$SU(3) \times SU(3)$ nonlocal quark model and QCD phase transition

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Critical Point and Onset of Deconfinement
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In collaboration with Michael Buballa

1. Motivation
2. Nonlocal chiral quark model
   1. Thermodynamic potential at mean field
   2. Beyond mean field
3. Comparison with lattice QCD calculations
   1. Order parameters
   2. Pressure
   3. Entropy
   4. Interaction measure
4. Conclusion

The quark part of Lagrangian of the nonlocal model has the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{4q} + \mathcal{L}_{tH}$$

$$\mathcal{L}_{free} = \bar{q}(x)(i\hat{\partial} - m_c)q(x)$$

m_c is a current quark mass matrix with diagonal elements $m_c^u = m_c^d$, m_c^s.

$$\mathcal{L}_{4q} = \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)]$$

nonlocal quark currents are

$$J_M^a(x) = \int d^4(x_1x_2)\, f(x_1)f(x_2)\, \bar{q}(x-x_1)\, \Gamma_M q(x+x_2),$$

$$\Gamma_S = \lambda^a, \Gamma_P = i\gamma^5\lambda^a$$

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$$\mathcal{L}_{tH} = H[\mathrm{det}\bar{q}(1+\gamma_5)q + \mathrm{det}\bar{q}(1-\gamma_5)q]$$

nonlocal quark currents are

$$J_M^a(x) = \int d^4(x_1 x_2)\, f(x_1) f(x_2)\, \bar{q}(x - x_1)\, \Gamma_M q(x + x_2),$$
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Lagrangian

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$$\mathcal{L}_{4q} = \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)]$$

$$\mathcal{L}_{tH} = -\frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_P^a(x)J_P^b(x)J_P^c(x)]$$

nonlocal quark currents are

$$J_M^a(x) = \int d^4(x_1x_2)\, f(x_1)f(x_2)\, \bar{q}(x-x_1)\, \Gamma_M q(x+x_2),$$

$$\Gamma_S = \lambda^a, \Gamma_P = i\gamma^5\lambda^a$$

Thermodynamic potential

The mean-field part of the thermodynamic potential reads

$$\Omega_{\mathrm{mf}}(T,\mu) = -2T\sum_f \sum_{i=0,\pm}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left[(\omega_n^i)^2 + \vec{k}^2 + M_f^2((\omega_n^i)^2+\vec{k}^2)\right] + -\frac{1}{2}\left[\sum_f (m_d^f \bar{S}^f + \frac{G}{2}\bar{S}^f\bar{S}^f) + \frac{H}{2}\bar{S}^u\bar{S}^d\bar{S}^s\right] + \mathcal{U}(\Phi,\bar{\Phi})$$

where $M_f(p^2) = m_c^f + m_d^f f^2(p^2)$. The Matsubara frequencies $\omega_n = (2n+1)\pi T$ are shifted due to the coupling to the Polyakov loop according to

$$\omega_n^{\pm} = \omega_n \pm \phi_3 - i\mu_f, \quad \omega_n^0 = \omega_n - i\mu_f,$$

where $A_0 = -iA_4$ and $A_4 = \phi_i\lambda^i$, $\Phi = \bar{\Phi} = (1 + 2\cos(\phi_3/T))/3$.
In stationary phase approximation

$$\bar{S}^f = 8\sum_{i=0,\pm}\sum_n \int \frac{d^3k}{(2\pi)^3} \frac{f^2((\omega_n^i)^2) M_f((\omega_n^i)^2+\vec{k}^2)}{(\omega_n^i)^2 + \vec{k}^2 + M_f^2((\omega_n^i)^2+\vec{k}^2)}$$

Thermodynamic potential

$\mathcal{U}$ potential in logarithmic form [1]

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\,\bar{\Phi}\Phi + b(T)\,\ln\left[1-6\,\bar{\Phi}\Phi+4\left(\bar{\Phi}^3+\Phi^3\right)-3\left(\bar{\Phi}\Phi\right)^2\right]$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3, \quad T_0 = 270\,\mathrm{MeV}$$

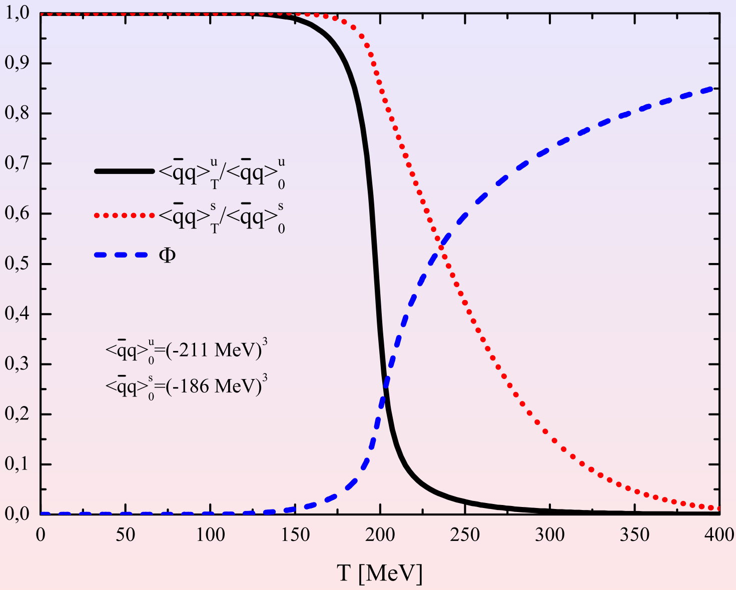
Equation of motion

$$\frac{\partial\Omega_{\mathrm{mf}}}{\partial m_d^f} = 0 \quad , \frac{\partial\Omega_{\mathrm{mf}}}{\partial\phi_3} = 0$$

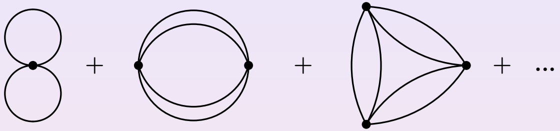
Model parameters for $f(p^2) = \exp(-p^2/\Lambda^2)$
"large" mass[2]
m_c^u =8.5 MeV, m_d^u =304.5 MeV, m_c^s =110 MeV, m_d^s =390 MeV, Λ = 1.0 GeV
"small" mass
m_c^u =5.5 MeV, m_d^u =250.0 MeV, m_c^s =151 MeV, m_d^s =353 MeV, Λ = 1.3 GeV

[1] S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75** (2007) 034007.
[2] A. Scarpettini, D. Gomez Dumm and N. N. Scoccola, Phys. Rev. D **69** (2004) 114018.

Quark condensate and Polyakov loop at mean field



Thermodynamic potential beyond mean field



The contribution of the mesonic correlations is given by (this is result of ring sum)

$$\Omega_{\mathrm{Nc}} \quad = \quad -\sum_{\mathrm{M}} \frac{d_{\mathrm{M}}}{2} i \mathbf{Tr} \ln \left[1 - G^{\mathrm{M}} \Pi^{\mathrm{M}}\right],$$

where G^{M} and Π^{M} are the matrices of coupling constants and polarization loops.

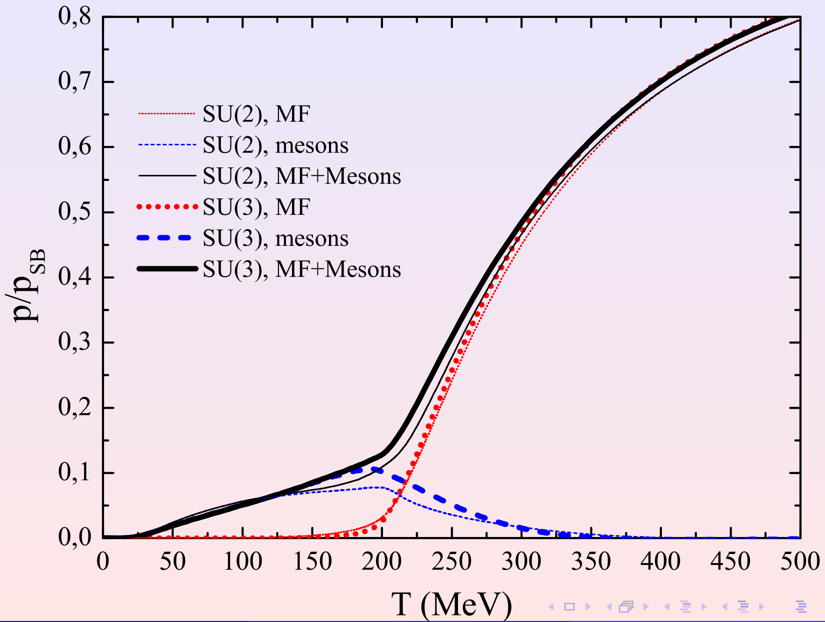
Observables

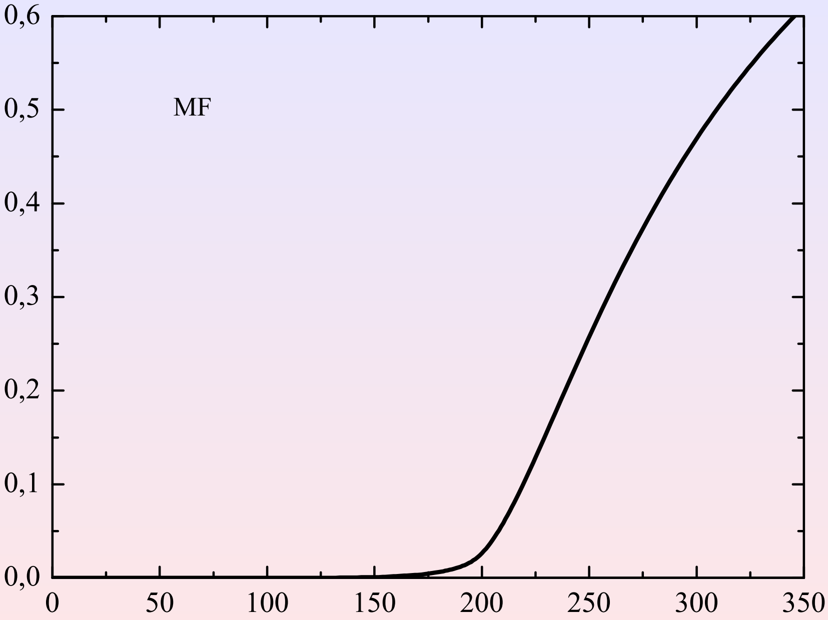
- subtracted condensate

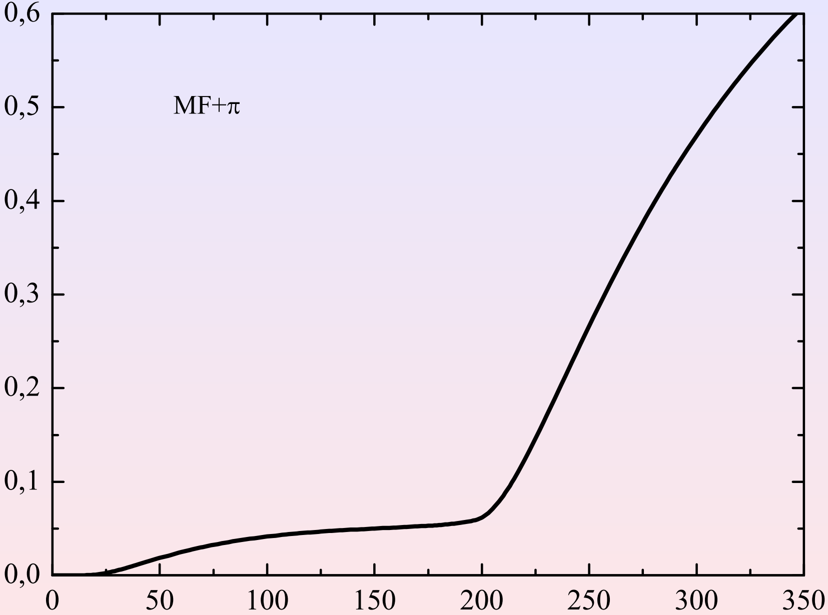
$$\Delta_{l,s} = \frac{\langle \bar{u}u \rangle^T - \frac{m_c^u}{m_c^s} \langle \bar{s}s \rangle^T}{\langle \bar{u}u \rangle^0 - \frac{m_c^u}{m_c^s} \langle \bar{s}s \rangle^0}$$

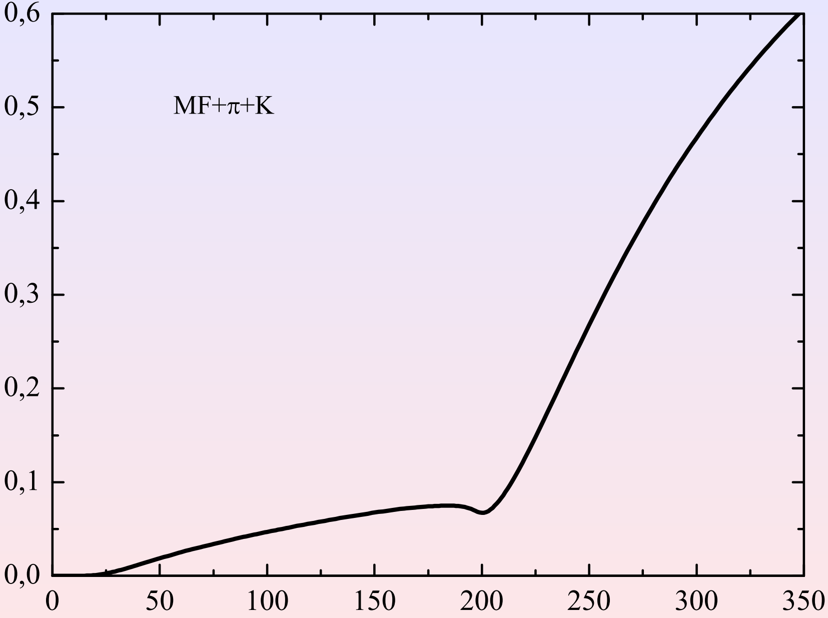
- Polyakov loop Φ
- pressure $p = -\Omega$
- entropy density $s = -\partial\Omega/\partial T$
- energy density $\epsilon = -p + Ts$
- interaction measure $\epsilon - 3p$
- non-strange(l) and strange(s) quark number susceptibilities

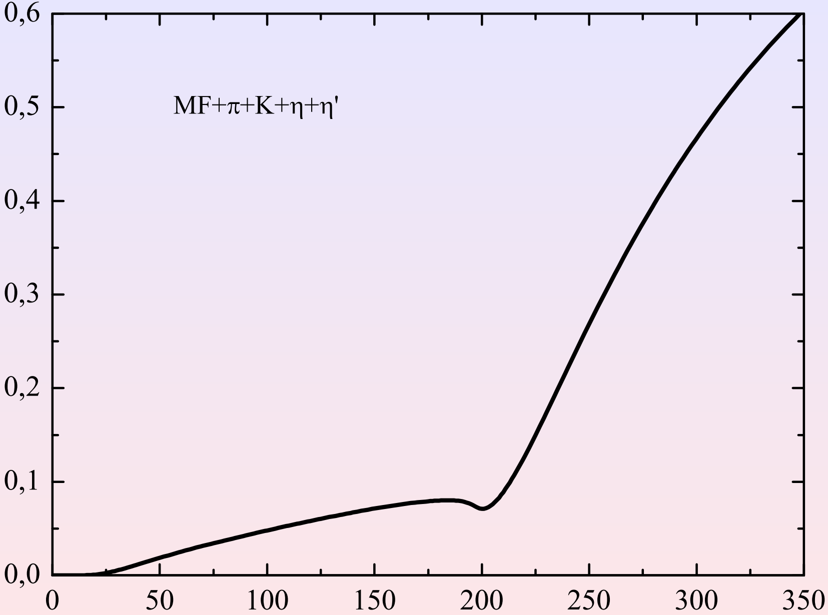
$$\chi_l = -\frac{\partial^2 \Omega}{\partial \mu_l^2}, \quad \chi_s = -\frac{\partial^2 \Omega}{\partial \mu_s^2}$$

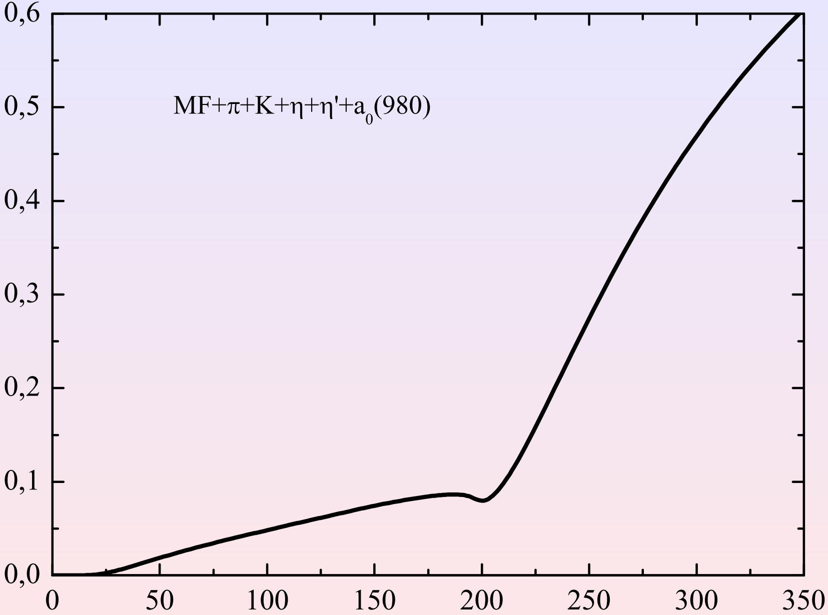

0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
p/p_SB
0
50
100
150
200
250
300
350
400
450
500
T (MeV)
SU(2), MF
SU(2), mesons
SU(2), MF+Mesons
SU(3), MF
SU(3), mesons
SU(3), MF+Mesons

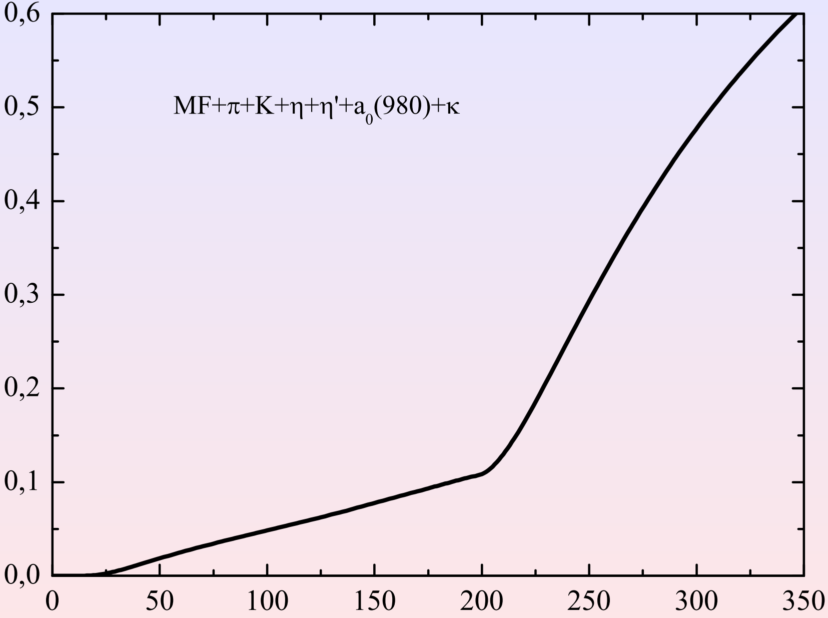

0,6
0,5
0,4
0,3
0,2
0,1
0,0
MF
0
50
100
150
200
250
300
350

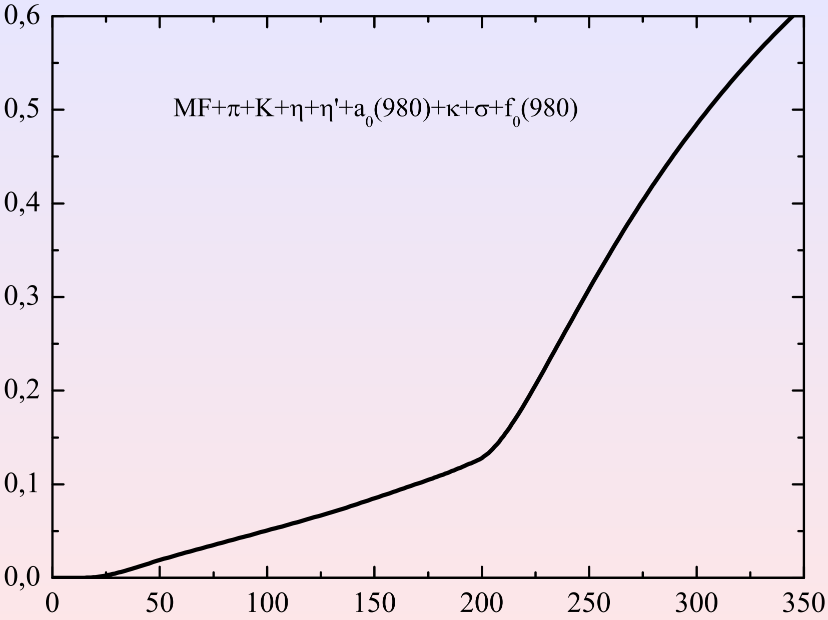

0,6
0,5
0,4
0,3
0,2
0,1
0,0
MF+π
0
50
100
150
200
250
300
350


MF+π+K
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0
50
100
150
200
250
300
350

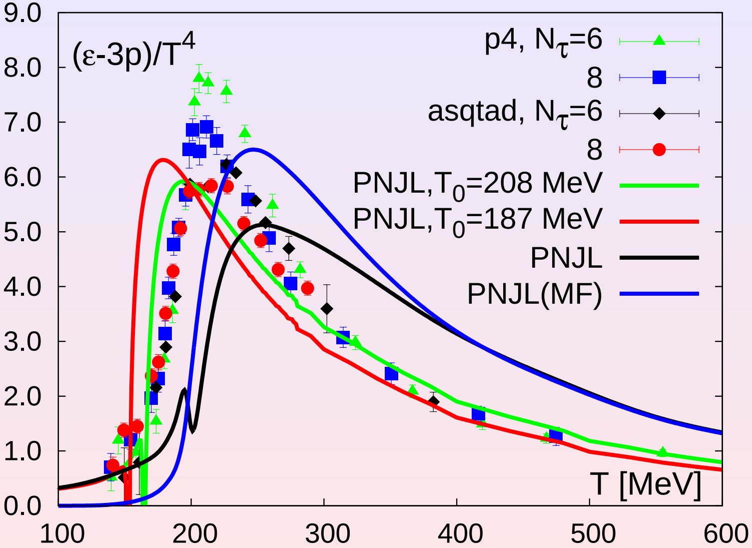

0,6
0,5
0,4
0,3
0,2
0,1
0,0
MF+π+K+η+η'
0
50
100
150
200
250
300
350


MF+π+K+η+η'+a0(980)
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0
50
100
150
200
250
300
350

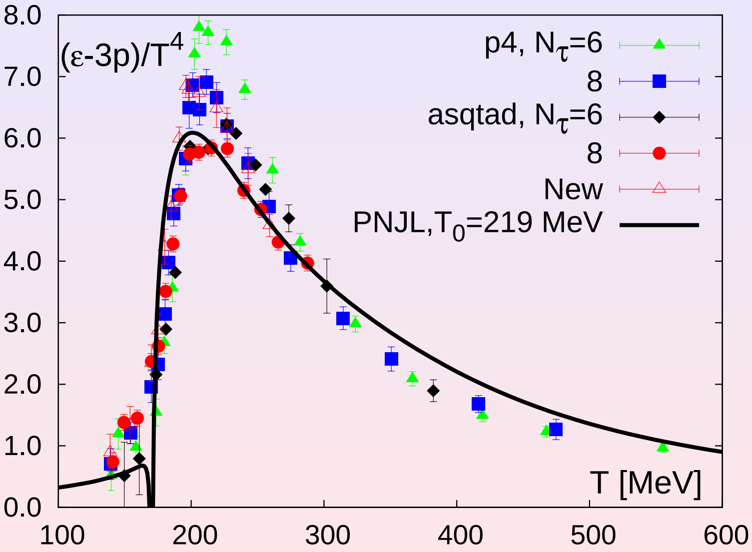

MF+π+K+η+η'+a0(980)+κ
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0
50
100
150
200
250
300
350

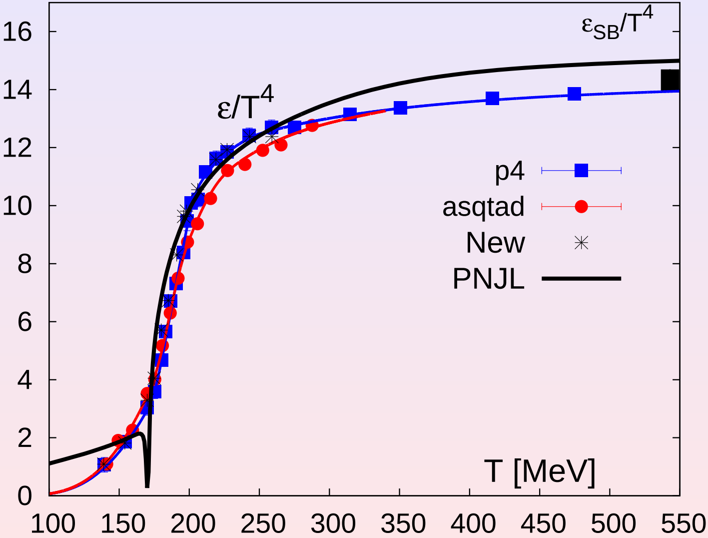


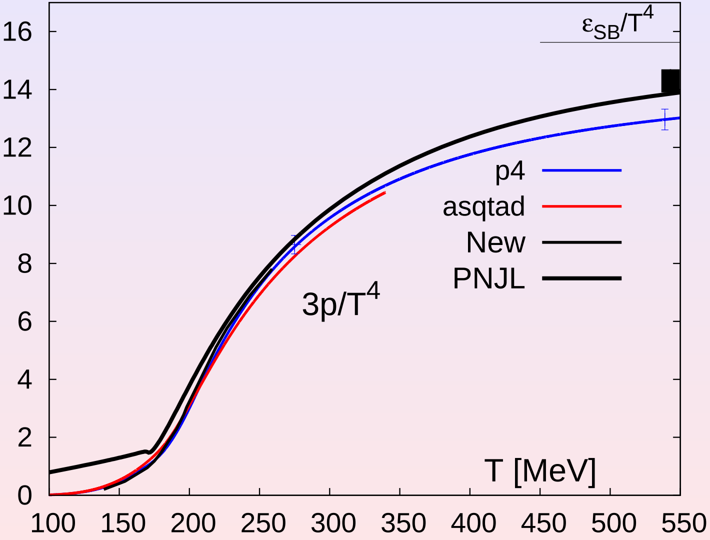
MF+π+K+η+η'+a0(980)+κ+σ+f0(980)
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0
50
100
150
200
250
300
350

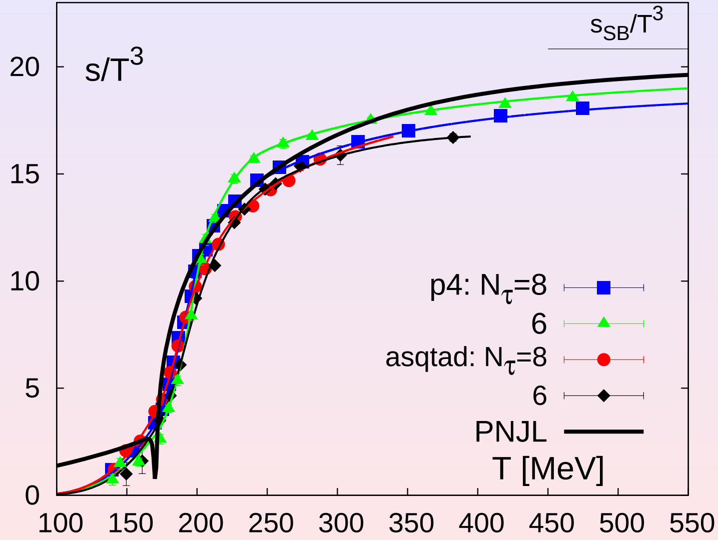

(ε-3p)/T^4
p4, Nτ=6
8
asqtad, Nτ=6
8
PNJL,T0=208 MeV
PNJL,T0=187 MeV
PNJL
PNJL(MF)
T [MeV]
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
100
200
300
400
500
600

Interaction measure for $T_0 = 219$ MeV



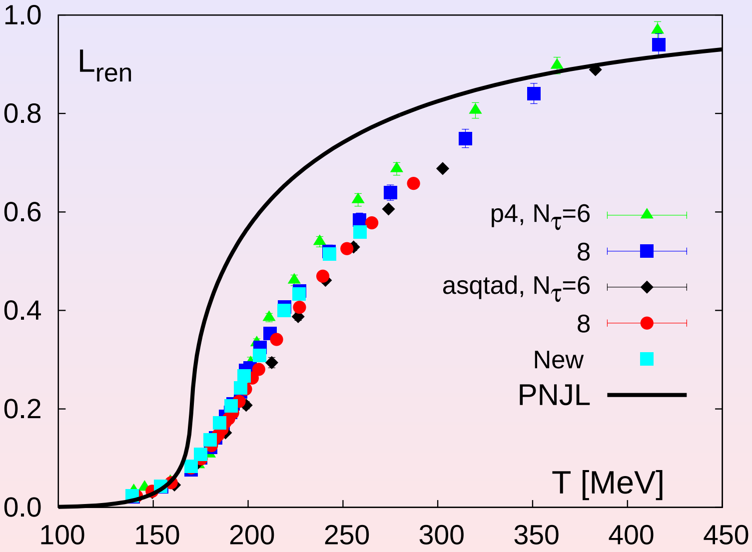

16
14
12
10
8
6
4
2
0
100 150 200 250 300 350 400 450 500 550
T [MeV]
ε/T^4
ε_SB/T^4
p4
asqtad
New
PNJL

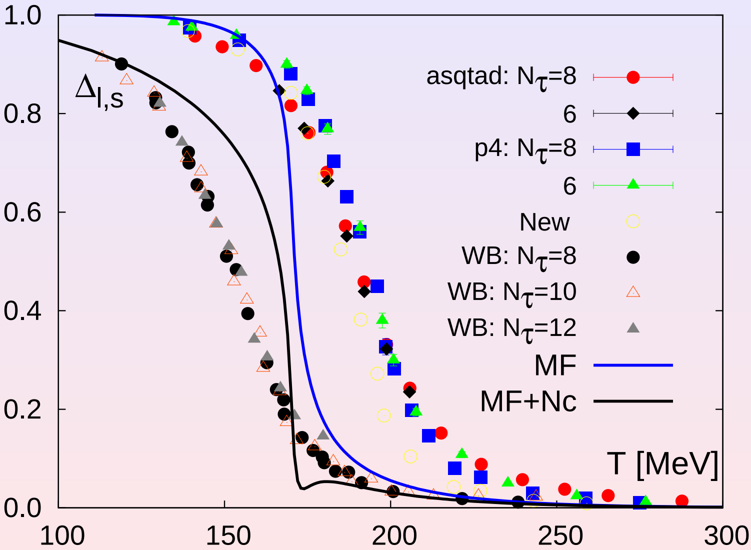

εSB/T4
3p/T4
p4
asqtad
New
PNJL
T [MeV]
0
2
4
6
8
10
12
14
16
100
150
200
250
300
350
400
450
500
550



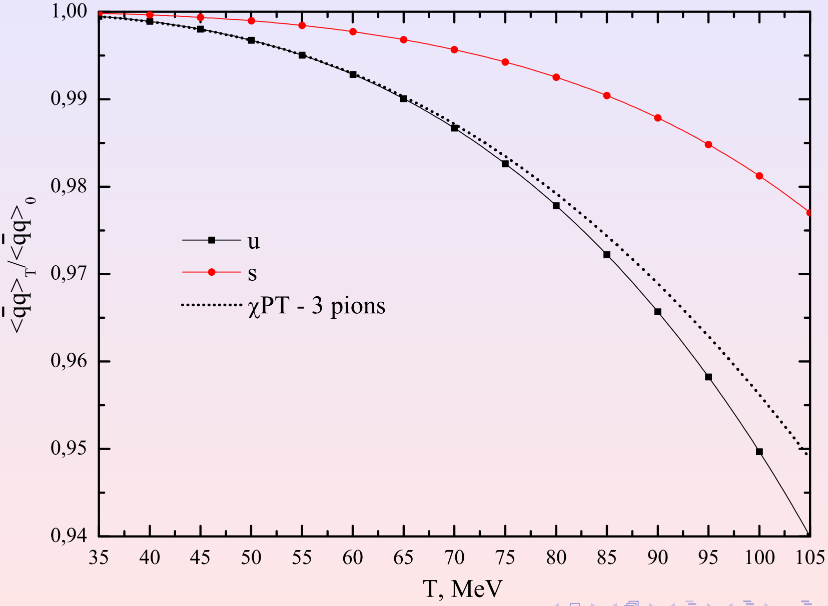
s_{SB}/T^3
s/T^3
20
15
10
5
0
p4: N_τ=8
6
asqtad: N_τ=8
6
PNJL
T [MeV]
100
150
200
250
300
350
400
450
500
550

Polyakov loop

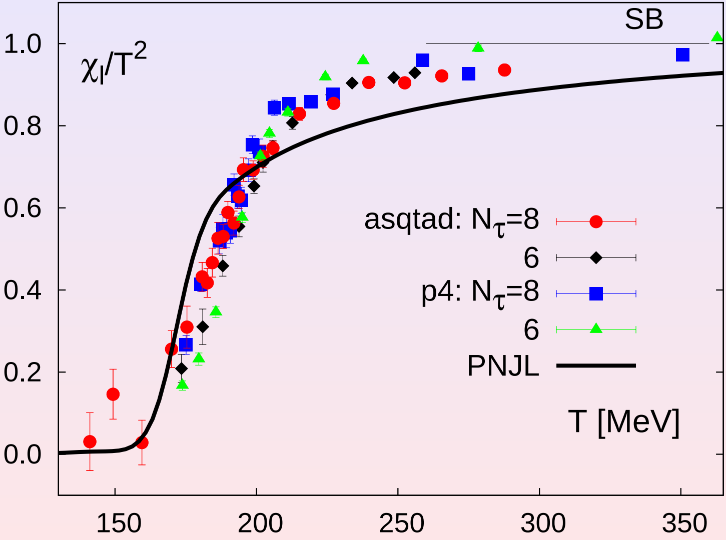



1.0
0.8
0.6
0.4
0.2
0.0
$\Delta_{l,s}$
asqtad: N_τ=8
6
p4: N_τ=8
6
New
WB: N_τ=8
WB: N_τ=10
WB: N_τ=12
MF
MF+Nc
T [MeV]
100
150
200
250
300

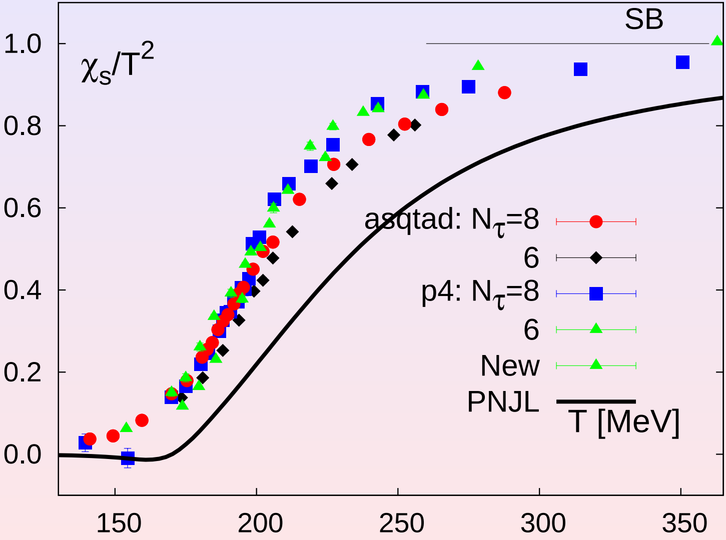
Condensate at low temperatures



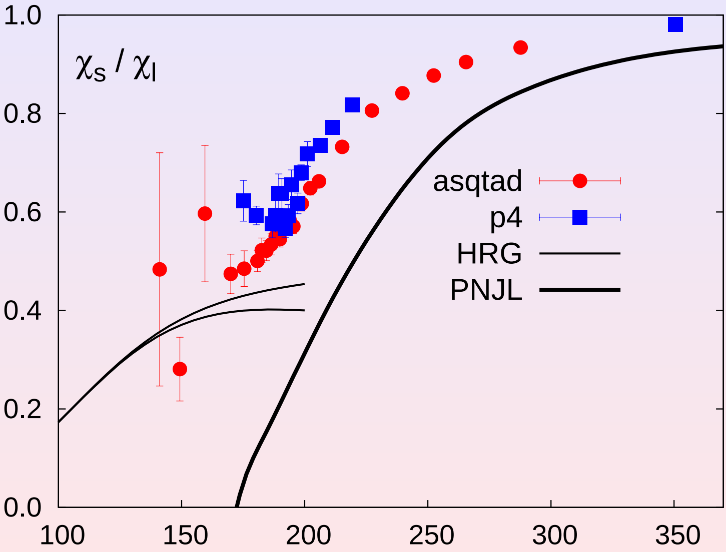
non-strange quark number susceptibility. “large” mass parametrization



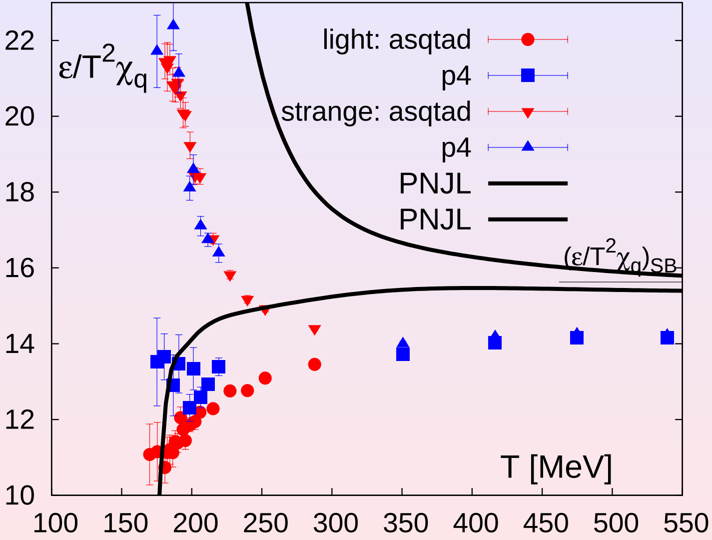
strange quark number susceptibility. “large” mass parametrization



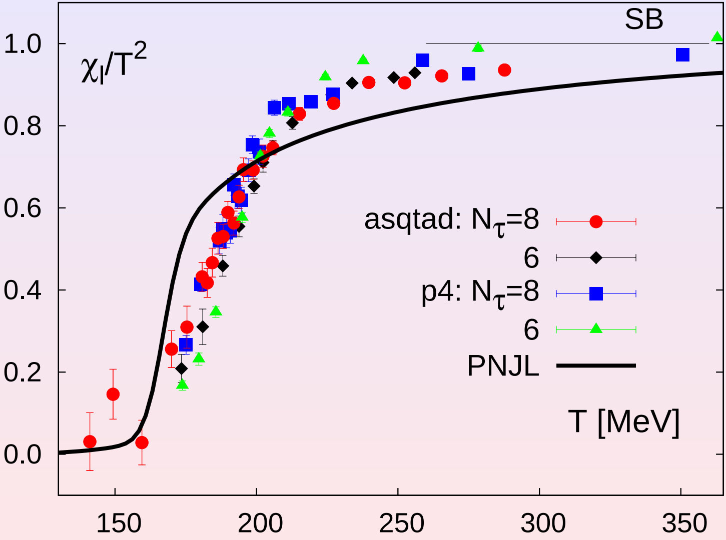
χ_l/χ_s. "large" mass parametrization



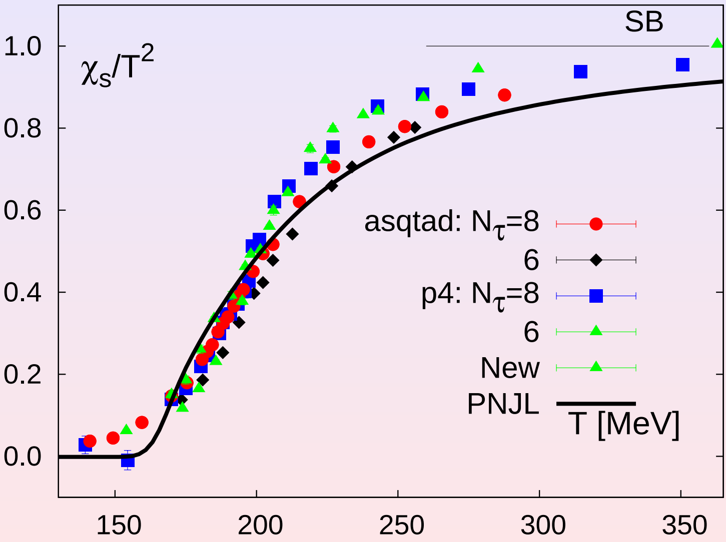
$\epsilon/T^2\chi_l$ and $\epsilon/T^2\chi_s$. "large" mass parametrization

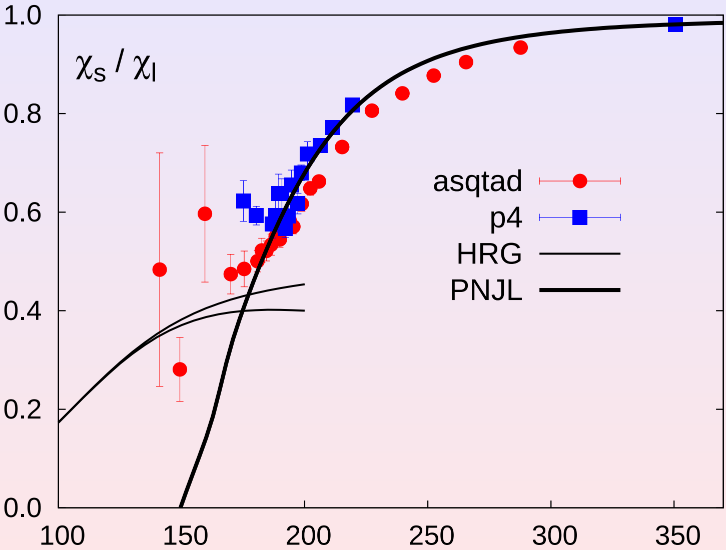


non-strange quark number susceptibility. “small” mass parametrization

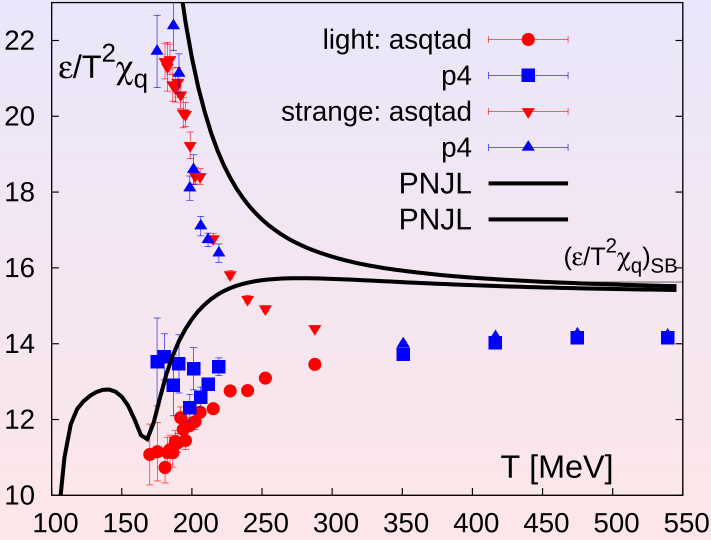


strange quark number susceptibility. “small” mass parametrization




1.0
0.8
0.6
0.4
0.2
0.0
100
150
200
250
300
350
χs / χl
asqtad
p4
HRG
PNJL

$\epsilon/T^2\chi_l$ and $\epsilon/T^2\chi_s$. "small" mass parametrization



Conclusions

1. The nonlocal quark model is extended beyond mean field using ring sum contribution (strict $1/N_c$ expansion)
2. It seems that T_0 parameter of Polyakov loop potential should be lowered ($T_0 = 219$ MeV from fit of lattice data)
3. Ring sum contribution is enough for correct description of pressure and quark condensate at low T
4. Ring sum contribution is not enough for correct description of quark number susceptibilities at low T
5. Mesonic correlations lead to the lowering of the temperature of phase transition
6. Near the phase transition $1/N_c$ expansion breaks out – non-perturbative scheme is needed

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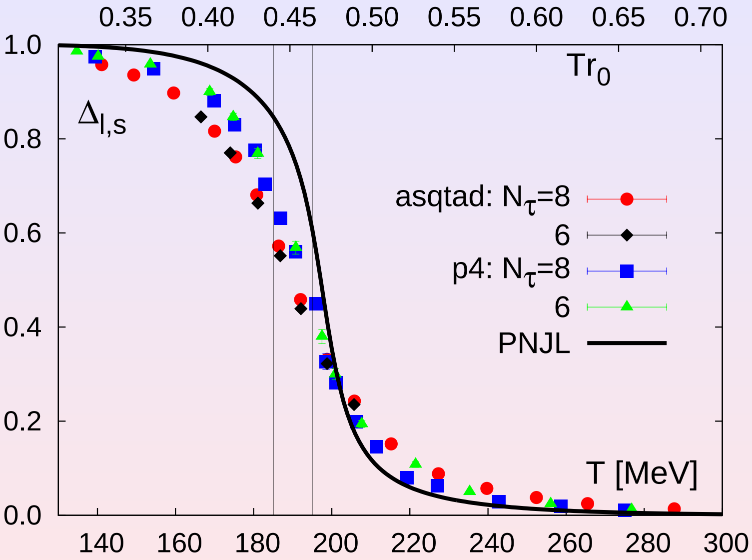
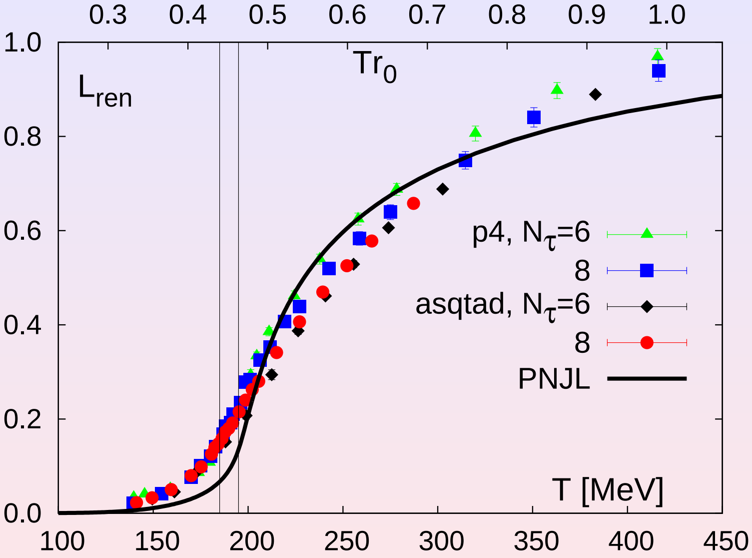

0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70
Tr_0
1.0
0.8
0.6
0.4
0.2
0.0
$\Delta_{l,s}$
asqtad: N_τ=8
6
p4: N_τ=8
6
PNJL
T [MeV]
140 160 180 200 220 240 260 280 300


0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Tr0
Lren
1.0
0.8
0.6
0.4
0.2
0.0
p4, Nτ=6
8
asqtad, Nτ=6
8
PNJL
T [MeV]
100
150
200
250
300
350
400
450

Polyakov loop

Untraced Polyakov loop(Polyakov line)

$$L(\vec{x}) = \mathcal{P}\exp\left\{ i\int_0^\beta d\tau A_4(\tau,\vec{x}) \right\}$$

transforms under SU(3) gauge transformations as : $L \to ULU^\dagger$
Gauge invariant object

$$\Phi = \frac{1}{3}\mathrm{Tr}_c L \qquad \bar{\Phi} = \frac{1}{3}\mathrm{Tr}_c L^+$$

transform under global Z(3) transformations :

$$\Phi \to e^{i\frac{2\pi n}{3}}\Phi \qquad \bar{\Phi} \to e^{-i\frac{2\pi n}{3}}\bar{\Phi}$$

$$\langle\Phi\rangle = e^{-F_q/T}$$

confinement $\langle\Phi\rangle = 0$
deconfinement $\langle\Phi\rangle \neq 0$

$\mathcal{U}$ potential

$\mathcal{U}$ potential I

$$\frac{\mathcal{U}(\Phi,T)}{T^4}(\Phi,\bar{\Phi}) = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2,$$

$$b_2(T) = a_0 + a_1\left[\frac{T_0}{T}\right] + a_2\left[\frac{T_0}{T}\right]^2 + a_3\left[\frac{T_0}{T}\right]^3$$

C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019.

$\mathcal{U}$ potential II

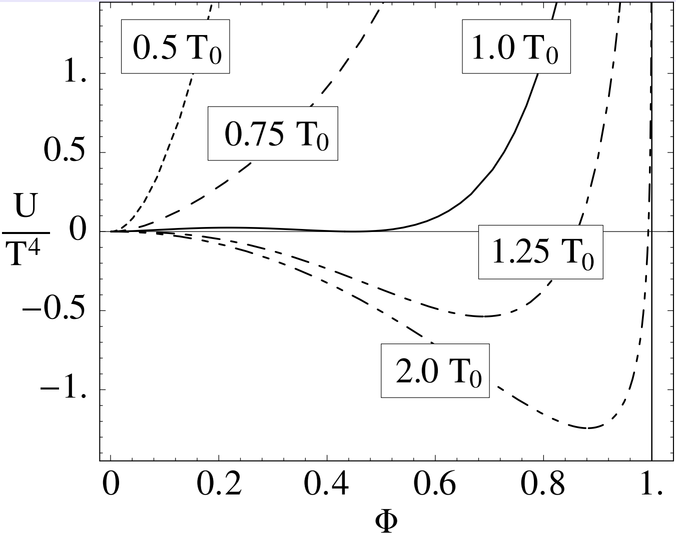
$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\,\bar{\Phi}\Phi + b(T)\,\ln\left[1 - 6\,\bar{\Phi}\Phi + 4\left(\bar{\Phi}^3+\Phi^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \qquad b(T) = b_3\left(\frac{T_0}{T}\right)^3.$$

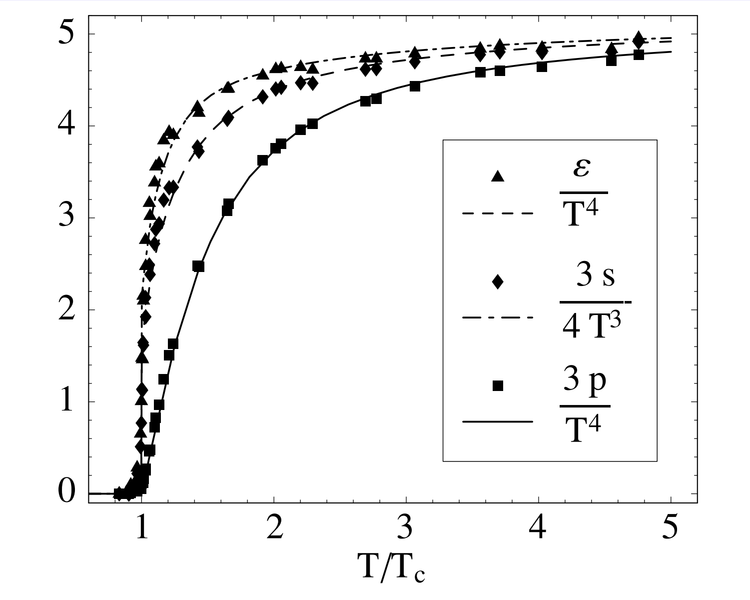
S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75** (2007) 034007.

$\mathcal{U}$ potential. Effective potential for different temperatures



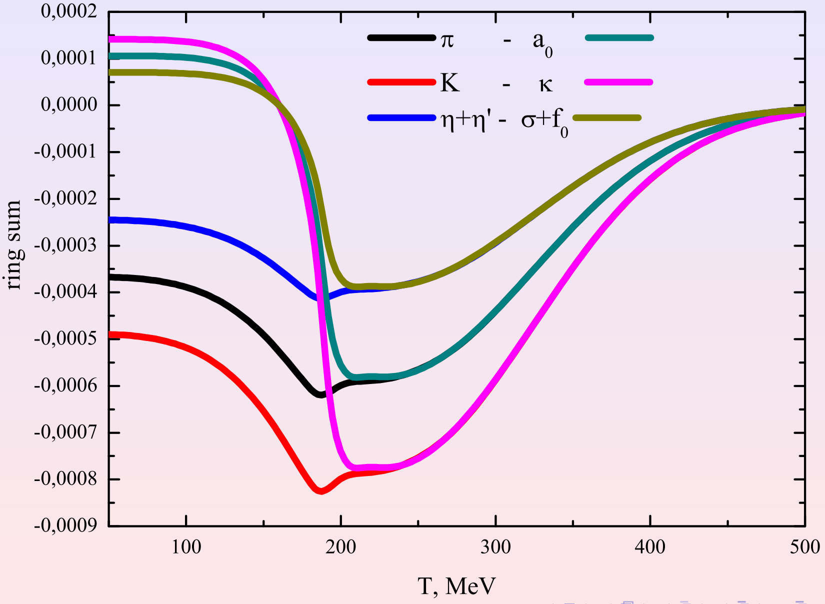
S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75** (2007) 034007

$\mathcal{U}$ potential

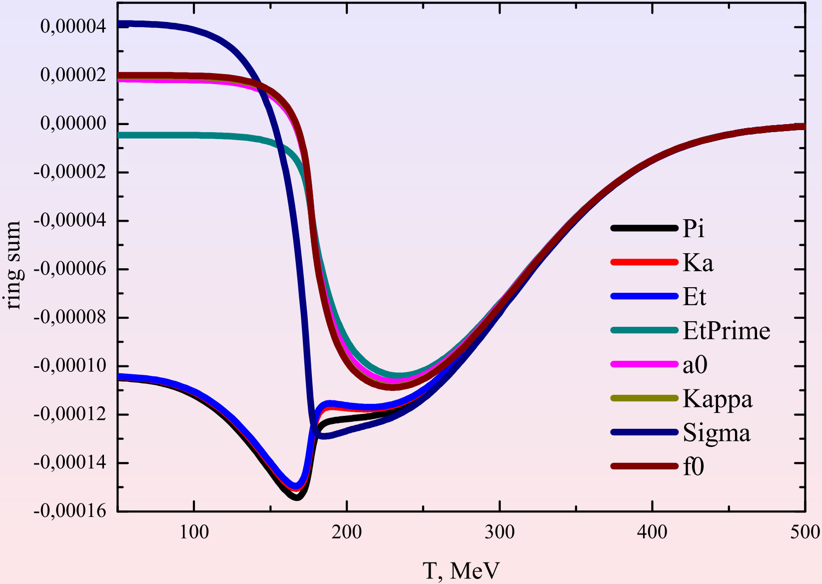


Fit[3] to scaled pressure, entropy density and energy density as functions of the temperature in the pure gauge sector, compared to the corresponding lattice data (G. Boyd *et al.*, Nucl. Phys. B **469**, 419 (1996)).

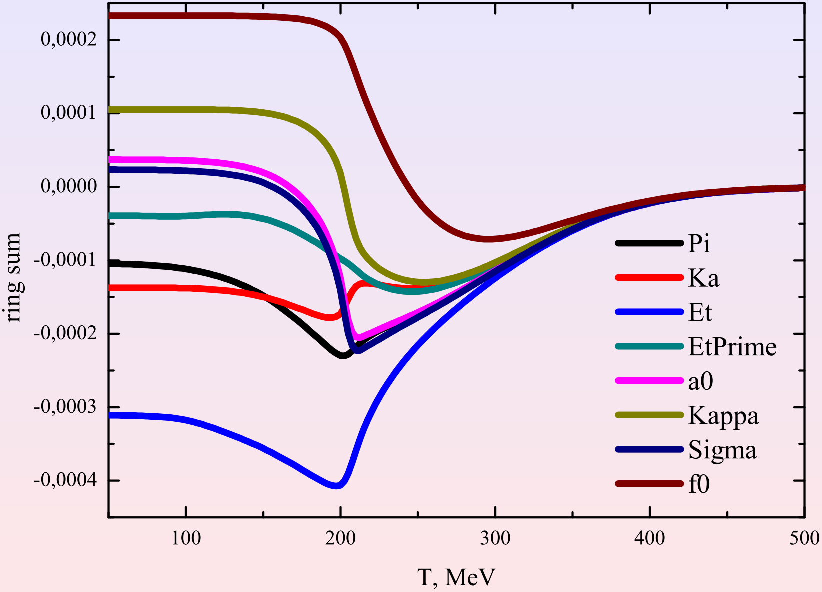
RingSum $m_u = m_s$, $H = 0$



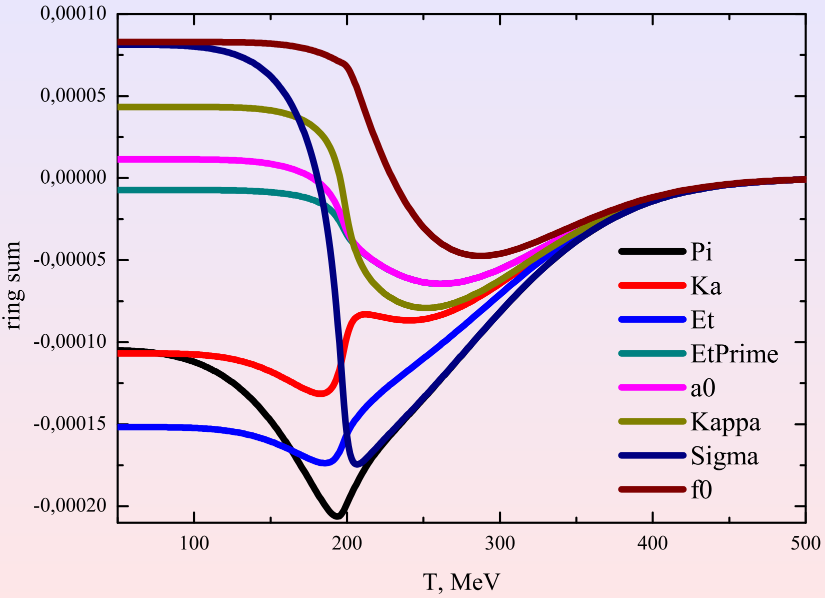
RingSum $m_u = m_s$, $H \neq 0$

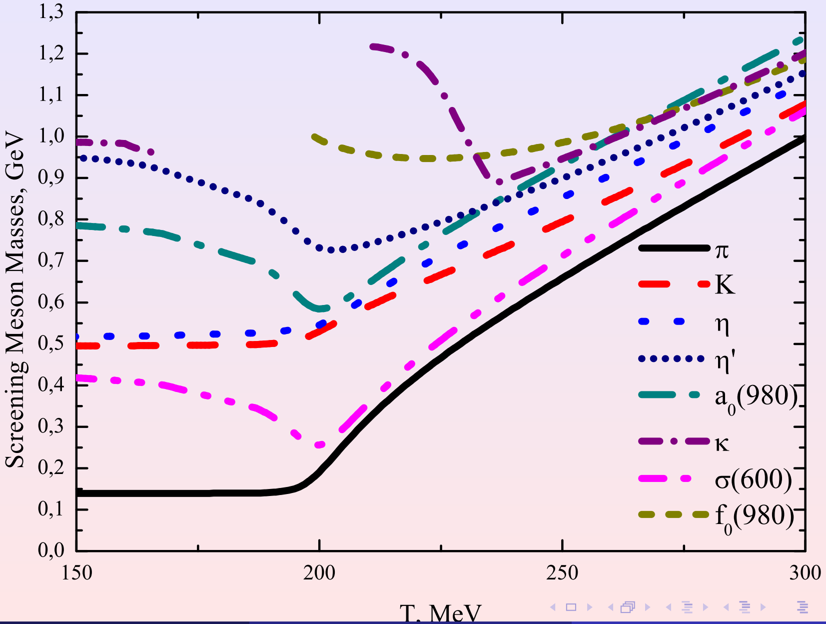


RingSum $m_u \neq m_s$, $H = 0$



RingSum $m_u \neq m_s$, $H \neq 0$




Screening Meson Masses, GeV
T, MeV
1,3
1,2
1,1
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
150
200
250
300
π
K
η
η'
a0(980)
κ
σ(600)
f0(980)