Elements of QFT in Curved Space-Time

Ilya L. Shapiro

Universidade Federal de Juiz de Fora, MG, Brazil

JINR & Dubna University, February 2016

Lecture 3. Conformal anomaly and effective action

- **Some examples of** $4d$ **conformal theories.**
- **Conformal anomaly and its ambiguities.**
- **Anomaly induced effective action.**
- **Light massive fields case.**
- **Applications:**
  Vacuum states near black holes and Starobinsky model.

Examples of *4d* conformal theories

- **General scalar action with ξ term**

$$S_{scal} = \int d^4x \sqrt{-g}\, \left\{ \frac{1}{2}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\,\xi R\phi^2 - \frac{f}{4!}\,\phi^4 \right\}$$

is invariant under global but not local conformal transformation.

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu}e^{2\lambda}\,, \qquad \phi \to \phi' = \phi\, e^{-\lambda}\,, \qquad \lambda = \textbf{const}\,.$$

Only in the case $\xi = \dfrac{1}{6}$

one meets local conformal symmetry

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu}\, e^{2\sigma}, \qquad \phi \to \phi' = \phi\, e^{-\sigma},$$

$$\sigma = \sigma(x)\,.$$

- **General metric-dilaton theory** *Shapiro & Takata, PLB-1994*

$$S = \int d^4x\sqrt{-g}\,\{A(\phi)\,(\nabla\phi)^2 + B(\phi)R + C(\phi)\}\,.$$

Consider conformal transformation of the metric plus scalar reparametrization

$$g'_{\mu\nu} = g_{\mu\nu}e^{2\sigma(\phi)}, \qquad\qquad \Phi = \Phi(\phi)$$

The well-known particular case is

$$S = \int d^4x\sqrt{-g}\,\left\{-\frac{1}{2}\,\phi\,\Delta_2\,\phi - \frac{f}{4\,!}\,\phi^4\right\}$$

where

$$\Delta_2 = \Box + \frac{1}{6}\,R\,.$$

It is equivalent to Einstein-Hilbert action with a wrong sign

$$S_{EH} = +\,\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}\,.$$

Massless spinor and vector fields

$$S_{1/2} = \frac{i}{2}\int d^4x\sqrt{-g}\,\{\bar{\psi}\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\,\gamma^\mu\psi\}$$

and

$$S_1 = -\frac{1}{4}\int d^4x\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu}\,.$$

The transformation rules are

$$\psi \to \psi' = \psi\, e^{-3\sigma/2}\,, \quad \bar{\psi} \to \bar{\psi}' = \bar{\psi}\, e^{-3\sigma/2}\,, \quad A_\mu \to A'_\mu = A_\mu\,,$$

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu}\, e^{2\sigma} \qquad \sigma = \sigma(x)\,.$$

Note: the difference between conformal weight and dimension for the vector field is due to

$$A_\mu = A_b\, e^b_\mu\,, \quad e^b_\mu\, e^a_\nu\,\eta_{ab} = g_{\mu\nu}.$$

Direct relation between local & global conformal symmetries.

• **The conformal (Weyl) gravity in the dimension $n = 4$ includes only metric field**

$$S_W = \int d^4x \sqrt{-g}\, C^2 \,,$$

It can be easily generalized to an arbitrary dimension

$$C^2(n) = R^2_{\mu\nu\alpha\beta} - \frac{4}{n-2} R^2_{\mu\nu} + \frac{1}{(n-1)(n-2)} R^2 \,.$$

• **Fourth derivative scalar of the first kind**

$$S_4 = \int d^4x \sqrt{-g}\, \varphi \Delta_4 \varphi \,,$$

where $\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3} R\Box + \frac{1}{3} R_{;\mu} \nabla^\mu \,.$

The transformation law is $\varphi \to \varphi' \,.$

S.M. Paneitz, MIT preprint - 1983; SIGMA - 2008
R.J. Riegert; E.S. Fradkin & A.A. Tseytlin, PLB - 1984.

• • **General review of classical conformal theories** • •
V.Faraoni, E.Gunzig, P.Nardone, Fund.Cosm.Phys., gr-qc/9811047.

Quantum (Semiclassical) Theory

Introduction: *Birrell & Davies (1980); Buchbinder, Odintsov & I.Sh. (1992); L. Parker & D.J. Toms (2009).*

The most remarkable thing at the quantum level is that the classical conformal invariance is broken (trace anomaly).

Recent reviews: *I.Sh. et al. - gr-qc/0412113, hep-th/0610168 (both very technical), gr-qc/0801.0216.*

The first step is to consistently formulate the action on classical curved background.

In a conformal theory at 1-loop level it is sufficient to consider

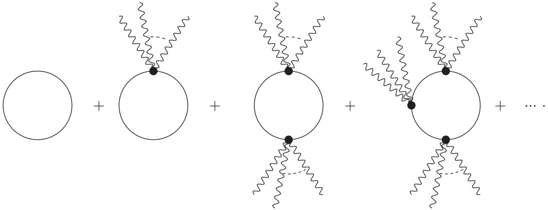
$$S_{conf.\ vac} = \int d^4x \sqrt{-g}\, \left\{ a_1 C^2 + a_2 E + a_3 \Box R \right\} .$$

QFT in curved space can be renormalizable if we define

$$S_t = S_{min} + S_{non.min} + S_{vac} \,.$$

Renormalization involves fields and parameters like couplings and masses, ξ and vacuum action parameters.

Relevant diagrams for the vacuum sector



All possible covariant counterterms have the same structure as

$$S_{vac} = S_{EH} + S_{HD}\,, \qquad S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R + 2\Lambda)\,,$$

$$S_{HD} = \int d^4x\sqrt{-g}\,\{a_1C^2 + a_2E + a_3\Box R + a_4R^2\}\,.$$

Conformal anomaly

k_Φ **is the conformal weight of the field** Φ.
The Noether identity for the local conformal symmetry

$$\left[-2\,g_{\mu\nu}\,\frac{\delta}{\delta g_{\mu\nu}} + k_\Phi\,\Phi\,\frac{\delta}{\delta\Phi}\right]\,S(g_{\mu\nu},\,\Phi)\;=\;0$$

produces on shell $$-\,\frac{2}{\sqrt{g}}\,g_{\mu\nu}\,\frac{\delta S_{vac}(g_{\mu\nu})}{\delta g_{\mu\nu}}\;=\;T_{(vac)}{}_\mu^{\;\mu}\;=\;T^\mu_\mu\;=\;0\,.$$

At quantum level $S_{vac}(g_{\mu\nu})$ **is replaced by the EA** $\Gamma_{vac}(g_{\mu\nu})$.

For free fields only 1-loop order is relevant [here $\varepsilon=(4\pi)^2(n-4)$**]**

$$\Gamma_{div}\;=\;-\,\frac{1}{\varepsilon}\,\int d^4x\sqrt{g}\,\left\{\beta_1 C^2+\beta_2 E+\beta_3\Box R\right\}\;.$$

For the global conf. symmetry the renormalization group tells us

$$\langle T^\mu_\mu\rangle\;=\;\left\{\beta_1 C^2+\beta_2 E+a'\Box R\right\}\;,$$

where $a'=\beta_3$. **In the local case** a' is **ambiguous**.

The simplest way to derive the conformal anomaly is using dimensional regularization (Duff, 1977).

The expression for divergences

$$\bar{\Gamma}_{div} = \frac{1}{\varepsilon} \int d^4x \sqrt{g} \left\{ \beta_1 C^2 + \beta_2 E + \beta_3 \Box R \right\} .$$

where

$$\begin{pmatrix} \beta_1 \\ -\beta_2 \\ \beta_3 \end{pmatrix} = \frac{1}{360(4\pi)^2} \begin{pmatrix} 3N_0 + 18N_{1/2} + 36N_1 \\ N_0 + 11N_{1/2} + 62N_1 \\ 2N_0 + 12N_{1/2} - 36N_1 \end{pmatrix}$$

The renormalized one-loop effective action has the form

$$\Gamma_R = S + \bar{\Gamma} + \Delta S ,$$

where $\bar{\Gamma} = \bar{\Gamma}_{div} + \bar{\Gamma}_{fin}$ **is the naive quantum correction to the classical action and** ΔS **is a counterterm.**

ΔS **is an infinite local counterterm which is called to cancel the divergence. It is the only source of non-invariance.**

The anomalous trace is

$$T \;=\; \langle T^\mu_\mu \rangle \;=\; -\,\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\;\frac{\delta\,\Gamma_R}{\delta\,g_{\mu\nu}}\Big|_{n=4} \;=\; -\,\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\;\frac{\delta\,\Delta S}{\delta\,g_{\mu\nu}}\Big|_{n=4}\;.$$

Conformal parametrization of the metric:

$$g_{\mu\nu} = \bar{g}_{\mu\nu}\cdot e^{2\sigma}\,, \qquad \sigma = \sigma(x)$$

where $\bar{g}_{\mu\nu}$ is the fiducial metric with fixed determinant.

There is a useful relation

$$\frac{2}{\sqrt{-g}}\,g_{\mu\nu}\,\frac{\delta\,A[g_{\mu\nu}]}{\delta\,g_{\mu\nu}} = \frac{1}{\sqrt{-\bar{g}}}\;\frac{\delta\,A[\bar{g}_{\mu\nu}\,e^{2\sigma}]}{\delta\sigma}\,\Big|_{\bar{g}_{\mu\nu}\to g_{\mu\nu}\,,\,\sigma\to 0,\,n\to 4} \qquad (*)$$

$$\int d^nx\,\sqrt{-g}\,C^2(n) \;=\; \int d^nx\,\sqrt{-\bar{g}}\,e^{(n-4)\sigma}\,\bar{C}^2(n)\,.$$

Then $$\frac{\delta}{\delta\sigma}\int\frac{d^4x\sqrt{-\bar{g}}}{n-4}\,e^{(n-4)\sigma}\,\bar{C}^2(n)\Big|_{n\to 4} \;=\; \sqrt{-g}\,C^2\,.$$

The derivatives of $\sigma(x)$in other terms are irrelevant.

In the simplest case $\sigma = \lambda = const$, we immediately arrive at the expression for T with $a' = \beta_3$.

For global conformal transform this procedure always works,

$$\langle T_\mu^\mu \rangle = \frac{1}{(4\pi)^2} \left(\omega C^2 + bE + c\Box R \right) .$$

However the local case $\sigma(x)$ it is more complicated, e.g.,

$$\frac{\delta}{\delta g_{\mu\nu}} \int \sqrt{-g}\Box R \equiv 0 .$$

We have a conflict between global and local conf. anomalies.

Or a conflict between formulas and intuitive expectations.
M.J. Duff, Class. Quantum. Grav. (1994)

Problem resolved:
M. Asorey, E. Gorbar & I.Sh., CQG 21 (2003).

- **Anomaly-induced Effective Action (EA) of vacuum**

One can use $\langle T^\mu_\mu \rangle$ to obtain equation for the finite 1-loop EA

$$\frac{2}{\sqrt{-g}}\, g_{\mu\nu}\frac{\delta\,\bar{\Gamma}_{ind}}{\delta g_{\mu\nu}} = \langle T^\mu_\mu \rangle = \frac{1}{(4\pi)^2}\,\big(\omega C^2 + bE + c\Box R\big)\,.$$

The solution is straightforward
Riegert; Fradkin & Tseytlin, PLB-1984.

It can be generalized for the theory with more background fields, e.g., with vector, torsion or scalar fields.

I.L. Buchbinder, S.D. Odintsov & I.Sh. Phys.Lett. B (1985).
J.A. Helayel-Neto, A. Penna-Firme & I.Sh. Phys.Lett. B (1998);
I.Sh., J. Solà, Phys.Lett. B (2002);
M. Giannotti, E. Mottola, Phys. Rev. D (2009).

The simplest possibility is to parameterize metric

$$g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot e^{2\sigma} , \qquad \sigma = \sigma(x) .$$

The solution for the effective action is

$$\bar{\Gamma}_{ind} \,=\, S_c[\bar{g}_{\mu\nu}] \,+\, \frac{1}{(4\pi)^2} \int d^4x \sqrt{-\bar{g}}\, \{\omega\sigma\bar{C}^2 \tag{1}$$

$$+\, b\sigma(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}) + 2b\sigma\bar{\Delta}_4\sigma \,-\, \frac{1}{12}\,(c + \frac{2}{3}b)[\bar{R} - 6(\bar{\nabla}\sigma)^2 - (\bar{\Box}\sigma)]^2)\} ,$$

where $S_c[\bar{g}_{\mu\nu}] = S_c[g_{\mu\nu}]$ is an unknown conformal functional, which serves as an integration constant in eq. for Γ_{ind}.

The solution (1) has serious merits:
1) Being simple, 2) Being exact in case $S_c[\bar{g}_{\mu\nu}]$ is irrelevant.
Example: FRW metrics.

An important disadvantage is that it is not covariant or, in other words, it is not expressed in terms of original metric $g_{\mu\nu}$.

Now we obtain the non-local covariant solution and after represent it in the local form using auxiliary fields.

First one has to establish the relations

$$\sqrt{-g}C^2 = \sqrt{-\bar{g}}\bar{C}^2\,, \qquad \sqrt{-\bar{g}}\,\bar{\Delta}_4 = \sqrt{-g}\,\Delta_4\,,$$

$$\sqrt{-g}(E - \frac{2}{3}\Box R) = \sqrt{-\bar{g}}(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R} + 4\bar{\Delta}_4\sigma)$$

and also introduce the Green function

$$\Delta_4\,G(x,y) = \delta(x,y)\,.$$

Using these formulas we find, for a functional $A(g_{\mu\nu}) = A(\bar{g}_{\mu\nu})$,

$$\frac{\delta}{\delta\sigma}\int_x A\ (E - \frac{2}{3}\Box R)\Big| = 4\sqrt{-g}\Delta_4\,A\,.$$

where $\displaystyle\int_x = \int d^4x\,\sqrt{-g(x)}\,, \qquad \Big| = \Big|_{\bar{g}_{\mu\nu}\to g_{\mu\nu}}$

As a consequence, we obtain

$$\frac{\delta}{\delta\sigma(y)} \iint\limits_{x\,y} \frac{1}{4} C^2(x)\, G(x,y) \, \Big(E - \frac{2}{3}\Box R\Big)_y \Big|$$

$$= \int d^4x \sqrt{-\bar{g}(x)}\, \bar{\Delta}_4(x)\, \bar{G}(x,y)\, \bar{C}^2(x) \Big| \; = \sqrt{-g}\, C^2(y)\,.$$

Hence, the part of Γ_{ind} which is responsible for $T_\omega = -\omega C^2$, is

$$\Gamma_\omega = \frac{\omega}{4} \iint\limits_{x\,y} C^2(x)\, G(x,y)\, (E - \frac{2}{3}\Box R)_y\,.$$

Similarly one can check that the variation $T_b = b\,(E - \frac{2}{3}\Box R)$ is produced by the term

$$\Gamma_b = \frac{b}{8} \iint\limits_{x\,y} (E - \frac{2}{3}\Box R)_x\, G(x,y)\, (E - \frac{2}{3}\Box R)_y\,.$$

Finally, we can use simple relation

$$g_{\mu\nu}\,\frac{\delta}{\delta g_{\mu\nu}}\,\int_x R^2(x)\,=\,-6\,\sqrt{-g}\,\Box R\,.$$

to establish the remaining local constituent of Γ_{ind}

$$\Gamma_c = -\frac{3c+2b}{36(4\pi)^2}\,\int_x R^2(x)\,.$$

The general covariant solution for Γ_{ind} is the sum,

$$\Gamma_{ind} = S_c[g_{\mu\nu}] \,-\, \frac{3c+2b}{36(4\pi)^2}\,\int_x R^2(x)$$

$$+\,\frac{\omega}{4}\,\int_x\int_y C^2(x)\,G(x,y)\,(E-\frac{2}{3}\Box R)_y$$

$$+\,\frac{b}{8}\,\int_x\int_y (E-\frac{2}{3}\Box R)_x\,G(x,y)\,(E-\frac{2}{3}\Box R)_y\,.$$

One can rewrite this expression using auxiliary scalars.

The nonlocal terms can be rewritten in a symmetric form

$$\Big(E - \frac{2}{3}\Box R\Big)_x G(x,y)\left[\frac{\omega}{4}C^2 - \frac{b}{8}(E - \frac{2}{3}\Box R)\right]_y$$

$$= \frac{b}{8}\int\!\!\int_{x\,y} \Big(E - \frac{2}{3}\Box R - \frac{\omega}{b}C^2\Big)_x G(x,y)\Big(E - \frac{2}{3}\Box R - \frac{\omega}{b}C^2\Big)_y$$

$$- \frac{\omega^2}{8b}\int\!\!\int_{x\,y} C_x^2\, G(x,y)\, C_y^2\,.$$

These form is appropriate for rewriting it via auxiliary fields.
Then we arrive at the local covariant expression for EA

$$\Gamma_{ind} = S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2}\int_x R^2(x) + \int_x \Big\{\frac{1}{2}\,\varphi\Delta_4\varphi - \frac{1}{2}\,\psi\Delta_4\psi$$

$$+ \frac{\omega}{8\pi\sqrt{-b}}\,\psi\,C^2 + \varphi\,\Big[\frac{\sqrt{-b}}{8\pi}\,(E - \frac{2}{3}\,\Box R) - \frac{\omega}{8\pi\sqrt{-b}}\,C^2\Big]\Big\}\,.$$

The above form of EA is the best one for Γ_{ind}.

I.Sh. and A.Jacksenaev, Phys. Lett. B (1994)

Similar expression has been independently introduced by

P. Mazur & E. Mottola, 1997-1998.

Comments:

1) Imposing boundary conditions on the two auxiliary fields φ **and** ψ **is equivalent to defining boundary conditions for the Green functions** $G(x,y)$.

2) Introducing the new term $\int C_x^2 G(x,y) C_y^2$ **into the action may be viewed as redefinition of the conformal functional** $S_c[g_{\mu\nu}]$.

However, writing the non-conformal terms in the symmetric form, essentially modifies the four-point function. Using ψ **we restore the structure generated by anomaly.**

Recent generalization.

Quantum effects of chiral fermion produce an imaginary contribution which violates parity,

$$\langle T^\mu_\mu \rangle \,=\, -\,\omega_1 C^2 - bE_4 - c\Box R - \epsilon P_4\,,$$

where the Pontryagin density term appears,

$$P_4 \,=\, \frac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta}\,R_{\mu\nu\rho\sigma}\,R_{\alpha\beta}{}^{\rho\sigma}\,, \qquad \epsilon = \frac{i}{48\cdot 16\pi^2}\,.$$

L.Bonora, S.Giaccari, B.de Souza, JHEP (2014), arXiv:1403.2606.

It is a relatively easy exercise to derive the corresponding anomaly-induced effective action.

S. Mauro, I.Sh., PLB (2015) arXiv:1412.5002.

First, one can prove the conformal symmetry of this term

$$P_4 = \frac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta}\,C_{\mu\nu\rho\sigma}\,C_{\alpha\beta}{}^{\rho\sigma}\,.$$

After that we immediately arrive at

$$\Gamma_{ind} = S_c[g_{\mu\nu}] - \frac{3c+2b}{36(4\pi)^2}\int_x R^2 + \int_x \Big\{\frac{1}{2}\,\varphi\Delta_4\varphi - \frac{1}{2}\,\psi\Delta_4\psi$$

$$+\varphi\,\Big[\,\frac{\sqrt{-b}}{8\pi}\,\Big(E-\frac{2}{3}\,\Box R\Big)\, - \,\frac{1}{8\pi\sqrt{-b}}\,\big(\omega C^2+\epsilon P_4\big)\Big] + \frac{1}{8\pi\sqrt{-b}}\,\psi\,\big(\omega C^2+\epsilon P_4\big)$$

It is natural to change variables,

$$\chi = \frac{\psi-\varphi}{\sqrt{2}}\,,\qquad \xi = \frac{\psi+\varphi}{\sqrt{2}}\,,$$

Then the total gravitational action becomes

$$\Gamma_{grav} = S_{EH} + S_{HD} + S_c[g_{\mu\nu}] + \int_x \Big\{\xi\Delta_4\chi + k_1\Big(E-\frac{2}{3}\,\Box R\Big)\big(\xi-\chi\big)$$

$$+k_2\,\chi\,\big(\omega C^2+\epsilon P_4\big) + k_3 R^2\Big\}\,.$$

The coefficients are, as before,

$$k_1 = \frac{1}{8\pi}\sqrt{-\frac{b}{2}}\,, \qquad k_2 = \frac{1}{8\pi\sqrt{-2b}}\,, \qquad k_3 = -\frac{2b+3c}{36\,(4\pi)^2}\,,$$

The action

$$\Gamma_{grav} = S_{EH} + S_{HD} + S_c[g_{\mu\nu}] + \int_x \Big\{ \underline{\xi\Delta_4\chi} + k_1\big(E - \frac{2}{3}\Box R\big)(\xi - \chi)$$

$$+ k_2\,\chi\,\big(\omega C^2 + \underline{\epsilon P_4}\big) + k_3 R^2 \Big\}\,.$$

is a special case of the Chern-Simons modified general relativity,

R. Jackiw and S.Y. Pi, Phys. Rev. D 68 (2003), gr-qc/0308071.

A. Lue, L. Wang, M. Kamionkowski, Phys.Rev.Lett. 83 (1999) 1506.

S. Alexander, N. Yunes, Phys.Rept. 480 (2009) 1.

with a special form of the kinetic term.

Applications of the anomaly-induced EA

- **Classification of vacuum states in the vicinity of a black hole**

Anomaly is, in part, responsible for the Hawking radiation
S.M. Christensen, S.A. Fulling, PRD (1977).

The anomaly-induced effective action of gravity enables one to perform a kind of systematic classification of the vacuum states for the quantum fields on the black hole background.

We can distinguish the different vacuum states by choosing different boundary conditions for the auxiliary fields φ and ψ.

R. Balbinot, A. Fabbri & I.Sh., PRL **83**; *NPB* **559** *(1999).*

Generalization for the Reissner-Nordstrom black hole,
P.R. Anderson, E. Mottola & R. Vaulin, PRD **76** *(2007).*

At the classical level, the black hole (BH) does not emit radiation, but such emission can take place if we take quantum effects into account.

After being discovered by Hawking (1975)**, the same result has been obtained from analytical estimates of $\langle T_{\mu\nu} \rangle$ for quantum matter fields in a fixed Schwarzschild BH geometry.**

S.M. Christensen & S.A. Fulling, PRD 15 (1977).

Detailed analytical and numerical study, based on the analysis of $\langle T_{\mu\nu} \rangle$ in the classical black hole background:

P. Candelas, PRD **21** *(1980);*
D.N. Page, PRD **25** *(1982);*
M.R. Brown, A.C. Ottewill and D.N. Page, PRD **33** *(1986);*
V.P. Frolov and A.I. Zelnikov, PRD **35** *(1987);*
P.R. Anderson, W.A. Hiscock and D.A. Samuel, PRD **51** *(1995). ...*

A fundamental property is the existence of three different vacuum quantum states.

i) **The Boulware $|B\rangle$ state reproduces the Minkowski vacuum $|M\rangle$ in the limit $r \to \infty$, where $\langle B|T_{\mu\nu}|B\rangle \sim r^{-6}$.**
On the horizon this quantity is divergent in a free falling frame.

ii) **For Unruh vacuum $|U\rangle$ the value $\langle U|T_{\mu\nu}|U\rangle$ is regular on the future event horizon but not on the past one. Asymptotically in the future $\langle U|T_{\mu\nu}|U\rangle$ has the form of a flux of radiation at the Hawking temperature $T_H = 1/8\pi M$.**
This vacuum state is the most appropriate to discuss evaporation of black holes formed by gravitational collapse of matter.

iii) **The Israel-Hartle-Hawking $|H\rangle$** state
$\langle H|T_{\mu\nu}|H\rangle$ **for $r \to \infty$ describes a thermal bath of radiation at T_H.**

The existence of three vacuum states reflects distinct positions of observers and the construction of different *in* **and** *out* **modes with respect to the corresponding coordinates.**

The main difference between classical and quantum theories is that, in the first case we know how to transform the relevant quantities when we change the coordinate system.

The natural question is how to perform a transition between different vacuum states $|H\rangle$, $|B\rangle$ **and** $|U\rangle$ **?**

The anomaly-induced effective action doesn't make any reference to a particular quantum state, but it includes the conformal invariant functional $S_c[g_{\mu\nu}]$ **– a source of uncertainty.**

Strategy: one has to fix the extended set of boundary conditions, including the ones for the auxiliary scalars φ and ψ.

The procedure for identifying the vacuum state is as follows:

1) Solving equations for φ and ψ.
The solutions always depend on the set of integration constants.

2) One has to find "appropriate" boundary conditions to identify $\langle V|T_{\mu\nu}|V\rangle$ for the given vacuum state $|V\rangle = (|B\rangle,\ |U\rangle,\ |H\rangle)$.

3) Use

$$\langle T_{\mu\nu}\rangle \quad \longrightarrow \quad \frac{2}{\sqrt{-g}}\frac{\delta\Gamma_{ind}}{\delta g_{\mu\nu}} = \langle S_{\mu\nu}\rangle\,,$$

where of course $\langle S\rangle = \langle T\rangle$.

The general solution is $\varphi(r,t) = d\cdot t + w(r)$, **where** $w(r)$ **satisfies the equation**

$$\frac{dw}{dr} = \frac{B}{3}r + \frac{2MB}{3} - \frac{A}{6} - \frac{\alpha}{72M} + \frac{1}{r-2M}\Big(\frac{4}{3}BM^2 + \frac{C}{2M} - AM - \frac{\alpha}{24}\Big) - \frac{C}{2Mr}$$

$$- \left[\frac{\alpha M}{r^3} + \frac{24AM-\alpha}{144\,M^2}\right]\,\frac{r^2\,\ln r}{r-2M} + \frac{(24AM-a)\big(r^3-8M^3\big)\ln(r-2M)}{3r(r-2M)48M^2}\,.$$

(d,A,B,C) **are constants which specify the homogeneous solution** $\Box^2\varphi = 0$ **and hence the quantum state.**

For ψ **we have a similar solution, but with** (d',A',B',C').

Due to the independence of φ **and** ψ. **the two sets are independent on each other.**

In case of a Boulware state $|B\rangle$ we request

$$|B\rangle \to |M\rangle \quad \textbf{when} \quad r \to \infty\,.$$

In the Minkowski vacuum we can safely set $\varphi = \psi = 0$.

This asymptotic conditions enables one to arrive at the asymptotic expressions

$$\langle B|S^{\nu}_{\mu}|B\rangle \;\to\; \frac{\alpha^2 - \beta^2}{2\,(24)^2\,(2M)^4\,\big(1 - 2M/r\big)^2} \times \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 \end{pmatrix}$$

for $r \to 2M$ **and**

$$\langle B|S^{\nu}_{\mu}|B\rangle \propto \mathcal{O}\big(r^{-6}\big) \qquad \textbf{for} \qquad r \to \infty\,.$$

This behavior fits perfectly will with the ones observed within other methods.

Unruh vacuum case

Choosing another values of the integration constants we meet the following asymptotic behavior near the horizon $r \to 2M$:

$$\langle U|S_a{}^b|U\rangle \sim \frac{\alpha^2 - \beta^2}{2(48M^2)^2} \begin{pmatrix} 1/f & -1 \\ 1/f^2 & -1/f \end{pmatrix},$$

regular on the future horizon, $a, b = r, t$. **The asymptotic form**

$$r \to \infty \qquad \langle U|S_\mu{}^\nu|U\rangle \to \frac{\alpha^2 - \beta^2}{2r^2(24M)^2} \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

These results are in exact agreement with the standard ones on the Hawking radiation: *B.S. DeWitt, Phys. Rep.* **C19** *(1975) 297.*

once the luminosity L **of the radiating BH is identified with**

$$\frac{L}{4\pi} = \frac{(\alpha^2 - \beta^2)}{2(24M)^2}.$$

Cosmological application: Starobinsky Model.

Starobinsky model based on quantum effects.

Fischetti, Hartle and Hu (1978);
Starobinsky, (1980-1983) ;
Mukhanov, Chibisov, (1982) ;
Anderson, Vilenkin, ... (1983-1986)
Hawking, Hertog and Real, (2001) .

Modified Starobinsky model

Fabris, Pelinson, Solà, I.Sh., ...

Cosmological Model based on the action

$$S_{total} \;=\; -\,\frac{M_P^2}{16\pi}\,\int d^4x\sqrt{-g}\,(R+2\Lambda)+S_{matter}+S_{vac}+\bar{\Gamma}_{ind}\,.$$

Equation of motion for $a(t),\quad dt=a\,(\eta)\,d\eta,\quad k=0$

$$\frac{\ddot{\ddot{a}}}{a}+\frac{3\dot{a}\dddot{a}}{a^2}+\frac{\ddot{a}^2}{a^2}-\left(5+\frac{4b}{c}\right)\frac{\ddot{a}\dot{a}^2}{a^3}-\frac{M_P^2}{8\pi c}\left(\frac{\ddot{a}}{a}+\frac{\ddot{a}^2}{a^2}-\frac{2\Lambda}{3}\right)\;=\;0\,,$$

$k=0,\pm 1$. **Particular solutions (Starobinsky, PLB-1980)**

$$a(t)\;=\;a_0e^{Ht}\,,\qquad\quad k=0\,,$$

where Hubble parameter $H=\dot{a}/a$ **is**

$$H^2\;=\;-\,\frac{M_P^2}{32\pi b}\left(1\pm\sqrt{1+\frac{64\pi b}{3}\,\frac{\Lambda}{M_P^2}}\,\right).$$

A. Pelinson, I.Sh., F. Takakura, NPB (2003).

For $0 < \Lambda \ll M_P^2$ there are two solutions:

$$H \approx \sqrt{\Lambda/3}\,; \qquad (IR)$$

$$H \approx \sqrt{-\frac{M_P^2}{16\pi b} - \frac{\Lambda}{3}} \approx \frac{M_P}{\sqrt{-16\pi b}}\,. \qquad (UV)$$

Perturbations of the conformal factor

$$\sigma(t) \to \sigma(t) + y(t).$$

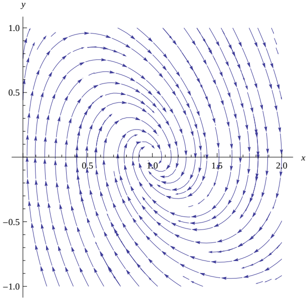
The criterion for a stable (UV) inflation is

$$c > 0 \quad \Longleftrightarrow \quad N_1 < \frac{1}{3}N_{1/2} + \frac{1}{18}N_0\,,$$

in agreement with Starobinsky (1980).

The original Starobinsky model is based on the unstable case and involves special choice of initial data. This situation can be improved further by using the stable version and an appropriate transition scheme.

In the unstable phase there are very different solutions, some of them violent (hyperinflation). How can we know that the transition from stable to unstable phase really happens? *A. Pelinson et al, NPB(PS) (2003).* **Phase portrait of a stable case:**



$$\text{Starobinsky (1980)}: \; x = \left(\frac{H}{H_0}\right)^{\frac{3}{2}}, \quad y = \frac{\dot{H}}{2\sqrt{H_0^3 H}}, \quad dt = \frac{dx}{3H_0 x^{2/3} y}.$$

Simple test of the unstable version of Starobinsky Model.
A.Pelinson, I.Sh. et al., IRGA-NPB(PS)- 2003.,

Consider late Universe, $k = 0,\ H_0 = \sqrt{\Lambda/3}$.

Only photon is active, $N_0 = 0\,, \qquad N_{1/2} = 0\,, \qquad N_1 = 1\,.$

Graviton typical energy is $H_0 \approx 10^{-42}\, GeV$, $\Longrightarrow$ **all massive particles (even neutrino)** $m_\nu \geq 10^{-12}\, GeV$ **decouple from gravity.** $c < 0 \quad \Longrightarrow$ **today inflation is unstable.**

Stability for the small $H = H_0$ **case:** $H \to H_0 + const \cdot e^{\lambda t}$

$$\lambda^3 + 7H_0\lambda^2 + \left[\frac{(3c-b)4{H_0}^2}{c} - \frac{M_P^2}{8\pi c}\right]\lambda - \frac{32\pi b{H_0}^3 + M_P^2 H_0}{2\pi c} = 0\,.$$

The solutions are $\lambda_1 = -4H_0\,, \quad \lambda_{2/3} = -\frac{3}{2}H_0 \pm \frac{M_P}{\sqrt{8\pi|c|}}\, i\,.$

$\Lambda > 0$ **protects our world from quantum corrections!**

Transition. **Suppose at UV** $(H \gg M_F)$ **there is SUSY, e.g. MSSM,**

$$N_1 = 12\,, \qquad N_{1/2} = 32\,, \qquad N_0 = 104\,.$$

This provides stable inflation, because

$$\frac{1}{3}\,N_{1/2} + \frac{1}{18}\,N_0 > N_1 \qquad \Longrightarrow \qquad c > 0\,.$$

For realistic SUSY model inflation is independent on initial data.

Fine!

But why should inflation end? **Already for MSM** $(N_{1,\,1/2,\,0} = 12, 24, 4)$, $c < 0$, **inflation is unstable.**

Natural interpretation:
I.Sh. Int.J.Mod.Ph.D. (2002); A. Pelinson et al NPB (2003).

All sparticles are heavy $\Rightarrow$ **decouple when** H **becomes smaller than their masses.**

Direct calculations confirmed that the transition $c > 0 \Longrightarrow c < 0$ **is smooth, indicating a possibility of a smooth graceful exit.**

- **Using anomaly for deriving EA of massive fields.**

Why the energy scale H decreases during inflation?

In the exponential phase Hubble parameter $H(t) = const$.

Another unclear point: Using anomaly-induced EA for massive fields is not a correct approximation.

Maybe all difficulties can be solved if taking masses of the fields into account?

Consider a reliable Ansatz for the EA of massive fields.
J.Solà, I.Sh. PLB - 2002;
also A.Pelinson, I.Sh. & F.Takakura, Nucl.Ph. 648B (2003).

In part, it is based on
R.D.Peccei, J.Solà, C.Wetterich, Ph.Lett. B 195 (1987) 183
and S. Deser, Ann. Phys. **59** *(1970) 248*.

The idea is to construct the conformal formulation of the SM and use it to derive EA for massive fields.

Conformal formulation of massive theory

The conformally non-invariant terms:

$$m_s^2\,\varphi^2\,,\quad m_f\,\bar{\psi}\psi\,,\quad \textbf{and}\quad L_{EH}\,=\,-\,\frac{1}{16\pi G}\,(R+2\Lambda)\,.$$

Replacing dimensional parameters by the new scalar χ:

$$m_{s,f}\to\frac{m_{s,f}}{M}\,\chi\,,\quad M_P^2\to\frac{M_P^2}{M^2}\,\chi^2\,,\quad \Lambda\to\frac{\Lambda}{M^2}\,\chi^2\,.$$

M **is related to a scale of conformal symmetry breaking. Massive terms get replaced by Yukawa and $(scalar)^4$ type interactions with χ. In the IR $\chi\sim M$.**

In the gravity sector

$$\mathcal{L}_{EH}^{*}=-\frac{M_P^2}{16\pi\,M^2}\,\Big\{[R\chi^2+6\,(\partial\chi)^2]\,+\,\frac{2\Lambda\,\chi^4}{M^2}\Big\}$$

in order to provide local conformal invariance.

The new theory is conformal invariant

$$\sigma = \sigma(x), \qquad \begin{cases} \chi \to \chi\, e^{-\sigma}\,, & g_{\mu\nu} \to g_{\mu\nu}\, e^{2\sigma} \\ \varphi \to \varphi\, e^{-\sigma}\,, & \psi \to \psi\, e^{-3/2\,\sigma} \end{cases}$$

The conformal symmetry comes together with a new scalar χ, absorbing conformal degree of freedom. Fixing $\chi \to M$ we come back to original formulation.
The conformal anomaly becomes

$$\langle T \rangle = -\Big\{ wC^2 + bE + c\Box R + \frac{f}{M^2}\,[R\chi^2 + 6(\partial\chi)^2] + \frac{g}{M^4}\,\chi^4 \Big\},$$

f **and** g **are** β**-functions for** $(16\pi G)^{-1}$ **and** $\rho_\Lambda = \Lambda/8\pi G$ **,**

$$f = \sum_i \frac{N_f}{3\,(4\pi)^2}\, m_f^2\,, \qquad \tilde{f} = \frac{16\pi f}{M_P^2}\,,$$

$$g = \frac{1}{2(4\pi)^2} \sum_s N_s\, m_s^4 - \frac{2}{(4\pi)^2} \sum_f N_f\, m_f^4\,,$$

N_f **and** N_s **are multiplicities of the fields.**

Anomaly-induced EA in terms of $g_{\mu\nu} = \bar{g}_{\mu\nu} \cdot e^{2\sigma}$ **and** $\chi = \bar{\chi} \cdot e^{-\sigma}$

$$\bar{\Gamma} \,=\, S_c[\bar{g}_{\mu\nu}, \bar{\chi}] \,+\, \int d^4x \sqrt{-\bar{g}}\Big\{ w\sigma\bar{C}^2 + b\sigma(\bar{E} - \frac{2}{3}\bar{\nabla}^2\bar{R}) + 2b\sigma\bar{\Delta}\sigma$$

$$+\, \frac{f}{M^4}\,\sigma[\bar{R}\bar{\chi}^2 + 6(\partial\bar{\chi})^2] \,+\, \frac{g}{M^4}\,\bar{\chi}^4\sigma\,\Big\} - \frac{3c+2b}{36}\int d^4x \sqrt{-g}R^2\,.$$

This may be seen as a local version of Renormalization Group. In curved space-time RG corresponds to the scaling

$$g_{\mu\nu} \to g_{\mu\nu} \cdot e^{-2\tau} \quad \Longrightarrow \quad \Gamma[e^{-2\tau}g_{\alpha\beta}, \Phi_i, P, \mu] = \Gamma[g_{\alpha\beta}, \Phi_i(\tau), P(\tau), \mu]\,.$$

In the leading-log approximation we meet the RG improved classical action of vacuum

$$S_{vac}[g_{\alpha\beta}, P(\tau), \mu]\,, \quad \textbf{where} \quad P(\tau) = P_0 + \beta_P\,\tau\,.$$

The equivalence in all terms which do not vanish for $\sigma = \tau$**.**

Cosmological implications

$$S_t = S_{matter} + S^*_{EH} + S_{vac} + \bar{\Gamma}\,.$$

The equation of motion for $\Lambda = 0,\ g = 0$

$$a^2\,\ddot{\ddot{a}} + 3\,a\,\dot{a}\,\dddot{a} - \left(5 + \frac{4b}{c}\right)\dot{a}^2\,\ddot{a} + a\,\ddot{a}^2 - \frac{M_P^2}{8\pi c}\left(a^2\ddot{a} + a\dot{a}^2\right)\,[1 - \tilde{f}\cdot\ln a] \;=\; 0\,,$$

Let us solve by $M_P^2 \to M_P^2\,[1 - \tilde{f}\cdot\ln a]$,

$$\dot{\sigma} = H = H_o\sqrt{1 - \tilde{f}\sigma(t)}\,, \qquad H_o = \frac{M_P}{\sqrt{-16b}}\,.$$

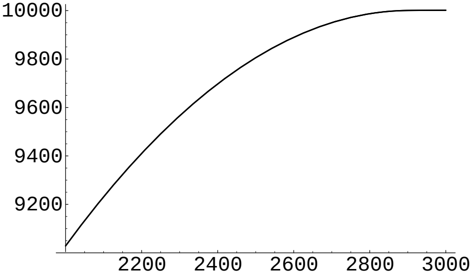
This leads to the simple solution

$$\sigma(t) \;=\; H_o\,t \;-\; \frac{H_0^2}{4}\,\tilde{f}\;t^2\,.$$

Remarkably, this formula fits with the numerical solution with a wonderful 10^{-6} **precision!**

$\tilde{f} > 0 \;\; \Rightarrow$ **we arrive at the tempered inflation!!**

Anomaly-induced inflation slows down if taking masses of quantum fields into account.



$$\sigma(t) = \ln a(t) \approx H_0 t - \frac{H_0^2}{4} \tilde{f}\, t^2 , \qquad H_0 \propto M_P$$

The total amount of e**-folds may be as large as** 10^{32}**, but only** 65 **last ones, where** $H \propto M_*$ **(SUSY breaking scale) are relevant.**

From the formal QFT viewpoint, there is no solution, because for the transition period, when

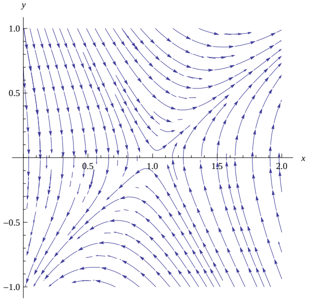
$$H \quad \sim \quad \text{masses of quantum matter fields}$$

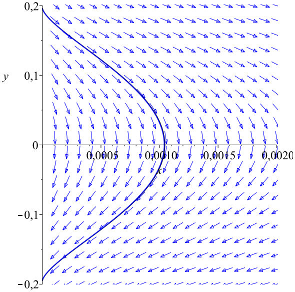
we have no method, approach, idea or approximation to perform calculations, except for dS space, which is useless here.

The simplest, purely phenomenological approach is to take a final point of the stable tempered inflation epoch ... and use it as initial point for the unstable phase. Where we are going to end up in this way?

A. Pelinson, Tiberio de Paula, I.Sh., A. Starobinsky, From stable to unstable anomaly-induced inflation. arXiv:1509.08882..

The qualitative output of this phenomenological approach is positive, in the sense that the final point of the stable inflation (related to SUSY breakdown) belongs to the “right” integration curve of the unstable inflation.





One can check that this curve really ends up at the classical radiation-dominated solution.

This result gives us a chance to have a consistent inflation based on QFT results.

We have seen that the anomaly-induced corrections remove initial cosmological singularity. What about other cases?

There are indications that the black hole singularity at $r = 0$ disappears if the semiclassical effects are taken into account.

Frolov & Vilkovisky, PLB (1980).

Frolov & I.Sh., PRD (2009).

Lu, Perkins, Pope, Stelle, PRL-2015, arXiv:1502.01028; PRD-2015.

Singularities represent a "WINDOW" to QG. The semiclassical effects may CLOSE IT, making observation of QG impossible.

Conclusions.

- **Integrating conformal anomaly is very efficient and extremely economic and explicit way to derive EA.**

- **The conformal symmetry can not be exact, it is only a useful approximation. And its effectiveness is mainly restricted to the one-loop level.**

- **In order to arrive at some applications one is forced to deal with the non-conformal massive quantum fields.**

- **The success of anomaly-based approach is closely related to the fact we know very well how to deal with divergences and hence control UV limit of QFT in curved space.**

- **Currently it is unclear how to go beyond the UV limit. This problem represents the most challenging and very difficult part of the semiclassical approach.**

Elements of QFT in Curved Space-Time

Ilya L. Shapiro

Universidade Federal de Juiz de Fora, MG, Brazil

JINR & Dubna University, February 2016

Lecture 4.

- **Cosmological Constant Problem.**

Contents:

- **Myth and Legends of Cosmological Constant.**
- **Cosmological Constant at classical level.**
- **Cosmological Constant (CC) Problem and trying to solve it.**
- **Renormalization Group Running of CC.**
- **Covariance and Physical Renormalization Group for CC.**
- **Applications to Cosmology and Astrophysics**

Recommended reading:

- *S. Weinberg, Rev. Mod. Phys.* ***61*** *(1989) 1.*

- *I.Sh., J. Solà, JHEP* **02** *(2002) 006; J. Phys.* **A40** *(2007) 6583; Phys. Lett.* **B682** *(2009) 105.*

- *E. Bianchi, C. Rovelli, arXiv:1002.3966 [astro-ph.CO].*

The history of the cosmological constant (CC) started when A. Einstein introduced a constant term into his equations.

Original purpose was to get a static cosmological solution.

Nowadays we know Universe is expanding according to the Hubble law. So, why do not we remove the CC from the scene?

Mathematically, the CC term comes to our mind first when we want to formulate covariant action for gravity

$$S_{grav} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,, \qquad \rho_\Lambda = \frac{\Lambda}{8\pi G}\,.$$

So, what is the problem? Is there some?

• Myth and Legends of Cosmological Constant.

The greatest one: The CC term can be calculated in the framework of Quantum Field Theory (QFT) or other Quantum Theory and, surprisingly, it has a strange value, 120 orders of magnitude larger than the one observed in cosmology.

Real deal: In QFT we can not derive any independent massive (or massless) parameter from the first principles.

The values of all massive parameter are defined through a process which includes experimental measurement.

And CC is not an exception.

More precisely: naive calculation always provide an infinite value for a massive parameter, with both potential and logarithmic-type divergences. After infinity is subtracted, we have to fix the finite value. And this involves a measurement.

Not all those quantities which are calculated to be infinite, are in fact equal to zero.

W. Pauli

The famous "120 orders of magnitude" correspond to the Planck-scale cut-off of quartic divergence in the CC sector.

Taking this naive cut-off as a physical result is an absurd.

With the same logic absolutely all masses should have Planck value. Since this is not the case, with the same logic we should have "m_e problem", "m_τ problem", "m_μ problem", "m_W problem", "m_Z problem", "m_H problem", "m_ν problem", etc.

In reality, there is no problem with neither one of them, since the corresponding values are fixed by renormalization conditions and eventually by a measurements.

For the case of CC term, the "measurement" means a full set of available experimental and observational data at the scale of the Universe. All of them (SN-Ia, CMB, LSS, ...) are likely converging to the nonzero, positive value

$$\rho_\Lambda^0 \approx 0.7\,\rho_c^0\,.$$

Definitely, at this level there is no problem with the CC term. We have a "measured" value and this is all the story. We see CC is positive and so it is. It is just fine.

So, where is the CC problem?

The answer is: The CC problem does exist, it is caused by finite and really big contributions to the CC in the QFT framework.

- Λ **-term at the classical level.** **Is it a Constant?**

The action of renormalizable theory (e.g., SM) in curved space is

$$S_{total} = -\frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,(R+2\Lambda) + S_{HD} + S_{matter}\,.$$

Higher derivative terms S_{HD} **are necessary in quantum theory.**

See, e.g., Birrell, Davies (1980); Buchbinder, Odintsov, Sh.(1992).

In the low-energy domain, one can in principle disregard S_{HD} **and the dynamical equations take on the Einstein form**

$$R_\mu^\nu - \frac{1}{2}R\,\delta_\mu^\nu = 8\pi G\,T_\mu^\nu + \Lambda\,\delta_\mu^\nu\,.$$

For isotropic fluid in the locally co-moving frame

$$T_\mu^\nu = \mathrm{diag}\,(\rho,\,-p,\,-p,\,-p)\,.$$

The Λ**-dependent term has exactly the form (1), with**

$$\rho_\Lambda^{vac} = \frac{\Lambda}{8\pi G} = -p_\Lambda^{vac}\,.$$

Definitely, it is a wrong idea to consider the Λ-term as a fluid with negative pressure, repulsive gravity and so on and so forth.

$$\rho_\Lambda^{vac} = \frac{\Lambda}{8\pi G} = -p_\Lambda^{vac}$$

is just a useful form to present the vacuum CC term.

The CC term is not a part of the action of matter, it is not a strange fluid. It is just the simplest possible covariant term.

Amazingly, it is not a constant term!

Without gravity the CC term is an irrelevant constant. However, it acquires dynamical significance through the Einstein equations.

Consider another parametrization of the metric

$$g_{\mu\nu} = \frac{\chi^2}{M_P^2}\,\bar{g}_{\mu\nu}\,,$$

where $\bar{g}_{\mu\nu}$ is some fiducial metric, for instance, it can be $\eta_{\mu\nu}$.

Furthermore, $\chi = \chi(x)$ is a new scalar field.

The CC term looks rather different in these new variables:

$$S_\Lambda = -\int d^4x\sqrt{-g}\,\rho_\Lambda = -\int d^4x\sqrt{-\bar{g}}\,f\chi^4\,, \qquad f = \frac{\Lambda}{8\pi M_P^2}\,.$$

This is quartic term in the potential for the scalar interaction.

The same change of variables transforms $\int\sqrt{-g}R$ - term into the action of a scalar field χ with the negative kinetic term and conformal coupling to curvature.

• Main CC Problem (I) and attempts to solve it.

Why we can not remove the CC from the scene, set it zero?

The reason is that, from the theoretical side, there are many sources of the CC, and simply set it to zero is very difficult.

These sources are as follows:

1) CC is necessary for the consistent QFT in curved space;

2) Induced CC (vacuum energy) always comes from the SSB in the SM of particle physics;

3) Possible variation of the Λ-term due to quantum effects.

Observation about general structure of renormalization in curved space.

Starting from the first paper
R. Utiyama & B.S. DeWitt, J. Math. Phys. 3 (1962) 608.
we know that the divergences and counterterms in QFT in curved space-time satisfy two conditions:

- **They are covariant if the regularization is consistent with covariance.**

- **They are local functionals of the metric.**

See the book
I.L. Buchbinder, S.D. Odintsov & I.Sh., Effective Action in Quantum Gravity (IOPP, 1992).
for introduction and recent papers
I.Sh. Class.Quant.Grav. (2008 - Topical review). arXiv: 0801.0216.
P. Lavrov & I.Sh., Phys.Rev. D81 (2010) 044026.
for a more simple consideration and more rigid proof.

What may happen if we use a non-covariant regularization?

Example: cut-off regularization for the Energy-Momentum Tensor of vacuum.

B.S. DeWitt, Phys. Reports. (1975)

E.K. Akhmedov, arXiv: hep-th/0204048.

$$\rho_{vac} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\sqrt{\vec{k}^2+m^2}\,,$$

$$p_{vac} = \frac{1}{6}\int \frac{d^3k}{(2\pi)^3}\frac{\vec{k}^2}{\sqrt{\vec{k}^2+m^2}}\,,$$

For each mode we have, in the massless limit, EOS of radiation. Naturally, after integration with cut-off we will get the EOS for the radiation in the quartic divergences.

But, Lorentz invariance requires the EOS to be $p_{vac} = -\rho_{vac}$.

Indeed, this discrepancy only reflects the non-covariant nature of the momentum cut-off regularization.

Similarly, the quadratic divergences must have the EOS identical to the one of the Einstein tensor. But it can be, instead, any other EOS in a non-covariant regularization scheme.

Usually, only logarithmic divergences are stable even under non-covariant regularization.

In order to have the covariant cut-off, one has to choose, e.g, Schwinger-DeWitt proper-time representation with the cut-off on the lower limit of the integral.

New discussion of this issue:
M.Asorey, P.Lavrov, B.Ribeiro & I.Sh., Vacuum stress-tensor in SSB theories. arXive: 1202.4235

Reminder about QFT in curved space-time:

Renormalizable theory of matter fields on classical curved background requires classical action of vacuum

$$S_{vac} = S_{HE} + S_{HD}\,, \qquad S_{HE} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R + 2\Lambda)\,.$$

Important remark: **Without independent vacuum parameter $\Lambda = \Lambda_{vac}$ the theory is inconsistent.**

Loops of massive particle give divergences of the Λ_{vac} -type.

If $\Lambda_{vac} \equiv 0$, these divergences can not be removed by renormalization, and we have a kind of theoretical disaster.

Of course the same is true for all other terms in S_{vac}, including Hilbert term and higher derivative terms.

RG equations for CC and G:

$$(4\pi)^2\,\mu\frac{d\,\rho_\Lambda^{vac}}{d\mu} = (4\pi)^2\,\mu\,\frac{d}{d\mu}\left(\frac{\Lambda_{vac}}{8\pi G_{vac}}\right) = \frac{N_s m_s^4}{2} - 2N_f m_f^4\,.$$

$$(4\pi)^2\,\mu\,\frac{d}{d\mu}\left(\frac{1}{16\pi G_{vac}}\right) = \frac{N_s m_s^2}{2}\left(\xi - \frac{1}{6}\right) + \frac{N_f m_f^2}{3}\,.$$

It is not clear how these equations can be used in cosmology, where the typical energies are very small.

However, even the UV running means the ρ_Λ^{vac} can not be much smaller then the fourth power of the typical mass of the theory.

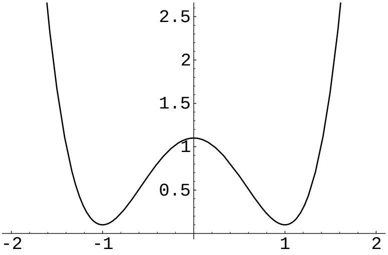
Consequence: the natural value from the MSM perspective is

$$\rho_\Lambda^{vac} \sim M_F^4 \sim 10^8\,GeV^4\,.$$

Induced CC from SSB in the Standard Model.

In the stable point of the Higgs potential $V = -m^2\phi^2 + f\phi^4$ we meet $\Lambda_{ind} = \langle V \rangle \approx 10^8\, GeV^4$ – same order of magnitude as Λ_{vac} !

This is induced CC, similar to the one found by Zeldovich (1968).



The observed CC is a sum $\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind}$. Since ρ_Λ^{vac} is an independent parameter, the renormalization condition is

$$\rho_\Lambda^{vac}(\mu_c) = \rho_\Lambda^{obs} - \rho_\Lambda^{ind}(\mu_c)\,.$$

Here μ_c is the energy scale where ρ_Λ^{obs} is "measured".

Finally, the main CC relation is

$$\rho_\Lambda^{obs} = \rho_\Lambda^{vac}(\mu_c) + \rho_\Lambda^{ind}(\mu_c)\,.$$

The ρ_Λ^{obs} which is likely observed in SN-Ia, LSS and CMB is

$$\rho_\Lambda^{obs}(\mu_c) \;\approx\; 0.7\,\rho_c^0 \;\propto\; 10^{-47}\,GeV^4.$$

The CC Problem is that the magnitudes of $\rho_\Lambda^{vac}(\mu_c)$ and $\rho_\Lambda^{ind}(\mu_c)$ are a huge **55** orders of magnitude greater than the sum!

Obviously, these two huge terms do cancel.

"Why they cancel so nicely" is the CC Problem (Weinberg, 1989).

The origin of the problem is the difference between the M_F scale of ρ_Λ^{ind} and ρ_Λ^{vac} and the μ_c scale of ρ_Λ^{obs}.

Obviously, CC Problem is nothing else but a sort of hierarchy problem, perhaps the most difficult one.

There were attempts to fix the overall CC value to zero and replace it by quintessence, Chaplygin gas, k-essence etc.

Warning: 5-th element looks nice only due to Milla Jovovich.

In reality we have to trade 55-orders fine-tuning to the ∞-orders one, plus another 55 for the quintessence.

Further aspects of the CC Problem are as follows:

1) The Universe is not static, hence both ρ_Λ^{vac} and ρ_Λ^{ind} can independently run, at least in the Early Universe.

2) There are also possible abrupt changes of the overall observed CC due to the phase transitions in the Early Universe.

3) Finally, it looks like our Universe was somehow "prepared", from the initial moment of its "creation", with a 55 -order precision, such that today $\rho_\Lambda^{obs} \sim \rho_c$.

4) This fine-tuning, up to now, is impossible to explain.

5) The last observation on the CC Problem.

ρ_Λ^{vac} is an independent parameter which has to be adjusted, with at least 55 orders of magnitude precision, to cancel ρ_Λ^{ind} .

Therefore, a solution of the CC problem has to start by explaining the value of ρ_Λ^{ind} from the first principles.

However this quantity depends on the VEV of the Higgs field, on scalar coupling, on W and Z masses, all other couplings, on the EW phase transition, on chiral phase transition and also on higher loop (up to 21 loops!!) corrections within MSM. And also on the details of possible physics beyond MSM, of course.

55 orders of precision require all this. So, we can see that "solving" the CC problem from the first principles requires, as a preliminary step, deriving the particle mass spectrum of the Standard Model (and its extensions) from the first principles.

We are currently far from this level of knowledge in fundamental physics. For this reason it is right to call it the great CC problem.

• Can symmetries help to solve the CC Problem?

There were many attempts to solve the CC problem introducing more symmetries. A remarkable example is SUSY.

However, the CC problem emerges at very low energies, where SUSY is broken.

Thus, SUSY may solve the problem, but only at high energies, where CC problem does not exist.

In (super)string theory, situation is even more complicated because the choice of a vacuum is not definite.

Furthermore, even if some string vacuum would “indicate” zero CC, it is unclear how this can affect the low-energy physics.

At low energies, we know that the appropriate theory is QFT (specifically the SM, with SSB etc) and not a string theory.

- **Auto-relaxation mechanisms**

There was a number of interesting attempts to create a sort of automatic mechanism for relaxing the CC

Dolgov; Peccei, Solà, Wetterich; Hawking; Ford et al.
Recently: *Štefančić; Grande, Solà, Štefančić.*

Weinberg (1989) discussed some of these approaches: they merely move fine tuning from CC to other parameter(s).

At the same time, it seems no comprehensive proof of this "no-go theorem" was given.

The only visible way to a solution:
Maybe one can modify SM or Einstein equations in such a way that gravity does not "feel" induced CC.

- **Antropic arguments.**

Weinberg, Garriga & Vilenkin, Donoghue, ...

This approach may be the most realistic, it also agrees with the QFT principles.

The idea is to study the limits on the CC and other parameters (e.g. neutrino mass) from the fact that the universe is compatible with the human life and civilization.

For example, negative CC does not let the cosmic structure form sufficiently fast, too large positive CC leads to other problems.

The "shortcoming" of this approach is that we never learn why the two counterparts of the CC do cancel.

• Renormalization Group (RG) solutions.

At low energies the quantum effects of some kind is supposed to produce an efficient screening of the observable CC.

Some realizations of this idea:

1) **IR effects of quantum gravity. Qualitative discussion -** *Polyakov, 1982, 2001.*

2) **Attempt to support this idea by direct calculations on fixed** dS **background** – *Tsamis & Woodard et al, 1995-2010.*

3) **More real thing: IR quantum effects of the conformal factor in** $4d$ *– Antoniadis and Mottola, 1992.*

4) **Using the assumed non-Gaussian UV fixed point in Quantum Gravity, assymptotic safety** *– Reuter, Percacci et al, from 2000.*

5) **Driving induced CC between the GUT scale** M_X **and the cosmic scale** μ_c **by the quantum effects of GUT's.** *– I.Sh., 1994; Jackiw et al, 2005.*

General Situation and effective approach to the CC Problem.

- **There are vacuum and induced contributions to CC. Both of them are ≥ 55 orders of magnitude greater than the observed sum. the vacuum part ρ_Λ^{vac} is unique independent part of CC.**

- **The main CC problem (I) is a hierarchy problem due to the conflict between particle physics scale $\sim 100\, GeV$ and the cosmic scale $\mu_c \sim 10^{-42}\, GeV$. That is why we need 55-order (at least!) fine-tuning.**

- **From the QFT viewpoint vanishing overall CC would be much worst thing. In this case we would need ∞-order fine-tuning.**

- **The coincidence problem (II) is: Why $\rho_\Lambda^{obs} \propto \rho_c$ at the present epoch. The two problems are closely related.**

We take a phenomenological point of view and don't try solving problems (I) & (II). Instead we consider problem (III): whether CC may vary due to IR quantum effects, e.g., of matter fields.

CC can vary due to the RG running?

At high energies scalar m_s and fermion m_f lead to RG equation

$$(4\pi)^2 \, \mu \, \frac{d\rho_\Lambda}{d\mu} \, = \, \frac{m_s^4}{2} \, - \, 2m_f^4 \, + \, \dots \, . \qquad (1)$$

To use this RG in cosmology, we have to answer two questions:

- **What is** μ?

- •• **At which energy scale Eq.** (1) **can be used?**

The answer to • is almost obvious: in the late Universe $\mu \sim H$.

The answer to •• is not that simple.

If applied to the late Universe, (1) **results in a very fast running of CC, breaking the standard cosmological model.**

This does not happen, because in QFT there is a phenomenon called decoupling.

Decoupling at the classical level.

Consider propagator of massive field at very low energy

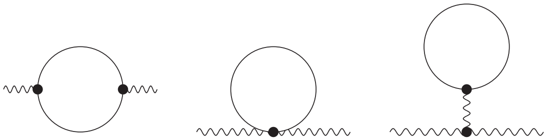
$$\frac{1}{k^2+m^2} = \frac{1}{m^2}\left(1 - \frac{k^2}{m^2} + \frac{k^4}{m^4} + ...\right) .$$

In case of $k^2 \ll m^2$ there is no propagation of particle.

What about quantum theory, loop corrections?

Formally, in loops integration goes over all values of momenta.

Is it true that the effects of heavy fields always become irrelevant at low energies?



For simplicity, consider a fermion loop effect in QED.

In the UV, the mass of quantum fermion is negligible, this simplifies the form factor, and we arrive at

$$\tilde{\beta}\, F^{\mu\nu}\, \ln\left(\frac{\Box}{\mu^2}\right) F_{\mu\nu}\,.$$

The momentum-subtraction β-function

$$\beta_e^1 \,=\, \frac{e^3}{6a^3\,(4\pi)^2}\,\left[20a^3 - 48a + 3(a^2-4)^2\,\ln\left(\frac{2+a}{2-a}\right)\right]\,,$$

$$a^2 = \frac{4\Box}{\Box - 4m^2}\,.$$

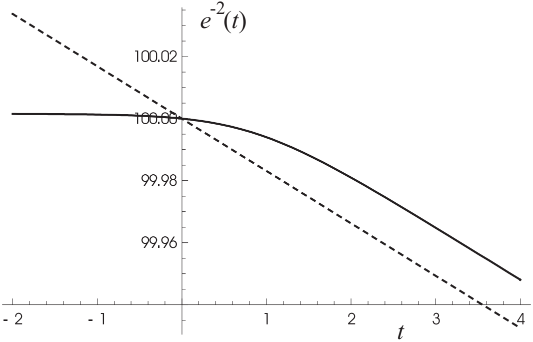
Special cases:

UV limit $\quad p^2 \gg m^2 \implies \beta_e^{1\;UV} \,=\, \frac{4\,e^3}{3\,(4\pi)^2} + \mathcal{O}\left(\frac{m^2}{p^2}\right).$

IR limit $\quad p^2 \ll m^2 \implies \beta_e^{1\;IR} \,=\, \frac{e^3}{(4\pi)^2}\cdot\,\frac{4\,p^2}{15\,m^2} \,+\, \mathcal{O}\left(\frac{p^4}{m^4}\right).$

This is the standard form of the Appelquist and Carazzone decoupling theorem (PRD, 1977).

One can obtain the general expression which interpolates between the UV and IR limits.



These plots show the effective electron charge as a function of $\log(\mu/\mu_0)$ in the case of the MS-scheme, and for the momentum-subtraction scheme, with $\ln(p/\mu_0)$.

An interesting high-energy effect is a small apparent shift of the initial value of the effective charge.

In the gravitational sector we meet Appelquist and Carazzone - like decoupling, but only in the higher derivative sectors.
In the perturbative approach, with $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we do not see running for the cosmological and inverse Newton constants.
Why do we get $\beta_\Lambda = \beta_{1/G} = 0$?

Momentum subtraction running corresponds to the insertion of, e.g., $\ln(\Box/\mu^2)$ formfactors into effective action.

Say, in QED: $$-\frac{e^2}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{3(4\pi)^2}\,F_{\mu\nu}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,F^{\mu\nu}\,.$$

Similarly, one can insert formfactors into

$$C_{\mu\nu\alpha\beta}\,\ln\Big(-\frac{\Box}{\mu^2}\Big)\,C_{\mu\nu\alpha\beta}\,.$$

However, such insertion is impossible for Λ and for $1/G$, because $\Box\Lambda \equiv 0$ and $\Box R$ is a full derivative.

Further discussion:
Ed. Gorbar & I.Sh., JHEP (2003); J. Solà & I.Sh., PLB (2010).

Is it true that physical $\beta_\Lambda = \beta_{1/G} = 0$ **?**

Probably not. Perhaps the linearized gravity approach is simply not an appropriate tool for the CC and Einstein terms.

Let us use the covariance arguments. The EA can not include odd terms in metric derivatives. In the cosmological setting this means no $\mathcal{O}(H)$ **and also no** $\mathcal{O}(H^3)$ **terms, etc. Hence**

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{16\pi G(H)} = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\left(H^2 - H_0^2\right), \qquad \nu = const\,.$$

Then the conservation law for $G(H;\nu)$ **gives**

$$G(H;\nu) = \frac{G_0}{1 + \nu\,\ln\left(H^2/H_0^2\right)}, \quad \textbf{where} \quad G(H_0) = G_0 = \frac{1}{M_P^2}\,.$$

Here we used the identification

$$\mu \sim H \qquad \text{in the cosmological setting.}$$

The same $\rho_\Lambda(\mu)$ follows from the assumption of the Appelquist and Carazzone - like decoupling for CC.

A.Babic, B.Guberina, R.Horvat, H.Štefančić, PRD 65 (2002); I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB 574 (2003).

We know that for a single particle

$$\beta_\Lambda^{MS}(m) \sim m^4 \,,$$

hence the quadratic decoupling gives

$$\beta_\Lambda^{IR}(m) = \frac{\mu^2}{m^2}\,\beta_\Lambda^{MS}(m) \sim \mu^2 m^2 \,.$$

The total beta-function will be given by algebraic sum

$$\beta_\Lambda^{IR} = \sum k_i \mu^2 m_i^2 = \sigma M^2\,\mu^2 \propto \frac{3\nu}{8\pi}\,M_P^2\,H^2 \,.$$

This leads to the same result in the cosmological setting,

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi}\,M_p^2\left(H^2 - H_0^2\right).$$

One can obtain the same $G(\mu)$ in one more independent way.

I.Sh., J. Solà, JHEP (2002); C. Farina, I.Sh. et al, PRD (2011).

Consider $\overline{\rm MS}$-based renormalization group equation for $G(\mu)$:

$$\mu\frac{dG^{-1}}{d\mu} = \sum_{particles} A_{ij}\, m_i\, m_j = 2\nu\, M_P^2\,, \qquad G^{-1}(\mu_0) = G_0^{-1} = M_P^2\,.$$

Here the coefficients A_{ij} depend on the coupling constants, m_i are masses of all particles. In particular, at one loop,

$$\sum_{particles} A_{ij}\, m_i\, m_j = \sum_{fermions} \frac{m_f^2}{3(4\pi)^2} - \sum_{scalars} \frac{m_s^2}{(4\pi)^2}\Big(\xi_s - \frac{1}{6}\Big)\,.$$

One can rewrite it as

$$\mu\frac{d(G/G_0)}{d\mu} = -2\nu\,(G/G_0)^2 \quad\Longrightarrow\quad G(\mu) = \frac{G_0}{1+\nu\,\ln\left(\mu^2/\mu_0^2\right)}\,. \qquad (*)$$

It is the same formula which results from covariance and/or from AC-like quadratic decoupling for the CC plus conservation law.
$(*)$ seems to be a unique possible form of a relevant $G(\mu)$.

All in all, it is not a surprise that the eq.

$$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)}\,.$$

emerges in different approaches to renorm. group in gravity:

- **Higher derivative quantum gravity.**

A. Salam & J. Strathdee, PRD (1978);
E.S. Fradkin & A. Tseytlin, NPB (1982).

- **Non-perturbative quantum gravity with (hipothetic) UV-stable fixed point.**

A. Bonanno & M. Reuter, PRD (2002).

- **Semiclassical gravity.**

B.L. Nelson & P. Panangaden, PRD (1982).

So, we arrived at the two relations:

$$\rho_\Lambda(H) = \rho_\Lambda(H_0) + \frac{3\nu}{8\pi} M_p^2 \left(\mu^2 - \mu_0^2\right) \qquad (1)$$

and

$$G(\mu) = \frac{G_0}{1 + \nu \ln\left(\mu^2/\mu_0^2\right)} \,. \qquad (2)$$

Remember the standard identification

$\mu \sim H$ **in the cosmological setting.**

A. Babic, B. Guberina, R. Horvat, H. Štefančić, PRD (2005).

Cosmological models based on the assumption of the standard AC-like decoupling for the cosmological constant:

● **Models with (1) and energy matter-vacuum exchange:**
I.Sh., J.Solà, Nucl.Phys. (PS), IRGA-2003;
I.Sh., J.Solà, C.España-Bonet, P.Ruiz-Lapuente, PLB (2003).

● ● **Models with (1), (2) and without matter-vacuum exchange:**
I.Sh., J.Solà, H.Štefančić, JCAP (2005).

- **Models with constant $G \equiv G_0$ and permitted energy exchange between vacuum and matter sectors.**

For the equation of state $P = \alpha\rho$ the solution is analytical,

$$\rho(z\,;\nu) \,=\, \rho_0\,(1+z)^r\,,$$

$$\rho^\Lambda(z\,;\nu) \,=\, \rho_0^\Lambda \,+\, \frac{\nu}{1-\nu}\,\left[\,\rho(z\,;\nu) - \rho_0\,\right]\,,$$

The limits from density perturbations / LSS data: $|\nu| < 10^{-6}$.

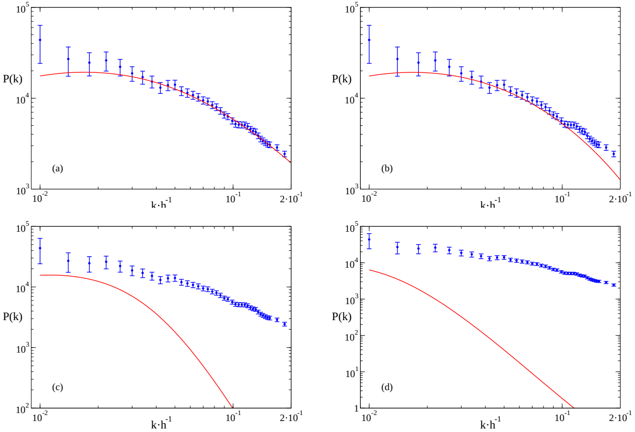
Analog models:
Opher & Pelinson, PRD (2004); Wang & Meng, Cl.Q.Gr. 22 (2005).

Direct analysis of cosmic perturbations:
J. Fabris, I.Sh., J. Solà, JCAP 0702 (2007).

Given the Harrison-Zeldovich initial spectrum, the power spectrum today can be obtained by integrating the eqs. for perturbations.

Initial data based on $w(z)$ **from** *J.M. Bardeen et al, Astr.J. (1986).*

Results of numerical analysis for the ● model:



The ν-dependent power spectrum vs the LSS data from the 2dfFGRS. The ordinate axis represents $P(k) = |\delta_m(k)|^2$ **where** $\delta_m(k)$ **is the solution at** $z = 0$. $\quad \nu = 10^{-8},\ 10^{-6},\ 10^{-4},\ 10^{-3}$.
In all cases $\Omega_B^0, \Omega_{DM}^0, \Omega_\Lambda^0 = 0.04, 0.21, 0.75$.

●● Models with variable $G = G(H)$ but without energy exchange between vacuum and matter sectors.

Theoretically this looks much better!

$$\rho_\Lambda(H) \,=\, \rho_\Lambda(H_0) \,+\, \frac{3\nu}{8\pi}\, M_p^2\left(H^2 - H_0^2\right).$$

By using the energy-momentum tensor conservation we find

$$G(H;\nu) = \frac{G_0}{1+\nu\,\ln\left(H^2/H_0^2\right)}\,, \quad \textbf{where} \quad G(H_0) = \frac{1}{M_P^2}\,.$$

These relations exactly correspond to the RG approach discussed above, with $\mu = H$.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

The limits on ν from density perturbations, etc.

J.Grande, J.Solà, J.Fabris & I.Sh., Cl. Q. Grav. 27 (2010) .

An important general result is: In the models with variable Λ and G in which matter is covariantly conserved, the solutions of perturbation equations *do not* depend on the wavenumber k.

As a consequence we meet relatively weak modifications of the spectrum compared to ΛCDM.

The bound $\nu < 10^{-3}$ comes just from the "F-test". It is related only to the modification of the function $H(z)$.

R. Opher & A. Pelinson, astro-ph/0703779.
J.Grande, R.Opher, A.Pelinson, J.Solà, JCAP 0712 (2007).

One can obtain the same restriction for ν also from the primordial nucleosynthesis (BBN).

Can we apply the running $G(\mu)$ to other physical problems?

In the renormalization group framework the relation

$$G(\mu) = \frac{G_0}{1 + \nu \,\ln\left(\mu^2/\mu_0^2\right)}\,, \quad \textbf{where} \quad \mu = H$$

in the cosmological setting.

What could be an interpretation of μ in astrophysics?

Consider the rotation curves of galaxies. The simplest assumption is $\mu \propto 1/r$.

Applications for the point-like model of galaxy:

J.T.Goldman, J.Perez-Mercader, F.Cooper & M.M.Nieto, PLB (1992).
O. Bertolami, J.M. Mourao & J. Perez-Mercader, PLB 311 (1993).
M. Reuter & H. Weyer, PRD **70** *(2004); JCAP* **0412** *(2004).*
I.Sh., J.Solà, H.Štefančić, JCAP (2005).

We can safely restrict the consideration by a weakly varying G,

$$G = G_0 + \delta G = G_0(1+\kappa)\,, \qquad |\kappa| \ll 1\,.$$

The value of ν is small, the same should be with $\kappa = \delta G/G_0$.
Perform a conformal transformation

$$\bar{g}_{\mu\nu} = \frac{G_0}{G}\, g_{\mu\nu} = (1-\kappa) g_{\mu\nu}\,.$$

In $\mathcal{O}(\kappa)$, metric $\bar{g}_{\mu\nu}$ obeys Einstein equations with $G_0 = const$.

The nonrelativistic limits of the two metrics

$$g_{00} = -1 - \frac{2\Phi}{c^2} \quad \text{and} \quad \bar{g}_{00} = -1 - \frac{2\Phi_{\text{Newt}}}{c^2}\,,$$

Φ_{Newt} being Newton potential and Φ is a modified potential.

$$g_{00} \,=\, -1 - \frac{2\Phi}{c^2} \approx -1 - \frac{2\Phi_{\text{Newt}}}{c^2} - \kappa \implies \Phi = \Phi_{\text{Newt}} + \frac{c^2\,\delta G}{2\,G_0}\,.$$

For the nonrelativistic limit of the modified gravitaty we obtain

$$-\Phi^{,i} \,=\, -\Phi^{,i}_{\text{Newt}} - \frac{c^2\,G^{,i}}{2\,G_0}\,, \quad \text{where we used} \quad G^{,i} = (\delta G)^{,i}\,.$$

The last formula $$-\Phi^{,i} = -\Phi^{,i}_{\text{Newt}} - \frac{c^2\,G^{,i}}{2\,G_0}$$ is very instructive.

- Quantum correction comes with the factor of $c^2 \Longrightarrow$ can make real effect at the typical galaxy scale.

E.g., for a point-like model of galaxy and $\mu \propto 1/r$ it is sufficient to have $\nu \approx 10^{-6}$ to provide flat rotation curves.

I.Sh., J.Solà, H.Štefančić, JCAP (2005).

- - $\mu \propto 1/r$ is, obviously, not a really good choice for a non-point-like model of the galaxy.

The reason is that this identification produces the "quantum-gravitational" force even if there is no mass at all !!

What would be the "right" identification of μ ?

Let us come back to QFT, which offers a good hint:
μ **must be** $\sim$ **energy of the external gravitational line in the Feynman diagram in the almost-Newtonian regime.**

The phenomenologically good choice is

$$\frac{\mu}{\mu_0} = \left(\frac{\Phi_{\text{Newt}}}{\Phi_0}\right)^{\alpha},$$

where α **is a phenomenological parameter We have found that** α **is generally growing with the mass of the galaxy.**

D. Rodrigues, P. Letelier & I.Sh., JCAP (2010).

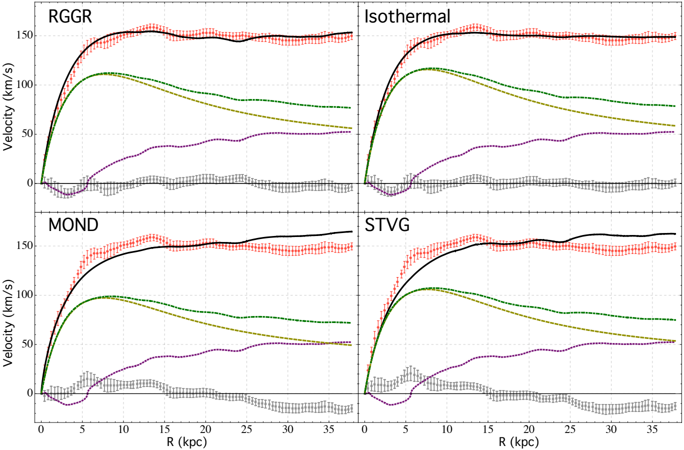
QFT viewpoint: α **reflects** $\mu \sim \Phi_{\text{Newt}}$ **is not an ultimate choice.**

With greater mass of the galaxy the "error" in identification becomes greater too, hence we need a greater α **to correct this.**
α **must be very small at the scale of the Solar system.**

Regular scale-setting procedure gives the same result:
S. Domazet & H. Štefančić, PLB (2011).

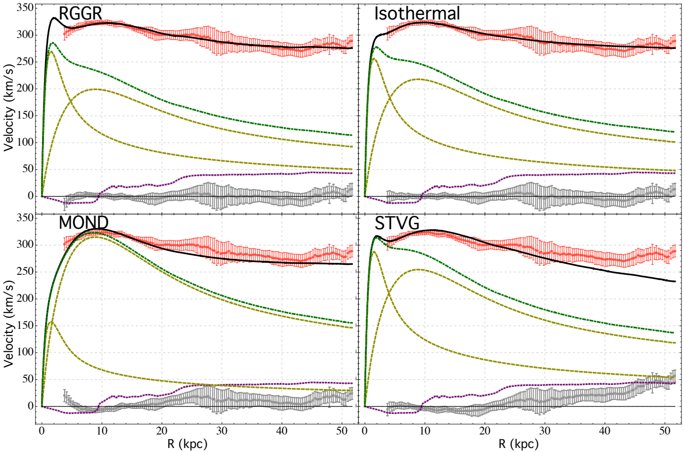
Last, but not least, the astro-ph application is impressively successful

D. Rodrigues, P. Letelier & I.Sh., JCAP (2010). *(9 samples)*



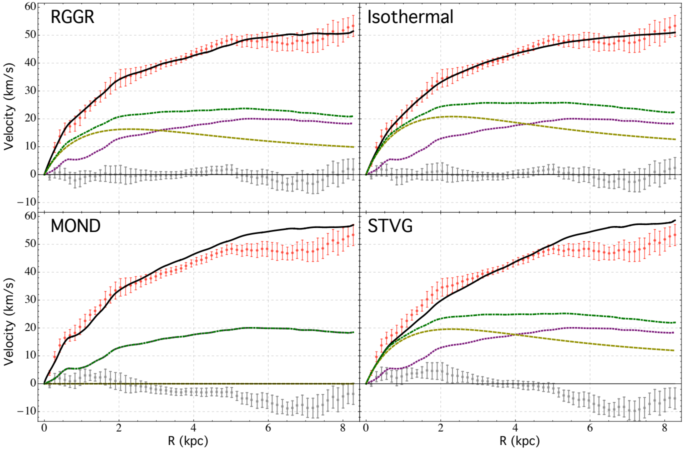
Rotation curve of the spiral galaxy *NGC 3198.* $\alpha\nu = 1.7 \times 10^{-7}$.
[Collaboration THINGS (2008)].

One more example, this time with descendent rotation curve.
$\alpha\nu = 6.7 \times 10^{-7}$.



Rotation curve of the galaxy *NGC 2841*. **RGGR is based on hypothetical covariant quantum corrections without DM.**

One more example: low-surface brightness galaxy with ascendent rotation curve. $\alpha\nu = 0.2 \times 10^{-7}$.



Rotation curve of the galaxy *DDO 154*. RGGR is based on hypothetical covariant quantum corrections without DM.

What about the Solar System?

C. Farina, W. Kort-Kamp, S. Mauro & I.Sh., PRD 83 (2011).

We used the dynamics of the Laplace-Runge-Lenz vector in the $G(\mu) = G_0/(1 + \mu \log(\mu/\mu_0))$ - **corrected Newton gravity.**

Upper bound for the Solar System: $\alpha\nu \leq 10^{-17}$.

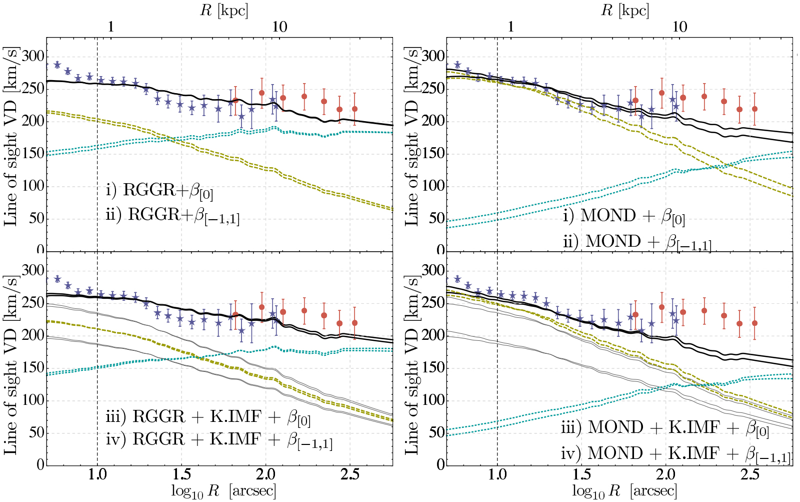
One of the works now on track: extending the galaxies sample.

P. Louzada, D. Rodrigues, J. Fabris, ..., in work: **50+ disk galaxies.**

Davi Rodrigues, ..., in progress: **elliptical galaxies.**

The general tendency which we observe so far is greater α **needed to for larger mass of the astrophysical object: from Solar System (upper bound) to biggest tested galaxies.**

Very new, yet unpublished, example.



Rotation curve of the giant elliptic galaxy NGC 4374: RGGR vs MOND. $\alpha\nu = 17 \times 10^{-7}$. **Special thanks to PN.S. Collaboration.**

It looks like we do not need CDM to explain the rotation curves of the galaxies. However, does it really mean that we can really go on with one less dark component?

Maybe not, but it is worthwhile to check it. It is well known that the main requests for the DM come from the fitting of the LSS, CMB, BAO, lenthing etc.

However there is certain hope to relpace, e.g., ΛCDM by a ΛWDM (e.g. sterile neutrino) with much smaller Ω_{DM}.

The idea to trade $0.04, 0.23, 0.73 \quad \Longrightarrow \quad 0.04,\ 0.0x,\ 0.9(1\text{-}x)$

Such a new concordance model would have less relevant coincidence problem, and in general such a possibility is interesting to verify.

First move:

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275; PRD (2012).

We are using “our” Reduced Relativistic Gas model.

The Reduced Relativistic Gas model is a Simple cosmological model with relativistic gas.

G. de Berredo-Peixoto, I.Sh., F. Sobreira, Mod.Ph.Lett. A (2005);
J. Fabris, I.Sh., F.Sobreira, JCAP (2009).

The model describes ideal gas of massive relativistic particles with all of them have the same kinetic energy.

The Equation of State (EOS) of such gas is

$$P = \frac{\rho}{3}\left[1 - \left(\frac{mc^2}{\varepsilon}\right)\right]^2 = \frac{\rho}{3}\left(1 - \frac{\rho_d^2}{\rho^2}\right).$$

In this formula ε is the kinetic energy of the individual particle, $\varepsilon = mc^2/\sqrt{1-\beta^2}$. Furthermore, $\rho_d = \rho_{d0}^2(1+z)^3$ is the mass (static energy) density. One can use one or another form of the equation of state (1), depending on the situation.

The deviation from Maxwell or relativistic Fermi-Dirac distribution is less than 2.5%. The nice thing is that one can solve the Friedmann equation in this model analytically.

The model was successfully used to impose an upper bound to the warmness of DM from LSS data, providing the same results as more complicated models.

J. Fabris, I.Sh., F.Sobreira, JCAP (2009).

So, why it is "our" and not just our model?

Because we were not first. The same EOS has been used by A.D. Sakharov in 1965. to predict the oscillations in the CMB spectrum for the first time!!

A.D. Sakharov, Soviet Physics JETP, 49 (1965) 345.

In the recent paper

J. Fabris, A. Toribio & I.Sh., Testing DM warmness and quantity via the RRG model. arXiv:1105.2275 [astro-ph.CO]; PRD-2012

we have used RRG without quantum effects to fit
Supernova type Ia (Union2 sample), $H(z)$**, CMB (**R **factor), BAO, LSS (2dfGRS data)**
In this way we confirm that Λ**CDM is the most favored model.**

However, for the LSS data alone we met the possibility of an alternative model with a small quantity of a WDM.

This output is potentially relevant due to the fact the LSS is the test which is not affected by the possible quantum RG running in the low-energy gravitational action.

Such a model almost has no issue with the coincidence CC problem (II), because $\Omega_\Lambda^0 \simeq 0.95$ **.**

Conclusions

- **CC term is a very natural and in fact necessary concept, which should be separated from myths and legends.**

- **It looks like there is no real chance to solve the great CC problem from the "first principles", especially because for this end one needs the real knowledge of these principles.**

- **We can learn a lot by thinking about the CC problem, such thinking is definitely not "forbidden".**

- **The question of whether CC can be variable is, to some extent, reduced to existing-nonexisting paradigm.**

- **In the positive case we arrive at the very rich cosmological and astrophysical model with one free parameter ν plus certain freedom of scale identification.**