Non-Perturbative Effects for the Quark Gluon Plasma Equation of State

Viktor. V. Begun, Mark I. Gorenstein

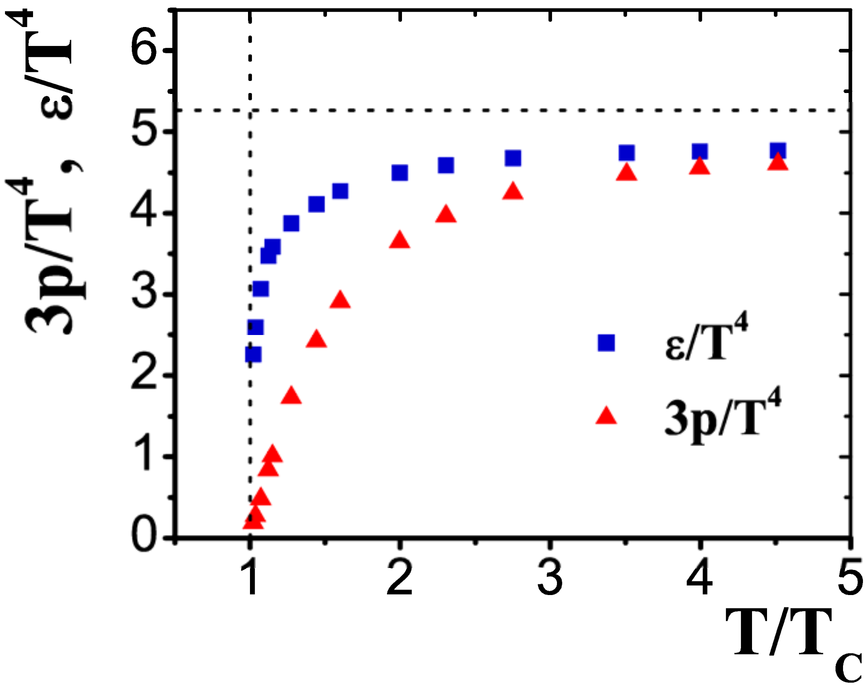
Oleg A. Mogilevsky

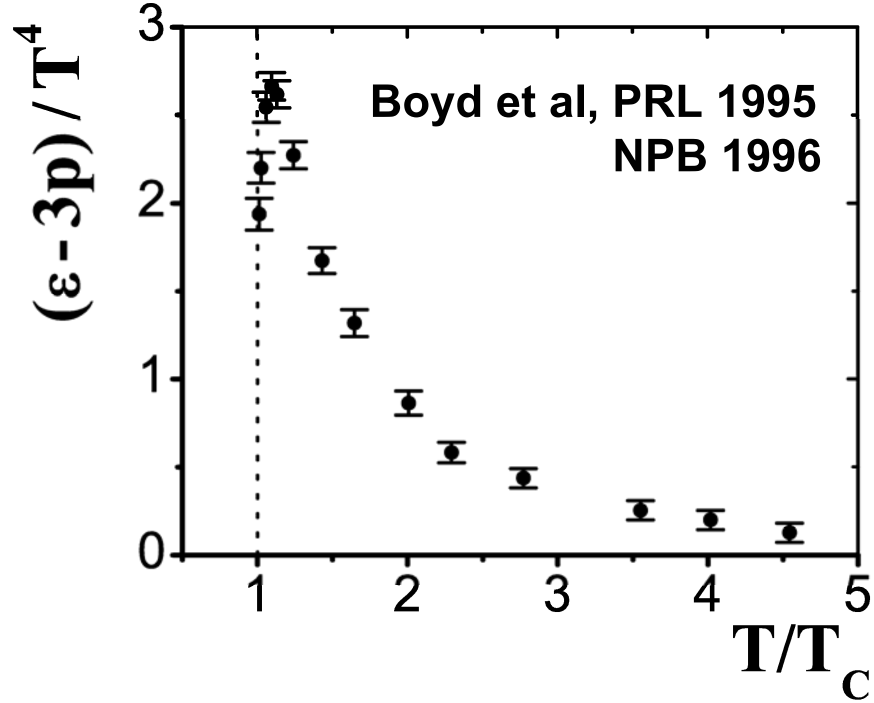
Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

V.V. Begun, M.I. Gorenstein, O.A.Mogilevsky

arXiv:1001.3139v2, arXiv:1004.0953v1 [hep-ph]

Lattice results for the QCD EoS in the SU(3) gluodynamics





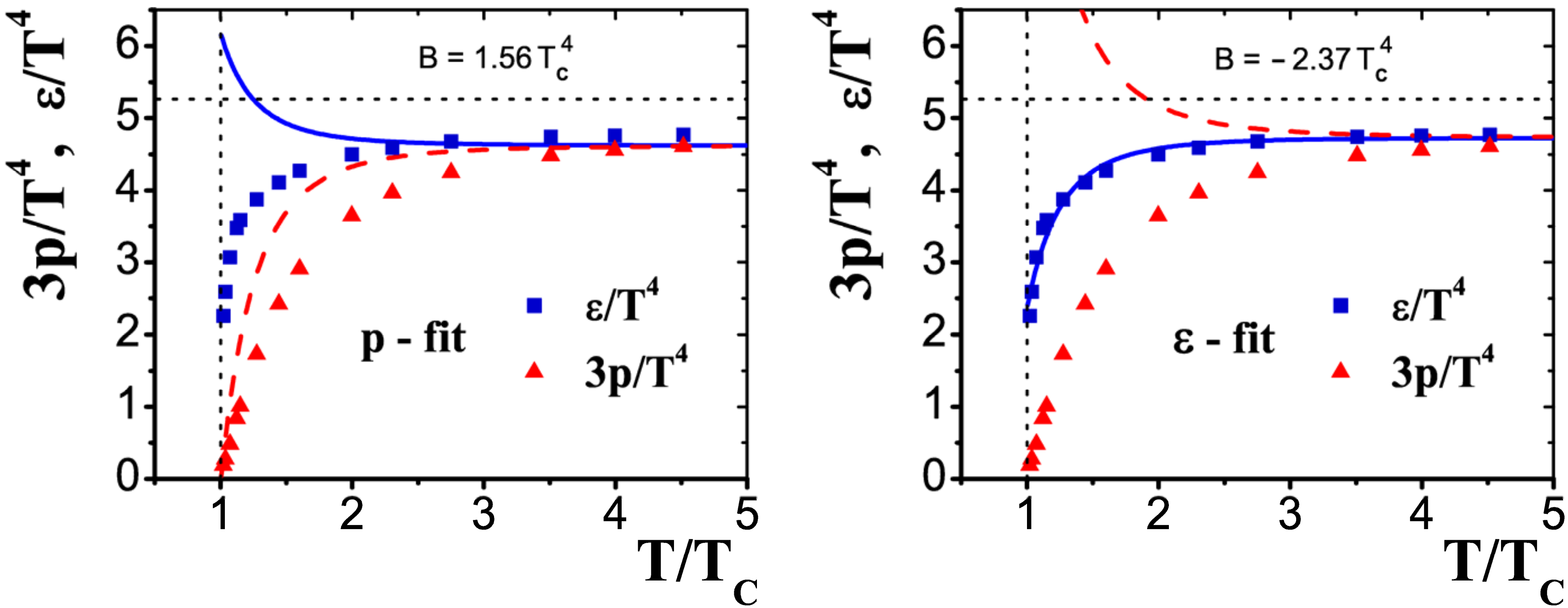
1) The p(T) is very small at Tc and rapidly increases at T > Tc
2) At high T the system behaves as ideal massless gas $p \simeq \varepsilon/3 \simeq \sigma T^4/3$
3) The constant σ is about 10% smaller than the SB limit
4) Both ε/T^4 and $3p/T^4$ approach their limiting value from below
5) The interaction measure demonstrates a prominent maximum at T = 1.1 Tc

Bag Model

For non-interacting massless constituents and zero values of all conserved charges:

$$\varepsilon(\mathrm{T}) = \sigma\, \mathrm{T}^4 + \mathrm{B}$$

$$\mathrm{p}(\mathrm{T}) = \frac{\sigma}{3}\, \mathrm{T}^4 - \mathrm{B}$$



With negative values of **B** one obtains a good fit of ε/T^4 for **T > Tc**, but finds a disagreement for $3\mathrm{p}/\mathrm{T}^4$

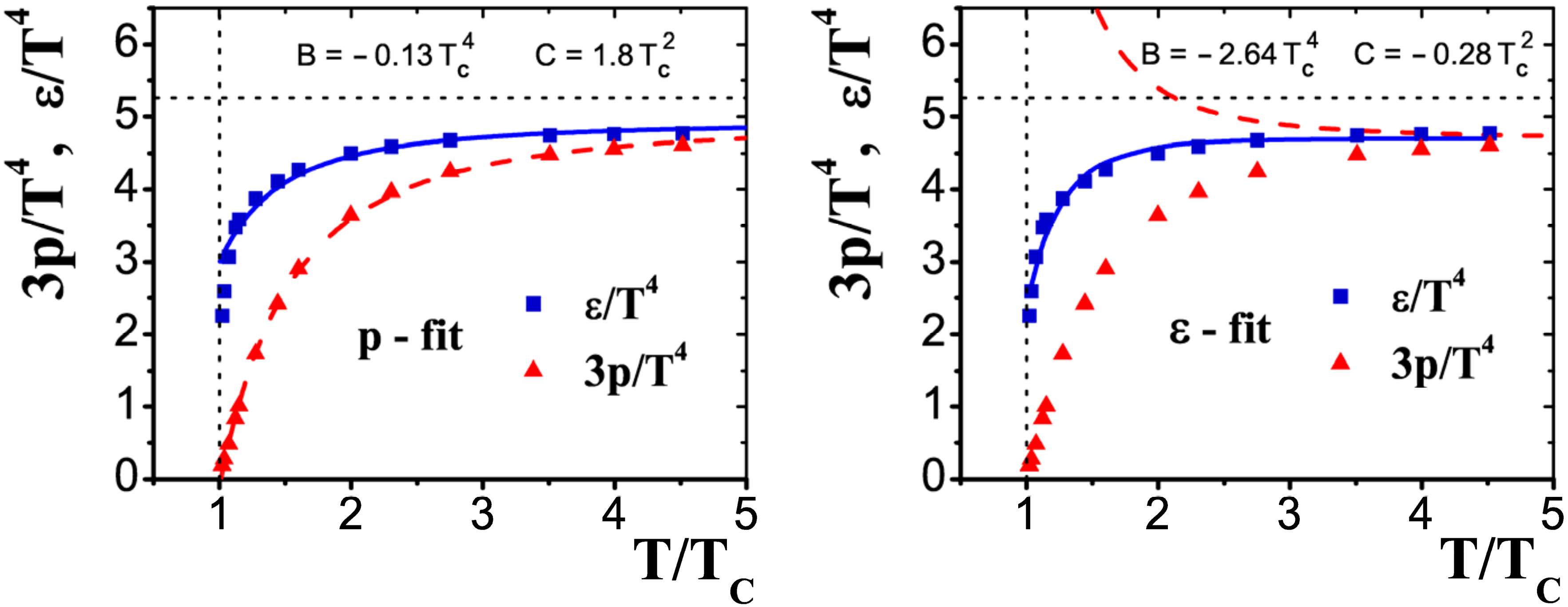
C – Bag Model

R.D. Pisarsky, PRD 2006

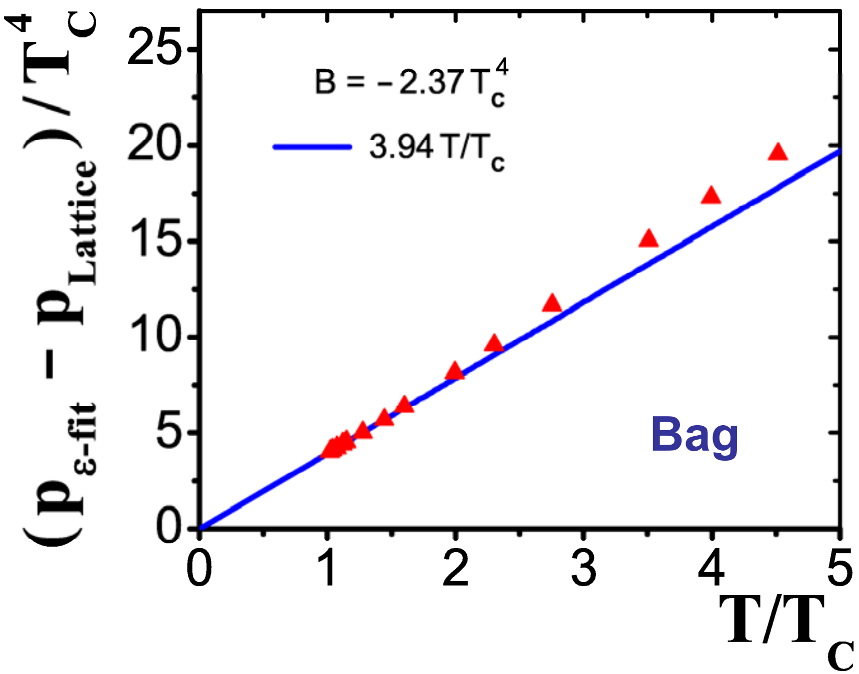
Prog. Theor. Phys. Suppl. 2007

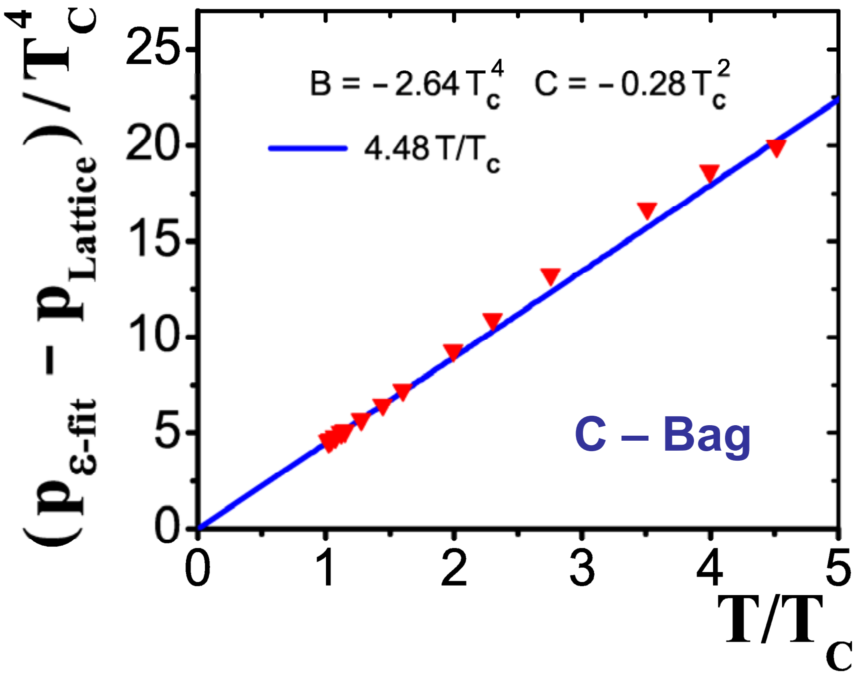
$$\varepsilon(\mathrm{T}) = \sigma\,\mathrm{T}^4 - \mathrm{C}\,\mathrm{T}^2 + \mathrm{B}$$

$$\mathrm{p}(\mathrm{T}) = \frac{\sigma}{3}\mathrm{T}^4 - \mathrm{C}\,\mathrm{T}^2 - \mathrm{B}$$



Linear Term in pressure





The thermodynamical relation

$$\varepsilon = T\frac{dp}{dT} - p \quad \Rightarrow \quad \begin{aligned} \varepsilon(T) &= \sigma T^4 + B \\ p(T) &= \frac{\sigma}{3}T^4 - B - A\,T \end{aligned}$$

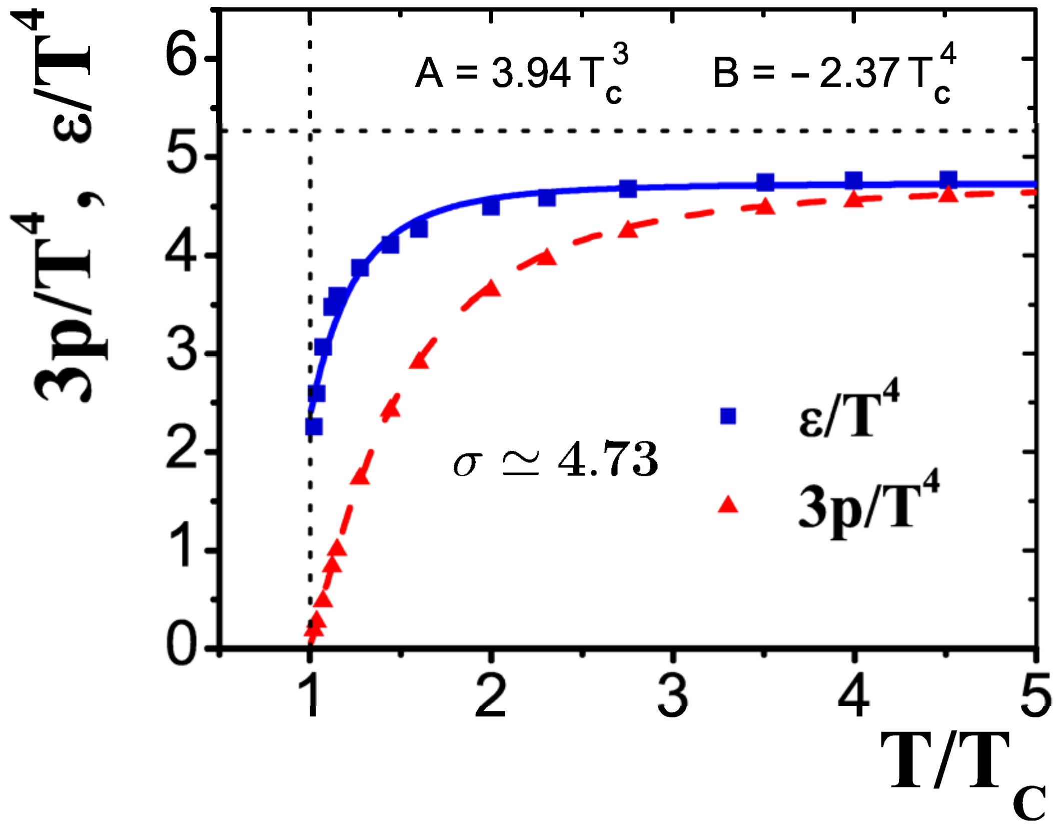
is the 1st order differential equation. Its general solution involves an arbitrary integration constant A

A – Bag Model

The term −AT gives a negative contribution to p(T) and guarantees both a correct high temperature behavior of $3p/T^4$ and its strong drop at T near Tc.

$$\varepsilon(T) = \sigma T^4 + B$$

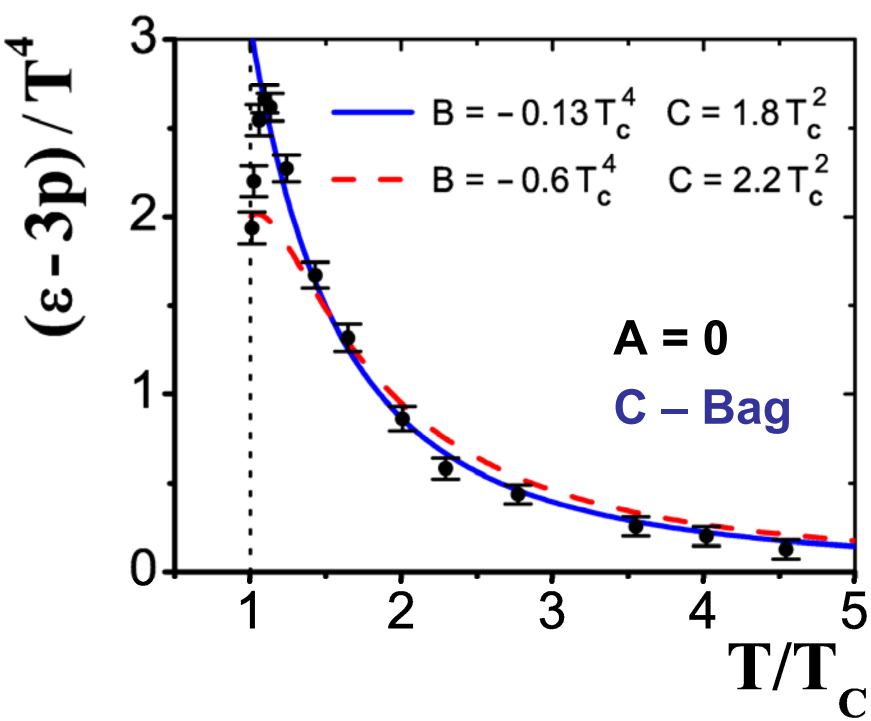
$$p(T) = \frac{\sigma}{3} T^4 - B - A\,T$$

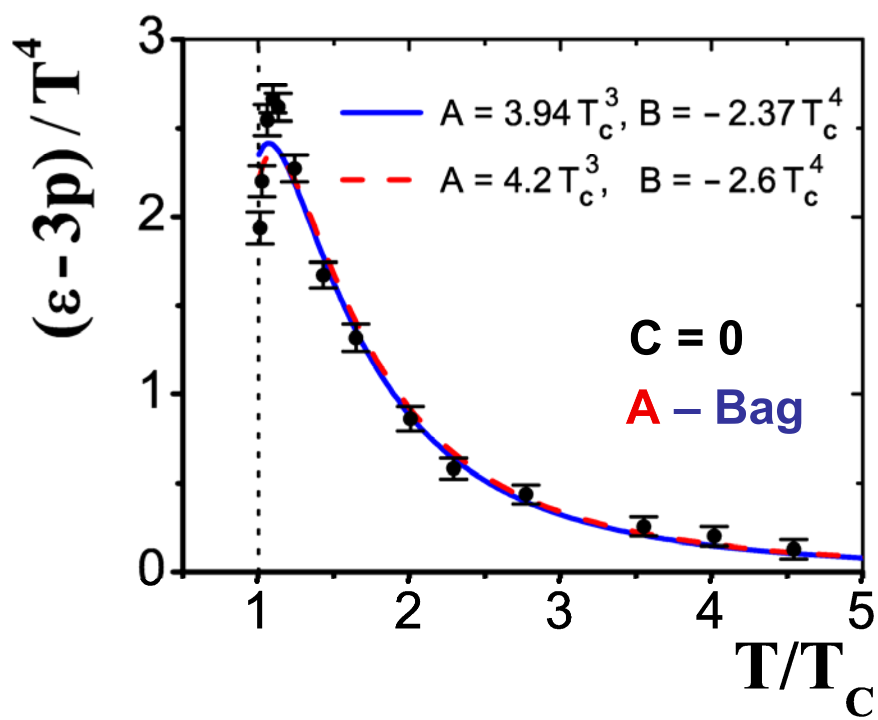


The A-BM, gives essentially the same values of the model parameters σ, B, and A either one starts from fitting of $3p/T^4$ or from ε/T^4

Interaction Measure

$$\frac{\varepsilon - 3p}{T^4} = \frac{2C}{T^2} + \frac{3A}{T^3} + \frac{4B}{T^4}$$





The C-BM gives no maximum. The requirement of a maximum makes the fit worse at T > 2Tc, whereas the A-BM gives the maximum either one fits the pressure or the interaction measure.

Possible Physical Origin of the A-Bag Model

the modified gluon dispersion relation

$$\omega(\mathrm{k}) = \sqrt{\mathrm{k}^2 + \frac{\mathrm{M}^4}{\mathrm{k}^2}}$$

where M is a QCD mass scale corresponds to effective mass

$$\mathrm{m} = \mathrm{M}^2/\mathrm{k}$$

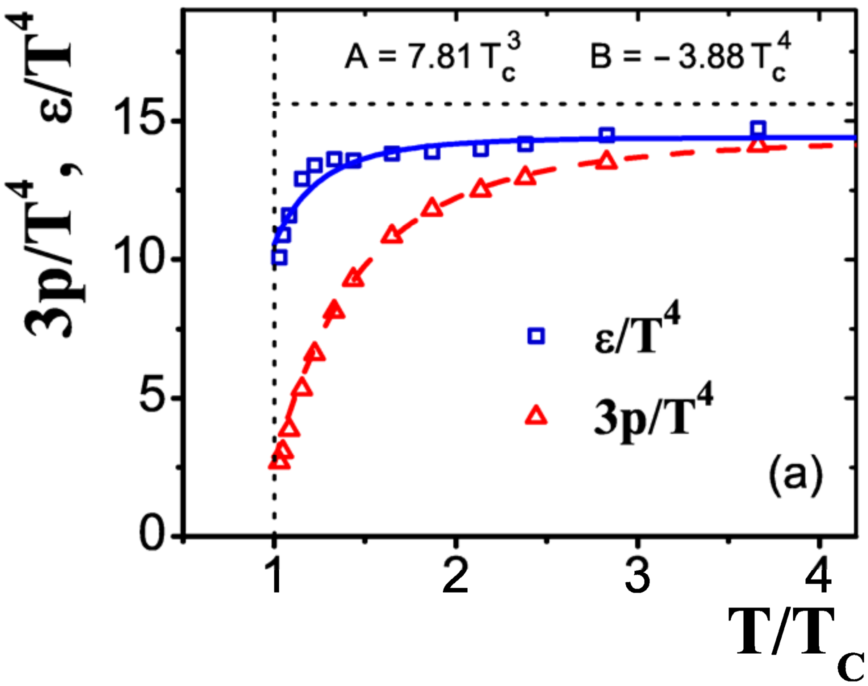
which is large at low k, and provides an infrared cut-off at k ~ M

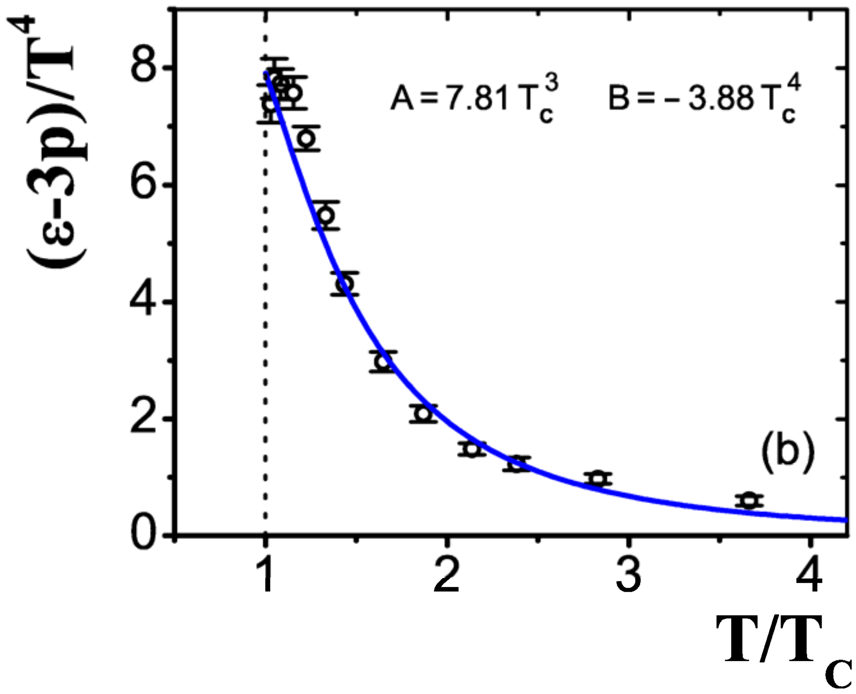
$$\omega(\mathrm{k}) = \mathrm{k}\,\theta(\mathrm{k} - \mathrm{K})$$

It gives power corrections of relative order
$1/\mathrm{T}^3$ for p/T^4 and $1/\mathrm{T}^4$ for ε/T^4

Gribov, Nucl. Phys. B 1978; Zwanziger, Phys. Rev. Lett. 2005
Karsch, Z. Phys. C 1988; Rischke, et. al. Phys. Lett. B 1992.

EoS in QCD with 2+1 quarks





Lattice data from M. Cheng et al. Phys.Rev.D 2008

Summary:

1. A linear in T pressure term is admitted by the thermodynamic relation between $\varepsilon(T)$ and $p(T)$

2. We find that the A-BM with negative bag constant B leads to the best agreement with the lattice results.

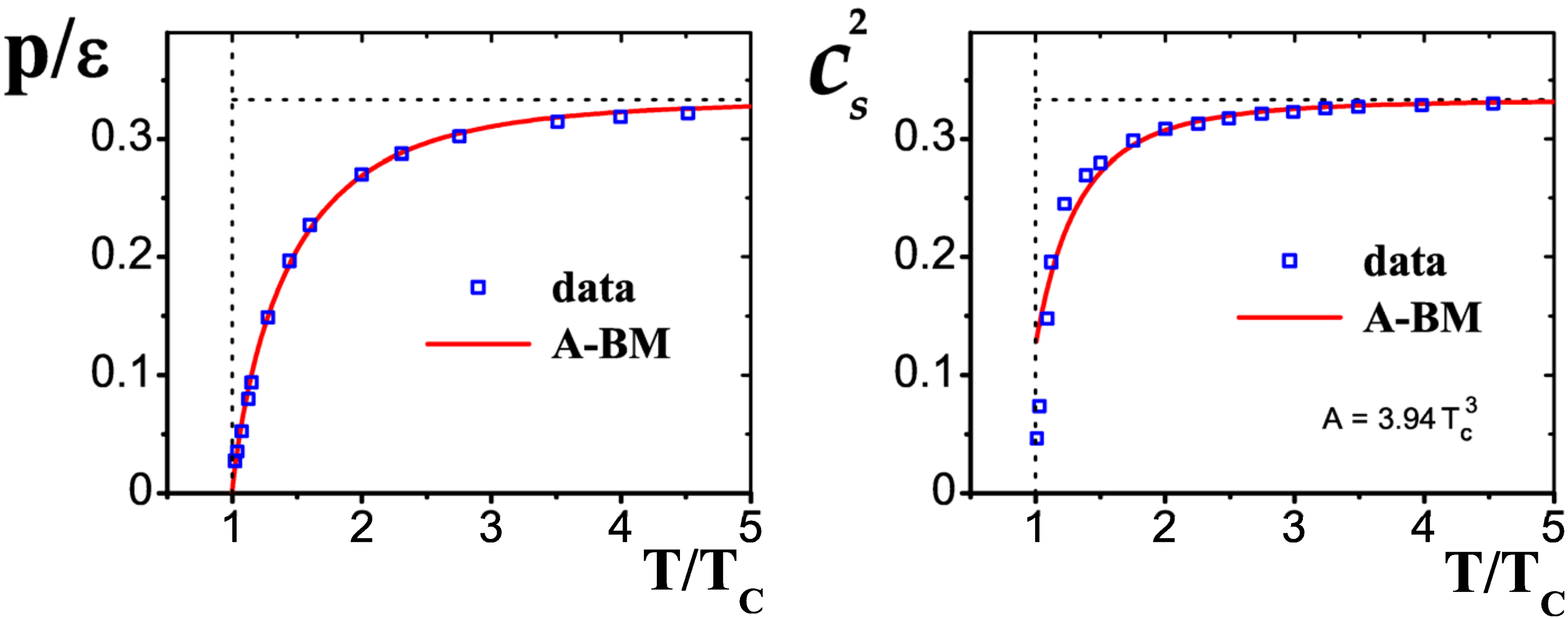
3. The A-BM gives a simple analytical parameterization of the QGP EoS. This opens new possibilities for its applications.

Why $B < 0$?

No answer yet ☹

It is allowed for $T>T_c$ ☺

Pressure over energy density, velocity of sound



We did not fit p/ε or c_s^2 , however, the correspondence to data is good.

$$c_s^2 \equiv \frac{dp}{d\varepsilon} = \frac{dp/dT}{d\varepsilon/dT} = \frac{1}{3} - \frac{A}{4\ \sigma\ T^3}$$

How to change σ ?

Quasi-particle approach

Interacting gluons are treated as a gas of non-interacting quasi-particles with gluon quantum numbers, but with mass m(T) and particle energy ω

$$\omega = \left[k^2 + m^2(T)\right]^{1/2}$$

$$\varepsilon(T) = \frac{16}{2\pi^2}\int_0^\infty k^2 dk \, \frac{\omega}{\exp(\omega/T) - 1} + B^*(T) \equiv \varepsilon_0(T,\omega) + B^*(T)$$

$$p(T) = \frac{16}{6\pi^2}\int_0^\infty k^2 dk \, \frac{k^2}{\omega} \frac{1}{\exp(\omega/T) - 1} - B^*(T) \equiv p_0(T,\omega) - B^*(T)$$

The thermodynamic relation

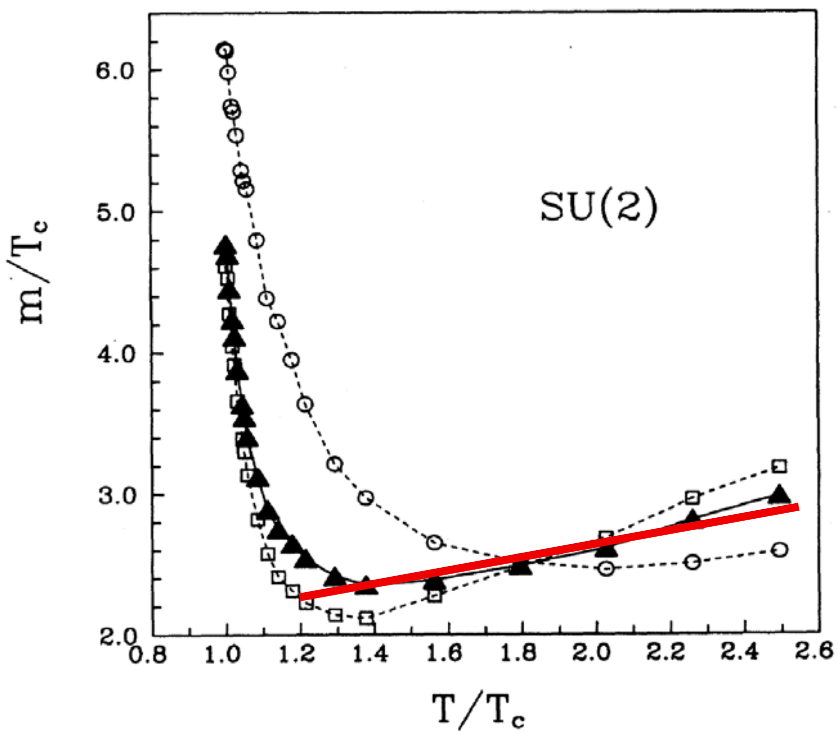
$$\varepsilon = T\frac{dp}{dT} - p$$

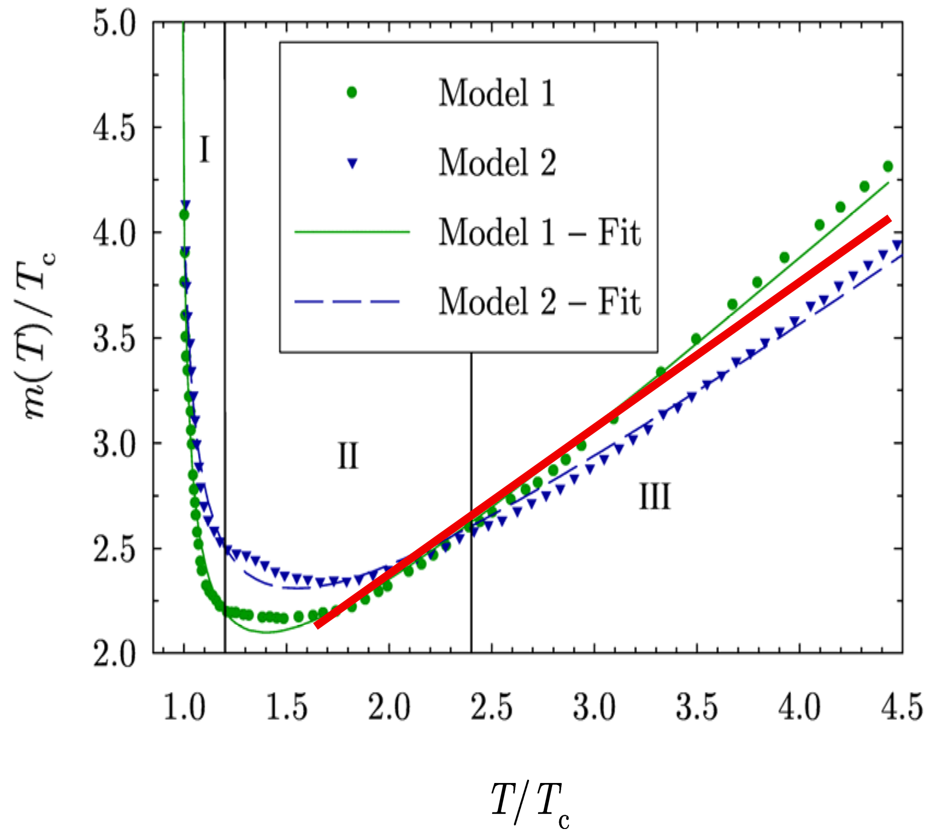
leads to the equation for B*

$$\frac{dB^*}{dT} = - \frac{\Delta_0(T,\omega)}{m}\frac{dm}{dT}$$

where $\Delta_0 \equiv \varepsilon_0 - 3p_0$

Mass of Quasi-particles





Gorenstein, Yang Phys.Rev.D 1995

Brau, Buisseret, Phys.Rev.D 2009

m ~ aT for T > 1.2 Tc

The modified SB constant

$$\sigma = \frac{3d}{2\pi^2} \sum_{n=1}^{\infty} \left[\frac{a^2}{n^2} K_2(na) + \frac{a^3}{4n} K_1(na) \right] \equiv \kappa(a)\ \sigma_{SB}$$

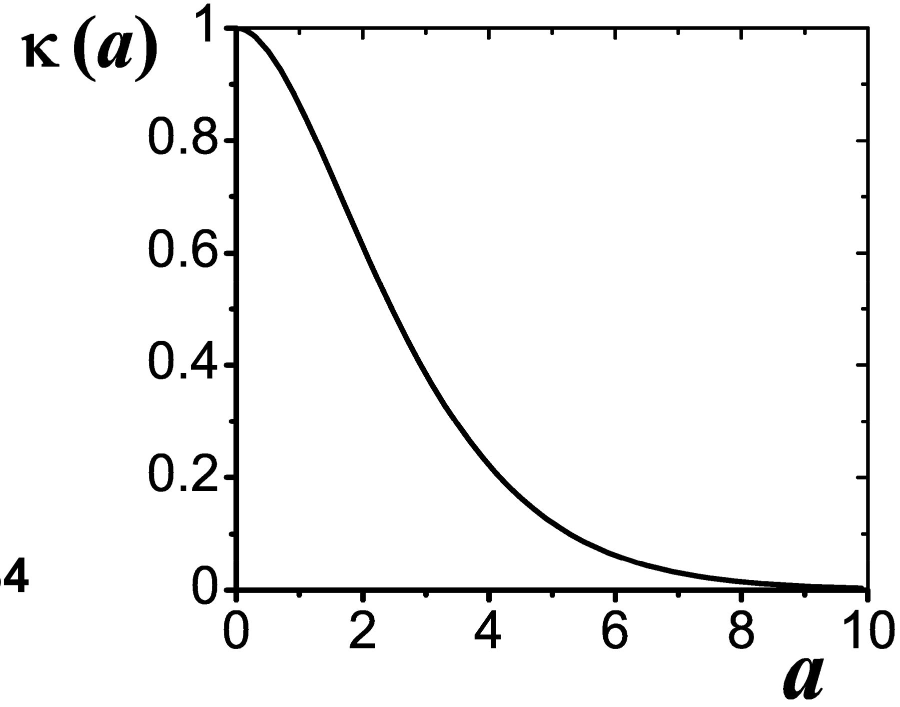
The modified SB constant

$\sigma = 4.73 < \sigma_{SB}$ allows to fit the

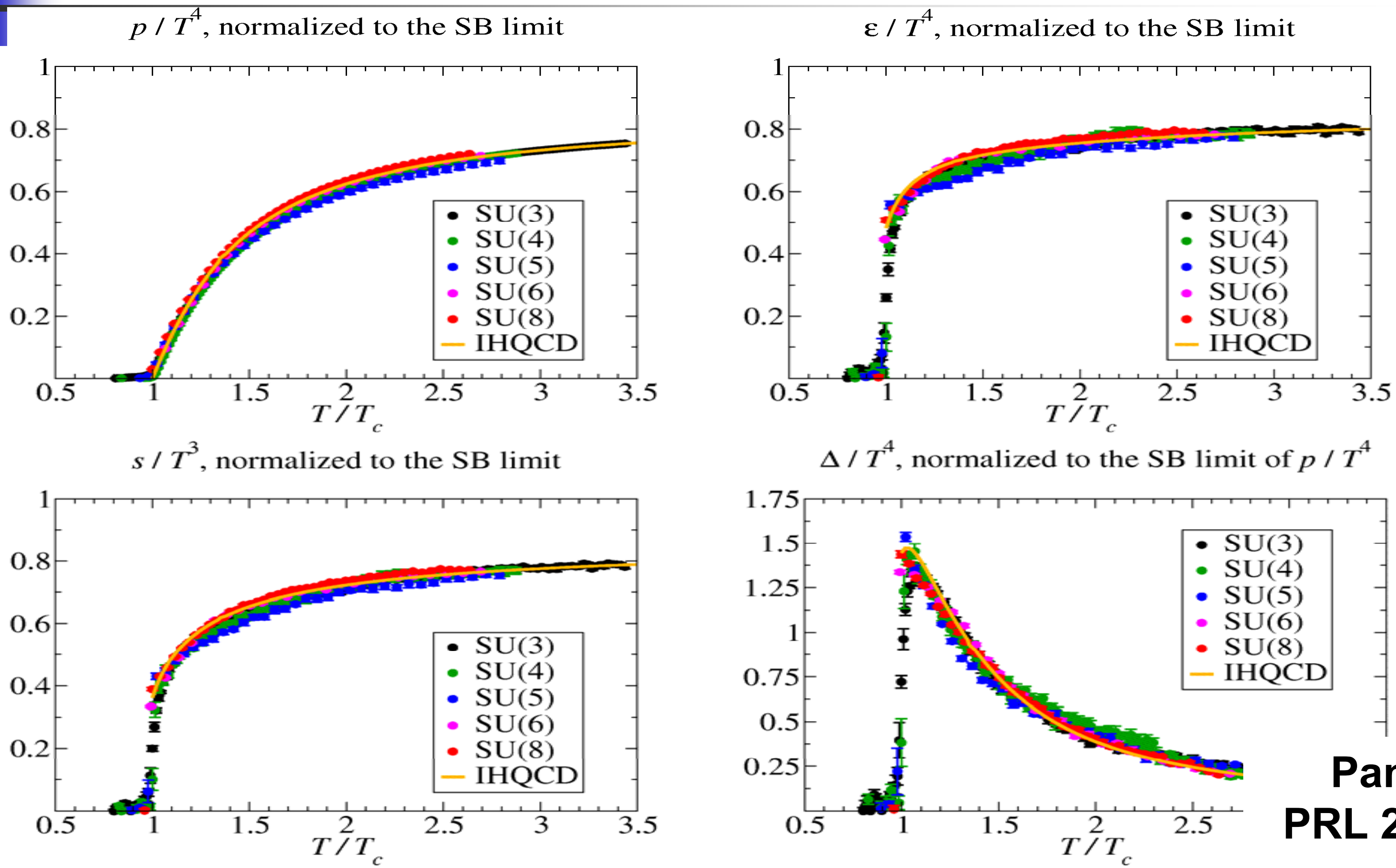
high temperature behavior of

pressure and energy density.

This requires $\kappa(a) \sim 0.9$ and $a \sim 0.84$



SU(N_c) gluodynamics

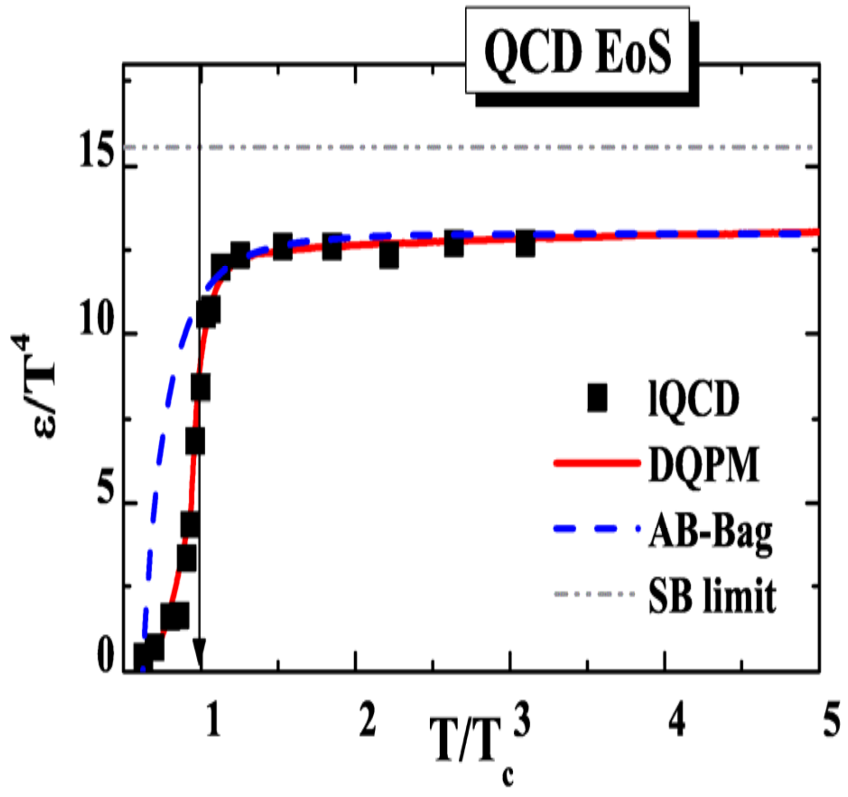


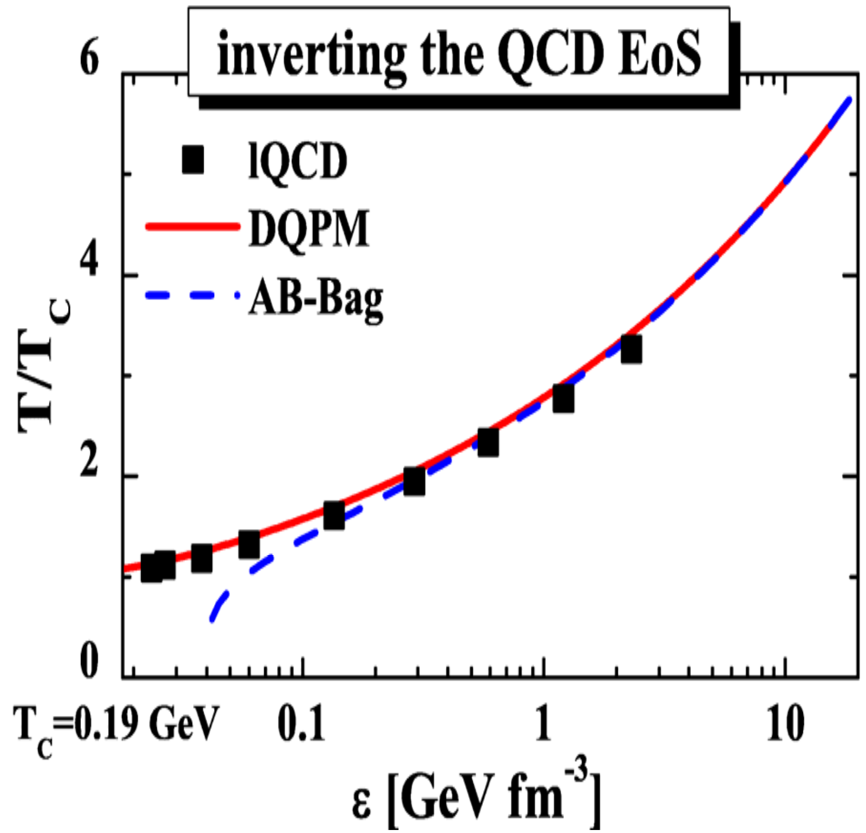
Panero PRL 2009

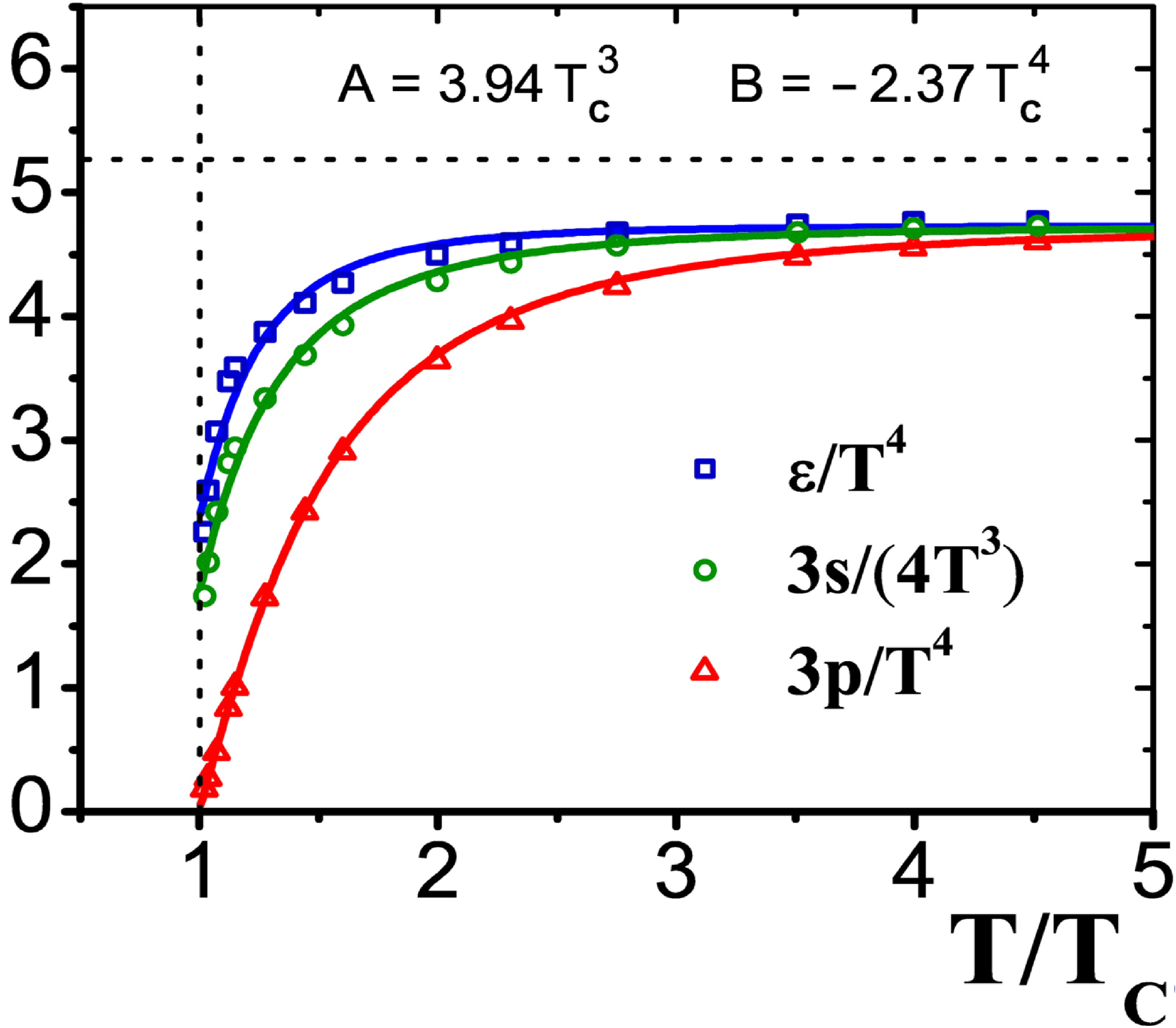
All thermodynamic quantities follow essentially the same curves for different N_C. Thus, our SU(3) fit of the A-BM can be extrapolated for SU(N_C) with $A \propto (N_c^2 - 1)T_c^3$ and $B \propto -(N_c^2 - 1)T_c^4$

Applications

Dilepton production by dynamical quasiparticles in the strongly interacting quark gluon plasma. arXiv:1004.2591 O. Linnyk.







A = 3.94 T_c^3
B = – 2.37 T_c^4
ε/T^4
3s/(4T^3)
3p/T^4
T/T_C

Summary:

1. The good fits of ε/T^4 lead to a wrong behavior of $3p/T^4$
   This happens because of a linear in T term admitted by the thermodynamic relation between $\varepsilon(T)$ and $p(T)$

2. We find that the A-BM with negative bag constant B leads to the best agreement with the lattice results.

3. The A-BM gives a simple analytical parameterization of the QGP EoS. This opens new possibilities for its applications.