Microscopic calculation of the ⟨low-temperature⟩ equation of state of dense matter and neutron star structure

FHNC/CBF & AFDMC

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S. Gandolfi, AYuI, . . . Mon. Not. Roy. Astron. Soc. **404** (2010) L35–L39.

S. Gandolfi, AYuI, . . . Phys. Rev. **C79** (2009) 054005.

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Motivations

- $\rho < 4\rho_0$ and $T < 40$ MeV is at present completely unaccesible to QCD.
- Quantum Monte Carlo methods (GFMC and AFDMC) provide very accurate results for nuclear systems with $A \sim 100$.
- NN scattering data and few-body theory $\rightarrow$ nuclear Hamiltonians. Few-body $\rightarrow$ many-body $\rightarrow$ experiments/observations?
- EoS of nuclear and neutron matter relevant for nuclear astrophysics (neutron stars and supernovae).
- Temperature effects on the equation of state?!

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Many-body methods used: AFMDC & CBF/FHNC

AFDMC ($T = 0$)

- Diffusion Monte Carlo
- Hubbard-Startonovich transformation

$$G(\mathbf{R}, \mathbf{R}', \Delta\tau) = \left(\frac{m}{2\pi\hbar^2\Delta\tau}\right)^{\frac{3A}{2}} \exp\left[-\frac{m|\mathbf{R}-\mathbf{R}'|^2}{2\hbar^2\Delta\tau}\right] e^{-V_{SI}(\mathbf{R})\Delta\tau} \times \prod_{n=1}^{3A} \int \frac{dx_n}{\sqrt{2\pi}} \exp\left[-\frac{x_n^2}{2}\right] \exp\left[\sqrt{-\lambda_n\Delta\tau}x_n\widehat{O}_n\right]$$

- Convergence: Importance sampling
- Size problem: fixed-phase approximation
- Finite box (finite N): solved (TABC)
- v_{LS}: only neutron matter at present

CBF/FHNC ($T \geq 0$)

$$\Psi_i[n_i(\mathbf{k})] = \mathcal{S}\left(\prod_{i<j}\mathcal{F}_{ij}\right)\Phi_i[n_i(\mathbf{k})]$$

The pair correlation operator $\mathcal{F}_{ij}$:

$$\mathcal{F}_{ij} = \sum_{c,c\tau,s,s\tau,t,t\tau} f_p(r_{ij})O_{ij}^p .$$

The Gibbs-Bogoliubov variational principle

$$F(\rho,T) \leq F_V(\rho,T) = \mathrm{Tr}(\rho_V H) - TS_V(\rho,T),$$

Fermi-Hypernetted chain equations used to evaluate

$$\frac{E_V(\rho,T)}{A} = \frac{\hbar^2 k_{av}^2}{2m} + \sum \text{diagrams}(V, \mathcal{F}, \ell(r,\rho,T))$$

- SOC $\sim \rho^2$: variational violation
- Elem. diagrams $\sim \rho^3$: sum rule control
- v_{LS} at the *second order* only

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$$v^p_{\mathrm{DD6}'} = v^p_{\mathrm{OPEP}} + v^p_I e^{-\gamma_1\rho} + v^p_S + \mathrm{TNA}(\rho)\,,$$
$$\mathrm{TNA}(\rho) = 3\gamma_2\rho^2 e^{-\gamma_3\rho}\left(1 - \frac{2}{3}\left(\frac{\rho_n-\rho_p}{\rho_n+\rho_p}\right)^2\right)$$

with γ_1, γ_2 and γ_3 being fixed by means of AFDMC method, so as to reproduce the experimental values of the saturation density $\rho_0 = 0.16$ fm^{-3}, the binding energy per particle $E_0 = -16$ MeV and the compressibility $K = 9\rho_0^2 \left(\partial^2 E(\rho)/\partial\rho^2\right)_{\rho_0} \approx 240$ MeV.
The available scattering data in S-, P-, D-, F-waves are well reproduced.

parameter	*FHNC/SOC*	*AFDMC*
γ_1	0.15 fm^3	0.10 fm^3
γ_2	-700 fm^6	-750 fm^6
γ_3	13.6 fm^3	13.9 fm^3

The $\gamma_1\rho$ term simulates the effect of the three(many)-body repulsion.
TNA(ρ) simulates an attractive many-body contribution via correlations.

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S. Gandolfi, AYuI, S. Fantoni, J.C. Miller, F. Pederiva and K.E. Schmidt
Mon. Not. R. Astron. Soc. 404, L35 (2010) [arXiv:0909.3487]

$$E_{\mathrm{SNM}}(\rho)/A = E_0 + a(\rho-\rho_0)^2 + b(\rho-\rho_0)^3 e^{\gamma(\rho-\rho_0)}\,,$$

$$E(\rho,x_p)/A = E_{\mathrm{SNM}}(\rho)/A + C_s\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}(1-2x_p)^2\,.$$

Parameters: $E_0 = -16.0$ MeV, $\rho_0 = 0.16$ fm^{-3}, $a = 520.0$ MeVfm^6, $b = -1297.4$ MeVfm^9 and $\gamma = -2.213$ fm^3. $C_s = 31.3$ MeV and $\gamma_s = 0.64$.

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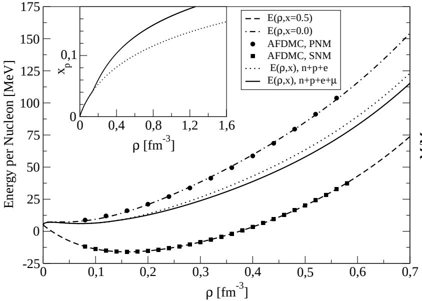

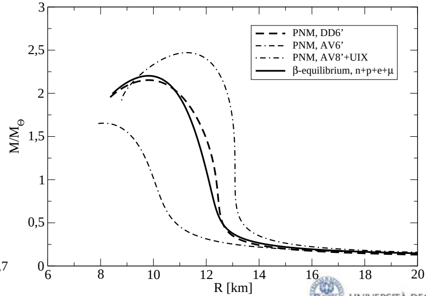

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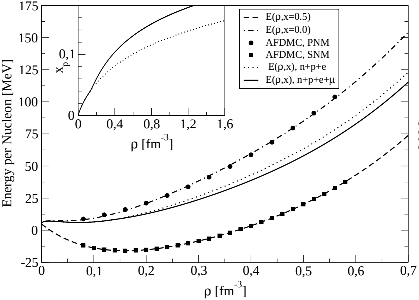

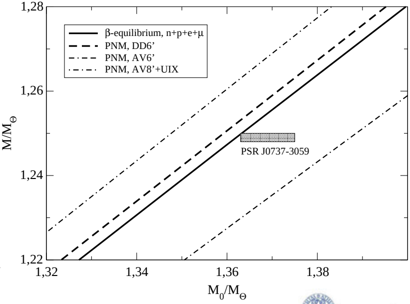

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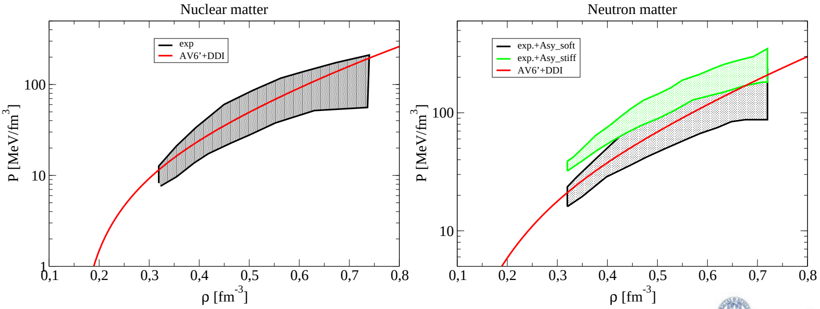

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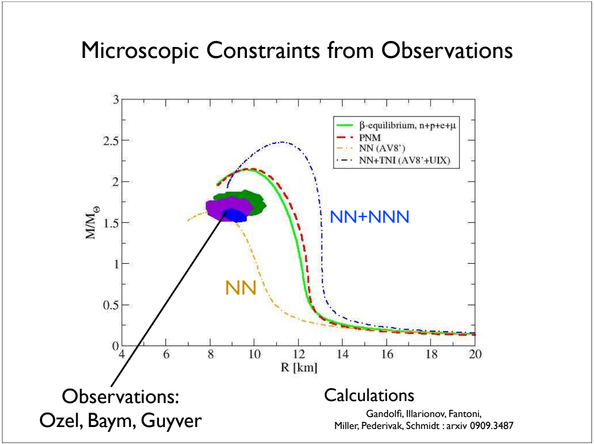
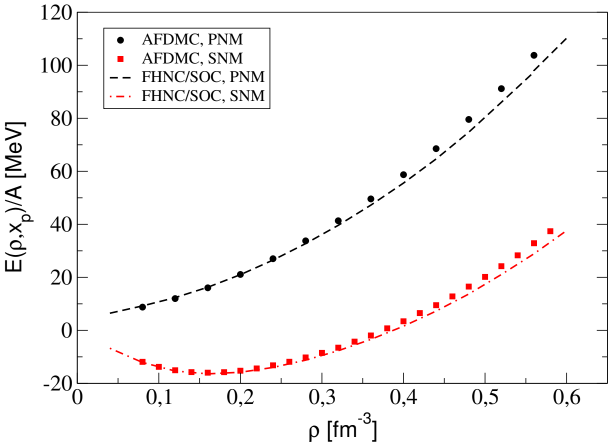


The constraned variational free energy:

$$F_{\text{con}}(\rho, T)/A = F(\rho, T)/A + \rho\Lambda\left[(I_c - 1)^2 + (I_\tau/3 + 1)^2\right] .$$



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AFDMC+FHNC/CBF: Hot nucleon matter / Free energy & Entropy

$$F(\rho, T, x_p)/A \approx E(\rho, x_p)/A - \alpha(x_p)\left(\frac{\rho_0}{\rho}\right)^{\beta(x_p)} T^2 ,$$

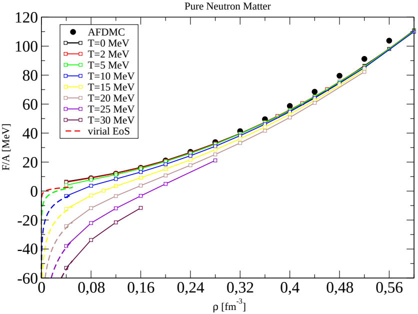
$$S(\rho, T, x_p) = -\left(\frac{\partial F/A}{\partial T}\right)_V \approx 2\alpha(x_p)\left(\frac{\rho_0}{\rho}\right)^{\beta(x_p)} T^2$$

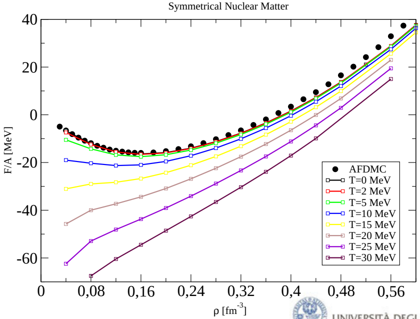
PNM: $x_p = 0$

$\alpha(0)/2^{1/3} = 0.0227, \beta(0) = 0.666$

SNM: $x_p = 1/2$

$\alpha(1/2)/4^{1/3} = 0.0253, \beta(1/2) = 0.647$





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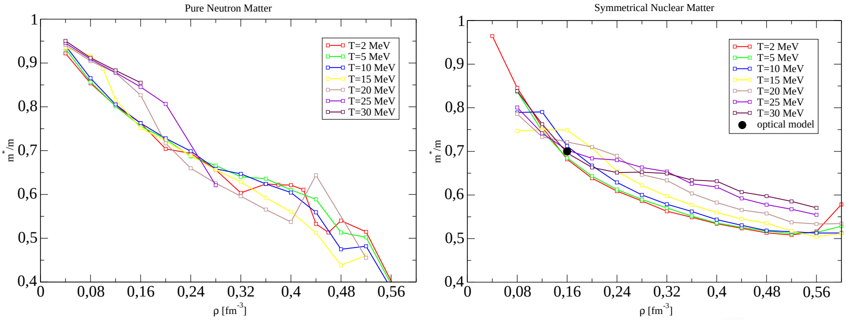
$$\epsilon(\mathbf{k},\rho,T) = \frac{\hbar^2 k^2}{2m\left[1 + A(\rho,T)\exp\left(-B(\rho,T)k^2\right)\right]}, \quad \frac{m^\star(\rho,T)}{m} = \frac{\hbar^2}{m}\left(\frac{1}{k}\frac{d\epsilon}{dk}\right)^{-1}_{k_F},$$

$$\bar{n}(\mathbf{k},\rho,T) = \frac{1}{\exp\left[\beta\left(\epsilon(\mathbf{k},\rho,T) - \mu(\rho,T)\right)\right] + 1}, \quad A = \sum_{\mathbf{k}} \bar{n}(\mathbf{k},\rho,T).$$



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Conclusions

- new EoS from microscopic calculations using the Auxiliary Field Diffusion Monte Carlo technique with nucleons interacting via a semi-phenomenological Hamiltonian (realistic 2-body + pheno many-body). LFP model revised.
- – observational constraints passed.
- first time an elementary diagrams contribution in FHNC/CBF is fully estimated $\rightarrow$ variational principle restored.
- new low-temperatire EoS from microscopic calculations using the restored FHNC/CBF technique.
  low-density limit $\rightarrow$ virial EoS, high-density limit $\rightarrow$ T^2/ρ^β.
- thermodynamics