Phase Structure of the Polyakov–Quark-Meson Model beyond Mean Field

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6$^{\mathrm{th}}$ International Workshop

on

Critical Point and Onset of Deconfinement (CPOD)

JINR, Dubna, Russia

QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$

   order parameter:

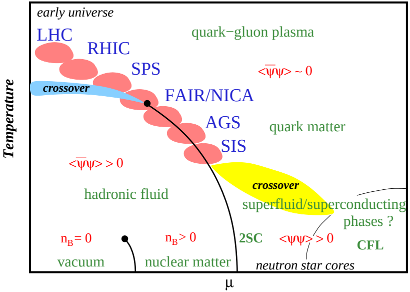
   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

2. de/confinement

   order parameter: Polyakov loop variable

   $$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase, } T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase, } T > T_c \end{cases}$$

   $$\Phi = \left\langle \mathrm{tr}_c \mathcal{P} \exp\left(i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle / N_c$$



At densities/temperatures of interest only model calculations available

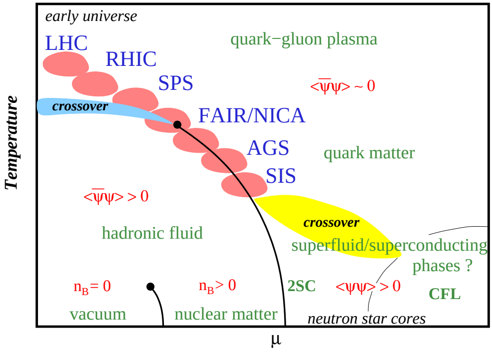
alternative: ➜ dressed Polyakov loop (or dual condensate)

it relates chiral and deconfinement transition to spectral properties of Dirac operator

effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

The conjectured QCD Phase Diagram



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Open issues:

related to chiral & deconfinement transition

- ▷ existence of CEP?
- ▷ its location?
- ▷ additional CEPs? How many?
- ▷ coincidence of both transitions at $\mu = 0$?
- ▷ quarkyonic phase at $\mu > 0$?
- ▷ chiral CEP/ deconfinement CEP?
- ▷ so far only MFA results effect of fluctuations (e.g. size of crit. reg.)?
- ▷ ...

Outline

- **Polyakov–Quark-Meson Model**

- **Functional renormalization group (FRG)**

$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling h:

$$\mathcal{L}_{\text{quark}} = \bar{q}\left(i\partial\!\!\!/ - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a)\right)q$$

Meson part: scalar σ_a and pseudoscalar π_a nonet

fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2\text{tr}[M^\dagger M] - \lambda_1(\text{tr}[M^\dagger M])^2 - \lambda_2\text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)]$$
$$+\text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with $b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$

- **logarithmic potential:** **Rößner et al. 2007**

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

with $a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2$ and $b(T) = b_3(T_0/T)^3$

- **Fukushima** **Fukushima 2008**

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

a controls deconfinement $\quad$ b strength of mixing chiral & deconfinement

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with $b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$

in presence of dynamical quarks: $T_0 = T_0(N_f, \mu)$ **BJS, Pawlowski, Wambach, 2007**

N_f	0	1	2	$2+1$	3
T_0 [MeV]	270	240	208	187	178

$\mu \neq 0:$ $\bar{\phi} > \phi$

since $\bar{\phi}$ is related to free energy gain of antiquarks

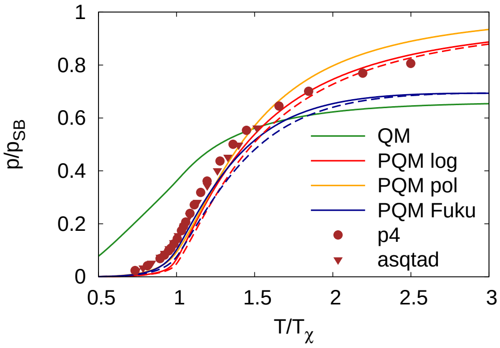
in medium with more quarks → antiquarks are more easily screened.

QCD Thermodynamics $N_f = 2+1$

[BJS, M. Wagner, J. Wambach '10]

SB limit: $\frac{p_{SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations



- ▷ solid lines:
  PQM with lattice masses (HotQCD)
  $m_\pi \sim 220, m_K \sim 503$ MeV
- ▷ dashed lines:
  (P)QM with realistic masses

lattice data: [Bazavov et al. '09]

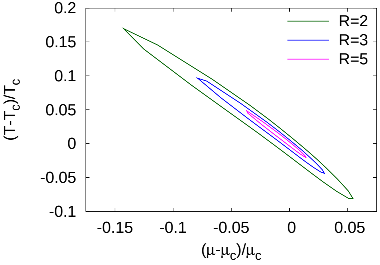
Critical region

contour plot of **size of the critical region** around CEP in the phase diagram

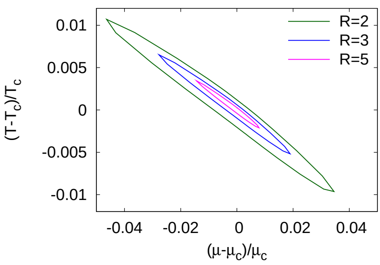
Contour lines are defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop

QM model (2+1)



PQM model (2+1)



[BJS, M. Wagner; in preparation]

Outline

○ **Polyakov–Quark-Meson Model**

○ **Functional renormalization group (FRG)**

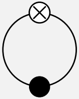
Functional RG Approach

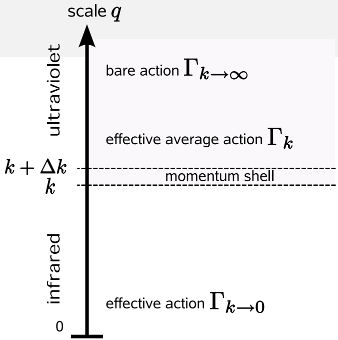
$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) **[Wetterich '93]**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$





Isentropes $s/n = \text{const}$ and Focussing

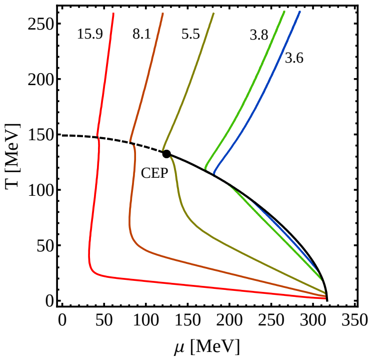
[E. Nakano, BJS, B.Stokic, B.Friman, K.Redlich '10]

here: $N_f = 2$ QM model: kink in MFA are washed out in FRG

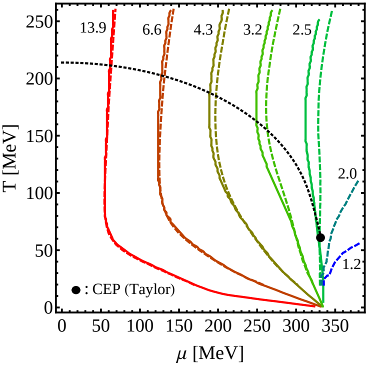
$\rightarrow$ **no focussing if fluctuations taken into account**

smallnest of critical region

MFA (no Dirac term)



FRG (dashed: full grid; solid: Taylor expansion)



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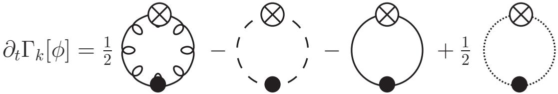
kink structure at boundary in mean field approximation

$\Rightarrow$ remnant of first-order transition in chiral limit

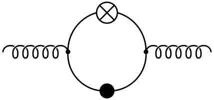
if Dirac term neglected

$\mathbf{T}_0(N_f, \mu)$ modification

full QCD FRG flow: gluon , ghosts, quark and meson (via hadronization) fluctuations
[J. Braun, H. Gies, L.M. Haas, F. Marhauser, J.M. Pawlowski et al.]



in presence of dynamical quarks
gluonic contribution modified:



pure YM flow
($\rightarrow$ Polyakov loop potential):

$$T_0 \leftrightarrow \Lambda_{QCD} \qquad : \qquad T_0 \rightarrow T_0(N_f, \mu)$$

[BJS, Pawlowski, Wambach, 2007]
[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Functional Renormalization Group

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \qquad ; \qquad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

PQM truncation $\mathrm{N_f} = 2$ **[Herbst, Pawlowski,BJS; arXiv:1008.0081]**

$$\Gamma_k = \int d^4x \left\{ \bar\psi \left(\not{D} + \mu\gamma_0 + ih(\sigma + i\gamma_5 \vec\tau\vec\pi) \right) \psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec\pi)^2 + \Omega_k[\sigma, \vec\pi, \Phi, \bar\Phi] \right\}$$

Initial action at UV scale Λ:

$$\Omega_\Lambda[\sigma, \vec\pi, \Phi, \bar\Phi] = U(\sigma, \vec\pi) + \mathcal{U}(\Phi, \bar\Phi) + \Omega_\Lambda^\infty[\sigma, \vec\pi, \Phi, \bar\Phi]$$

$$U(\sigma, \vec\pi) = \frac{\lambda}{4}(\sigma^2 + \vec\pi^2 - v^2)^2 - c\sigma$$

Functional Renormalization Group

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\ \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \qquad ; \qquad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

Flow equation for PQM $\mathbf{N_f = 2}$ **[Herbst, Pawlowski,BJS; arXiv:1008.0081]**

$$\begin{aligned}\partial_t \Omega_k \quad &= \quad \frac{k^5}{12\pi^2} \left[-\frac{2N_f N_c}{E_q} \left\{ 1 - N_q(T,\mu;\Phi,\bar\Phi) + N_{\bar q}(T,\mu;\Phi,\bar\Phi) \right\} \right. \\ &\qquad \left. + \frac{1}{E_\sigma} \coth\left(\frac{E_\sigma}{2T}\right) + \frac{3}{E_\pi} \coth\left(\frac{E_\pi}{2T}\right) \right]\end{aligned}$$

with $\quad E_{\sigma,\pi,q} = \sqrt{k^2 + m^2_{\sigma,\pi,q}}\ ,\ m_\sigma^2 = 2\Omega_k' + 4\sigma^2 \Omega_k''\ ,\ m_\pi^2 = 2\Omega_k'\ ,\ \ m_q^2 = g^2\sigma^2$

and

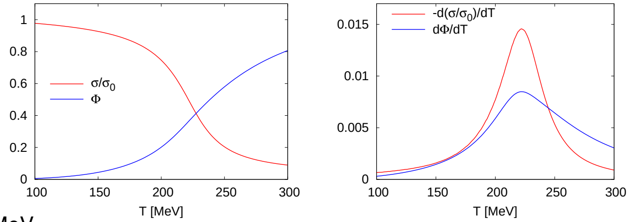
$$N_q(T,\mu;\Phi,\bar\Phi) \quad = \quad \frac{1 + 2\bar\Phi e^{\beta(E_q-\mu)} + \Phi e^{2\beta(E_q-\mu)}}{1 + 3\bar\Phi e^{\beta(E_q-\mu)} + 3\Phi e^{2\beta(E_q-\mu)} + e^{3\beta(E_q-\mu)}}$$

$$N_{\bar q}(T,\mu;\Phi,\bar\Phi) \quad = \quad N_q(T,-\mu;\Phi,\bar\Phi)|_{\mu\to-\mu}$$

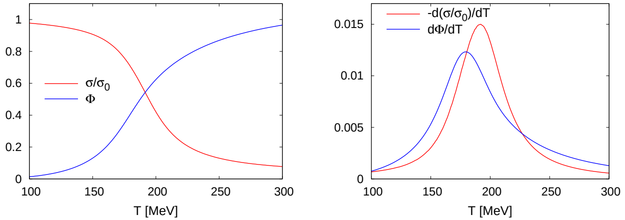
cf. [Skokov et al. arXiv:1004.2665]

$\mu = 0$: order parameters and T-derivatives

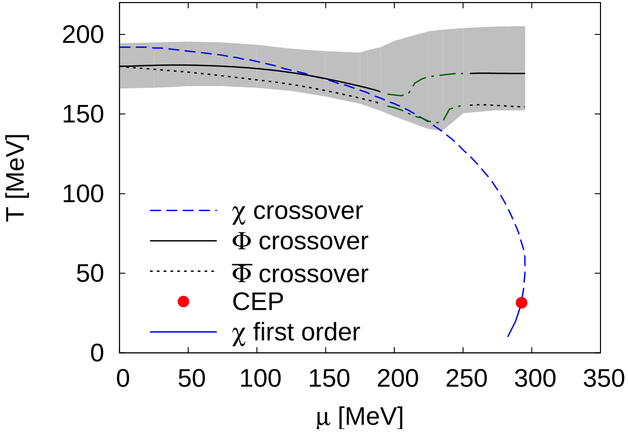
$T_0 = 270$ MeV



$T_0 = 208$ MeV

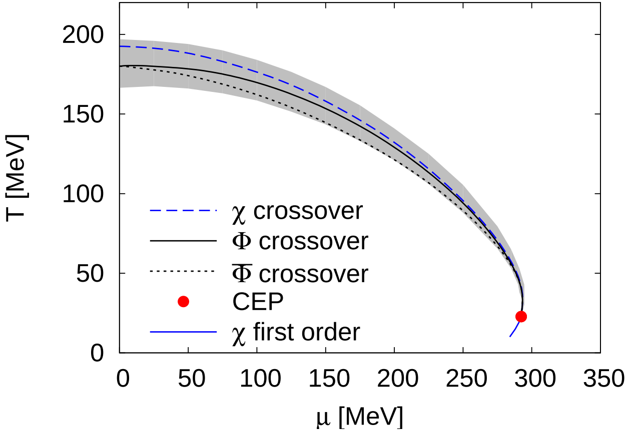


Phase diagram $T_0 = 208$ MeV



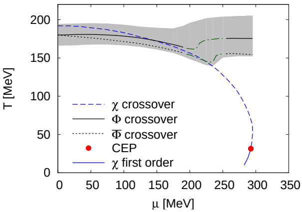
[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Phase diagram $T_0(\mu), T_0(0) = 208$ MeV



[Herbst, Pawlowski,BJS; arXiv:1008.0081]

Thermodynamics



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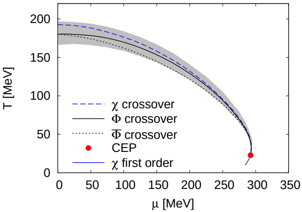
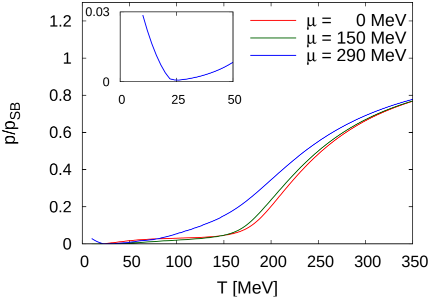
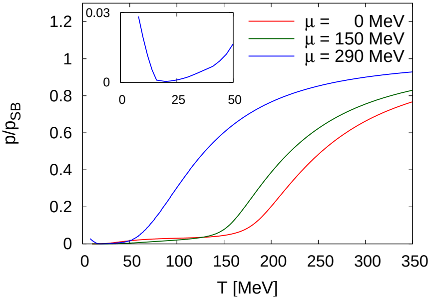


$T_0(\mu)$ MeV





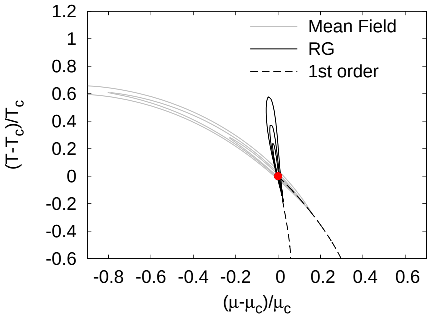
Critical region

similar conclusion if **fluctuations** are included

fluctuations via Functional Renormalization Group

comparison: $N_f = 2$ **QM model**

Mean Field $\leftrightarrow$ RG analysis



[BJS, J. Wambach '06]

Summary

- $N_f = 2$ and $N_f = 2+1$ chiral (Polyakov)-quark-meson model study

➜ Mean-field approximation and FRG

➜ **fluctuations are important**

Findings:

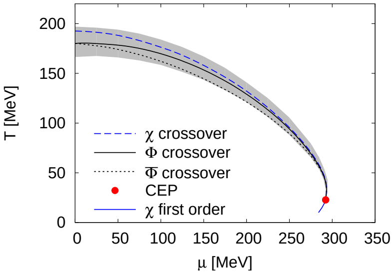
▷ matter **back-reaction to YM sector**: $T_0 \Rightarrow T_0(N_f, \mu)$

▷ **FRG with PQM truncation**: Chiral & deconfinement transition **coincide** for $N_f = 2$ with $T_0(\mu)$-corrections

▷ same conclusion for $N_f = 2+1$?

▷ **role of quantum fluctuations**

effects of Dirac term in a mean-field approximation



Outlook:

▷ include **glue dynamics** with FRG

$\longrightarrow$ towards full QCD

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