Inhomogeneous chiral symmetry breaking phases

Michael Buballa

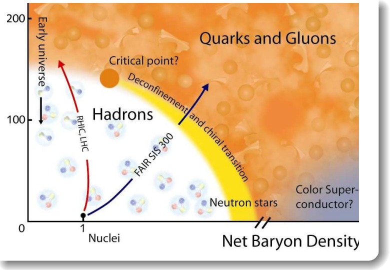
6^{th} International Conference on

Critical Point and Onset of Deconfinement

JINR Dubna, August 23 - 29, 2010

Motivation

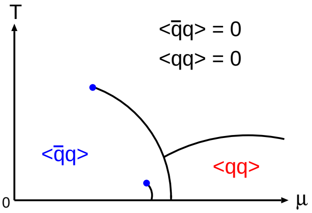
TECHNISCHE UNIVERSITÄT DARMSTADT



- QCD phase diagram

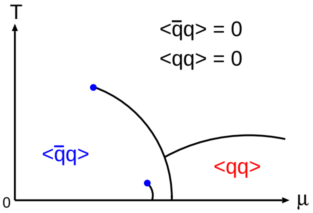
Motivation

TECHNISCHE
UNIVERSITÄT
DARMSTADT



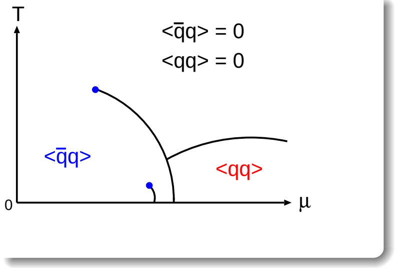
- QCD phase diagram

Motivation



- QCD phase diagram
- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space

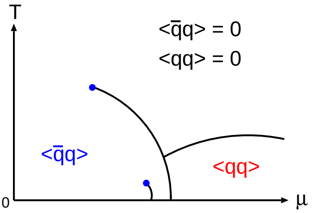
Motivation

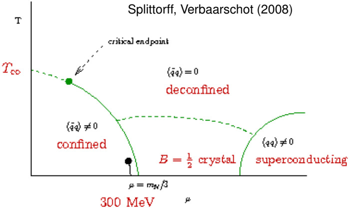


- QCD phase diagram
- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- inhomogeneous phases:

Motivation

TECHNISCHE UNIVERSITÄT DARMSTADT

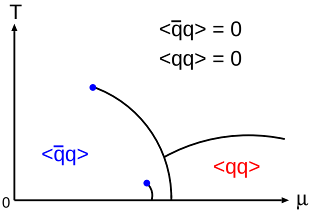


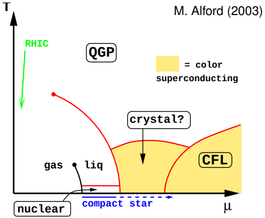


- QCD phase diagram
- frequent assumption:
  $\langle\bar{q}q\rangle$, $\langle qq\rangle$ constant in space
- inhomogeneous phases:
  - Skyrme crystal

Motivation

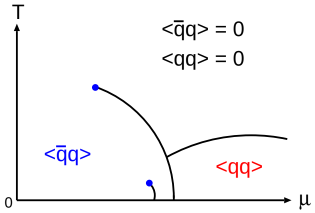
TECHNISCHE
UNIVERSITÄT
DARMSTADT

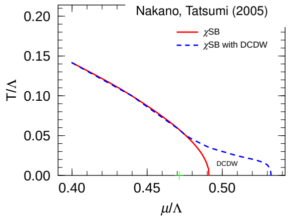




- QCD phase diagram
- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- inhomogeneous phases:
  - Skyrme crystal
  - crystalline color superconductors

Motivation

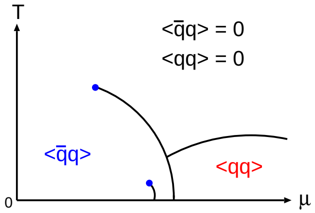




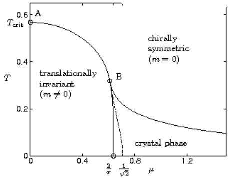
- QCD phase diagram
- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- inhomogeneous phases:
  - Skyrme crystal
  - crystalline color superconductors
  - chiral density wave

Motivation

TECHNISCHE
UNIVERSITÄT
DARMSTADT



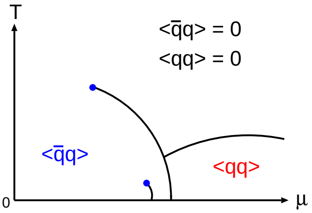
Thies, Urlichs (2003)



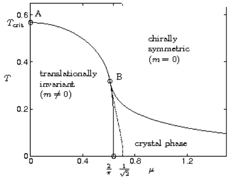
- QCD phase diagram
- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- inhomogeneous phases:
  - Skyrme crystal
  - crystalline color superconductors
  - chiral density wave
  - 1+1 D Gross-Neveu model

Motivation

TECHNISCHE UNIVERSITÄT DARMSTADT



Thies, Urlichs (2003)



- QCD phase diagram
- frequent assumption:
  $\langle\bar{q}q\rangle$, $\langle qq\rangle$ constant in space
- inhomogeneous phases:
  - Skyrme crystal
  - crystalline color superconductors
  - chiral density wave
  - 1+1 D Gross-Neveu model
- This talk:
  inhomogeneous χSB in the NJL model

Collaborators

- based on:

  Phys. Rev. D 82 (2010), in print [arXiv:1007.1397],

  together with

Stefano Carignano
(TU Darmstadt)

Dominik Nickel
(INT Seattle)

Model

- NJL model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right)$$

Model

- NJL model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right)$$

- bosonize: $\sigma(x) = \bar{\psi}(x)\psi(x), \quad \vec{\pi}(x) = \bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)$

$$\Rightarrow \quad \mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + 2G_S(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)\psi - G_S\left(\sigma^2 + \vec{\pi}^{\,2}\right)$$

Model

- NJL model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right)$$

- bosonize: $\sigma(x) = \bar{\psi}(x)\psi(x), \quad \vec{\pi}(x) = \bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)$

$$\Rightarrow \quad \mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + 2G_S(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)\psi - G_S\left(\sigma^2 + \vec{\pi}^{\,2}\right)$$

- mean-field approximation:

$$\sigma(x) \longrightarrow \langle\sigma(x)\rangle \equiv S(\vec{x}), \quad \pi_a(x) \longrightarrow \langle\pi_a(x)\rangle \equiv P(\vec{x})\,\delta_{a3}$$

  - $S(\vec{x})$, $P(\vec{x})$ time independent classical fields
  - retain space dependence !

Mean-field model

- mean-field Lagrangian: $\mathcal{L}_{MF} = \bar{\psi}(x)\mathcal{S}^{-1}(x)\psi(x) - G_S\left(S^2(\vec{x}) + P^2(\vec{x})\right)$
  - inverse dressed propagator:

$$\mathcal{S}^{-1}(x) = i\partial\!\!\!/ - m + 2G_S\left(S(\vec{x}) + i\gamma_5\tau_3 P(\vec{x})\right)$$

Mean-field model

TECHNISCHE UNIVERSITÄT DARMSTADT

- mean-field Lagrangian: $\mathcal{L}_{MF} = \bar{\psi}(x)\mathcal{S}^{-1}(x)\psi(x) - G_S\left(S^2(\vec{x}) + P^2(\vec{x})\right)$
  - inverse dressed propagator:

$$\mathcal{S}^{-1}(x) = i\partial\!\!\!/ - m + 2G_S\left(S(\vec{x}) + i\gamma_5\tau_3 P(\vec{x})\right) \equiv \gamma^0\,(i\partial_0 - \mathcal{H}_{MF})$$

  - $\mathcal{H}_{MF} = \mathcal{H}_{MF}[S, P]$: hermitean, time independent

Mean-field model

TECHNISCHE UNIVERSITÄT DARMSTADT

- mean-field Lagrangian: $\mathcal{L}_{MF} = \bar{\psi}(x)\mathcal{S}^{-1}(x)\psi(x) - G_S\left(S^2(\vec{x}) + P^2(\vec{x})\right)$
  - inverse dressed propagator:

$$\mathcal{S}^{-1}(x) = i\partial\!\!\!/ - m + 2G_S\left(S(\vec{x}) + i\gamma_5\tau_3 P(\vec{x})\right) \equiv \gamma^0\,(i\partial_0 - \mathcal{H}_{MF})$$

  - $\mathcal{H}_{MF} = \mathcal{H}_{MF}[S, P]$: hermitean, time independent
- thermodynamic potential:

$$\Omega_{MF}(T, \mu; S, P) = -\frac{T}{V}\mathbf{Tr}\ln\left(\frac{1}{T}(i\partial_0 - \mathcal{H}_{MF} + \mu)\right) + \frac{G_S}{V}\int_V d^3x\,\left(S^2(\vec{x}) + P^2(\vec{x})\right)$$

Mean-field model

TECHNISCHE UNIVERSITÄT DARMSTADT

- mean-field Lagrangian: $\mathcal{L}_{MF} = \bar{\psi}(x)\mathcal{S}^{-1}(x)\psi(x) - G_S\left(S^2(\vec{x}) + P^2(\vec{x})\right)$
  - inverse dressed propagator:

$$\mathcal{S}^{-1}(x) = i\partial\!\!\!/ - m + 2G_S\left(S(\vec{x}) + i\gamma_5\tau_3 P(\vec{x})\right) \equiv \gamma^0\,(i\partial_0 - \mathcal{H}_{MF})$$

  - $\mathcal{H}_{MF} = \mathcal{H}_{MF}[S, P]$: hermitean, time independent

- thermodynamic potential:

$$\Omega_{MF}(T,\mu;S,P) = -\frac{T}{V}\mathbf{Tr}\ln\left(\frac{1}{T}(i\partial_0 - \mathcal{H}_{MF} + \mu)\right) + \frac{G_S}{V}\int_V d^3x\,\left(S^2(\vec{x}) + P^2(\vec{x})\right)$$

$$= -\frac{1}{V}\sum_\lambda\left[\frac{E_\lambda - \mu}{2} + T\ln\left(1 + e^{\frac{E_\lambda - \mu}{T}}\right)\right] + \frac{1}{V}\int_V d^3x\,\frac{|M(\vec{x}) - m|^2}{4G_s}$$

  - mass function: $M(\vec{x}) = m - 2G_S(S(\vec{x}) + iP(\vec{x}))$
  - $E_\lambda = E_\lambda[M(\vec{x})]$ = eigenvalues of $\mathcal{H}_{MF}$

One dimensional modulations

- remaining tasks:
  - calculate eigenvalue spectrum of $\mathcal{H}_{MF}$ for given mass function $M(\vec{x})$
  - minimize w.r.t. $M(\vec{x})$

extremely difficult!

One dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- remaining tasks:
  - calculate eigenvalue spectrum of $\mathcal{H}_{MF}$ for given mass function $M(\vec{x})$
  - minimize w.r.t. $M(\vec{x})$

  extremely difficult!
- simplification:
  - consider only one-dimensional modulations $M(z)$

One dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- remaining tasks:
  - calculate eigenvalue spectrum of $\mathcal{H}_{MF}$ for given mass function $M(\vec{x})$
  - minimize w.r.t. $M(\vec{x})$

  extremely difficult!
- simplification:
  - consider only one-dimensional modulations $M(z)$
  - important observation: [D. Nickel, PRD (2009)]

    problem can be mapped to the 1 + 1 dimensional case

One dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- remaining tasks:
  - calculate eigenvalue spectrum of $\mathcal{H}_{MF}$ for given mass function $M(\vec{x})$
  - minimize w.r.t. $M(\vec{x})$

  extremely difficult!

- simplification:
  - consider only one-dimensional modulations $M(z)$
  - important observation: [D. Nickel, PRD (2009)]

    problem can be mapped to the $1 + 1$ dimensional case
  - solutions known analytically: [M. Thies, J. Phys. A (2006)]

    $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu)$ (chiral limit),

    $\mathrm{sn}(\xi|\nu)$: Jacobi elliptic functions

One dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- remaining tasks:
  - calculate eigenvalue spectrum of $\mathcal{H}_{MF}$ for given mass function $M(\vec{x})$
  - minimize w.r.t. $M(\vec{x})$

  extremely difficult!

- simplification:
  - consider only one-dimensional modulations $M(z)$
  - important observation: [D. Nickel, PRD (2009)]

    problem can be mapped to the 1 + 1 dimensional case
  - solutions known analytically: [M. Thies, J. Phys. A (2006)]

    $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu)$ (chiral limit),

    $\mathrm{sn}(\xi|\nu)$: Jacobi elliptic functions
  - remaining task:

    minimize w.r.t. 2 parameters ($m \neq 0$: 3 parameters)

One dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- remaining tasks:
  - calculate eigenvalue spectrum of $\mathcal{H}_{MF}$ for given mass function $M(\vec{x})$
  - minimize w.r.t. $M(\vec{x})$

  extremely difficult!

- simplification:
  - consider only one-dimensional modulations $M(z)$
  - important observation: [D. Nickel, PRD (2009)]

    problem can be mapped to the $1+1$ dimensional case
  - solutions known analytically: [M. Thies, J. Phys. A (2006)]

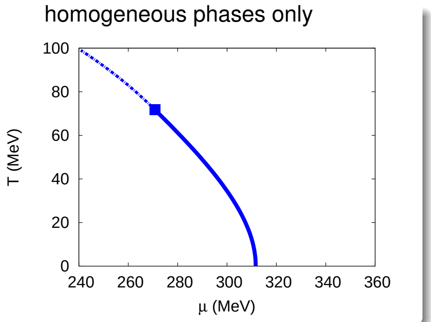
    $M(z) = \sqrt{\nu}\Delta\, \mathrm{sn}(\Delta z|\nu)$ (chiral limit),

    $\mathrm{sn}(\xi|\nu)$: Jacobi elliptic functions
  - remaining task:

    minimize w.r.t. 2 parameters ($m \neq 0$: 3 parameters) doable!

Phase diagram (chiral limit)

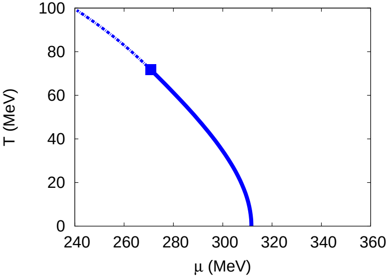
[D. Nickel, PRD (2009)]



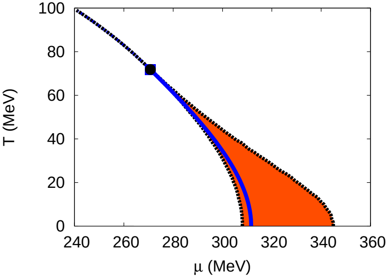
Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]

homogeneous phases only



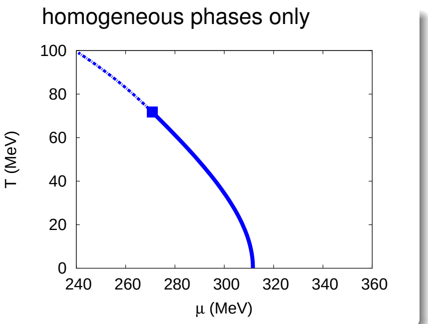
including inhomogeneous phase

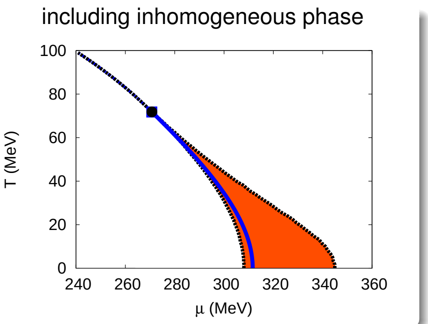


Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]

TECHNISCHE
UNIVERSITÄT
DARMSTADT

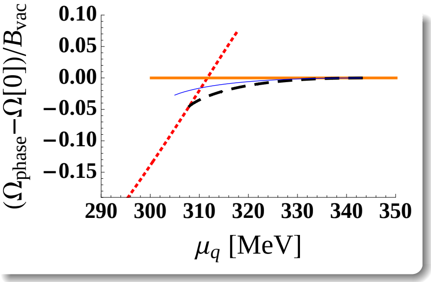




- 1st-order line completely covered by the inhomogeneous phase!
- all phase boundaries 2nd order
- critical point coincides with Lifshitz point

Free energy difference

[D. Nickel, PRD (2009)]



- homogeneous chirally broken
- solitons
- chiral density wave:
  $M_{CDW}(z) = \Delta\, e^{iqz}$
  ("chiral spiral")

- soliton phase favored, when it exists
- $\delta\Omega_{soliton} \approx 2\delta\Omega_{CDW} \quad \Rightarrow$ chiral spiral never favored

Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

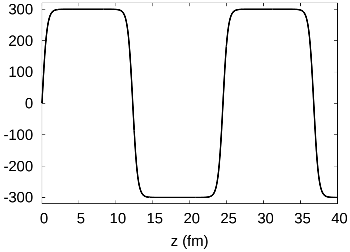
- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

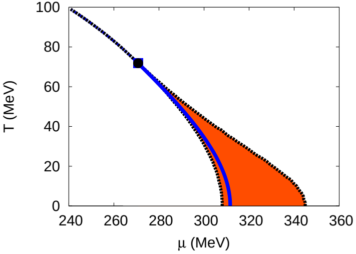
Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

$M(z)$ (μ = 307.5 MeV)

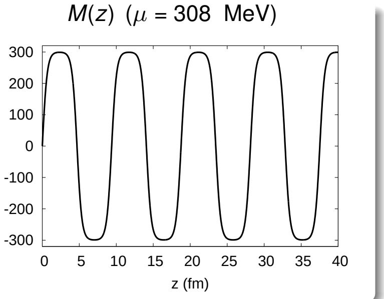


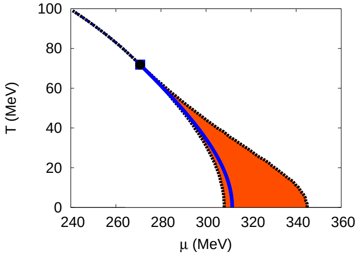


Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

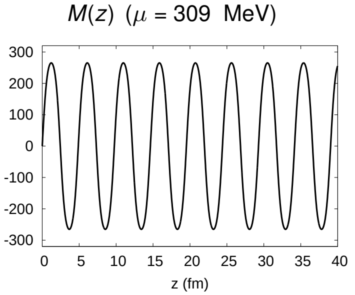


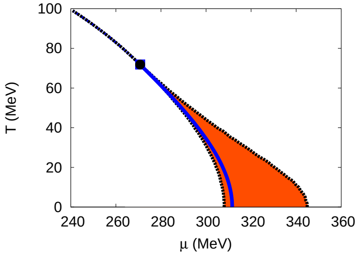


Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$



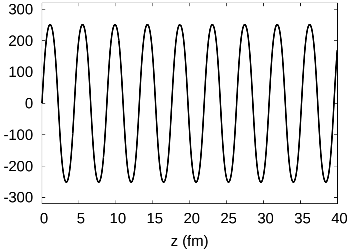


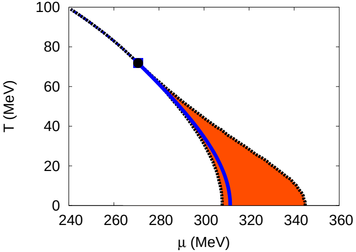
Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

$M(z)$ ($\mu = 310$ MeV)

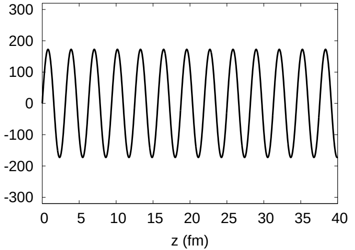


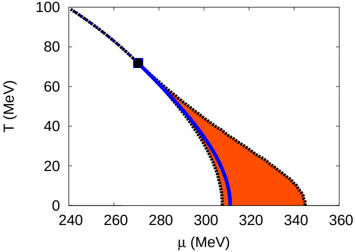


Mass functions and density profiles ($T = 0$)

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

$M(z)$ ($\mu =$ 320 MeV)

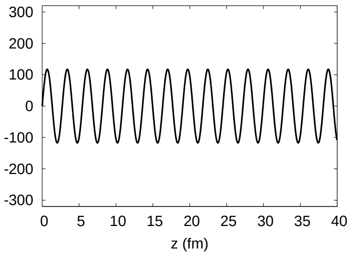


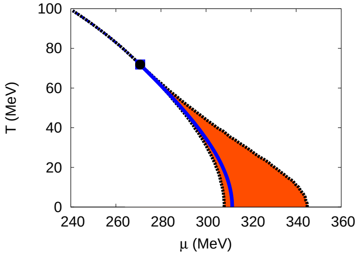


Mass functions and density profiles ($T = 0$)

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

$M(z)$ (μ = 330 MeV)



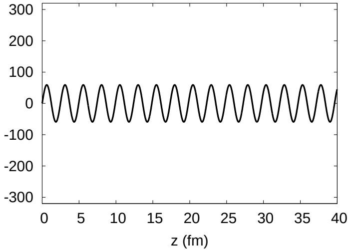


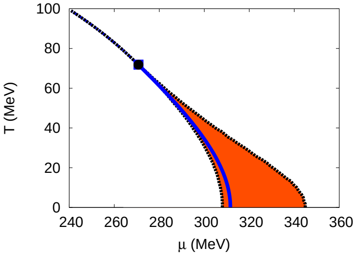
Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

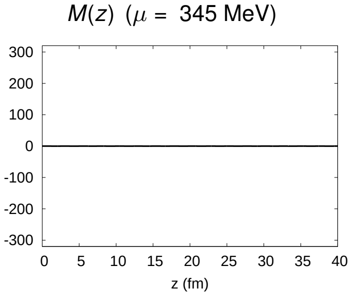
$M(z)$ ($\mu =$ 340 MeV)

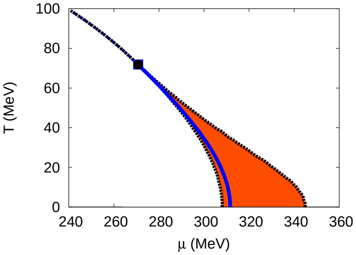




Mass functions and density profiles ($T = 0$)

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

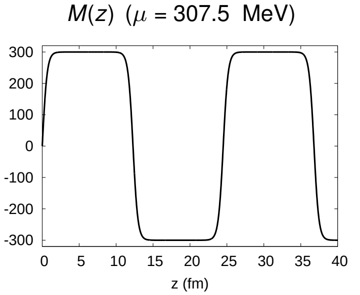


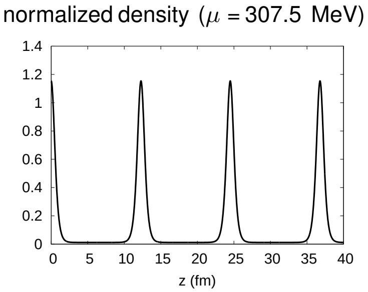


Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

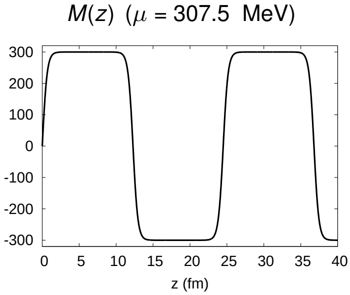


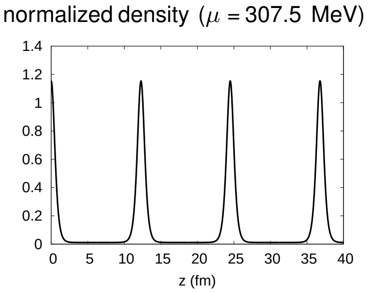


Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$



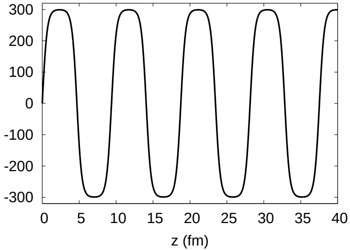


- Quarks reside in the chirally restored regions.

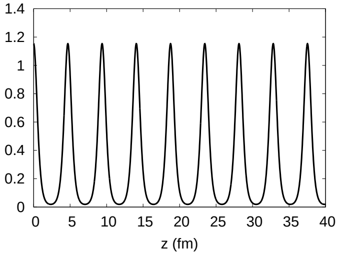
Mass functions and density profiles ($T = 0$)

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$

$M(z)$ ($\mu = 308$ MeV)



normalized density ($\mu = 308$ MeV)

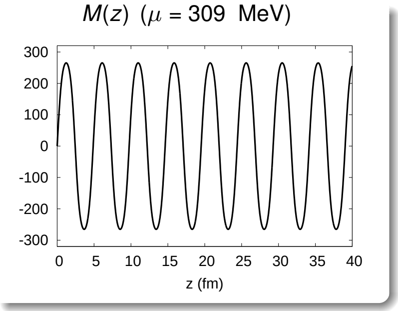


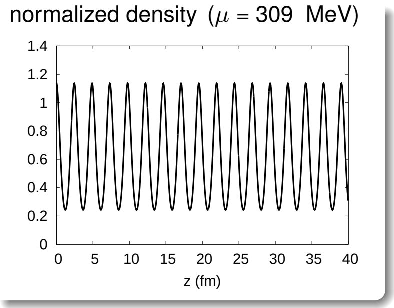
- Quarks reside in the chirally restored regions.

Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\, \mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$



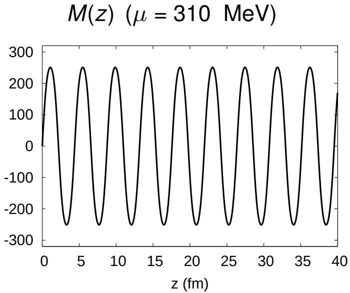


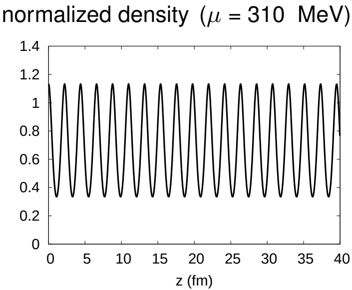
- Quarks reside in the chirally restored regions.

Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$



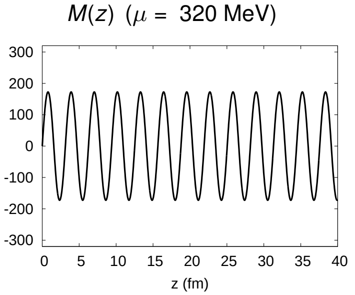


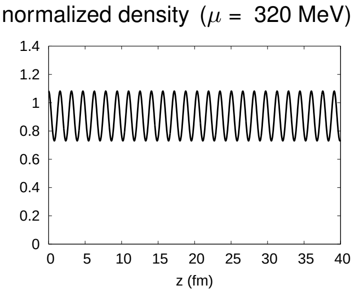
- Quarks reside in the chirally restored regions.

Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta\tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta\sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$



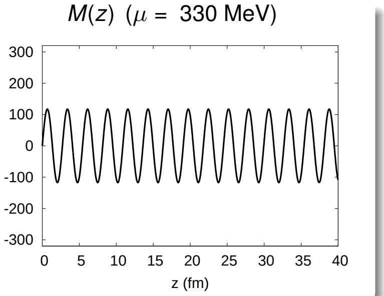


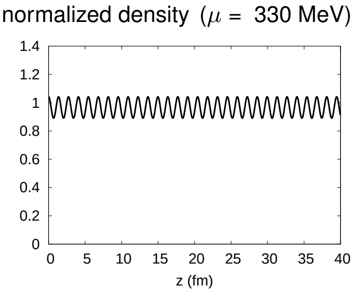
- Quarks reside in the chirally restored regions.
- Density gets smoothened with increasing μ and T.

Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$



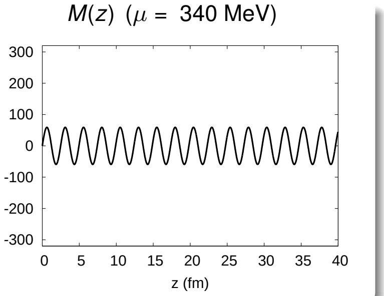


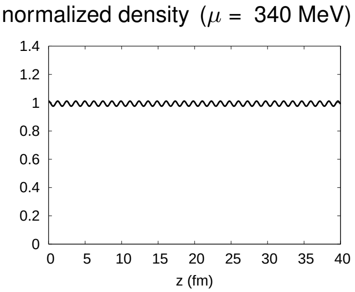
- Quarks reside in the chirally restored regions.
- Density gets smoothened with increasing μ and T.

Mass functions and density profiles ($T = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \longrightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

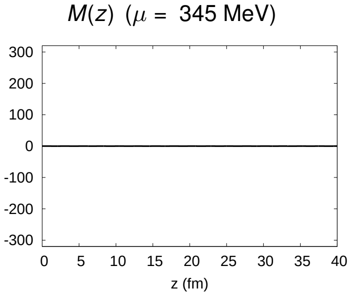


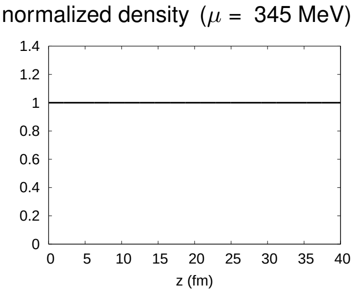


- Quarks reside in the chirally restored regions.
- Density gets smoothened with increasing μ and T.

Mass functions and density profiles ($T = 0$)

- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \rightarrow 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \rightarrow 0 \end{cases}$





- Quarks reside in the chirally restored regions.
- Density gets smoothened with increasing μ and T.

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^{\mu}\psi)^2$

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field: $\bar{\psi}\gamma^\mu\psi \rightarrow \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field: $\bar{\psi}\gamma^\mu\psi \to \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu = \mathcal{H}_{MF}|_{G_V=0} - \tilde{\mu}(\vec{x})$
  - $\tilde{\mu}(\vec{x}) = \mu - 2G_V\, n(\vec{x})$ "shifted chemical potential"

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field: $\bar{\psi}\gamma^\mu\psi \to \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu = \mathcal{H}_{MF}|_{G_V=0} - \tilde{\mu}(\vec{x})$
  - $\tilde{\mu}(\vec{x}) = \mu - 2G_V\, n(\vec{x})$ "shifted chemical potential"
- further approximation: $n(\vec{x}) \to \langle n \rangle = const. \quad \Rightarrow \quad \tilde{\mu} = const.$

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field: $\bar{\psi}\gamma^\mu\psi \rightarrow \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu = \mathcal{H}_{MF}|_{G_V=0} - \tilde{\mu}(\vec{x})$
  - $\tilde{\mu}(\vec{x}) = \mu - 2G_V\,n(\vec{x})$ "shifted chemical potential"
- further approximation: $n(\vec{x}) \rightarrow \langle n\rangle = const. \quad \Rightarrow \quad \tilde{\mu} = const.$
  - questionable in the inhomogeneous phase at low μ and T
  - ok near the restored phase (including the Lifshitz point)

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

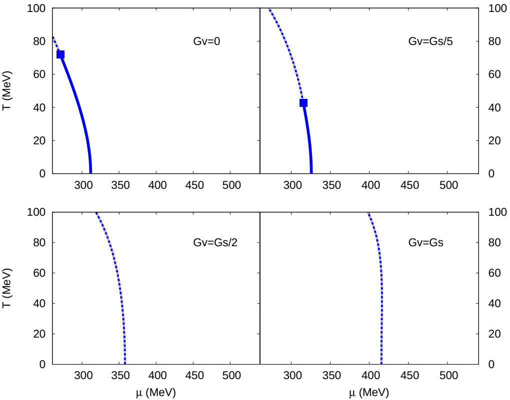
- additional vector term: $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$
- additional mean field: $\bar{\psi}\gamma^\mu\psi \rightarrow \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu \;=\; \mathcal{H}_{MF}\big|_{G_V=0} - \tilde{\mu}(\vec{x})$
  - $\tilde{\mu}(\vec{x}) = \mu - 2G_V\, n(\vec{x})$ "shifted chemical potential"
- further approximation: $n(\vec{x}) \rightarrow \langle n\rangle = const. \quad \Rightarrow \quad \tilde{\mu} = const.$
  - questionable in the inhomogeneous phase at low μ and T
  - ok near the restored phase (including the Lifshitz point)
  - advantage: known analytic solutions can still be used

Including vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

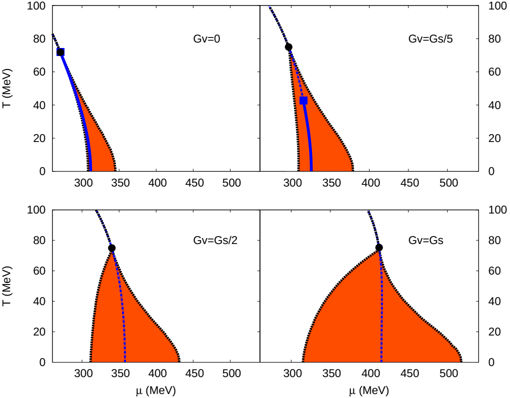
- additional vector term: $\mathcal{L}_V = -G_V(\bar\psi\gamma^\mu\psi)^2$
- additional mean field: $\bar\psi\gamma^\mu\psi \to \langle\bar\psi\gamma^\mu\psi\rangle \equiv n(\vec{x})\,\delta^{\mu 0}$ (*density*!)
- mean-field Hamiltonian: $\mathcal{H}_{MF} - \mu = \mathcal{H}_{MF}|_{G_V=0} - \tilde\mu(\vec{x})$
  - $\tilde\mu(\vec{x}) = \mu - 2G_V\,n(\vec{x})$ "shifted chemical potential"
- further approximation: $n(\vec{x}) \to \langle n\rangle = const. \quad \Rightarrow \quad \tilde\mu = const.$
  - questionable in the inhomogeneous phase at low μ and T
  - ok near the restored phase (including the Lifshitz point)
  - advantage: known analytic solutions can still be used
  - additional parameter: $\tilde\mu$, fixed by constraint $\frac{\partial\Omega_{MF}}{\partial\tilde\mu} = 0$

Phase diagram



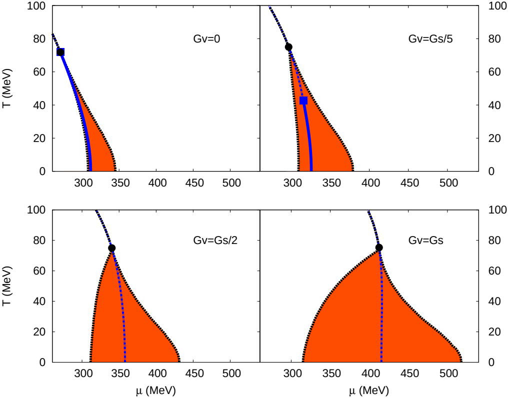
- homogeneous phases: strong G_V-dependence of the critical point

Phase diagram

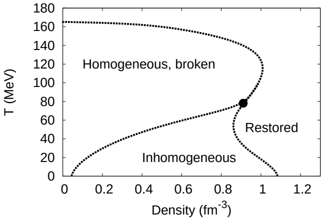


- homogeneous phases: strong G_V-dependence of the critical point
- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

Phase diagram



T-$\langle n \rangle$ phase diagram:



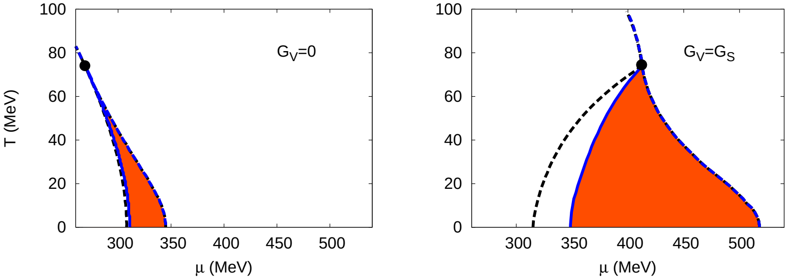
- independent of G_V!

- homogeneous phases: strong G_V-dependence of the critical point
- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

Chiral density wave

TECHNISCHE UNIVERSITÄT DARMSTADT

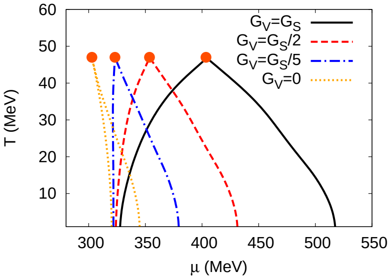
- How much can we trust the approximation $\tilde{\mu} = \mu - 2G_V\bar{n}$?
- Chiral density wave: $M(z) = \Delta\, e^{iqz} \quad \Rightarrow \quad n(z) = const.$



- CDW $\longrightarrow$ restored and Lifshitz point agree with soliton solution
- chirally broken $\longrightarrow$ CDW: 1st order and at higher μ
- exact phase boundary somewhere in between

Finite current quark masses

- phase diagrams for m = 5 MeV:



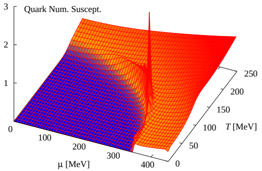
- same qualitative behavior

Susceptibilities

- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

homogeneous phases only:



[K. Fukushima, PRD (2008)]

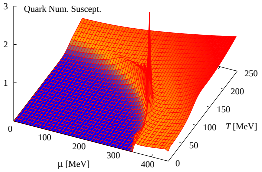
Susceptibilities

- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

- including inhomogeneous phases?

homogeneous phases only:



[K. Fukushima, PRD (2008)]

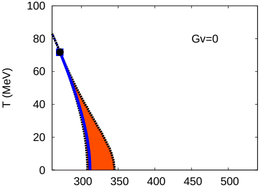
Susceptibilities

TECHNISCHE UNIVERSITÄT DARMSTADT

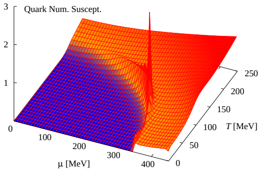
- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

- including inhomogeneous phases?
- expectations:



homogeneous phases only:



[K. Fukushima, PRD (2008)]

- $\underline{G_V = 0:}$

  CP = Lifshitz point

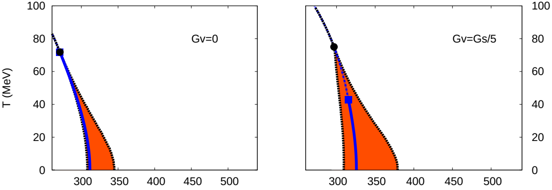
  $\longrightarrow$ no qualitative change

Susceptibilities

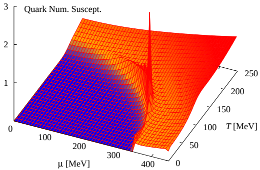
- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

- including inhomogeneous phases?
- expectations:



homogeneous phases only:



[K. Fukushima, PRD (2008)]

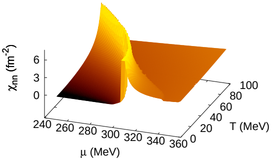
- $G_V = 0$:
  CP = Lifshitz point
  $\rightarrow$ no qualitative change
- $G_V > 0$:
  no CP $\rightarrow$ no divergence

Susceptibilities

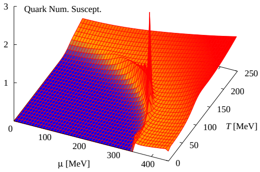
- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

- including inhomogeneous phases?
- results:



homogeneous phases only:



[K. Fukushima, PRD (2008)]

- $\underline{G_V = 0:}$

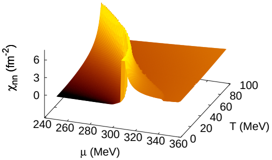
  χ_{nn} diverges at phase boundary (hom. broken - inhom.)

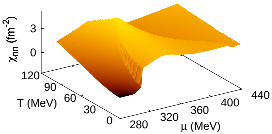
Susceptibilities

- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

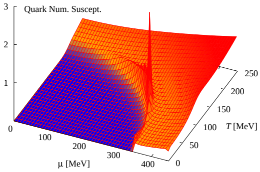
$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

- including inhomogeneous phases?
- results:





homogeneous phases only:

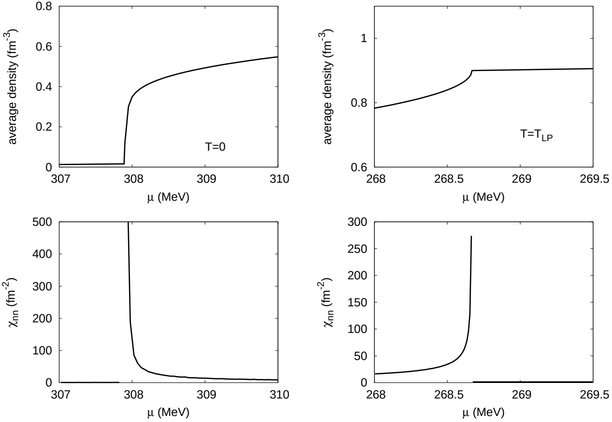


[K. Fukushima, PRD (2008)]

- $\underline{G_V = 0:}$
  χ_{nn} diverges at phase boundary (hom. broken - inhom.)
- $\underline{G_V > 0:}$
  no divergence

Susceptibilities

- densities and quark number susceptibilities for $G_V = 0$:



- $\underline{T = T_{CP},\ \mu < \mu_c}$:

  $\chi_{nn} \propto \frac{1}{\sqrt{\mu_c - \mu}}$

- $\underline{T = 0,\ \mu > \mu_{cr}}$:

  $\chi_{nn} \propto \frac{1}{(\mu - \mu_{cr}) \log^2(\mu - \mu_{cr})}$

- $\underline{G_V > 0}$:

  $\delta\chi_{nn}\big|_{T=0,\mu=\mu_{cr}} \approx \frac{1}{2G_V}$

Including Polyakov-loop dynamics

TECHNISCHE UNIVERSITÄT DARMSTADT

- PNJL model: $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + G_S\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right) + U(\ell, \bar{\ell})$
  - covariant derivative: $D_\mu = \partial_\mu + iA_0\delta_{\mu 0}$,
  - Polyakov loop: $L(\vec{x}) = \mathcal{P}\exp[i\int_0^{1/T} d\tau\, A_4(\tau, \vec{x})]$, $\quad A_4(\tau, \vec{x}) = iA_0(t = -i\tau, \vec{x})$
  - expectation values: $\ell = \frac{1}{N_c}\langle \mathrm{Tr}_c L\rangle$, $\quad \bar{\ell} = \frac{1}{N_c}\langle \mathrm{Tr}_c L^\dagger\rangle$
- assumption:

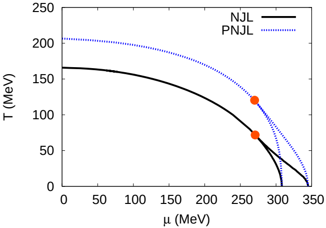
  ℓ, $\bar{\ell}$ space-time independent, even in inhomogeneous phases
- main effect:

  $$T\ln\left(1 + e^{-\frac{E-\mu}{T}}\right) \;\rightarrow\; T\ln\left(1 + e^{-3\frac{E-\mu}{T}} + 3\,\ell\, e^{-\frac{E-\mu}{T}} + 3\,\bar{\ell}\, e^{-2\frac{E-\mu}{T}}\right)$$

  $\rightarrow$ suppresion of thermally excited quarks at small ℓ, $\bar{\ell}$

Results

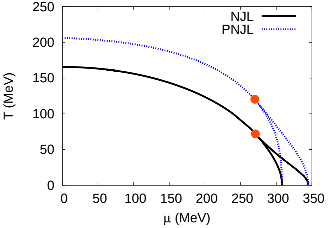
NJL vs. PNJL



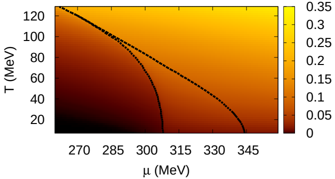
- Polyakov loop:
  - suppression of thermal effects
    → phase diagram stretched in T direction
  - no qualitative change

Results

NJL vs. PNJL



ℓ



- Polyakov loop:
  - suppression of thermal effects
    $\rightarrow$ phase diagram stretched in T direction
  - no qualitative change
- Polyakov-loop expectation value:
  - inhomogeneous regime:
    $\ell \lesssim 0.15, \quad \bar{\ell} \lesssim 0.2$
  - effects of neglecting spatial variations of $\ell, \bar{\ell}$ presumably small

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phases must be considered!

Conclusions

- Inhomogeneous phases must be considered!
- NJL model with one-dimensional modulations of $\langle \bar{q}q \rangle$:
  - 1st-order line and critical point covered by an inhomogeneous region
  - inhomogeneous phase rather stable w.r.t. vector interactions
  - number susceptibility always finite (for $G_V > 0$)
  - usual effect of the Polyakov loop

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phases must be considered!
- NJL model with one-dimensional modulations of $\langle \bar{q}q \rangle$:
  - 1st-order line and critical point covered by an inhomogeneous region
  - inhomogeneous phase rather stable w.r.t. vector interactions
  - number susceptibility always finite (for $G_V > 0$)
  - usual effect of the Polyakov loop
- outlook:
  - include strange quarks
  - include color superconductivity
  - relax approximations (constant density, constant Polyakov loop)
  - higher dimensional modulations