambda - \xi
\end{cases}
\quad \Rightarrow \quad 
\begin{cases}
\lambda &= \frac{v + u}{2} \\
\xi &= \frac{v - u}{2}
\end{cases}
\begin{align*}
U &= -\frac{c^2}{a} e^{-\frac{a}{c^2} u} \\
V &= \frac{c^2}{a} e^{\frac{a}{c^2} v}
\end{align*}
\end{align*}
Normal modes Ingoing component

\[ \hat{\phi}^R_+ (V) = \int_0^\infty d\omega \left[ \hat{a}^R_{+\omega} g_\omega (v) + \hat{a}^{R\dagger}_{+\omega} g^*_\omega (v) \right] \]

\[ g_\omega (v) = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v} = \frac{1}{\sqrt{4\pi\omega}} (a V)^{-\frac{i\omega}{a}} \]

\[ U = -\frac{1}{a} e^{-a u} \]
\[ V = \frac{1}{a} e^{a v} \]

\[ \hat{a}^{R\dagger}_{+\omega} e \hat{a}^R_{+\omega} \]

Creation and annihilation operators in the RW
Rindler LW

Rindler \((\bar{\xi}, \bar{\lambda})\)

\[
\bar{u} = \bar{\lambda} - \bar{\xi}
\]

\[
\bar{v} = \bar{\lambda} + \bar{\xi}
\]

\[
U = \frac{1}{a} e^{a \bar{u}}
\]

\[
V = -\frac{1}{a} e^{-a \bar{v}}
\]

\(V < 0;\)

\[
\hat{a}_{+\omega} \dagger \hat{a}_{+\omega}^L
\]

To cover this sector

FIG. 1: Trajectory of the Rindler observer as seen by the observer at rest.

\[
(U, V)_{LW} = \left( \frac{c^2}{a} e^{\frac{a}{c^2}} u, -\frac{c^2}{a} e^{-\frac{a}{c^2}} v \right)
\]

Creation and annihilation operators in the LW
\[ V \in (-\infty, \infty) \]

\[
\hat{\phi}^+_+ (V) = \theta (V) \hat{\phi}^R_+ (V) + \theta (-V) \hat{\phi}^L_+ (V) =
\]

\[
= \int_0^\infty d\omega \left\{ \theta (V) \left[ \hat{a}^R_{+\omega} g_\omega (v) + \hat{a}^{R\dagger}_{+\omega} g^*_\omega (v) \right] + \\
+ \theta (-V) \left[ \hat{a}^L_{+\omega} g_\omega (\nu) + \hat{a}^{L\dagger}_{+\omega} g^*\omega (\nu) \right] \right\}
\]

\[ g_\omega (v) = \frac{1}{\sqrt{4\pi \omega}} e^{-i\omega v} \]
set \( \{ f_k (V) \} \) \quad \text{Minkowski}

\( \{ g_\omega (v), g_\omega (\overline{v}) \} \) \quad \text{Rindler RW and LW}

\[ \theta (V) \ g_\omega (v) = \int_0^\infty dk \ [\alpha_{\omega k}^{R} f_k (V) + \beta_{\omega k}^{R} f^*_k (V)] \]

\[ \theta (-V) \ g_\omega (\overline{v}) = \int_0^\infty dk \ [\alpha_{\omega k}^{L} f_k (V) + \beta_{\omega k}^{L} f^*_k (V)] \]
Inversion

\[
\beta_{\omega k}^L = -e^{-\frac{\pi \omega}{a}} \alpha_{\omega k}^{R*} \\
\beta_{\omega k}^R = -e^{-\frac{\pi \omega}{a}} \alpha_{\omega k}^{L*}
\]

\[
\theta (V) \ g_\omega (v) = \int_0^\infty dk \left[ \alpha_{\omega k}^R f_k (V) + \beta_{\omega k}^R f_k^* (V) \right]
\]

\[
\theta (-V) \ g_\omega (\bar{v}) = \int_0^\infty dk \left[ \alpha_{\omega k}^L f_k (V) + \beta_{\omega k}^L f_k^* (V) \right]
\]

\[
\hat{\phi}_+ (V) = \theta (V) \hat{\phi}_+^R (V) + \theta (-V) \hat{\phi}_+^L (V) = \\
= \int_0^\infty d\omega \left\{ \theta (V) \left[ \hat{a}_{+\omega}^R g_\omega (v) + \hat{a}_{+\omega}^{R\dagger} g_\omega^* (v) \right] + \right. \\
+ \left. \theta (-V) \left[ \hat{a}_{+\omega}^L g_\omega (\bar{v}) + \hat{a}_{+\omega}^{L\dagger} g_\omega^* (\bar{v}) \right] \right\}
\]

\[
f_k (V) \equiv \frac{1}{\sqrt{4 \pi k}} e^{-i k V}
\]
\[ \hat{\phi}^+ (V) = \int_0^\infty d\omega \int_0^\infty \frac{dk}{\sqrt{2\pi k}} \times \]
\[ \alpha^R_{\omega_k} e^{-i k V} \left( \hat{a}^R_{+\omega} - e^{-\frac{\pi \omega}{a}} \hat{a}^L_{+\omega} \right) + \]
\[ \alpha^L_{\omega_k} e^{-i k V} \left( \hat{a}^L_{+\omega} - e^{-\frac{\pi \omega}{a}} \hat{a}^R_{+\omega} \right) \] + H.c.

\[ \hat{\phi}^+ (V) \left| 0_{(\mathcal{M})} \right\rangle = 0 \]

\[
\begin{align*}
\left( \hat{a}^R_{+\omega} - e^{-\frac{\pi \omega}{a}} \hat{a}^L_{+\omega} \right) \left| 0_{(\mathcal{M})} \right\rangle &= 0 \\
\left( \hat{a}^L_{+\omega} - e^{-\frac{\pi \omega}{a}} \hat{a}^R_{+\omega} \right) \left| 0_{(\mathcal{M})} \right\rangle &= 0
\end{align*}
\]
\[ \langle 0_{(M)} | \hat{N}_{\omega_i}^R | 0_{(M)} \rangle = \langle 0_{(M)} | \hat{a}_{\omega_i}^R \hat{a}_{\omega_i}^R | 0_{(M)} \rangle = e^{-\frac{2\pi\omega}{a}} \langle 0_{(M)} | \hat{a}_{\omega_i}^L \hat{a}_{\omega_i}^L | 0_{(M)} \rangle = \]

\[ e^{-\frac{2\pi\omega}{a}} \left[ \langle 0_{(M)} | \hat{N}_{\omega_i}^L | 0_{(M)} \rangle + 1 \right] \]

\[ \langle 0_{(M)} | \hat{N}_{\omega_i}^L | 0_{(M)} \rangle = e^{-\frac{2\pi\omega}{a}} \left[ \langle 0_{(M)} | \hat{N}_{\omega_i}^R | 0_{(M)} \rangle + 1 \right] \]
The expectation value of the number operator of Rindler particle in the RW or LW in the Minkowski vacuum is a Bose-Einstein Distribution with temperature

\[
\langle 0(\mathcal{M}) | \hat{N}^R_{\omega_i} | 0(\mathcal{M}) \rangle = \langle 0(\mathcal{M}) | \hat{N}^L_{\omega_i} | 0(\mathcal{M}) \rangle = \frac{1}{\frac{2\pi\omega}{e^\frac{a}{\hbar} - 1}}
\]

\[
T = \frac{a}{2\pi}
\]


Acceleration radiation in interacting field theories
William G. Unruh and Nathan Weiss

Unruh-Wald PRD 29(1984) 1047
hidden region

event horizon

Rindler horizon

accelerating rocket

\[ r_0 = \frac{c^2}{a} \]
Recall

In Classical Black-hole particles are confined $\Rightarrow$ event horizon $\Rightarrow$ no communication with outside, but...Hawking radiation [Hawking 1975]

Quantum effect $\sim$ uncertainty principle $\rightarrow$ vacuum fluctuation $e^+e^-$ outside event horizon, with $\Delta E \Delta t \sim 1$. If $e^+$ falls into black hole, then $e^-$ can escape; equivalent: $e^-$ tunnels through event horizon
There is no information about state of system beyond event horizon; $e^+$ on one side, $e^-$ on the other.

$\Rightarrow$ Hawking radiation must be thermal

$$\frac{dN}{dk} \sim \exp\left\{-\frac{k}{T_{BH}}\right\}$$

with black hole temperature

$$T_{BH} = \frac{\hbar}{8\pi c GM}$$

relativistic quantum effect: disappears for

$\hbar \rightarrow 0$ or $c \rightarrow \infty$

$\Rightarrow$ tunnelling through event horizon $\rightarrow$ thermal radiation
The correspondence with gravity

Unruh effect and the near horizon approximation

Rindler metric of an accelerated observer
(in spherical coordinates $\tau, \chi, \theta, \phi$)

$$ds^2 = \chi^2 a^2 d\tau^2 - d\chi^2 - \chi^2 \cosh^2 a\tau (d\theta^2 + \sin^2 \theta d\phi^2)$$

Schwarzschild BH metric; $\gamma = (1 - 2GM/r)$

$$ds^2 = \gamma dt^2 - \gamma^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

Coordinate transformation $\eta = \sqrt{\gamma}/k$, where $k = \text{surface gravity}$ and $r \to R = 2GM$

$$ds^2 = \eta^2 k^2 dt^2 - d\eta^2 - R^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$
Applications:

- for $F = \frac{GMm}{R^2}$ and Schwarzschild $R = 2M$, recover Hawking temperature

$$T_U = \frac{a}{2\pi} = \frac{GM}{2\pi R^2} = \frac{1}{8\pi GM}$$

- for $F = e\mathcal{E}$ recover Schwinger mechanism for production of pair (mass $m$) in strong field $\mathcal{E}$

$$T_U = \frac{a}{2\pi} = \frac{e\mathcal{E}}{\pi m}$$

$$P(m, \mathcal{E}) \sim \exp\{-m/T_U\} = \exp\{-\pi m^2/e\mathcal{E}\}$$

production probability $P(m, \mathcal{E})$

Unruh radiation as tunneling through the event horizons

Minkowski spacetime

\[ ds^2 = -dt^2 + dx^2 \]

Rindler observer

\[ t = (a^{-1} + x_R) \sinh(at_R) \]
\[ x = (a^{-1} + x_R) \cosh(at_R) \]

Rindler metric

\[ ds^2 = -(1 + ax_R)^2 dt_R^2 + dx_R^2 . \]

\[ \det(g_{ab}) \equiv g = -(1 + ax_R)^2, \text{ vanishes.} \]
\[ x_R = -\frac{1}{a}, \]

\[ ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2 , \]
alternative form of the Rindler metric

\[(1 + a x_R) = \sqrt{|1 + 2 a x_{R'}|} .\]

\[ds^2 = -(1 + 2 a x_{R'}) dt_{R'}^2 + (1 + 2 a x_{R'})^{-1} dx_{R'}^2 .\]

the substitution \(a \rightarrow GM/x_{R'}^2\)

the usual Schwarzschild metric.

\[ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2,\]

But also

\[ds^2 = - dt^2 + dx^2\]

\[t = \frac{\sqrt{1 + 2ax_{R'}}}{a} \sinh(at_{R'})\]

\[x = \frac{\sqrt{1 + 2ax_{R'}}}{a} \cosh(at_{R'})\]

\[t = \frac{\sqrt{|1 + 2ax_{R'}|}}{a} \cosh(at_{R'})\]

\[x = \frac{\sqrt{|1 + 2ax_{R'}|}}{a} \sinh(at_{R'})\]

for \(x_{R'} \geq - \frac{1}{2a}\)

\[\text{for } x_{R'} \leq - \frac{1}{2a}\]
Unruh radiation via WKB method
D.A. Singleton  http://dx.doi.org/10.5772/53898

THE WKB/TUNNELING METHOD

WKB approximation tells us how to find the transmission probability in terms of the incident wave and transmitted wave amplitudes. The transition probability is in turn given by the exponentially decaying part of the wave function over the non-classical (tunneling) region

\[ \Gamma_{QM} \propto e^{-1m \frac{1}{\hbar} \int p_x dx} \]

scalar field
metric \( g_{\mu\nu} \)

\[ \phi = \phi_0 e^{\frac{i}{\hbar} S(t, \vec{x})} \]

\[ g^{\mu\nu} \partial_\nu (S) \partial_\mu (S) + m^2 = 0 \]

Now for stationary spacetimes the action \( S \) can be split into a time and space part

\[ S(t, \vec{x}) = Et + S_0(\vec{x}) \]
\[ ds^2 = -(1 + 2a x_{R'}) dt_{R'}^2 + (1 + 2a x_{R'})^{-1} dx_{R'}^2 \]

\[ - \frac{1}{(1 + 2a x_{R'})} (\partial_t S)^2 + (1 + 2a x_{R'}) (\partial_x S)^2 + m^2 = 0 . \]

\[ S(t, \vec{x}) = Et + S_0(\vec{x}) \]

\[ - \frac{E}{(1 + 2a x_{R'})^2} + (\partial_x S_0(x_{R'}))^2 + \frac{m^2}{1 + 2a x_{R'}} = 0 \]

\[ S_0^\pm = \pm \int_{-\infty}^{\infty} \frac{\sqrt{E^2 - m^2(1 + 2a x_{R'})}}{1 + 2a x_{R'}} \, dx_{R'} . \]

the + sign corresponds to the ingoing particles (i.e., particles that move from right to left) and the − sign to the outgoing particles (i.e., particles that move left to right).

this integral has a pole along the path of integration at \( x_{R'} = -\frac{1}{2a} \).
FIG. 2: Contours of integration for (i) the ingoing and (ii) the outgoing particles.

A semi–circular contour which we parameterize as

\[ x_{R'} = -\frac{1}{2a} + \epsilon e^{i\theta}, \]

where \( \epsilon \ll 1 \) and \( \theta \) goes from 0 to \( \pi \) for the ingoing path and \( \pi \) to 0 for the outgoing path.

\[
S_0^\pm = \pm \int_{-\infty}^{\infty} \frac{\sqrt{E^2 - m^2(1 + 2a x_{R'})}}{1 + 2a x_{R'}} \, dx_{R'} .
\]

For ingoing (+) particles is

\[
S_0^+ = \int_0^\pi \frac{\sqrt{E^2 - m^2 \epsilon e^{i\theta}}}{2a\epsilon e^{i\theta}} \, i\epsilon e^{i\theta} \, d\theta = \frac{i\pi E}{2a} ,
\]
and for outgoing (−) particles,

\[ S_0^- = - \int_0^\pi \frac{\sqrt{E^2 - m^2 \epsilon e^{i\theta}}}{2 \epsilon e^{i\theta}} i \epsilon e^{i\theta} d\theta = \frac{i \pi E}{2a}. \]

the total action \( \Gamma(t, \vec{x}) = Et + S_0(\vec{x}) \).

\[ \Gamma \propto e^{-\frac{1}{\hbar} \left[ \text{Im}(\oint p_x dx) - E \text{Im}(\Delta t) \right]} . \]

\[ t = \frac{\sqrt{1 + 2ax_{R'}}}{a} \sinh(at_{R'}) \]

\[ x_{R'} \geq -\frac{1}{2a}. \]

\[ t_{R'} \rightarrow t_{R'} - \frac{i \pi}{2a} \]

\[ \sinh(at_{R'}) \rightarrow \sinh \left( at_{R'} - \frac{i \pi}{2} \right) = -i \cosh(at_{R'}) \]

\[ \sqrt{1 + 2ax_{R'}} \rightarrow i \sqrt{|1 + 2ax_{R'}|} \]
\[ S(t, \vec{x}) = S_0(\vec{x}) + Et \]

\[ E\Delta t = -\frac{i\pi E}{2a}. \]

When the horizon is crossed once, the total action \( S(t, \vec{x}) \) gets a contribution of \( E\Delta t = -\frac{iE\pi}{2a} \), and for a round trip, as implied by the spatial part \( \oint p_x dx \), the total contribution is \( E\Delta t_{total} = -\frac{iE\pi}{a} \).

\[
S_0^+ = \int_0^\pi \frac{\sqrt{E^2 - m^2 e^{i\theta}}}{2ae^{i\theta}} i\epsilon e^{i\theta} d\theta = \frac{i\pi E}{2a},
\]

\[
S_0^- = -\int_\pi^0 \frac{\sqrt{E^2 - m^2 e^{i\theta}}}{2ae^{i\theta}} i\epsilon e^{i\theta} d\theta = \frac{i\pi E}{2a}.
\]

\[ \Gamma \propto e^{-\frac{1}{\hbar}[\text{Im}(\oint p_x dx) - E\text{Im}(\Delta t)]]} . \]

\[ \text{Im}(\oint p_x dx) - E\text{Im}(\Delta t) = \frac{\pi E}{a} + \frac{\pi E}{a} \]

\[ T = \frac{\hbar a}{2\pi k c} . \]
Uniform acceleration

Event Horizon

Universal thermal behavior

In QCD? 

Confinement $V \rightarrow \sigma r$
Conjecture

Physical vacuum \iff Event horizon for colored constituents

Thermal hadron production \iff Hawking-Unruh radiation in QCD

P.C., D.Kharzeev and H.Satz -- D.Kharzeev and Y.Tuchin (temperature)

F.Becattini, P.C., J.Manninen and H.Satz (strangeness suppression in e+e-)

P.C. and H.Satz (strangeness enhancement in heavy ion collisions)

P.C., A. Iorio and H.Satz (entropy and freeze-out)
Questions

1) *Why do elementary high energy collisions show a statistical behavior?*

2) *Why is strangeness production universally suppressed in elementary collisions?*

3) *Why (almost) no strangeness suppression in nuclear collisions?*

4) *Why hadron freeze-out for \( s/T^3 = 7 \) or \( E/N = 1.08 \text{ Gev} \)*

5) *Why thermalization in so short time (0.5-1 fm/c)*