Lecture:

Microscopic dynamical models for heavy-ion collisions

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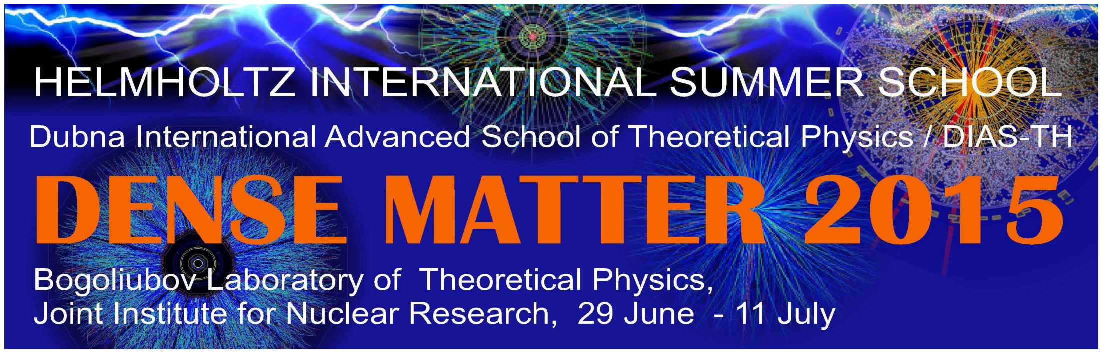
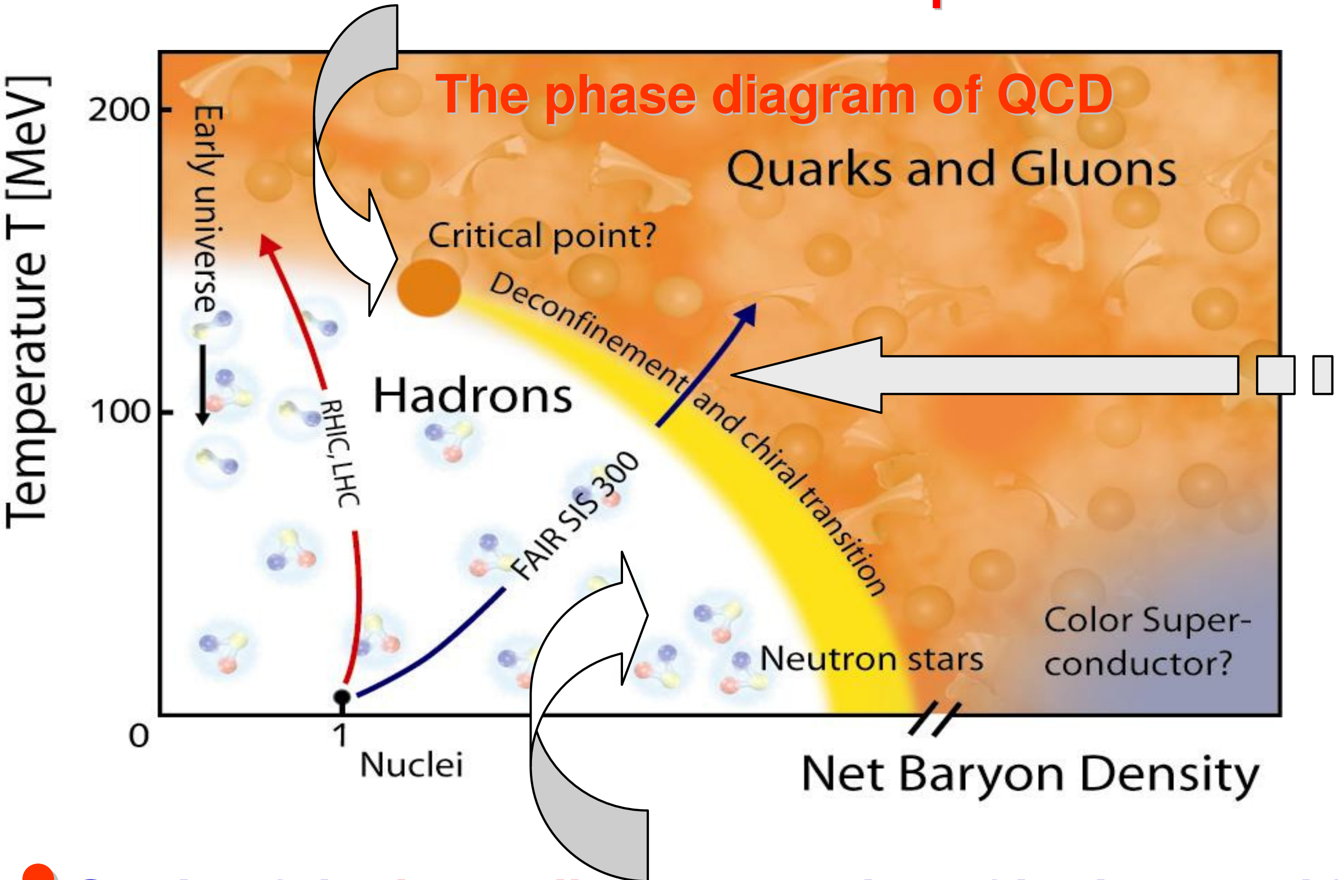
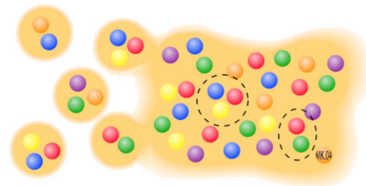


The ‚holy grail‘ of heavy-ion physics:

- Search for the **critical point**



- Study of the **phase transition from hadronic to partonic matter – Quark-Gluon-Plasma**

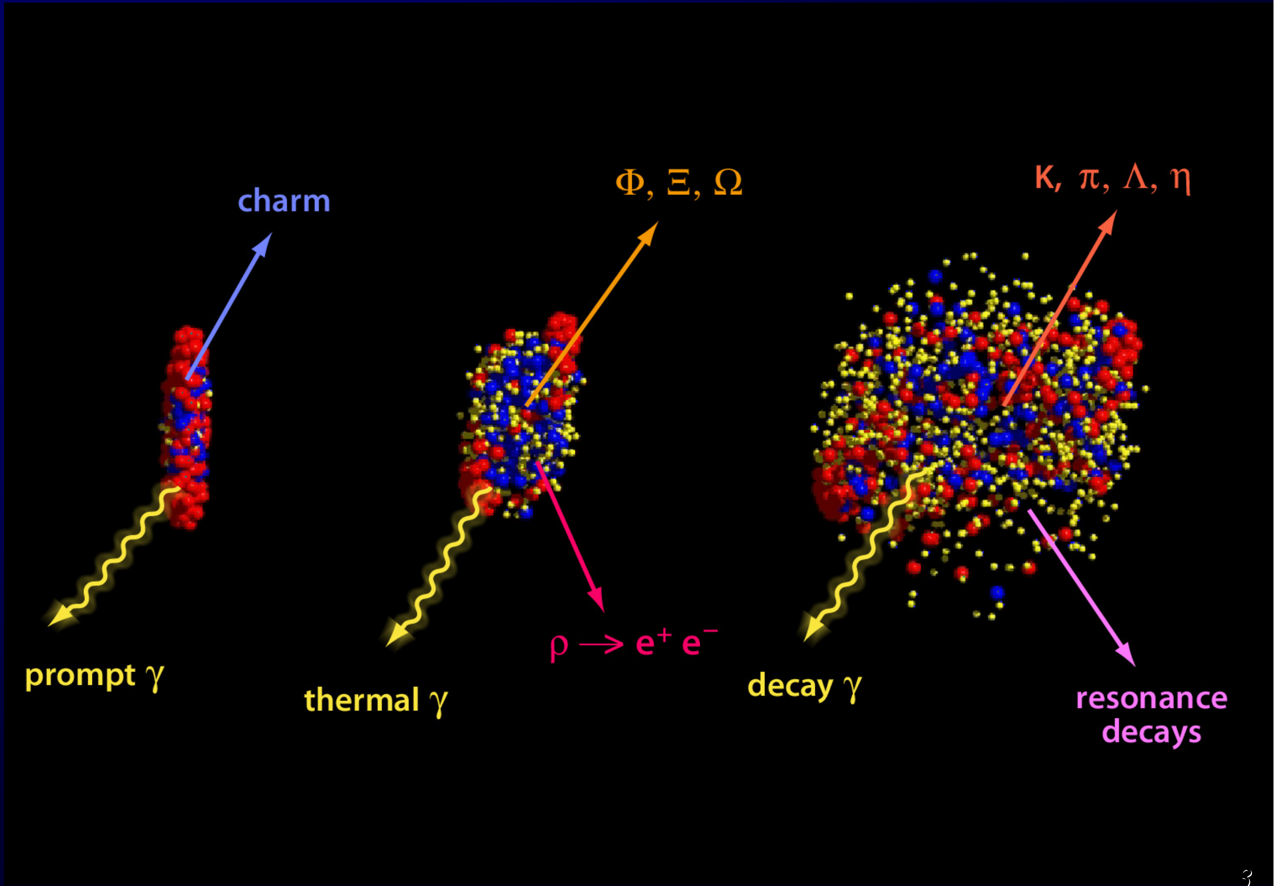


- Study of the **in-medium** properties of hadrons at high baryon density and temperature – chiral symmetry restoration

Signals of the phase transition:

- Multi-strange particle enhancement in A+A
- Charm suppression
- Collective flow (v_1, v_2)
- Thermal dileptons
- Jet quenching and angular correlations
- High p_T suppression of hadrons
- Nonstatistical event by event fluctuations and correlations
- ...

Experiment: measures final hadrons and leptons



How to learn about physics from data?

Compare with theory!

Basic models for heavy-ion collisions

- **Statistical models:**

basic assumption: system is described by a (grand) canonical ensemble of non-interacting fermions and bosons in thermal and chemical equilibrium

[-: no dynamics]

- **Ideal hydrodynamical models:**

basic assumption: conservation laws + equation of state; assumption of local thermal and chemical equilibrium

[-: - simplified dynamics]

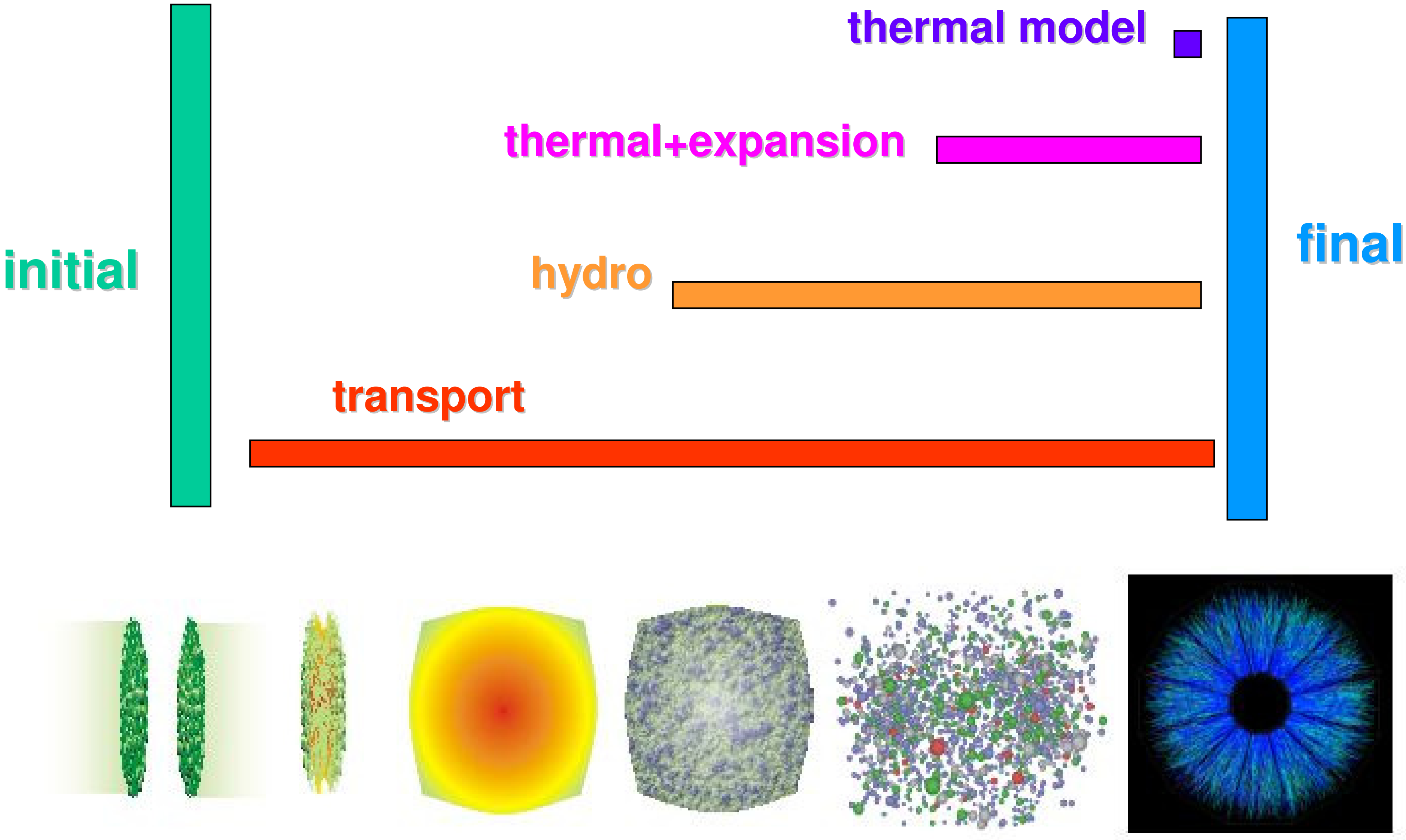
- **Transport models:**

based on transport theory of relativistic quantum many-body systems - nonequilibrium dynamics. Actual solutions: Monte Carlo simulations

[+: full dynamics | -: very complicated]

→ Microscopic transport models provide a unique dynamical description of nonequilibrium effects in heavy-ion collisions

Models of heavy-ion collisions



Dynamical description of heavy-ion collisions

The goal: to study the properties of strongly interacting matter under extreme conditions from a microscopic point of view

Realization: dynamical many-body transport models

This lecture:

1) Dynamical transport models (nonrelativistic formulation):
from the Schrödinger equation to Vlasov equation of motion ➔ BUU EoM

2) Density-matrix formalism: Correlation dynamics

3) Quantum field theory ➔ Kadanoff-Baym dynamics
➔ generalized off-shell transport equations

4) Microscopic description of the QGP, hadronization problem

5) Example of transport model: The PHSD transport approach, basic ideas

1. From the Schrödinger equation to the Vlasov equation of motion

Quantum mechanical description of the many-body system

Dynamics of heavy-ion collisions is a many-body problem!

Schrödinger equation for the system of N particles in three dimensions:

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t) = H(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t)\, \Psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t)$$

nonrelativistic formulation

Hartree-Fock approximation:

- many-body wave function → antisym. product of single-particle wave functions

$$\Psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t) = A \prod_{i=1}^{N} \psi_i(\vec{r}_i, t)$$

- many-body Hamiltonian → single-particle Hartree-Fock Hamiltonian

$$H(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t) = \sum_{i=1}^{N} T(\vec{r}_i) + V(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N, t)$$

kinetic term N-body potential

$$T(\vec{r}) = -\frac{\hbar}{2m} \vec{\nabla}_r^2$$

$$\approx \sum_{i=1}^{N} T(\vec{r}_i) + \sum_{i<j}^{N} V_{ij}(|\vec{r}_i - \vec{r}_j|, t) \approx \sum_{i=1}^{N} h_i(\vec{r}_i, t)$$

kinetic term 2-body potential

Hartree-Fock equation

Time-dependent Hartree-Fock equation for a single particle *i*:

$$i\hbar \frac{\partial}{\partial t}\psi_i(\vec{r},t) = \hat{h}\psi_i(\vec{r},t)$$

Single-particle Hartree-Fock Hamiltonian operator: $\hat{h} = \hat{T} + \hat{U}_H - \hat{U}_F$

- **Hartree term:** $\hat{U}_H = \sum_{i(occ)} \int d^3r' \psi_i^*(\vec{r}\,',t) V(\vec{r}-\vec{r}\,') \psi_i(\vec{r}\,',t)$ $\qquad \hat{T} = -\frac{\hbar}{2m}\vec{\nabla}_r^2$

  self-generated local mean-field potential (classical)

- **Fock term:** $\hat{U}_F = \sum_{i<N} \psi_i^*(\vec{r}\,',t) V(\vec{r},\vec{r}\,',t) \psi_i(\vec{r}\,',t)$

  non-local mean-field exchange potential (quantum statistics)

TDHF approximation describes only the interactions of particles with the time-dependent mean-field $U_{HF}(r,t)$!

➔ **EoM: propagation of particles in the self-generated mean-field**

$$i\hbar \frac{\partial}{\partial t}\psi_i(\vec{r},t) = \left(T(\vec{r},t) + U_H(\vec{r},t)\right)\ \psi_i(\vec{r},t) - \int d^3r'\ \ U_F(\vec{r},\vec{r}\,',t)\psi_i(\vec{r}\,',t)$$

➔ In order to describe the collisions between the individual(!) particles, one has to go beyond the mean-field level ! (see Part 2: Correlation dynamics)

Wigner transform of the density matrix

- Introduce the **single particle density matrix:**

$$\rho(\vec{r},\vec{r}',t) \equiv \sum_{\beta_{occ}} \psi^*_\beta(\vec{r}',t)\psi_\beta(\vec{r},t)$$

Thus, the **single-particle Hartree-Fock Hamiltonian** operator can be written as

$$h(\vec{r},t) = T(\vec{r}) + \sum_{\beta_{occ}} \int d^3r'\, V(\vec{r}-\vec{r}',t)\rho(\vec{r}',\vec{r}',t) = T(\vec{r}) + U(\vec{r},t)$$

local potential

➔ **EoM:**

$$\frac{\partial}{\partial t}\rho(\vec{r},\vec{r}',t) + \frac{i}{\hbar}\left[\frac{\hbar^2}{2m}\vec{\nabla}^2_r + U(\vec{r},t) - \frac{\hbar^2}{2m}\vec{\nabla}^2_{r'} - U(\vec{r}',t)\right]\rho(\vec{r},\vec{r}',t) = 0$$

Instead of considering the density matrix ρ, let's find the equation of motion for its **Fourier transform**, i.e. the **Wigner transform of the density matrix:**

$$f(\vec{r},\vec{p},t) = \int d^3s\ \exp\left(-\frac{i}{\hbar}\vec{p}\vec{s}\right)\ \rho\left(\vec{r}+\frac{\vec{s}}{2},\vec{r}-\frac{\vec{s}}{2},t\right)$$

new old

$$\vec{r} \rightarrow \frac{\vec{r}+\vec{r}'}{2}$$

$$\vec{s} \rightarrow \vec{r}-\vec{r}'$$

$f(\vec{r},\vec{p},t)$ is the **single-particle phase-space distribution function**

Density in coordinate space: $\rho(\vec{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3p\ f(\vec{r},\vec{p},t)$

Vlasov EoM

After the first order gradient expansion of the Wigner transformed EoM for *f* we obtain

Vlasov equation of motion
- free propagation of particles in the self-generated HF mean-field potential:

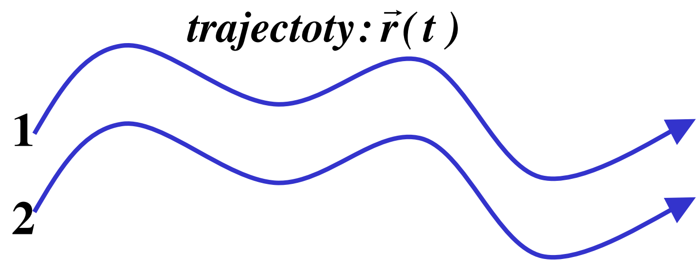
$$\frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\, f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\, \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = 0$$

Vlasov EoM is equivalent to: $$\frac{d}{dt} f(\vec{r},\vec{p},t) = 0 = \left[\frac{\partial}{\partial t} + \dot{\vec{r}}\vec{\nabla}_{\vec{r}} + \dot{\vec{p}}\vec{\nabla}_{\vec{p}}\right] f(\vec{r},\vec{p},t) = 0$$

➔ **Classical equations of motion :**

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{\vec{p}}{m}$$

$$\dot{\vec{p}} = \frac{d\vec{p}}{dt} = -\vec{\nabla}_{\vec{r}} U(\vec{r},t)$$

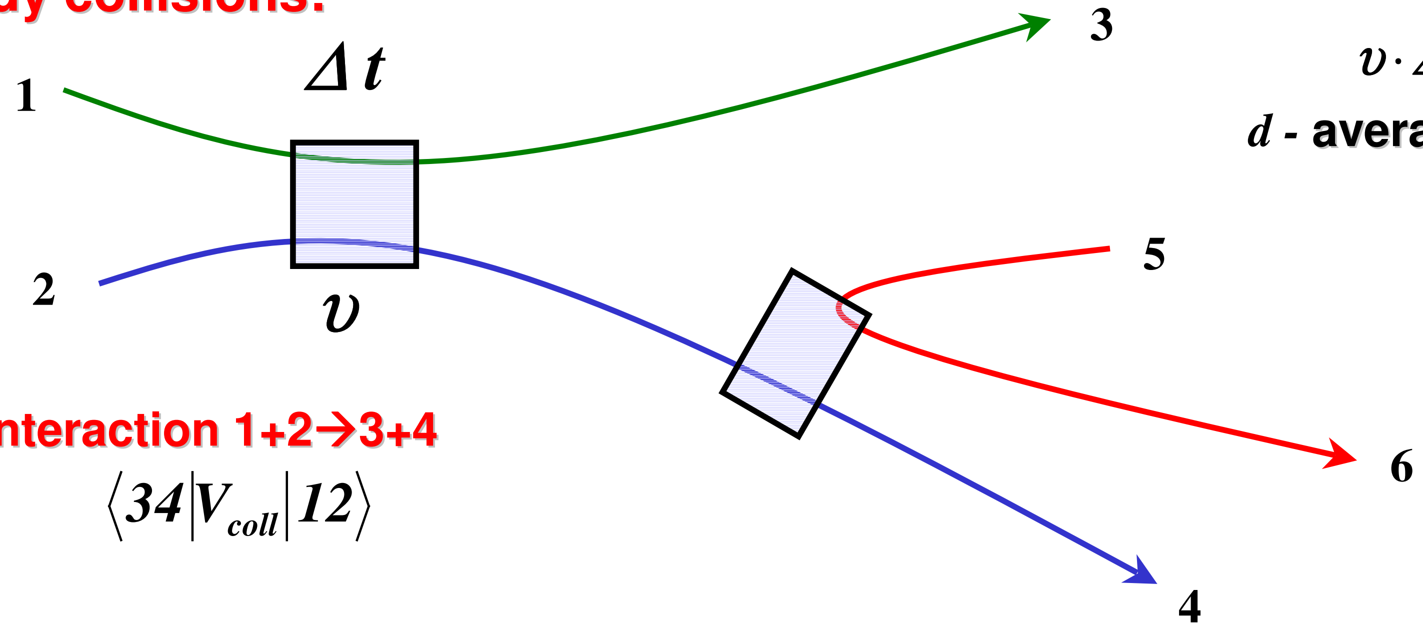


Note: the quantum physics plays a role in the initial conditions for *f*:
the initial *f in case of fermions* must respect the Pauli principle

Dynamical transport models with collisions

➔ In order to describe the collisions between the individual(!) particles, one has to go **beyond the mean-field level !** (see Part 2: Correlation dynamics)

add **2-body collisions:**



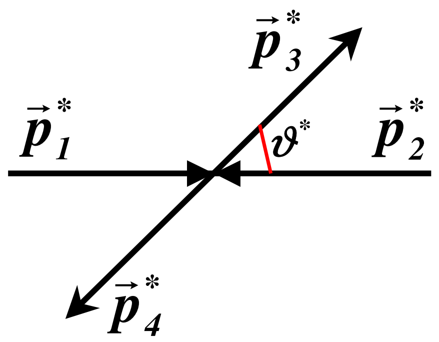
$$\upsilon \cdot \Delta t << \langle d \rangle$$

d - average distance

Interaction 1+2→3+4

$$\langle 34 | V_{coll} | 12 \rangle$$

In cms: $\vec{p}_1^{\,*} + \vec{p}_2^{\,*} = \vec{p}_3^{\,*} + \vec{p}_4^{\,*} = 0$



$$(\vec{r}_1, \vec{p}_1)\,(\vec{r}_2, \vec{p}_2) \rightarrow (\vec{r}_3, \vec{p}_3)\,(\vec{r}_4, \vec{p}_4)$$

- If the phase-space around $(\vec{r}_3, \vec{p}_3)$ *and* $(\vec{r}_4, \vec{p}_4)$ is essentially empty then the scattering is allowed,
- if the states are filled → Pauli suppression
  = **Pauli principle**

BUU (VUU) equation

Boltzmann (Vlasov)-Uehling-Uhlenbeck equation (NON-relativistic formulation!)
- free propagation of particles in the self-generated HF mean-field potential with an on-shell collision term:

$$\frac{d}{dt} f(\vec{r},\vec{p},t) \equiv \frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m} \vec{\nabla}_{\vec{r}} f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t) \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

Collision term for 1+2→3+4 (let‘s consider fermions) :

Probability including Pauli blocking of fermions

$$I_{coll} \equiv \left(\frac{\partial f}{\partial t}\right)_{coll} \Rightarrow \frac{1}{((2\pi)^3)^3} \int d^3p_2\, d^3p_3\, d^3p_4 \cdot w(1+2 \to 3+4) \cdot P$$

$$\times (2\pi)^3 \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\, (2\pi) \delta\left(\frac{\vec{p}_1}{2m_1} + \frac{\vec{p}_2}{2m_2} - \frac{\vec{p}_3}{2m_3} - \frac{\vec{p}_4}{2m_4}\right)$$

Transition probability for *1+2→3+4:* $w(1+2 \to 3+4) \Rightarrow \upsilon_{12} \cdot \frac{d^3\sigma}{d^3q}$

where $\upsilon_{12} = \frac{\hbar}{m} | \vec{p}_1 - \vec{p}_2 |$ **- relative velocity of the colliding nucleons**

$\frac{d^3\sigma}{d^3q}$ **- differential cross section, *q* – momentum transfer** $\vec{q} = \vec{p}_1 - \vec{p}_3$

BUU: Collision term

$$I_{coll} = \frac{4}{(2\pi)^3} \int d^3p_2\, d^3p_3 \int d\Omega\ |v_{12}|\, \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \cdot \frac{d\sigma}{d\Omega}(1+2 \to 3+4) \cdot P$$

Probability including Pauli blocking of fermions:

$$P = f(\vec{r},\vec{p}_3,t) f(\vec{r},\vec{p}_4,t) \left[1 - f(\vec{r},\vec{p}_1,t)\right] \left[1 - f(\vec{r},\vec{p}_2,t)\right]$$
$$- f(\vec{r},\vec{p}_1,t) f(\vec{r},\vec{p}_2,t) \left[1 - f(\vec{r},\vec{p}_3,t)\right] \left[1 - f(\vec{r},\vec{p}_4,t)\right]$$
$$\equiv f_3 f_4 (1-f_1)(1-f_2) - f_1 f_2 (1-f_3)(1-f_4)$$

Pauli blocking factors for fermions *

Gain term
3+4→1+2

Loss term
1+2→3+4

For particle 1 and 2:
Collision term = Gain term – Loss term

$$I_{coll} = G - L$$

***Note: for bosons – enhancement factor *1+f* (where *f<<1*);**
often one neglects bose enhancement for HIC, i.e. *1+f →1*

Dynamical transport model: collision terms

Collision terms for *(N,Δ,π)* system: $\Delta \leftrightarrow \pi N$

* *Relativistic formulation*

Eq. for Δ

$$Df_{\Delta} = \sum_{\pi,N} \frac{g}{(2\pi)^3} \int \frac{d^3p_\pi}{E_\pi} \frac{d^3p_N}{E_N} \, |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times f_\pi(p_\pi)\, f_N(p_N)(1 - f_\Delta(p_\Delta))$$

$$- \sum_{\pi,N} \frac{g}{(2\pi)^3} \int \frac{d^3p_\pi}{E_\pi} \frac{d^3p_N}{E_N} \, |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times f_\Delta(p_\Delta)\,(1 - f_N(p_N))(1 + f_\pi(p_\pi))$$

$$= \mathit{Gain}\,(\pi N \to \Delta) - \mathit{Loss}\,(\Delta \to \pi N) \qquad (21)$$

Δ production *Δ decay*

Eq. for π

$$Df_{\pi} = \sum_{N,\Delta} \frac{g}{(2\pi)^3} \int \frac{d^3p_\Delta}{E_\Delta} \frac{d^3p_N}{E_N} \, |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times f_\Delta(p_\Delta)\,(1 + f_\pi(p_\pi))(1 - f_N(p_N))$$

$$- \sum_{\pi,N} \frac{g}{(2\pi)^3} \int \frac{d^3p_\Delta}{E_\Delta} \frac{d^3p_N}{E_N} \, |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times f_\pi(p_\pi)\, f_N(p_N)(1 - f_\Delta(p_\Delta))$$

$$= \mathit{Gain}\,(\Delta \to \pi N) - \mathit{Loss}\,(\pi N \to \Delta) \qquad (22)$$

π production by Δdecay *π absorbtion by nucleon*

Dynamical transport model: collision terms

- BUU eq. for different particles of type $i=1,\ldots n$

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll}\left[f_1, f_2, \ldots, f_n\right] \qquad (20)$$

Drift term=Vlasov eq. collision term

$$i: \quad Baryons: \; p, n, \Delta(1232), N(1440), N(1535), \ldots, \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \; \Lambda_C$$

$$Mesons: \; \pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, \ldots, D, \bar{D}, J/\Psi, \Psi', \ldots$$

→ coupled set of BUU equations for different particles of type $i=1,\ldots n$

$$\begin{cases} Df_N = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \ldots, f_\pi, f_\rho, \ldots\right] \\ Df_\Delta = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \ldots, f_\pi, f_\rho, \ldots\right] \\ \ldots \\ Df_\pi = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \ldots, f_\pi, f_\rho, \ldots\right] \\ \ldots \end{cases}$$

Dynamical transport model: possible interactions

Consider possible interactions for the sytem of *(N,R,m),*
where *N*-nucleons, *R*- resonances, *m*-mesons

- **elastic collisions:**

Baryon-baryon (BB):

$$NN \to NN$$
$$NR \to NR$$
$$RR' \to RR'$$

meson-Baryon (mB)

$$mN \to mN$$
$$mR \to mR$$

meson-meson (mm)

$$m\, m' \to m\, m'$$

Detailed balance:

$$a + b \leftrightarrow c$$
$$a + b \leftrightarrow c + d$$

- **inelastic collisions:**

Baryon-baryon (BB):

$$NN \leftrightarrow NR$$
$$NR \leftrightarrow NR'$$
$$NN \leftrightarrow RR'$$
$$\ldots$$
$$BB \to X$$

meson-Baryon (mB)

$$mN \leftrightarrow R$$
$$mR \leftrightarrow R'$$
$$mB \leftrightarrow m'B'$$
$$\ldots$$
$$mB \to X$$

meson-meson (mm)

$$m\, m' \leftrightarrow \tilde{m}$$
$$m\, m' \leftrightarrow m'' m'''$$
$$\ldots$$
$$m\, m' \to X$$

X - multi-particle state

2. Density-matrix formalism: Correlation dynamics

Density-matrix formalism

- Schrödinger equation for a system of N fermions:

$$i\hbar \frac{\partial}{\partial t} |\Psi_N(t)> = H_N |\Psi_N(t)>$$

Hamiltonian operator: $$H_N = \sum_{i=1}^{N} h^0(i) + \sum_{i<j}^{N-1} v(ij),$$ $$h^0(i) = t(i)$$

kinetic energy operator

2-body potential

- Schrödinger eq. in density-operator representation

→ von Neumann (or Liouville) eq.:

$$i\hbar \frac{\partial}{\partial t} \hat{\rho}_N = [H_N, \hat{\rho}_N] \quad (1)$$

Density operator for N-body system summed over all possible quantum state *k* with amplitude P_k (P_k^2 → interpretation of probability):

$$\hat{\rho}_N = \sum_k P_k |\psi_{Nk}><\psi_{Nk}|$$

(for any possible quantum state *k* of N-body system)

$$|\psi_{Nk}> \equiv |\xi_1, \xi_2, ..., \xi_j, ..., \xi_N>_k$$

Notation: *j* – particle index of many body system (*j=1,…,N*) in different representations :

$$\xi_j \equiv (\vec{r}_j, \sigma_j, \tau_j, ...)$$

$$\text{or} \quad \xi_j \equiv (\vec{p}_j, \sigma_j, \tau_j, ...)$$

$$\text{or} \quad \xi_j \equiv (l_j, m_j, n_j, \sigma_j, \tau_j, ...)$$

discrete state

Density-matrix formalism

- von Neumann (or Liouville) eq. in matrix representation describes an N-particle system in- or out-off equilibrium

$$i\hbar\frac{\partial}{\partial t}\rho_N(1,..,N;1'..N';t) = [H_N,\rho_N] \quad (2)$$

Notation:

$$\rho_N(1,...,N;1',...,N',t) \equiv \rho_N(\xi_1,\xi_2,...,\xi_N;\xi_1',\xi_2',...\xi_N';t) = <\xi_1',\xi_2',...,\xi_N'|\hat{\rho}_N(t)|\xi_1,\xi_2,...,\xi_N>$$

$$\rho_1(1,1',t) \equiv \rho_1(\xi_1,\xi_1',t) \equiv> \rho_{11'}(t)$$

$$\rho_2(1,2,1',2',t) \equiv \rho_2(\xi_1,\xi_2,\xi_1',\xi_2',t) \equiv> \rho_{12,1'2'}(t)$$

- Introduce a reduced density matrices $\rho_n(1...n,1'...n';t)$ by taking the trace (integrate) over particles $n+1,...N$: (tensor of rank 2n): $n<N$

$$\rho_n = \frac{1}{(N-n)!}\mathrm{Tr}_{n+1,...,N}\rho_N = \frac{1}{n+1}\mathrm{Tr}_{n+1}\{\rho_{n+1}\} \quad (3)$$

← Recurrence

Normalization: $Tr_{(1,...N)}\rho_N = N!$ such that $\mathrm{Tr}_{1=1'}\rho(11';t) = \sum_i \langle a_i^\dagger a_i\rangle = N$

$$\mathrm{Tr}_{(1,2)}\rho_2 = \sum_{i,j}\langle a_i^\dagger a_j^\dagger a_j a_i\rangle = -\sum_{i,j}\langle a_i^\dagger a_j^\dagger a_i a_j\rangle = \sum_{i,j}\{\langle a_i^\dagger a_i a_j^\dagger a_j\rangle - \langle a_i^\dagger a_j\rangle\delta_{ij}\} = (N-1)\sum_j\langle a_j^\dagger a_j\rangle = N(N-1)$$

From recurrence (3):

$$n = N-1: \quad \rho_{N-1} \equiv \frac{1}{N}Tr_{\xi_N=\xi_N'}\rho_N(\xi_1,...\xi_N,\xi_1',...,\xi_N',t)$$

$$n = N-2: \quad \rho_{N-2} \equiv \frac{1}{N-1}Tr_{\xi_{N-1}=\xi_{N-1}'}\rho_{N-1}(\xi_1,...\xi_{N-1},\xi_1',...,\xi_{N-1}',t)$$

$$Tr_{(1,...,n)}\rho_n = \frac{N!}{(N-n)!}$$

Density matrix formalism: BBGKY-Hierarchy

Taking corresponding traces (i.e. $Tr_{(n+1,\dots N)}$) of the von-Neumann equation we obtain the BBGKY-Hierarchy (Bogolyubov, Born, Green, Kirkwood and Yvon)

$$i\frac{\partial}{\partial t}\,\rho_n = [\sum_{i=1}^{n} h^0(i), \rho_n] + [\sum_{1=i\langle j}^{n-1} v(ij), \rho_n] + \sum_{i=1}^{n} \mathrm{Tr}_{n+1}[v(i, n+1), \rho_{n+1}] \qquad (4)$$

for $1 \le n \le N$ with $\rho_{N+1} = 0$.

- **This set of equations is equivalent to the von-Neumann equation**
- **The approximations or truncations of this set will reduce the information about the system**

❑ **The explicit equations for *n=1, n=2* read:**

$$i\frac{\partial}{\partial t}\,\rho_1 = [h^0(1), \rho_1] + \mathrm{Tr}_2[v(12), \rho_2], \qquad (5)$$

$$i\frac{\partial}{\partial t}\,\rho_2 = [\sum_{i=1}^{2} h^0(i), \rho_2] + [v(12), \rho_2] + \mathrm{Tr}_3[v(13) + v(23), \rho_3] \qquad (6)$$

Eqs. (5,6) are not closed since eq. (6) for ρ_2 requires information from ρ_3. Its equation reads:

$$i\frac{\partial}{\partial t}\,\rho_3 = [\sum_{i=1}^{3} h^0(i), \rho_3] + [v(12) + v(13) + v(23), \rho_3] + Tr_4[v(14) + v(24) + v(34), \rho_4] \qquad (7)$$

Density matrix formalism: BBGKY-Hierarchy

Introduce the cluster expansion → Correlation dynamics:

- 1-body density matrix: $\rho_1(11') = \rho(11'),$

1 – initial state of particle „1"
1' – final state of the same particle „1"

- 2-body density matrix (consider fermions):

(8) $$\rho_2(12,1'2') = \rho(11')\rho(22') - \rho(12')\rho(21') + c_2(12,1'2') = \rho_{20}(12,1'2') + c_2(12,1'2')$$

2PI= 2-particle-irreducible approach

(9) $$\rho_2(12,1'2') = \mathcal{A}_{12}\rho(11')\rho(22') + c_2(12,1'2')$$

2-body antisymmetrization operator:

$$\mathcal{A}_{ij} = 1 - P_{ij}$$

Permutation operator

1PI = 1-particle-irreducible approach (TDHF approximation) + 2-body correlations

By neglecting c_2 in (9) we get the limit of independent particles (Time-Dependent Hartree-Fock). This implies that all effects from collisions or correlations are incorporated in c_2 and higher orders in c_2 etc.

- 3-body density matrix:

$$\begin{aligned}\rho_3(123,1'2'3') = &\rho(11')\rho(22')\rho(33') - \rho(12')\rho(21')\rho(33')\\ &-\rho(13')\rho(22')\rho(31') - \rho(11')\rho(32')\rho(23') + \rho(13')\rho(21')\rho(32') + \rho(12')\rho(31')\rho(23')\\ &+\rho(11')c_2(23,2'3') - \rho(12')c_2(23,1'3') - \rho(13')c_2(23,2'1') + \rho(22')c_2(13,1'3')\\ &-\rho(21')c_2(13,2'3') - \rho(23')c_2(13,1'2') + \rho(33')c_2(12,1'2') - \rho(31')c_2(12,3'2')\\ &-\rho(32')c_2(12,1'3') + c_3(123,1'2'3').\end{aligned} \quad (10)$$

Correlation dynamics

- From eq. (5) for ρ_1 (by substitution of eq. (8) for ρ_2), we obtain , **EoM for the one-body density matrix**:

$$i\frac{\partial}{\partial t}\rho(11';t) = [h^0(1) - h^0(1')]\rho(11';t)$$
$$+\mathrm{Tr}_{(2=2')}[v(12)\mathcal{A}_{12} - v(1'2')\mathcal{A}_{1'2'}]\rho(11';t)\rho(22';t) + \mathrm{Tr}_{(2=2')}[v(12) - v(1'2')]c_2(12,1'2';t) \quad (11)$$

- From eq. (6) for ρ_2 (by substitution of eq. (10) for ρ_3) and discarding explicit 3-body correlations c_3, we obtain **EoM for the two-body correlation matrix c_2** :

$$i\frac{\partial}{\partial t}c_2(12,1'2';t) = [h^0(1) + h^0(2) - h^0(1') - h^0(2')]c_2(12,1'2';t) \quad (12)$$
$$+\mathrm{Tr}_{(3=3')}[v(13)\mathcal{A}_{13} + v(23)\mathcal{A}_{23} - v(1'3')\mathcal{A}_{1'3'} - v(2'3')\mathcal{A}_{2'3'}]\rho(33';t)c_2(12,1'2';t)$$
$$+[v(12) - v(1'2')]\rho_{20}(12,1'2')$$
$$-\mathrm{Tr}_{(3=3')}\{v(13)\rho(23';t)\rho_{20}(13,1'2';t) - v(1'3')\rho(32';t)\rho_{20}(12,1'3';t)$$
$$+v(23)\rho(13';t)\rho_{20}(32,1'2';t) - v(2'3')\rho(31';t)\rho_{20}(12,3'2';t)\}$$

$$\rho(11')\rho(22') - \rho(12')\rho(21') = \rho_{20}(12,1'2')$$

$$+[v(12) - v(1'2')]c_2(12,1'2';t)$$
$$-\mathrm{Tr}_{(3=3')}\{v(13)\rho(23';t)c_2(13,1'2';t) - v(1'3')\rho(32';t)c_2(12,1'3';t)$$
$$+v(23)\rho(13';t)c_2(32,1'2';t) - v(2'3')\rho(31';t)c_2(12,3'2';t)\}$$

$$+\mathrm{Tr}_{(3=3')}\{[v(13)\mathcal{A}_{13}\mathcal{A}_{1'2'} - v(1'3')\mathcal{A}_{1'3'}\mathcal{A}_{12}]\ \rho(11';t)c_2(32,3'2';t)$$
$$+[v(23)\mathcal{A}_{23}\mathcal{A}_{1'2'} - v(2'3')\mathcal{A}_{2'3'}\mathcal{A}_{12}]\ \rho(22';t)c_2(13,1'3';t)\}.$$

Correlation dynamics

To reduce the complexity we introduce:

- a **one-body Hamiltonian** by

$$h(i) = t(i) + U^s(i) = t(i) + \mathrm{Tr}_{(n=n')} v(in) \mathcal{A}_{in} \rho(nn';t), \qquad (13)$$

$$h(i') = t(i') + U^s(i') = t(i') + \mathrm{Tr}_{(n=n')} v(i'n') \mathcal{A}_{i'n'} \rho(nn';t)$$

kinetic term + interaction with the self-generated time-dependent mean field

- **Pauli-blocking operator** is uniquely defined by (14)

$$Q_{ij}^{=} = 1 - \mathrm{Tr}_{(n=n')} (P_{in} + P_{jn}) \rho(nn';t); \qquad Q_{i'j'}^{=} = 1 - \mathrm{Tr}_{(n=n')} (P_{i'n'} + P_{j'n'}) \rho(nn';t),$$

- **Effective 2-body interaction in the medium:**

$$V^{=}(ij) = Q_{ij}^{=} v(ij); \qquad V^{=}(i'j') = Q_{i'j'}^{=} v(i'j'), \qquad (15)$$

Resummed interaction ➔ G-matrix approach

Correlation dynamics

❑* EoM for the **one-body density matrix**:

$$i\frac{\partial}{\partial t}\rho(11';t) = [h(1)-h(1')]\rho(11';t) + \mathrm{Tr}_{(2=2')}[v(12)-v(1'2')]c_2(12,1'2';t) \quad (16)$$

TDHF — 2-body correlations

EoM (16) describes the propagation of a particle in the self-generated mean field $U^s(i)$ with additional 2-body correlations that are further specified in the EoM (17) for c_2 :

❑* EoM for the **2-body correlation matrix**:

$$\begin{aligned}
i\frac{\partial}{\partial t}c_2(12,1'2';t) &= [\sum_{i=1}^{2}h(i)-\sum_{i'=1'}^{2'}h(i')]c_2(12,1'2';t) \\
&+[V^{=}(12)-V^{=}(1'2')]\rho_{20}(12,1'2';t) \\
&+[V^{=}(12)-V^{=}(1'2')]c_2(12,1'2';t) \\
&+\mathrm{Tr}_{(3=3')}\{[v(13)\mathcal{A}_{13}\mathcal{A}_{1'2'}-v(1'3')\mathcal{A}_{1'3'}\mathcal{A}_{12}]\ \rho(11';t)c_2(32,3'2';t) \\
&+[v(23)\mathcal{A}_{23}\mathcal{A}_{1'2'}-v(2'3')\mathcal{A}_{2'3'}\mathcal{A}_{12}]\ \rho(22';t)c_2(13,1'3';t)\}.
\end{aligned} \quad (17)$$

- Propagation of two particles 1 and 2 in the mean field U^s
- Born term: bare 2-body scattering
- resummed in-medium interaction with intermediate Pauli blocking (G-matrix theory)
- 2-Particle-2-hole interactions (important for groundstate correlations) and damping of low energy modes

Note: Time evolution of c_2 depends on the distribution of a third particle, which is integrated out in the trace! The third particle is interacting as well!

*: EoM is obtained after the ‚cluster' expansion and neglecting the explicit 3-body correlations c_3

Vlasov equation

BBGKY-Hierarchie - 1PI →
eq.(11) with $c_2(1,2,1',2')=0$

$$i\hbar\frac{\partial}{\partial t}\rho(11';t) = [h^0(1) - h^0(1')]\rho(11';t)$$

TDHF

$$\frac{\partial}{\partial t}\rho(\vec{r},\vec{r}',t) + \frac{i}{\hbar}\left[\frac{\hbar^2}{2m}\vec{\nabla}_r^2 + U(\vec{r},t) - \frac{\hbar^2}{2m}\vec{\nabla}_{r'}^2 - U(\vec{r}',t)\right]\rho(\vec{r},\vec{r}',t) = 0$$

- **perform Wigner transformation of one-body density distribution function ρ(r,r',t)→**

$$f(\vec{r},\vec{p},t) = \int d^3s\ \exp\left(-\frac{i}{\hbar}\vec{p}\vec{s}\right)\rho\left(\vec{r}+\frac{\vec{s}}{2},\vec{r}-\frac{\vec{s}}{2},t\right) \qquad (18)$$

***f(r,p,t)* is the single particle phase-space distribution function**

After the 1st order gradient expansion → Vlasov equation of motion
- free propagation of particles in the self-generated HF mean-field potential *U(r,t)*:

$$\frac{\partial}{\partial t}f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\ f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}}U(\vec{r},t)\ \vec{\nabla}_{\vec{p}}f(\vec{r},\vec{p},t) = 0 \qquad (19)$$

$$U(\vec{r},t) = \frac{1}{(2\pi\hbar)^3}\sum_{\beta_{occ}}\int d^3r' d^3p\, V(\vec{r}-\vec{r}',t)f(\vec{r}',\vec{p},t)$$

Uehling-Uhlenbeck equation: collision term

$$i\frac{\partial}{\partial t}\rho(11';t) = [h(1) - h(1')]\rho(11';t) + \mathrm{Tr}_{(2=2')}[v(12) - v(1'2')]c_2(12, 1'2';t) \qquad (21)$$

TDHF – Vlasov equation

2-body correlations

Collision term:

$$I(11',t) = \mathrm{Tr}_{(2=2')}[v(12) - v(1'2')]c_2(12, 1'2';t) \qquad (22)$$

- perform Wigner transformation
- Formally solve the EoM for c_2 (with some approximations in momentum space):

$$\begin{aligned} i\frac{\partial}{\partial t}c_2(12, 1'2';t) &= [\sum_{i=1}^{2} h(i) - \sum_{i'=1'}^{2'} h(i')]c_2(12, 1'2';t) \\ &+[V^{=}(12) - V^{=}(1'2')]\rho_{20}(12, 1'2';t) \\ &+[V^{=}(12) - V^{=}(1'2')]c_2(12, 1'2';t) \\ &+\mathrm{Tr}_{(3=3')}\{[v(13)\mathcal{A}_{13}\mathcal{A}_{1'2'} - v(1'3')\mathcal{A}_{1'3'}\mathcal{A}_{12}]\ \rho(11';t)c_2(32, 3'2';t) \\ &+[v(23)\mathcal{A}_{23}\mathcal{A}_{1'2'} - v(2'3')\mathcal{A}_{2'3'}\mathcal{A}_{12}]\ \rho(22';t)c_2(13, 1'3';t)\}. \end{aligned} \qquad (23)$$

- and insert obtained c_2 in the expression (22) for *I(11´,t)* : → BUU EoM

Boltzmann (Vlasov)-Uehling-Uhlenbeck (B(V)UU) equation : Collision term

$$\frac{d}{dt} f(\vec{r},\vec{p},t) \equiv \frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\, f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\, \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll} \qquad (24)$$

Collision term for 1+2→3+4 (let‘s consider fermions) :

$$I_{coll} = \frac{4}{(2\pi)^3} \int d^3p_2\, d^3p_3 \int d\Omega\ |v_{12}|\, \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \cdot \frac{d\sigma}{d\Omega}(1+2 \rightarrow 3+4) \cdot P \qquad (25)$$

Probability including Pauli blocking of fermions:

$$P = f_3 f_4 (1-f_1)(1-f_2) - f_1 f_2 (1-f_3)(1-f_4) \qquad (26)$$

Gain term ($f_3 f_4 (1-f_1)(1-f_2)$): 3+4→1+2

Loss term ($f_1 f_2 (1-f_3)(1-f_4)$): 1+2→3+4

For particle 1 and 2:
Collision term = Gain term – Loss term

$$I_{coll} = G - L$$

The BUU equations (24) describes the propagation in the self-generated mean-field U(r,t) as well as mutual two-body interactions respecting the Pauli-principle

3. Quantum field theory

➔ Kadanoff-Baym dynamics

➔ generalized off-shell transport equations

Theoretical description of 'in-medium effects'

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium
Example: hadronic medium - vector mesons, strange mesons
QGP – dressing of partons

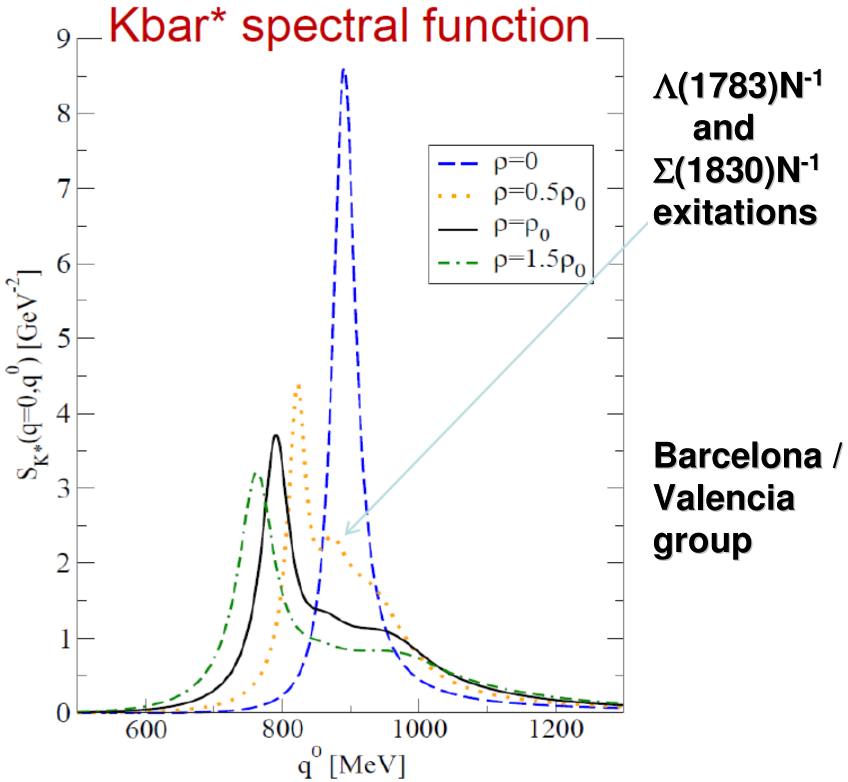
Many-body theory:
Strong interaction ➔ **large width** = short life-time
➔ broad spectral function ➔ **quantum object**

▪ **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

- ❑ semi-classical BUU

first order gradient expansion of quantum Kadanoff-Baym equations

- ❑ generalized transport equations



Dynamical description of strongly interacting systems

- **Semi-classical on-shell BUU:** applies for small collisional width, i.e. for a weakly interacting systems of particles

How to describe strongly interacting systems?!

- **Quantum field theory** ➔
Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

$$\hat{S}_{0x}^{-1}\, S_{xy}^{<} \;=\; \Sigma_{xz}^{ret} \odot S_{zy}^{<} \;+\; \Sigma_{xz}^{<} \odot S_{zy}^{adv}$$

(1962)

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ:

$$iS_{xy}^{<} = \eta \langle \{ \Phi^{+}(y)\Phi(x) \} \rangle$$

$$iS_{xy}^{>} = \langle \{ \Phi(y)\Phi^{+}(x) \} \rangle$$

$$iS_{xy}^{c} = \langle T^{c} \{ \Phi(x)\Phi^{+}(y) \} \rangle \quad - causal$$

$$iS_{xy}^{a} = \langle T^{a} \{ \Phi(x)\Phi^{+}(y) \} \rangle \quad - anticausal$$

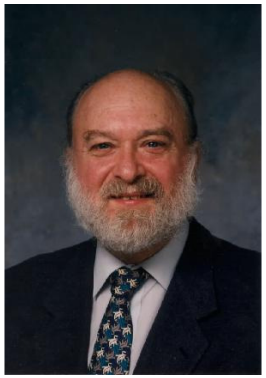
$$S_{xy}^{ret} = S_{xy}^{c} - S_{xy}^{<} = S_{xy}^{>} - S_{xy}^{a} \quad - retarded$$

$$S_{xy}^{adv} = S_{xy}^{c} - S_{xy}^{>} = S_{xy}^{<} - S_{xy}^{a} \quad - advanced$$

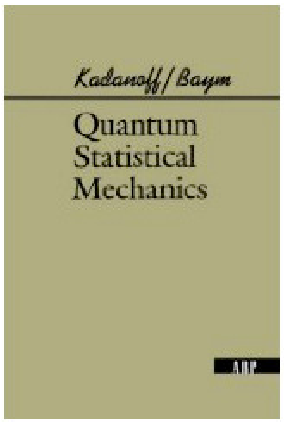
$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^{\mu}\partial_{\mu}^{x} + M_0^2)$$

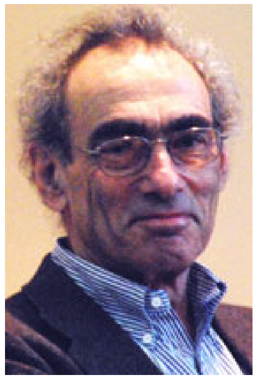
$$\eta = \pm 1 (bosons \,/\, fermions)$$

$$T^{a}(T^{c}) - (anti-)time - ordering \ operator$$



Leo Kadanoff





Gordon Baym

Wigner transformation of the Kadanoff-Baym equation

➢ do **Wigner transformation** of the Kadanoff-Baym equation

$$F_{XP} = \int d^4(x-y)\ e^{iP_\mu(x^\mu - y^\mu)}\ F_{xy}$$

For any function F_{XY} with X=(*x*+y)/2 – space-time coordinate, P – 4-momentum

Convolution integrals convert under Wigner transformation as

$$\int d^4(x-y)\ e^{iP_\mu(x^\mu - y^\mu)}\ F_{1,xz} \odot F_{2,zy} = e^{-i\diamond}\ F_{1,PX}\ F_{2,PX}$$

an infinite series in the differential operator ◇

Operator ◇ is a 4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

➢ consider only contribution up to **first order in the gradients**

= a standard approximation of kinetic theory which is justified if the gradients in the mean spacial coordinate X are small

From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term | Vlasov term | backflow term | collision term = ‚gain' - ‚loss' term

$$\diamond\{P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\}\{S^<_{XP}\} - \diamond\{\Sigma^<_{XP}\}\{ReS^{ret}_{XP}\} = \frac{i}{2}\left[\Sigma^>_{XP}\, S^<_{XP} - \Sigma^<_{XP}\, S^>_{XP}\right]$$

Backflow term incorporates the off-shell behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2-M^2)$

- GTE: Propagation of the Green's function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})

- Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

$\Gamma_{XP} = -Im\Sigma^{ret}_{XP} = 2p_0\Gamma$ – ‚width' of spectral function
= reaction rate of particle (at space-time position X)

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

- Life time $\tau = \frac{\hbar c}{\Gamma}$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

- Employ testparticle Ansatz for the real valued quantity $i\, S^{<}_{XP}$ -

$$F_{XP} = A_{XP} N_{XP} = i\, S^{<}_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

➔ **General testparticle ‚Cassing off-shell equations of motion'**
for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime' of particle (i) !

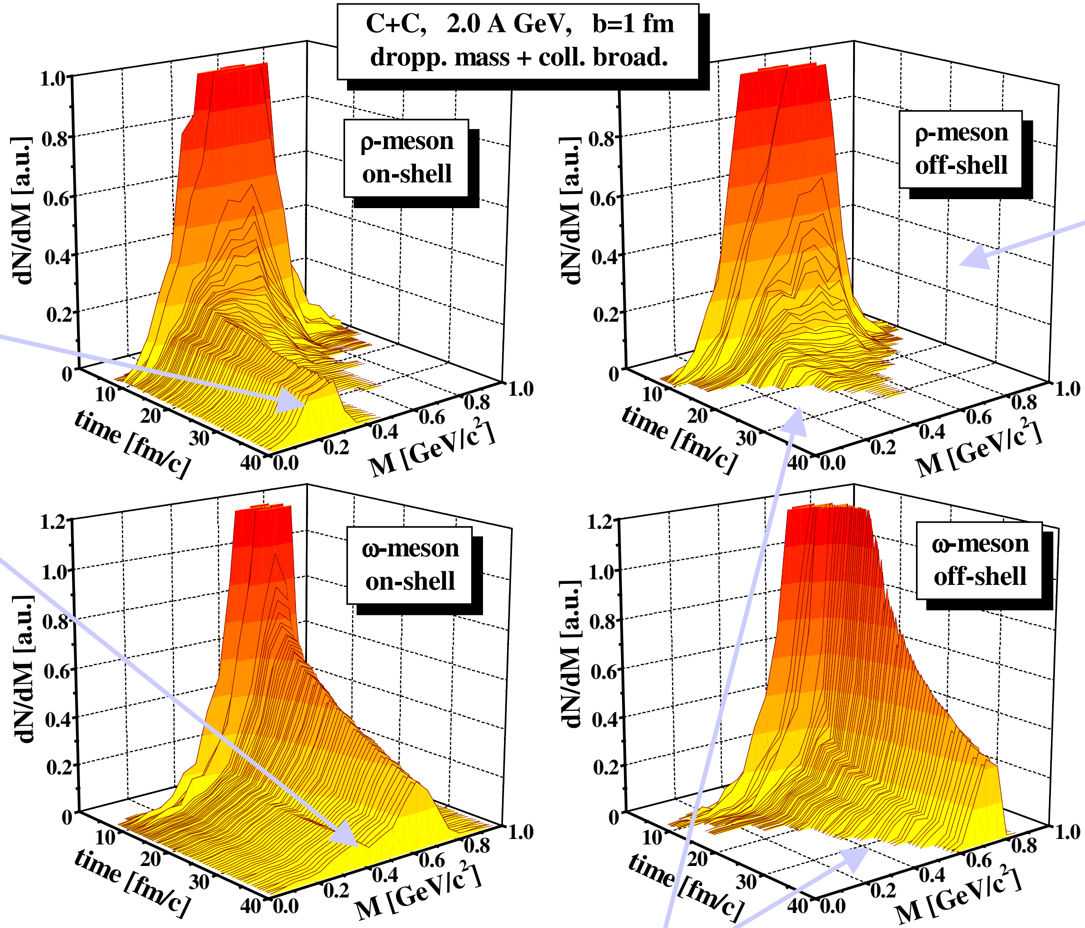
Off-shell vs. on-shell transport dynamics

Time evolution of the mass distribution of ρ and ω mesons for central C+C collisions (b=1 fm) at 2 A GeV for dropping mass + collisional broadening scenario

E.L.B. &W. Cassing, NPA 807 (2008) 214

On-shell

On-shell model: low mass ρ and ω mesons live forever (and e.g. shine fake dileptons)!



Off-shell

The off-shell spectral function becomes on-shell in the vacuum dynamically by propagation through the medium!

Collision term in off-shell transport models

Collision term for reaction 1+2->3+4:

$$I_{coll}(X,\vec{P},M^2) = Tr_2 Tr_3 Tr_4 A(X,\vec{P},M^2) A(X,\vec{P}_2,M_2^2) A(X,\vec{P}_3,M_3^2) A(X,\vec{P}_4,M_4^2)$$

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2) \rightarrow (\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}} \; \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[\,N_{X\vec{P}_3M_3^2}\, N_{X\vec{P}_4M_4^2}\, \bar{f}_{X\vec{P}M^2}\, \bar{f}_{X\vec{P}_2M_2^2} \;-\; N_{X\vec{P}M^2}\, N_{X\vec{P}_2M_2^2}\, \bar{f}_{X\vec{P}_3M_3^2}\, \bar{f}_{X\vec{P}_4M_4^2}\,]$$

'gain' term — 'loss' term

with $\bar{f}_{X\vec{P}M^2} = 1 + \eta\, N_{X\vec{P}M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dP_{0,2}^2}{2}$$

additional integration

The transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!

In-medium transition rates: G-matrix approach

Need to know in-medium transition amplitudes G and their off-shell dependence

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2)\rightarrow(\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}}$$

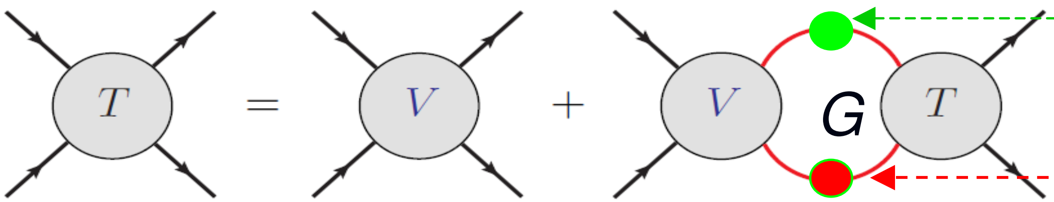
↓

Coupled channel G-matrix approach

Transition probability :

$$P_{1+2\rightarrow3+4}(s)=\int d\cos(\theta)\;\frac{1}{(2s_1+1)(2s_2+1)}\sum_i\sum_\alpha\;G^\dagger G$$

with G(p,ρ,T) - G-matrix from the solution of coupled-channel equations:



Meson selfenergy and spectral function

Baryons: Pauli blocking and potential dressing

- $\mathcal{T}_{ij}(\rho,T) \;=\; V_{ij} \;+\; V_{il}\;G_l(\rho,T)\;\mathcal{T}_{lj}(\rho,T)$

For strangeness:
D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC C90 (2014) 055207; W. Cassing, L. Tolos, E.B., A. Ramos, NPA727 (2003) 59

Mean-field potential in off-shell transport models

- **Many-body theory:** Interacting relativistic particles have a **complex self-energy:**

$$\Sigma_{XP}^{ret} = Re\Sigma_{XP}^{ret} + i\, Im\Sigma_{XP}^{ret}$$

The neg. imaginary part $\Gamma_{XP} = -Im\Sigma_{XP}^{ret} = 2p_0\Gamma$ is related via $\Gamma = \Gamma_{coll} + \Gamma_{dec}$ to the inverse livetime of the particle $\tau \sim 1/\Gamma$.

- The **collision width** Γ_{coll} is determined from the **loss term** of the collision integral I_{coll}

$$-I_{coll}(loss) = \Gamma_{coll}(X,\vec{P},M^2)N_{X\vec{P}M^2}$$

- By **dispersion relation** we get a contribution to the **real part of self-energy**:

$$Re\Sigma_{XP}^{ret}(p_0) = P\int_0^{\infty} dq \frac{Im\Sigma_{XP}^{ret}(q)}{(q-p_0)}$$

which gives **a mean-field potential U_{XP}** via:

$$Re\Sigma_{XP}^{ret}(p_0) = 2p_0 U_{XP}$$

→ The **complex self-energy** relates in a self-consistent way to the self-generated mean-field potential and collision width (inverse lifetime)

Detailed balance on the level of 2<->n: treatment of multi-particle collisions in transport approaches

W. Cassing, NPA 700 (2002) 618

Generalized collision integral for *n <->m* reactions:

$$I_{coll} = \sum_n \sum_m I_{coll}[n \leftrightarrow m]$$

$$I^i_{coll}[n \leftrightarrow m] =$$
$$\frac{1}{2} N^m_n \sum_\nu \sum_\lambda \left(\frac{1}{(2\pi)^4}\right)^{n+m-1} \int \left(\prod_{j=2}^{n} d^4p_j \;\; A_j(x,p_j)\right) \left(\prod_{k=1}^{m} d^4p_k \; A_k(x,p_k)\right)$$
$$\times A_i(x,p) \; W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda) \; (2\pi)^4 \; \delta^4(p^\mu + \sum_{j=2}^{n} p_j^\mu - \sum_{k=1}^{m} p_k^\mu)$$
$$\times [\tilde{f}_i(x,p) \prod_{k=1}^{m} f_k(x,p_k) \prod_{j=2}^{n} \tilde{f}_j(x,p_j) - f_i(x,p) \prod_{j=2}^{n} f_j(x,p_j) \prod_{k=1}^{m} \tilde{f}_k(x,p_k)].$$

$\tilde{f} = 1 + \eta f$ **is Pauli-blocking or Bose-enhancement factors;**
η=1 for bosons and η=-1 for fermions

$W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda)$ **is a transition probability**

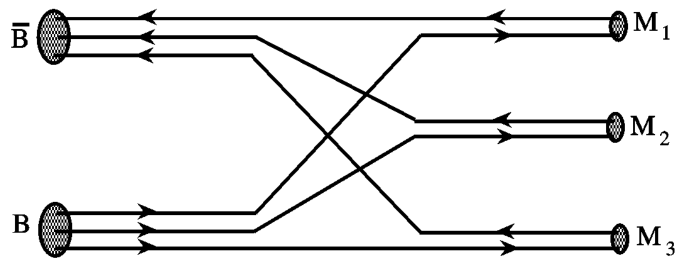
Antibaryon production in heavy-ion reactions

Multi-meson fusion reactions

$m_1+m_2+...+m_n \leftrightarrow B+\bar{B}$

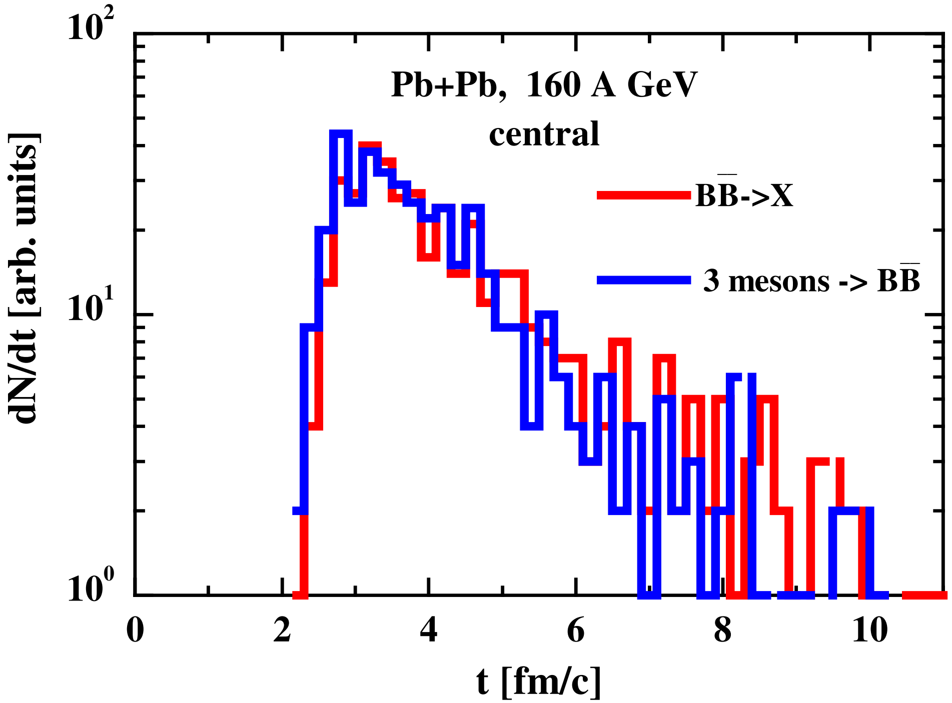
$(m=\pi,\rho,\omega,..)$

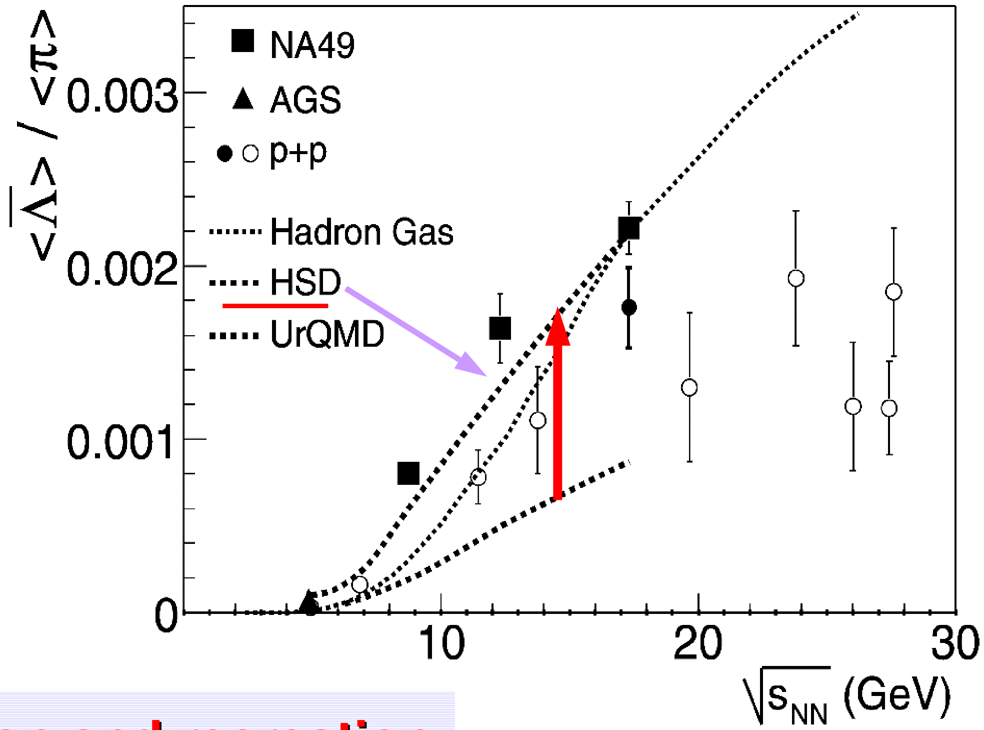
W. Cassing, NPA 700 (2002) 618



2<->3

- important for anti-proton, anti-lambda, anti-Xi, anti-Omega dynamics !



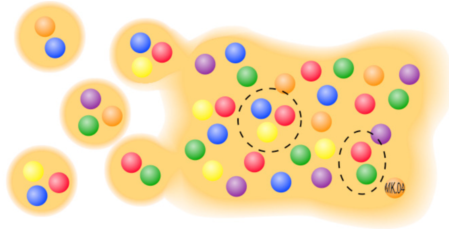


→ approximate equilibrium of annihilation and recreation

4. Microscopic description of the QGP, hadronization problem

5. Example of transport model: PHSD transport approach

Goal: microscopic transport description of the partonic and hadronic phase



Problems:

- How to model a QGP phase in line with lQCD data?
- How to solve the hadronization problem?

Ways to go:

pQCD based models:

- QGP phase: pQCD cascade
- hadronization: quark coalescence

➔ AMPT, HIJING, BAMPS

‚Hybrid' models:

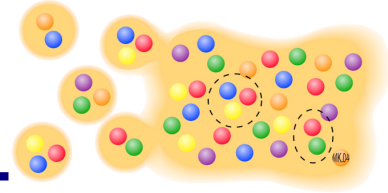
- QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

➔ Hybrid-UrQMD

- microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

➔ PHSD

From SIS to LHC: from hadrons to partons



The goal: to study of the phase transition from hadronic to partonic matter and properties of the Quark-Gluon-Plasma from a microscopic origin

➔ need a consistent non-equilibrium transport model

- with explicit parton-parton interactions (i.e. between quarks and gluons)
- explicit phase transition from hadronic to partonic degrees of freedom
- lQCD EoS for partonic phase (‚crossover' at μ_q=0)

- Transport theory: off-shell Kadanoff-Baym equations for the Green-functions $S^<_h(x,p)$ in phase-space representation for the partonic and hadronic phase

⇨ **Parton-Hadron-String-Dynamics (PHSD)**

QGP phase described by

Dynamical QuasiParticle Model (DQPM)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Dynamical QuasiParticle Model (DQPM) - Basic ideas:

DQPM describes QCD properties in terms of ‚resummed' single-particle Green's functions – in the sense of a two-particle irreducible (2PI) approach:

Gluon propagator: $\Delta^{-1} = P^2 - \Pi$ gluon self-energy: $\Pi = M_g^2 - i2\Gamma_g\omega$

Quark propagator: $S_q^{-1} = P^2 - \Sigma_q$ quark self-energy: $\Sigma_q = M_q^2 - i2\Gamma_q\omega$

- the resummed properties are specified by complex self-energies which depend on temperature:
  -- the real part of self-energies (Σ_q, Π) describes a dynamically generated mass (M_q,M_g);
  -- the imaginary part describes the interaction width of partons (Γ_q, Γ_g)
- space-like part of energy-momentum tensor $T_{\mu\nu}$ defines the potential energy density and the mean-field potential (1PI) for quarks and gluons (U_q, U_g)
- 2PI framework guaranties a consistent description of the system in- and out-off equilibrium on the basis of Kadanoff-Baym equations with proper states in equilibrium

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

The Dynamical QuasiParticle Model (DQPM)

- **Basic idea: interacting quasi-particles: massive quarks and gluons (g, q, q_{bar}) with Lorentzian spectral functions :**

$$\rho_i(\omega,T)=\frac{4\omega\Gamma_i(T)}{\left(\omega^2-\vec{p}^2-M_i^2(T)\right)^2+4\omega^2\Gamma_i^2(T)}$$

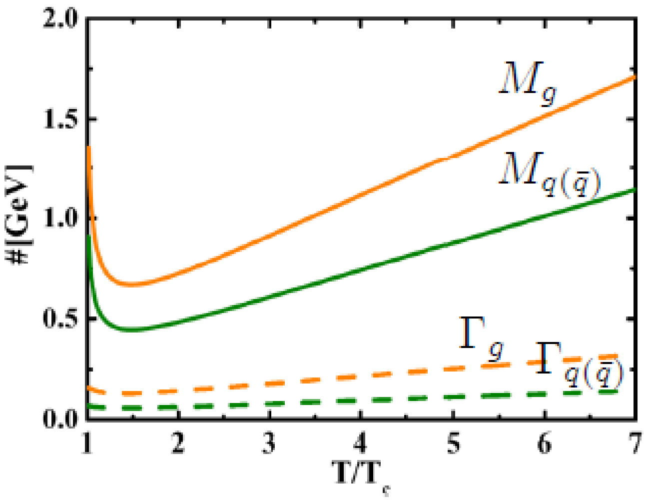
$(i=q,\bar{q},g)$

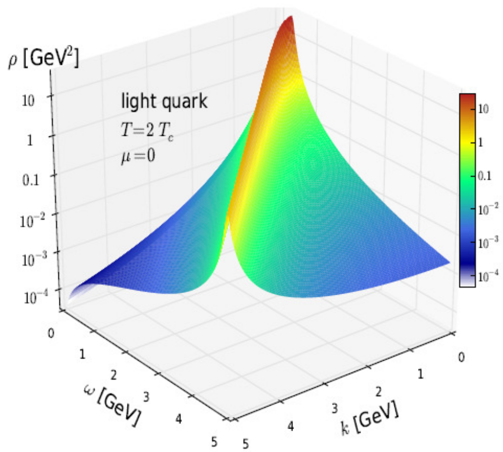
- **fit to lattice (lQCD) results**

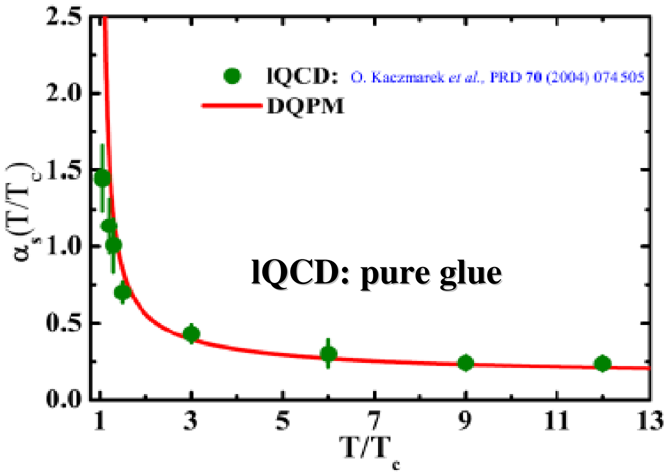
(e.g. entropy density) with 3 parameters

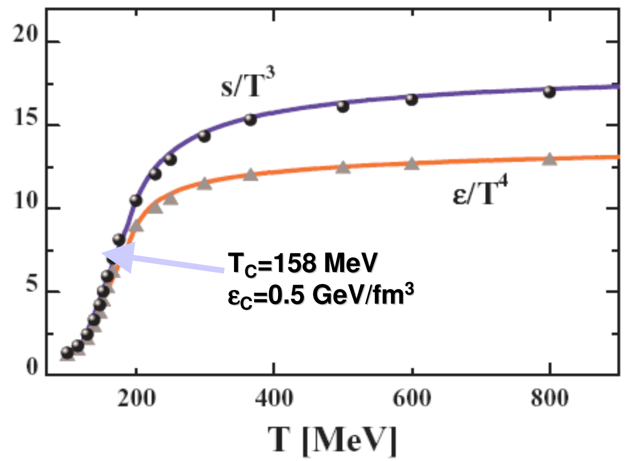
➔ **Quasi-particle properties:**

large width and mass for gluons and quarks





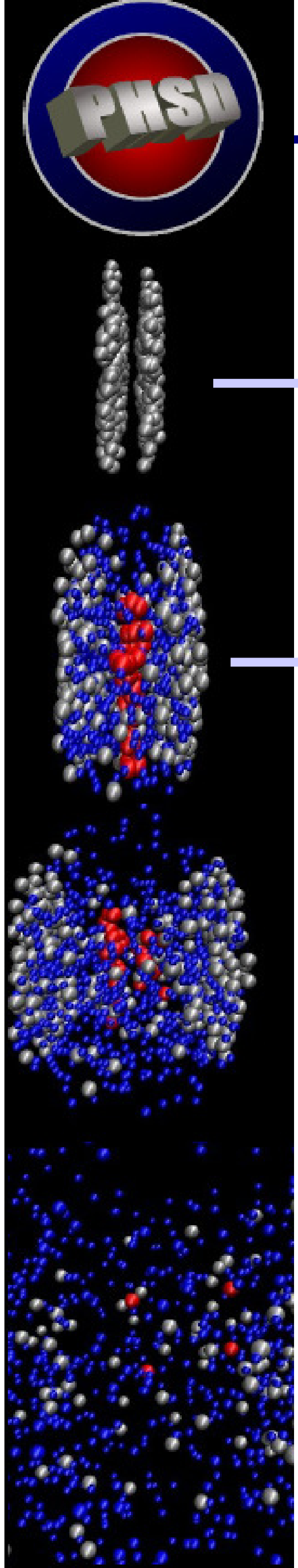




- **DQPM provides mean-fields (1PI) for gluons and quarks as well as effective 2-body interactions (2PI)**
- **DQPM gives transition rates for the formation of hadrons → PHSD**

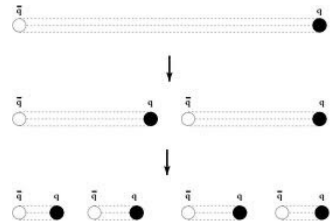
DQPM: Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

I. PHSD - basic concept



I. From hadrons to QGP:

LUND string model



- **Initial A+A collisions – as in HSD:**
  - string formation in primary NN collisions
  - string decay to pre-hadrons (*B* - baryons, *m* - mesons)

- **Formation of QGP stage by dissolution of pre-hadrons**
  (all new produced secondary hadrons)
  into massive colored quarks + mean-field energy

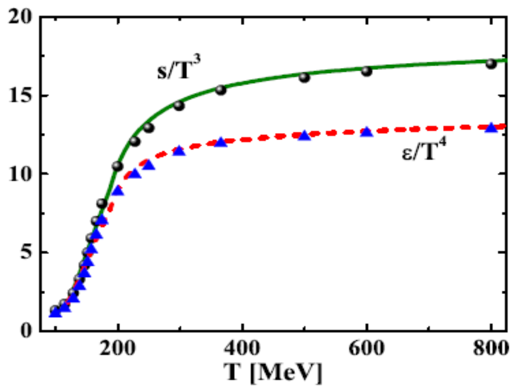
QGP phase:
$\varepsilon > \varepsilon_{critical}$

$$B \rightarrow qqq,\ m \rightarrow q\bar{q} \quad \forall \quad U_q$$

based on the Dynamical Quasi-Particle Model (DQPM) which defines quark spectral functions, i.e. masses $M_q(\varepsilon)$ and widths $\Gamma_q(\varepsilon)$

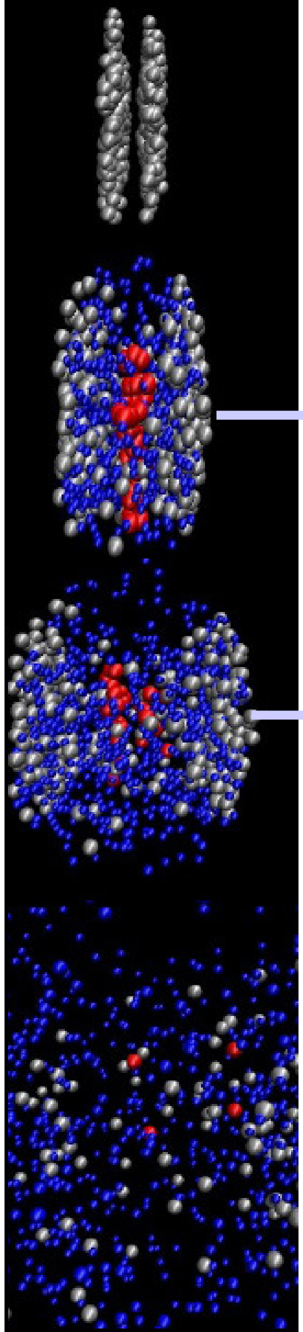
+ mean-field potential U_q at given ε – local energy density

(ε related by lQCD EoS to T - temperature in the local cell)



W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; EPJ ST 168 (2009) 3; NPA856 (2011) 162.

II. PHSD - basic concept



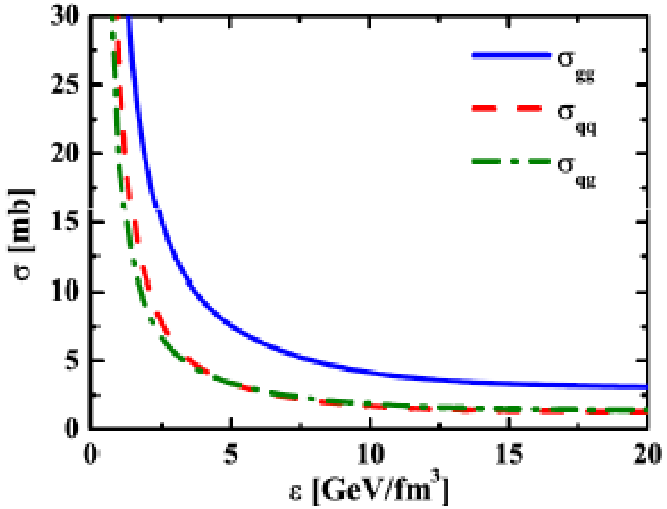
II. Partonic phase - QGP:

quarks and gluons (= ‚dynamical quasiparticles‘)
with off-shell spectral functions (width, mass) defined by the DQPM

- in self-generated mean-field potential for quarks and gluons U_q, U_g from the DQPM
- EoS of partonic phase: ‚crossover‘ from lattice QCD (fitted by DQPM)
- (quasi-) elastic and inelastic parton-parton interactions: using the effective cross sections from the DQPM

- (quasi-) elastic collisions:

$$q+q \to q+q \qquad g+q \to g+q$$
$$q+\bar{q} \to q+\bar{q} \qquad g+\bar{q} \to g+\bar{q}$$
$$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \qquad g+g \to g+g$$

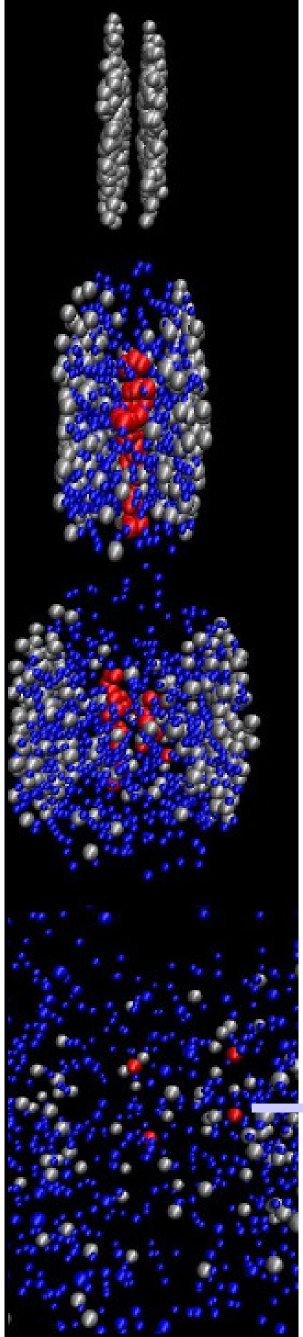


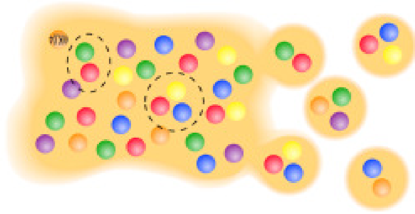
- inelastic collisions:
(Breight-Wigner cross sections)

$$\left\{ \begin{array}{l} q+\bar{q} \to g \\ g \to q+\bar{q} \end{array} \right.$$

$$\left. \begin{array}{l} q+\bar{q} \to g+g \\ g \to g+g \end{array} \right\}$$ suppressed (<1%) due to the large mass of gluons

III. PHSD - basic concept

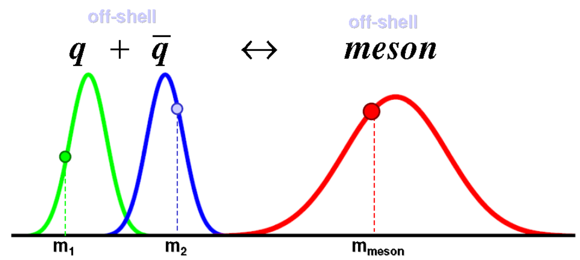




III. Hadronization:

Hadronization: based on DQPM
- massive, off-shell (anti-)quarks with broad spectral functions hadronize to off-shell mesons and baryons or color neutral excited states - ‚strings' (strings act as ‚doorway states' for hadrons)

$$g \to q+\bar{q}, \qquad q+\bar{q} \leftrightarrow meson \ ('\,string\,')$$
$$q+q+q \leftrightarrow baryon \ ('\,string\,')$$



- **Local covariant off-shell transition rate for q+qbar fusion**
  ➔ **meson formation:**

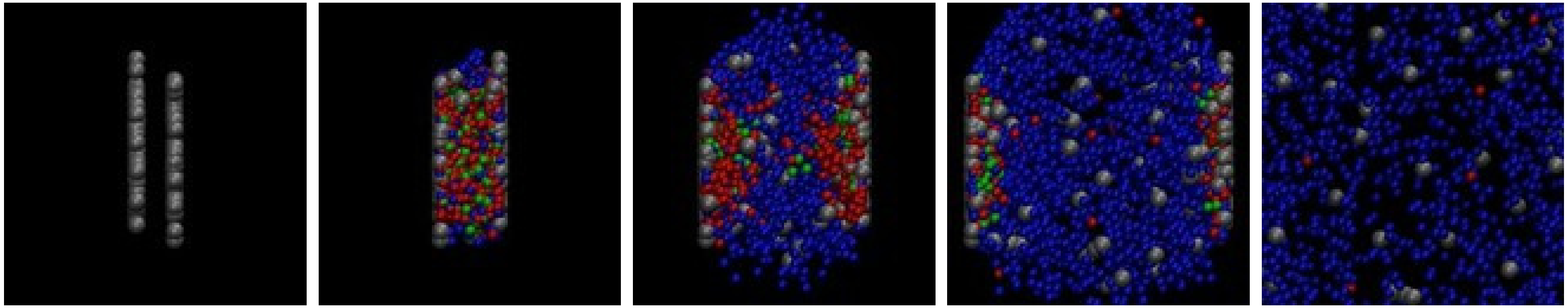
$$Tr_j = \sum_j \int d^4x_j d^4p_j/(2\pi)^4$$

$$\frac{dN^{q+\bar{q}\to m}}{d^4x\,d^4p} = Tr_q Tr_{\bar{q}}\,\delta^4(p-p_q-p_{\bar{q}})\,\delta^4\!\left(\frac{x_q+x_{\bar{q}}}{2}-x\right)\delta(flavor,color)$$
$$\cdot N_q(x_q,p_q)N_{\bar{q}}(x_{\bar{q}},p_{\bar{q}})\cdot\omega_q\rho_q(p_q)\cdot\omega_{\bar{q}}\rho_{\bar{q}}(p_{\bar{q}})\cdot|M_{q\bar{q}}|^2\,W_m\left(x_q-x_{\bar{q}},p_q-p_{\bar{q}}\right)$$

- $N_j(x,p)$ is the phase-space density of parton j at space-time position *x* and 4-momentum *p*
- W_m is the phase-space distribution of the formed ‚pre-hadrons' (Gaussian in phase space)
- $|M_{qq}|^2$ is the effective quark-antiquark interaction from the DQPM

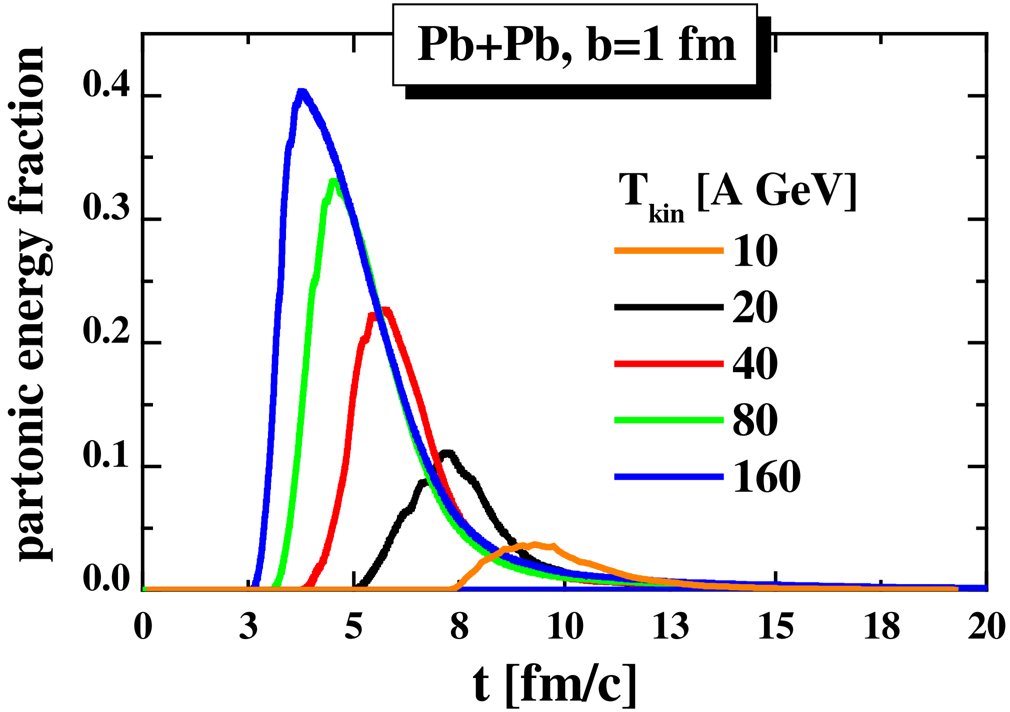
IV. Hadronic phase: hadron-string interactions – off-shell HSD

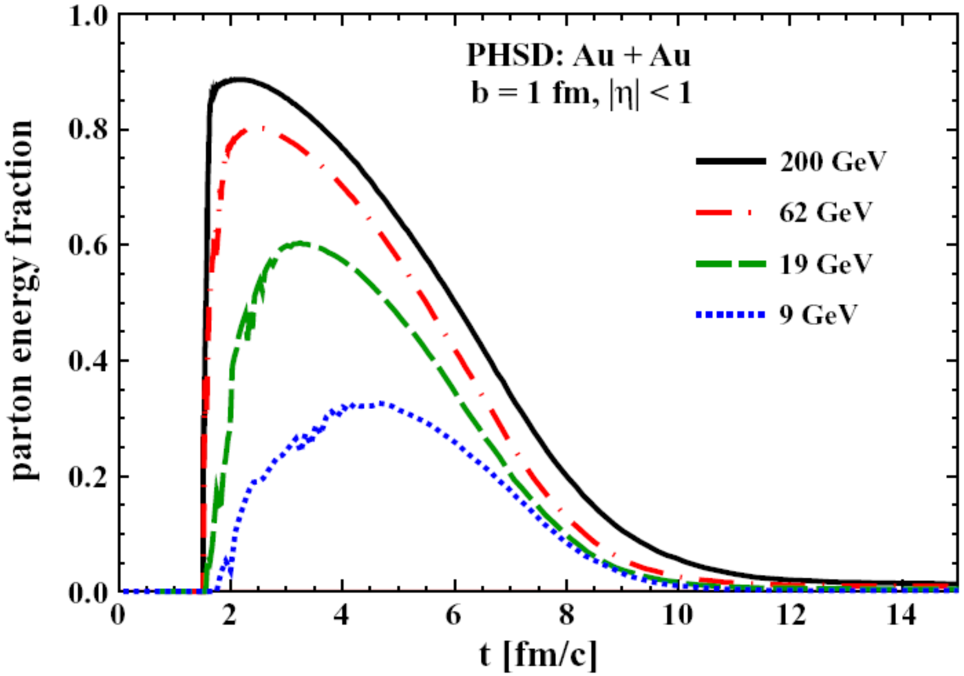
‚Bulk' properties in Au+Au



Partonic energy fraction in central A+A

Time evolution of the partonic energy fraction vs energy





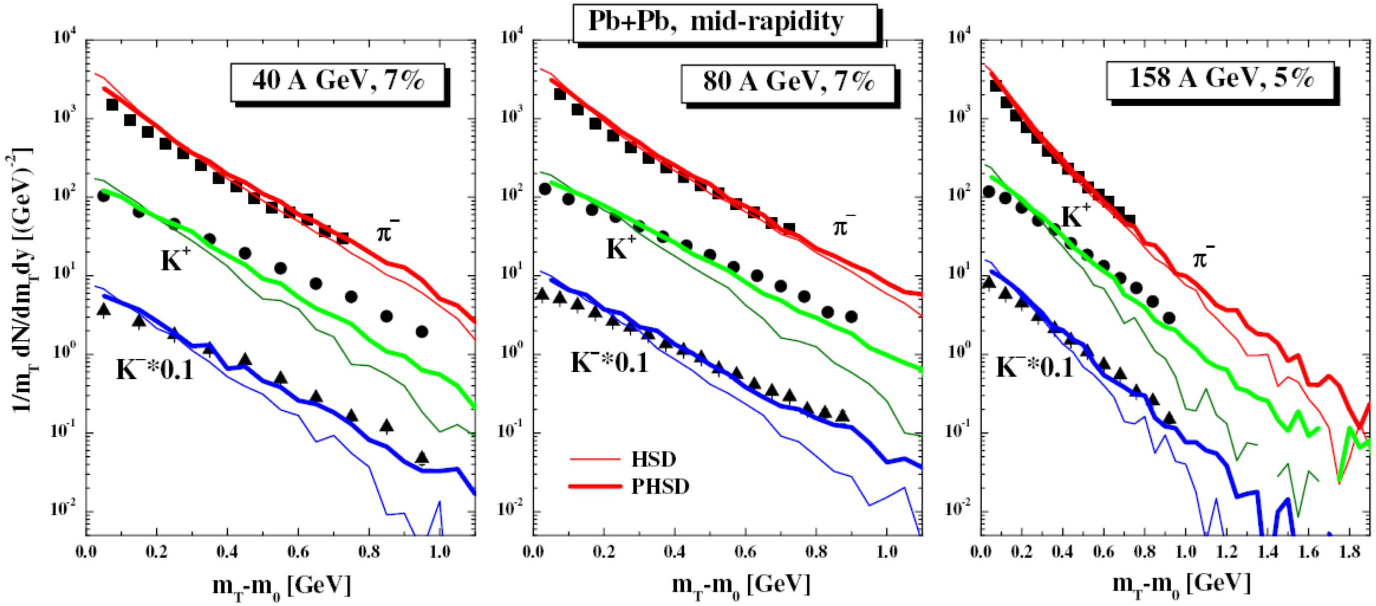
- Strong increase of partonic phase with energy from AGS to RHIC

- SPS: Pb+Pb, 160 A GeV: only about 40% of the converted energy goes to partons; the rest is contained in the large hadronic corona and leading partons
- RHIC: Au+Au, 21.3 A TeV: up to 90% - QGP at midrapidity

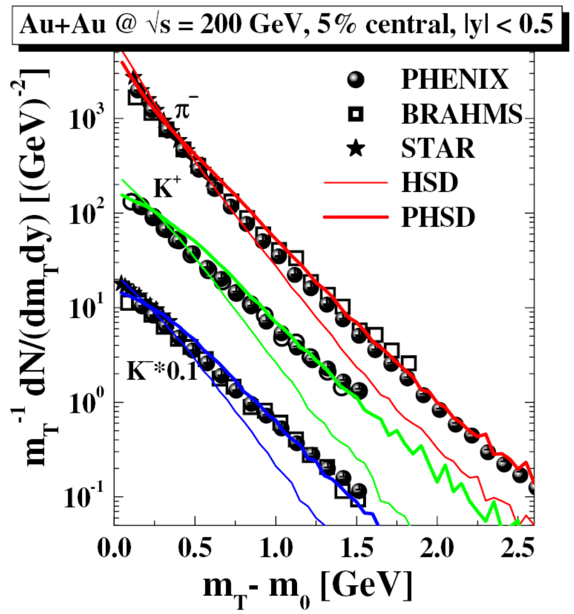
W. Cassing & E. Bratkovskaya, NPA 831 (2009) 215
V. Konchakovski et al., Phys. Rev. C 85 (2012) 011902

Transverse mass spectra from SPS to RHIC

Central Pb + Pb at SPS energies



Central Au+Au at RHIC



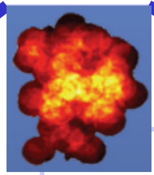
- PHSD gives harder m_T spectra and works better than HSD (wo QGP) at high energies – RHIC, SPS (and top FAIR, NICA)
- however, at low SPS (and low FAIR, NICA) energies the effect of the partonic phase decreases due to the decrease of the partonic fraction

W. Cassing & E. Bratkovskaya, NPA 831 (2009) 215
E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk, NPA856 (2011) 162

Dynamical models for HIC

Macroscopic

hydro-models:
- description of QGP and hadronic phase by hydrodanamical equations for fluid
- assumption of local equilibrium
- EoS with phase transition from QGP to HG
- initial conditions (e-b-e, fluctuating)



ideal
(Jyväskylä,SHASTA, TAMU, ...)

viscous
(Romachkke,(2+1)D VISH2+1, (3+1)D MUSIC,...)

fireball models:
- no explicit dynamics: parametrized time evolution (TAMU)

‚Hybrid'
QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

(‚hybrid'-UrQMD, EPOS, ...)

Microscopic

Non-equilibrium microscopic transport models – based on many-body theory

Hadron-string models
(UrQMD, IQMD, HSD, QGSM ...)

Partonic cascades
pQCD based
(Duke, BAMPS, ...)

Parton-hadron models:
- QGP: pQCD based cascade
- massless q, g
- hadronization: coalescence

(AMPT, HIJING)

PHSD
- QGP: lQCD EoS
- massive quasi-particles (q and g with spectral functions) in self-generated mean-field
- dynamical hadronization
- HG: off-shell dynamics

(applicable for strongly interacting systems)

Useful literature

L. P. Kadanoff, G. Baym, ‚*Quantum Statistical Mechanics*‘, Benjamin, 1962

M. Bonitz, ‚*Quantum kinetic theory*‘, B.G. Teubner Stuttgart, 1998

S.J. Wang and W. Cassing, Annals Phys. 159 (1985) 328

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006;
Nucl. Phys. A 743 (2004) 92

W. Cassing, Eur. Phys. J. ST 168 (2009) 3

W. Botermans and R. Malfliet, Phys. Rep. 198 (1990) 115

J. Berges, Phys.Rev.D7 (2006) 045022; AIP Conf. Proc. 739 (2005) 3

C.S. Fischer, J.Phys.G32 (2006) R253