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Staturation and Critical Phenomena in Deep-Inelastic Scattering

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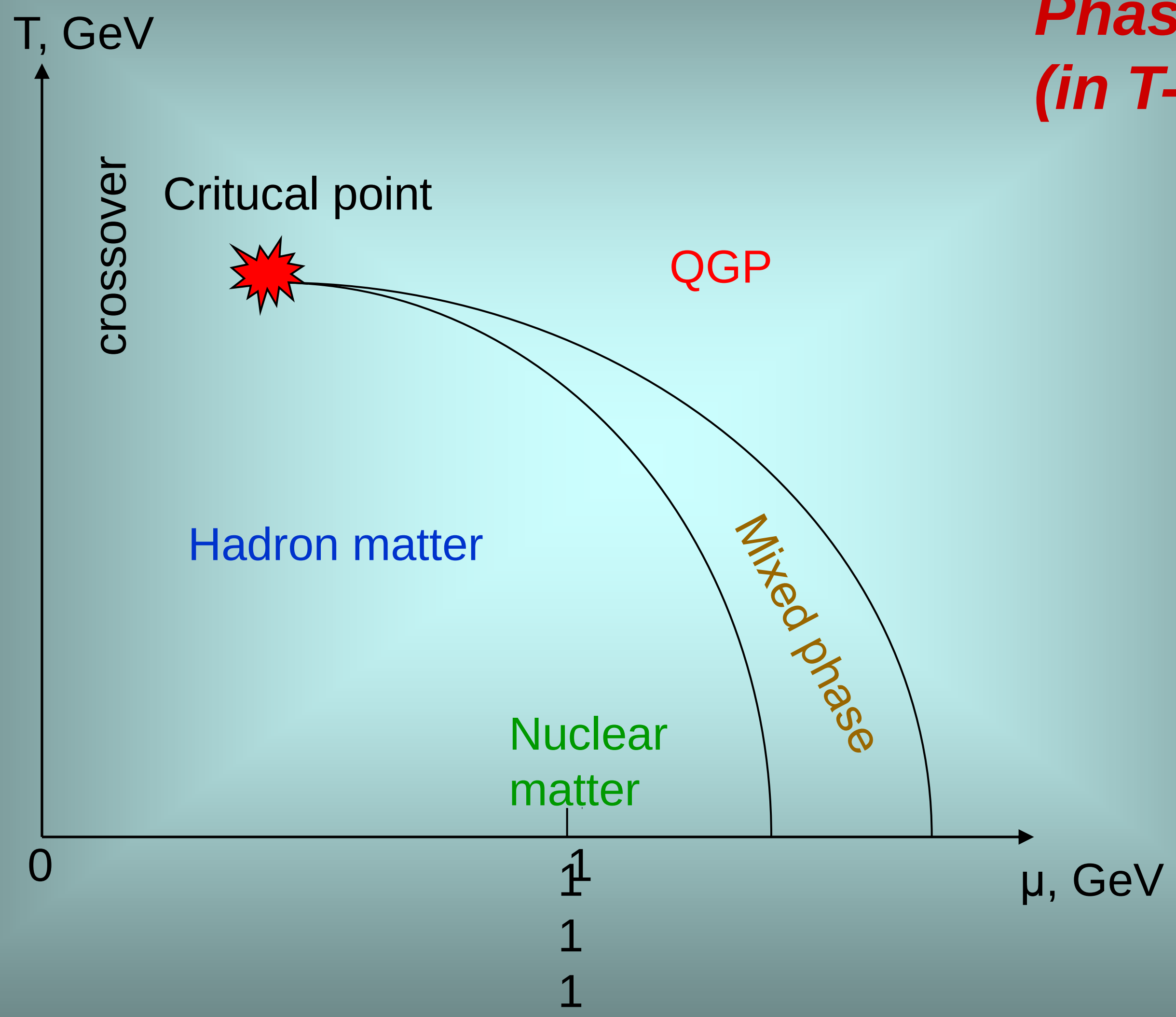
Based on work in collab. with S.Troshin and N.Tyurin, hep-ph/1002.3527 ("Blois" conf., CERN, 2009)

Phase diagram (in T-μ)



Basics of nuclear thermodynamics (Bag EoS), $p(T), \quad \mu = 0$:

$$p_q(T) = a_q T^4 - B, \quad p_h(T) = a_h T^4;$$

$$\epsilon_q(T) = 3a_q T^4 - B, \quad \epsilon_h = 3a_h T^4;$$

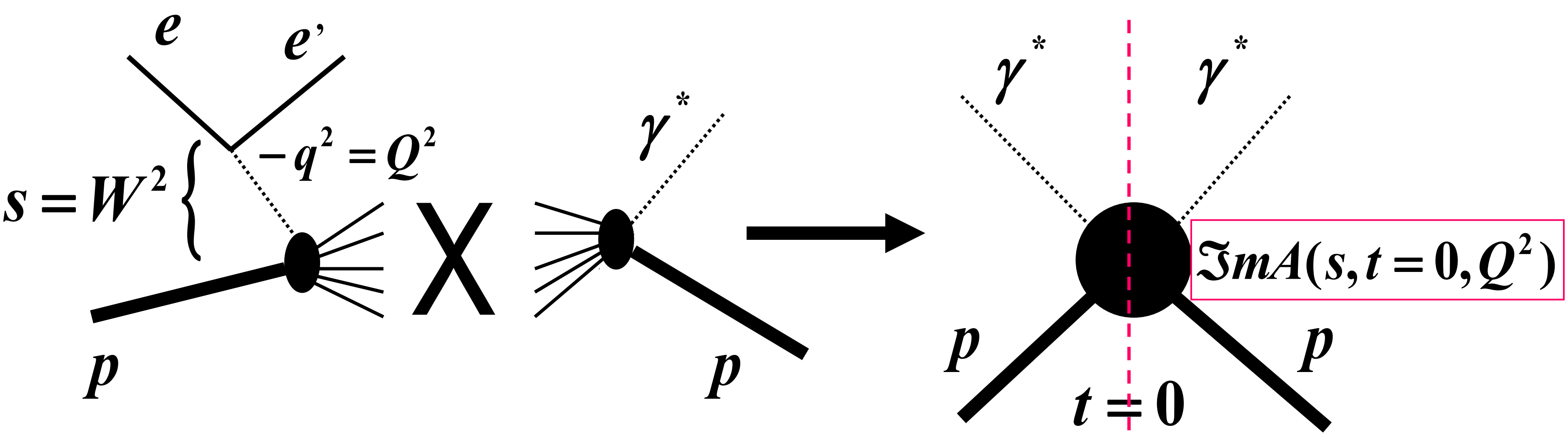
$$s_q(T) = 4a_q T^3, \quad s_h(T) = 4a_h T^3.$$

Further developments:

a) $B \to B(T, R)$; (Boyko, Jenkovszky, Sysoev, 90-ies);

b) $p \sim T^4 \to T^6$ (Jenkovszky, Trushevsky, '76).

DIS (and ordinary parton distributions)

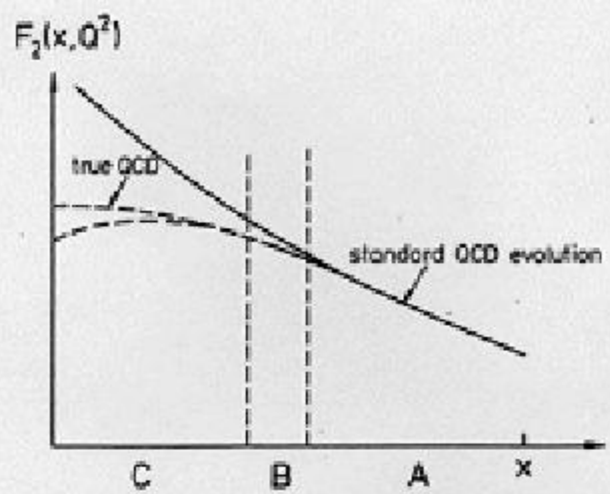


$$x = \frac{Q^2}{2pq}$$

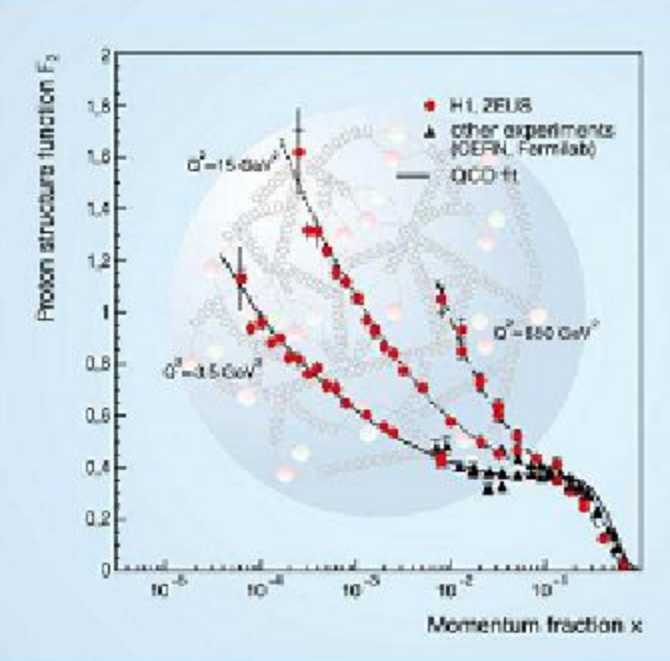
$$s = \frac{Q^2(1-x)}{x} + M^2 \approx \frac{Q^2}{x}, x << 1$$

The proton is smashed (completely destroyed)

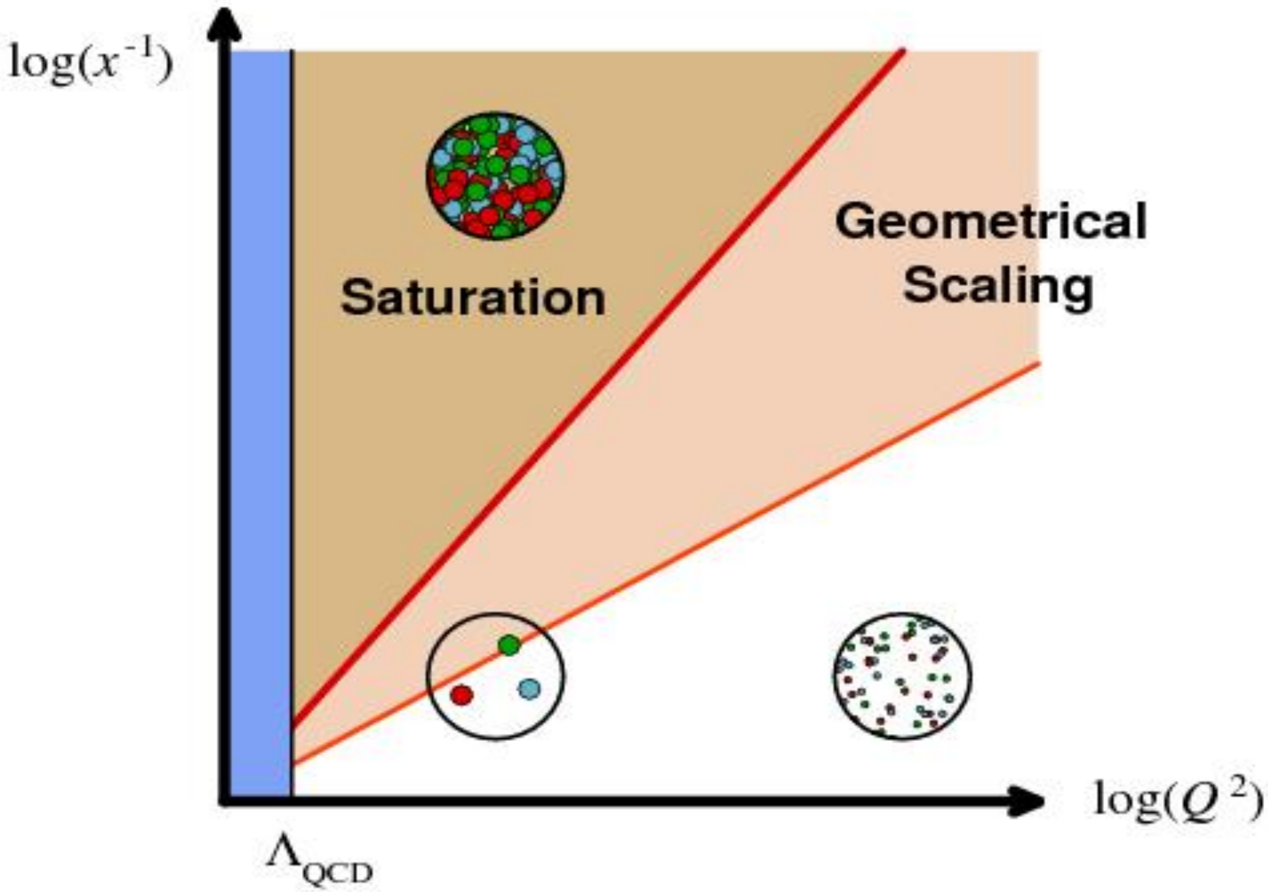
Scaling violation and saturation



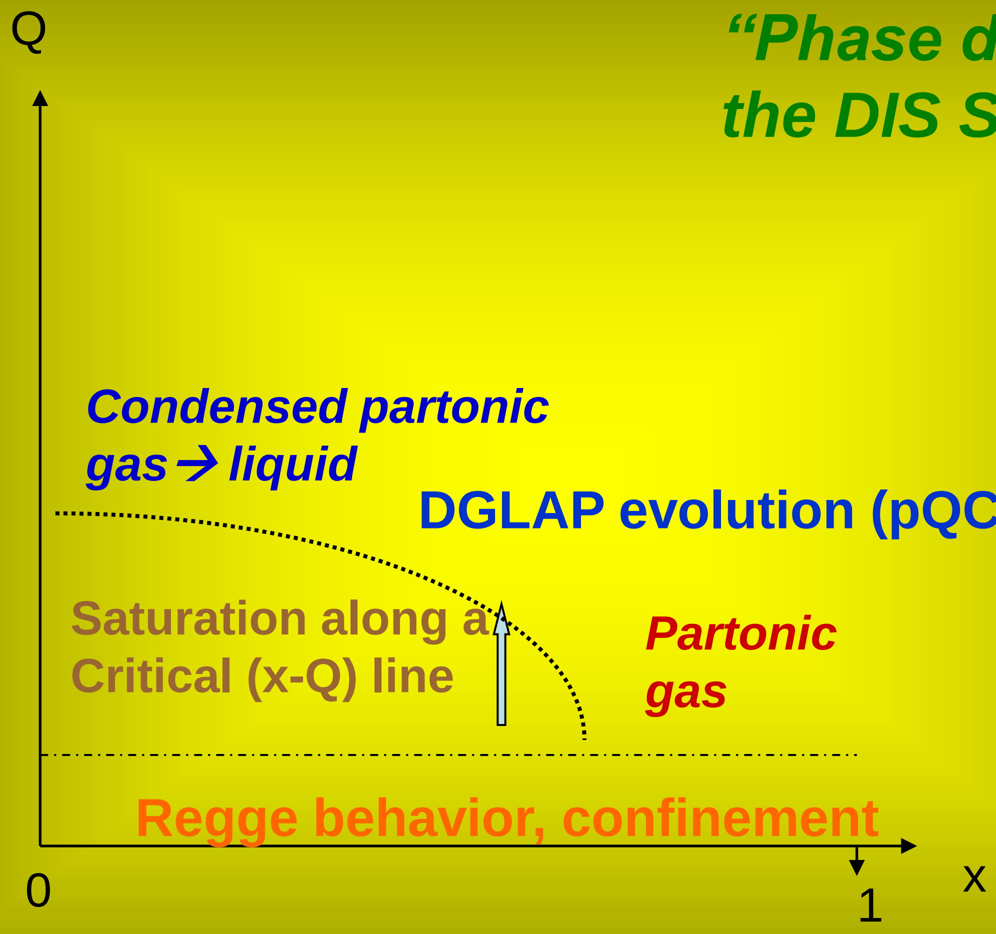
Small-x behavior of the structure function: standard QCD evolution versus "true QCD" evolution. A is the perturbative region, B the transition region, C the nonperturbative region.



Saturation and GS, Q_s~x^(-0.3)



"Phase diagram" of the DIS SF



A typical nucleon structure function (SF):

$$F_2(x, Q^2) = \Im A(s, t = 0, Q^2), \quad x \sim Q^2/s.$$

$$F_2(x, Q^2) = \int_0^1 x^{-\alpha(0)+1}(1 - x)^n.$$

$$F_2^{(S,0)}(x,Q^2) = A\left(\frac{Q^2}{Q^2+a}\right)^{1+\widetilde{\Delta}(Q^2)} e^{\Delta(x,Q^2)}, \tag{1'}$$

with the "effective power"

$$\widetilde{\Delta}(Q^2) = \epsilon + \gamma_1 \ell n\left(1 + \gamma_2 \ell n\left[1 + \frac{Q^2}{Q_0^2}\right]\right),$$

and

$$\Delta(x,Q^2) = \left(\widetilde{\Delta}(Q^2)\ell n\frac{x_0}{x}\right)^{f(Q^2)},$$

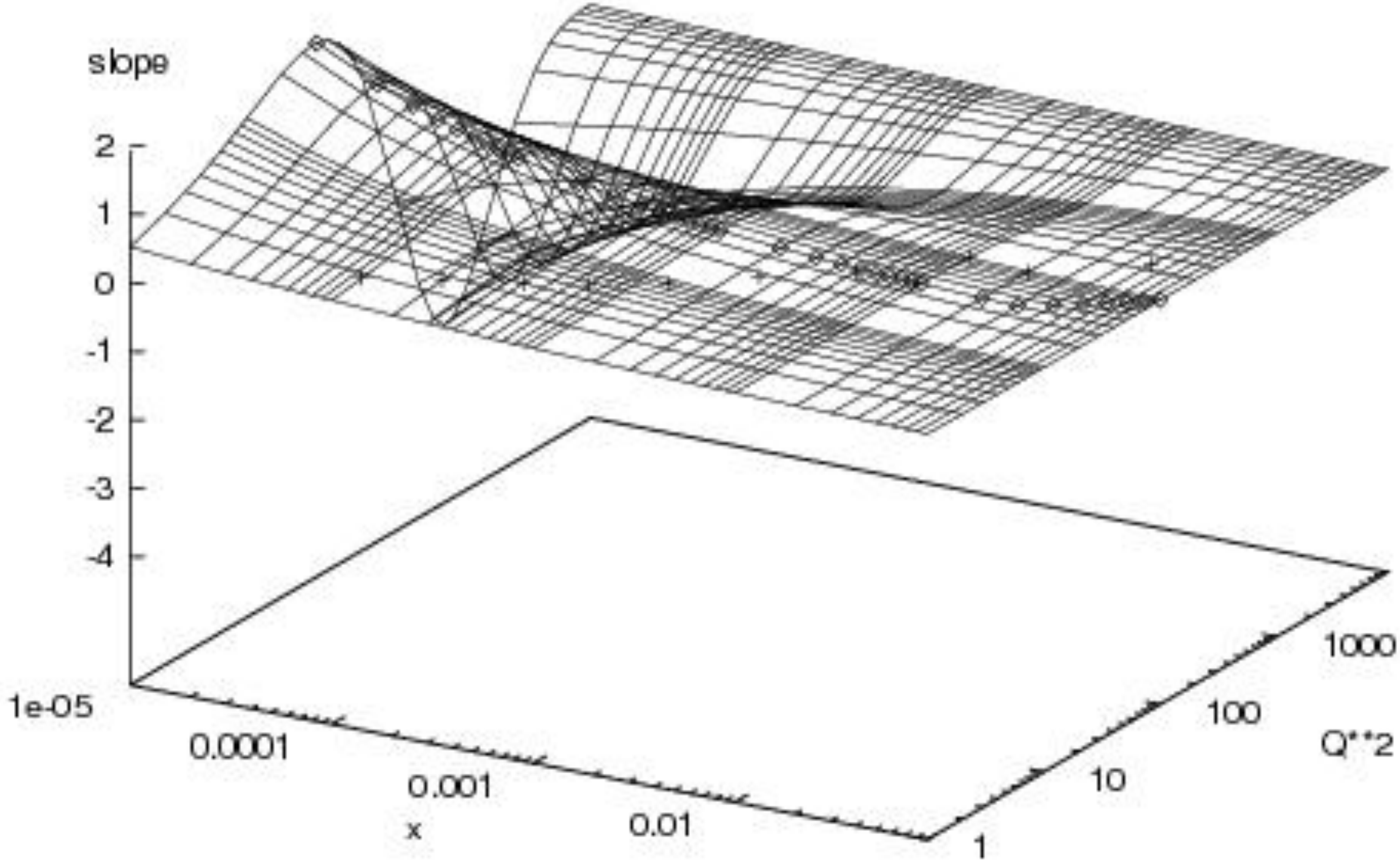
where

$$f(Q^2) = \frac{1}{2}\left(1 + e^{-Q^2/Q_1^2}\right).$$

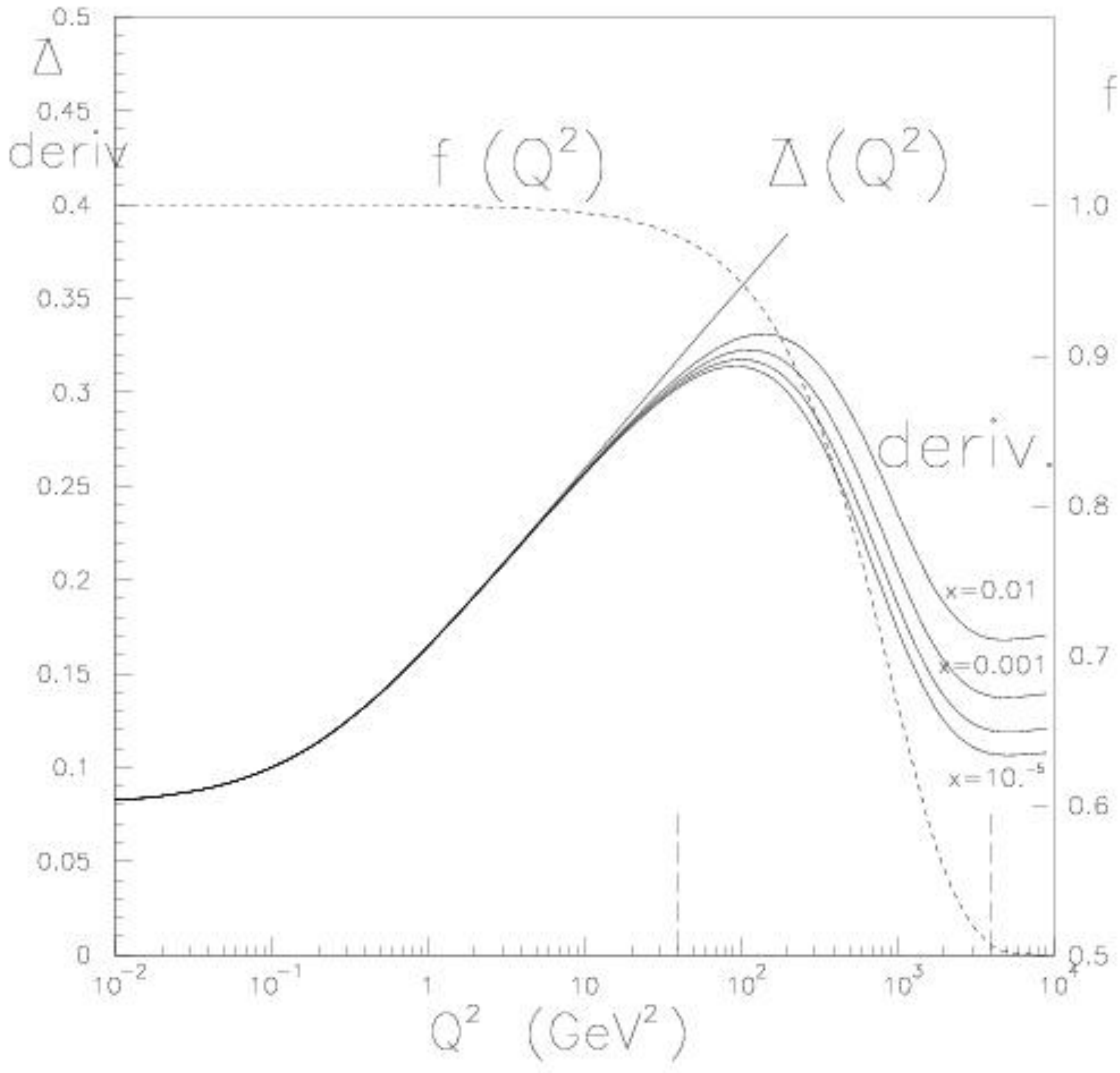
The saturation line in the $x - Q^2$ plane, the turning point (line) of the derivatives

$$B_Q(x, Q^2) = \frac{\partial F_2(x, Q^2)}{\partial(\ln Q^2)}, \quad B_x(x, Q^2) = \frac{\partial F_2(x, Q^2)}{(\partial \ln(1/x))},$$

called B_Q or B_x slopes, where $F_2(x, Q^2)$ is the structure function, satisfying the basic theoretical requirements, yet fitting the data.



P.Desgrolard, L.Jenkovszky and F. Paccanoni, EPJ, 1998



Δ
deriv
f (Q²)
Δ (Q²)
f
deriv.
x=0.01
x=0.001
x=10.⁻⁵
Q² (GeV²)

In the context of the BFKL equation the production of partons in a nucleon by splitting of partons resulting in the increase of their number N is described by the BFKL evolution equation

$$\frac{\partial N(x, k_T^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \bigotimes N(x, K_T^2),$$

where K_{BFKL} is the BFKL integral kernel (splitting function), resulting in the power-like increase of the SF toward smaller x, $F_2(x) \sim x^{-\alpha_P + 1}$, where α_P is the BFKL pomeron intercept, $\alpha_P - 1 = (4\alpha_s N_c \ln 2)/\pi > 0$, α_s is the ("running") QCD coupling and N_c is the number of colours.

At certain "saturation" values x and Q^2 an inverse process, namely the recombination of pairs of partons comes into play, and the BFKL equation is replaced by the following one

$$\frac{\partial N(x,k_T^2)}{\partial \ln(1/x)} =$$

$$\alpha_s K_{BFKL} \bigotimes N(x,K_T^2) - \alpha_s [K_{BFKL} \bigotimes N(x,K_T^2)]^2,$$

based on the idea that the number of recombinations is proportional to the the number of parton pairs, N^2. Saturation sets in when the second, quadratic term overshoots the first, linear one.

$$xG(x,Q^2) \sim \frac{X_0 x^b}{\exp\left[(x - X_0)/\bar{x}\right] + 1},$$

where x is the Bjorken (light-cone) variable, X_0 is the chemical potential.

Use a dimensionless "temperature" $\bar{x} = 2T/m$, where m is the proton mass, which is a consequence of the transition from the rest frame to the infinite-momentum frame (IMF). Accordingly,

$$G(x) \sim \exp\Bigl(-\tfrac{mx}{2T}\Bigr).$$

Statistical approach to DIS SFs (partial list of references):

R.S. Bhalerao: *Statistical model for the nucleon structure functions*, Phys. Lett. B **380**, 1 (1996), hep-ph/9607315; R.S. Bhalerao and R.K. Bhaduri, *Droplet formation of quark-gluon plasma atlow temperatures and high high densities*}, hep-ph/0009333; R.S.Bhalerao, N.G. Kelkar and B. Ram, *Model for polarized andunpolarized parton density functions in the nucleon*, Phys. Rev. C **63** 025208 (2001), hep-ph/9911286; K. Ganesamurtly, V. Devanathan and M. Rajasekaran, Z. Phys. C **52**, 589 (1991); V.Devanathan, S. Karthiyaini and K. Ganesamurthly, Mod. Phys. Lett.A **9** 3455 (1994); V. Devanathana and J.S. McCarthy, Mod.Phys. Lett. A **11** 147 (1996); Hai Lin, hep-ph/0105050, hep-ph/0105172, hep-ph/0106100. J. Cleymans, C. Angelini and R. Pazzi, Phys. Lett. B **135**, 473 (1984).E. Mac and E. Ugaz, Z. Phys. C {\bf 43} 655 (1989); J. Cleymans and R.L. Thews, Z. Phys. C **37**, 315 (1988); J. Cleymans, I.Dadic, and J. Joubert, Z. Phys. **68**, 275 (1994). C. Bourrely, F. Buccella, G. Miele, G. Migliore, J. Soffer, and V. Tibullo, Z.Phys. C **62** 431 (1994); C.Bourrely and J Soffer, Phys. Rev. D 51, 2108 (1995); C.Bourrely and J.Soffer, Nucl. Phys. B **445**, 341 (1995); A.V. Efremov et al., Phys. Rev. D**80**, 014021 (2009); J. Clyemans…O. Teryaev, *Duality of…* hep-ph/1004.2770.

A nucleon of mass M consists of a gas of massless particles (quarks, antiquarks and gluons) in equilibrium at temperature T in a spherical volume V with radius $R(s)$ increasing with squared c.m.s. energy s as $\ln s$ (or $\ln^2 s$). The invariant parton number density in phase space is given by

$$\frac{dn^i}{d^3p^i d^3r^i} = \frac{dn}{d^3p d^3r} = \frac{gf(E)}{(2\pi)^3},$$

where g is the degeneracy ($g = 16$ for gluons and $g = 6$ for q and $\bar{q}$ of a given flavor), E, p is the parton four-momentum and $f(E) = (\exp[\beta(E - \mu)] \pm 1)^{-1}$ is the Fermi or Bose distribution function with $\beta \equiv T^{-1}$.

The invariant parton density dn^i/dx in the IMF is related to dn/dE and $f(E)$ in the proton rest frame as follows

$$\frac{dn^i}{dx} = \frac{gV(s)M^2x}{(2\pi)^2}\int_{xM/2}^{M/2} dEf(E),$$

and the structure function

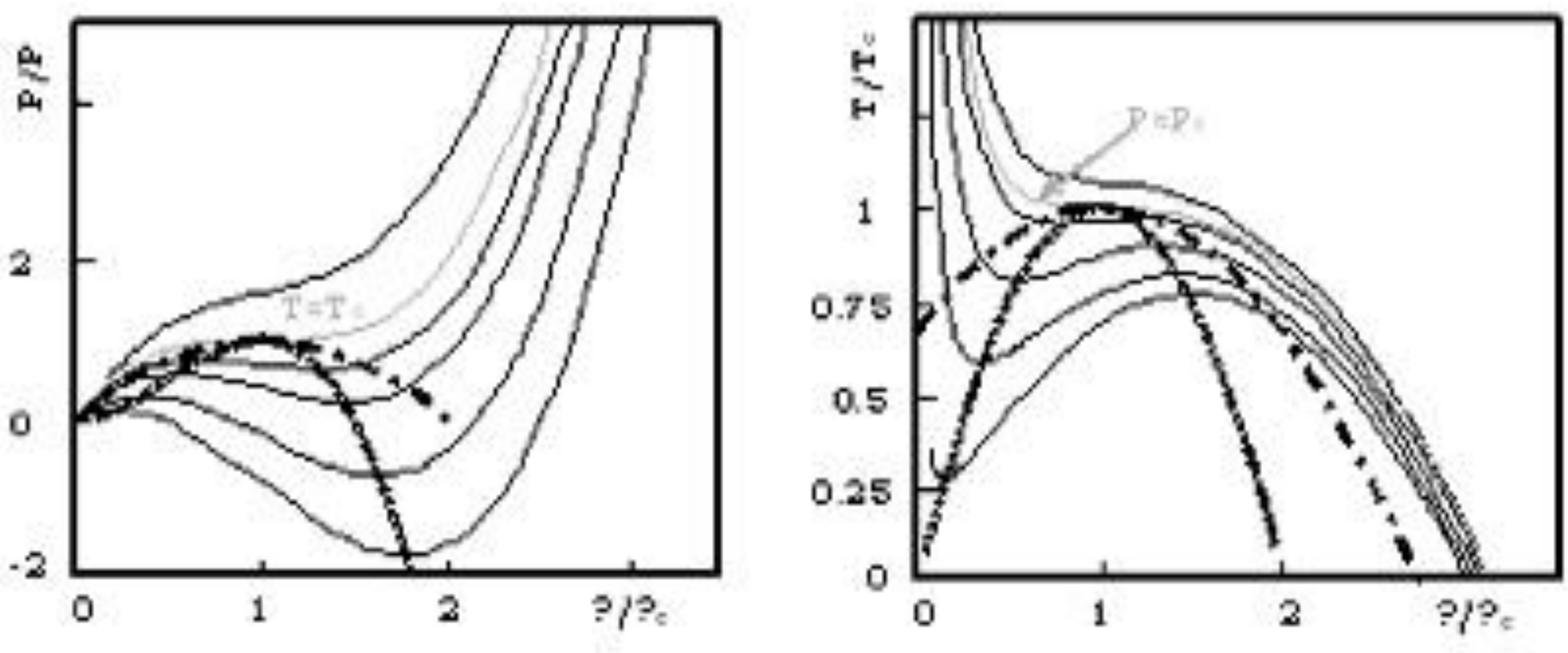
$$F_2(x) = x\sum_q e_q^2[(\frac{dn^i}{dx})_q + (\frac{dn^i}{dx})_{\bar{q}}].$$

Finite volume effects can be incorporated as

$$dn/dE = gf(E)(VE^2/2\pi^2 + aR^2E + bR),$$

where V and R are energy dependent and a, b, in front of the surface and curvature terms (CONFINENENT!).

Universality of the Van der Waals-like behavior (Jaqaman et al., 1983)

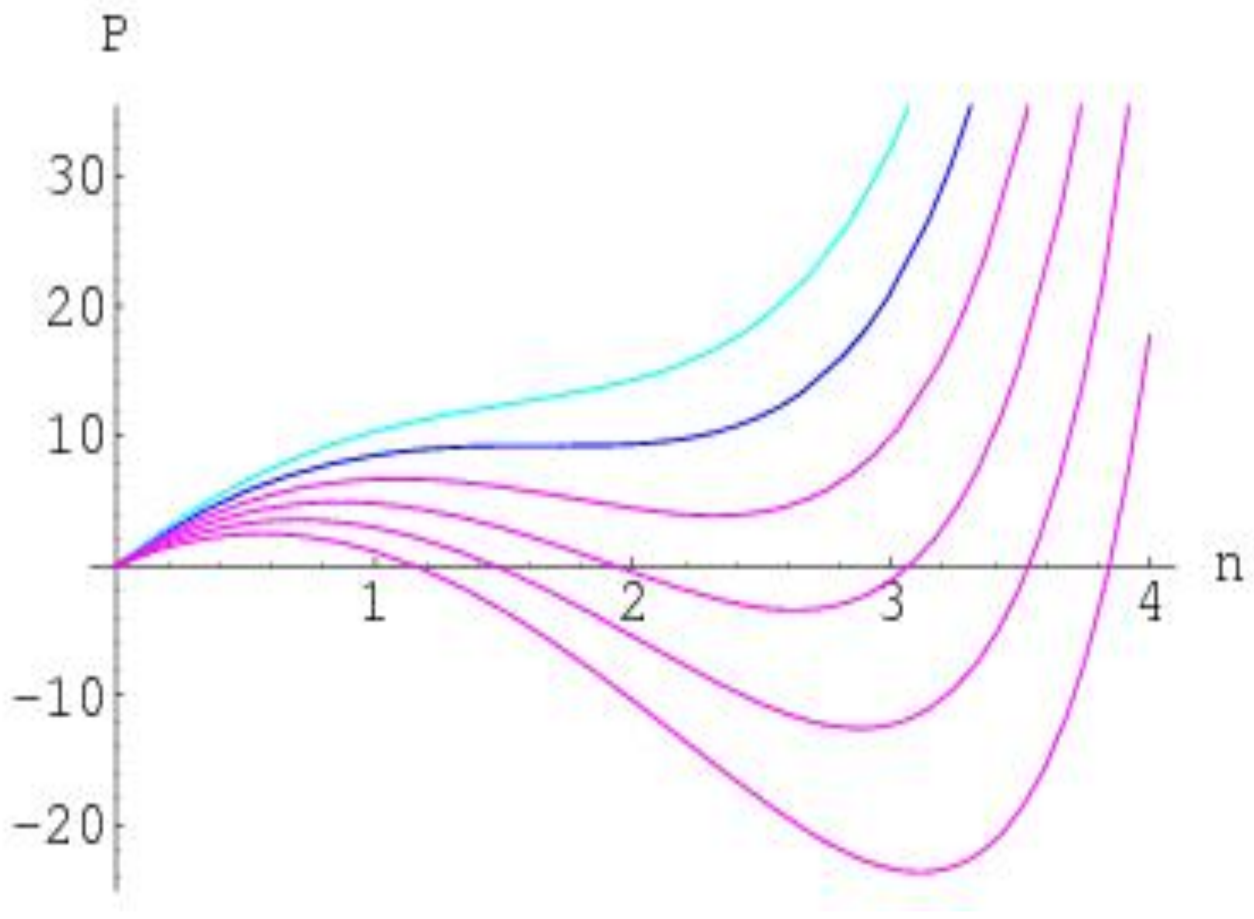


The Van der Waals EoS

$$P(T;N,V) = -(\frac{\partial F}{\partial V})_{TN} =$$

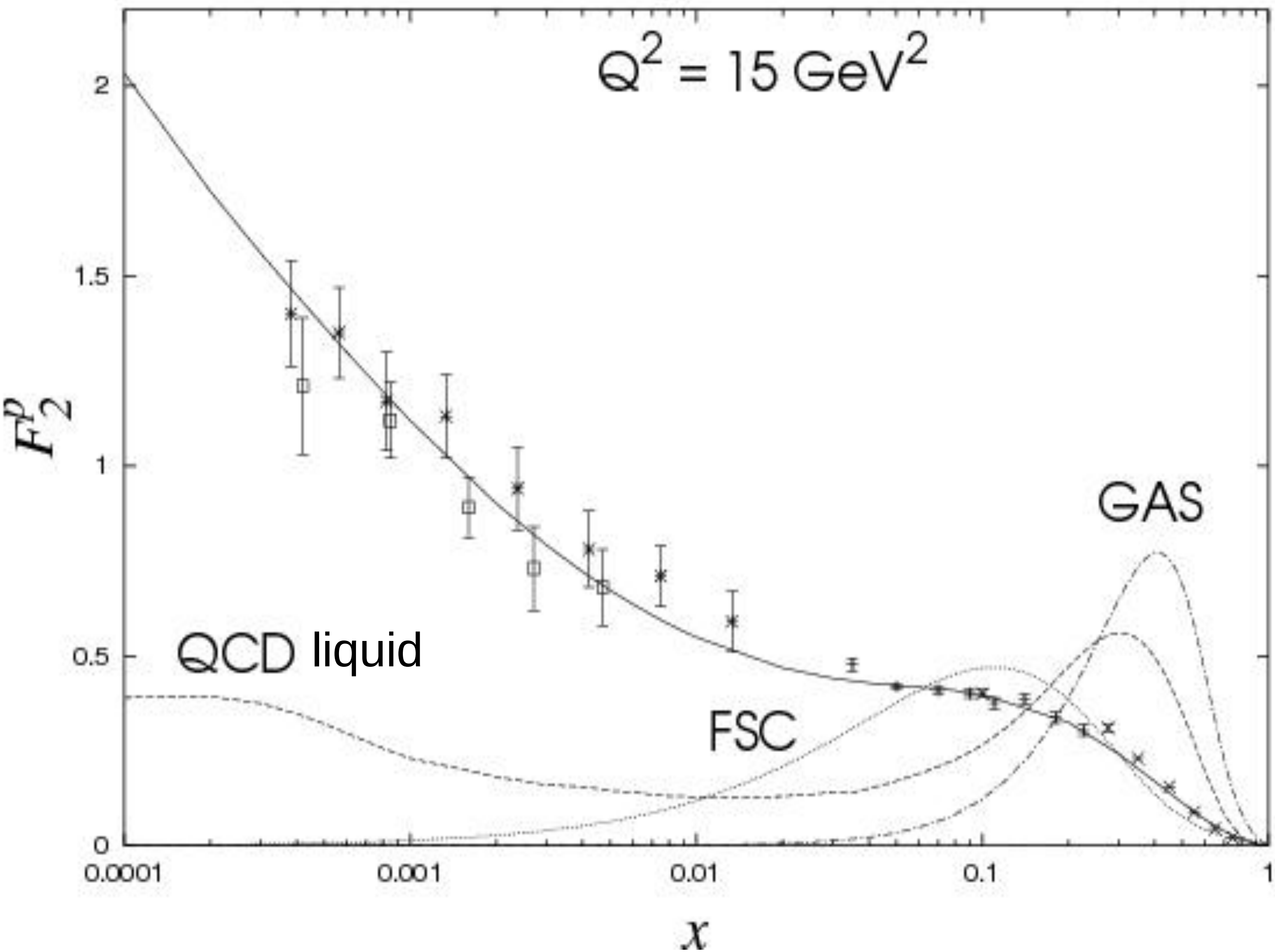
$$\frac{NT}{V-bN} - a(\frac{N}{V})^2 = \frac{nT}{1-bn} - an^2,$$

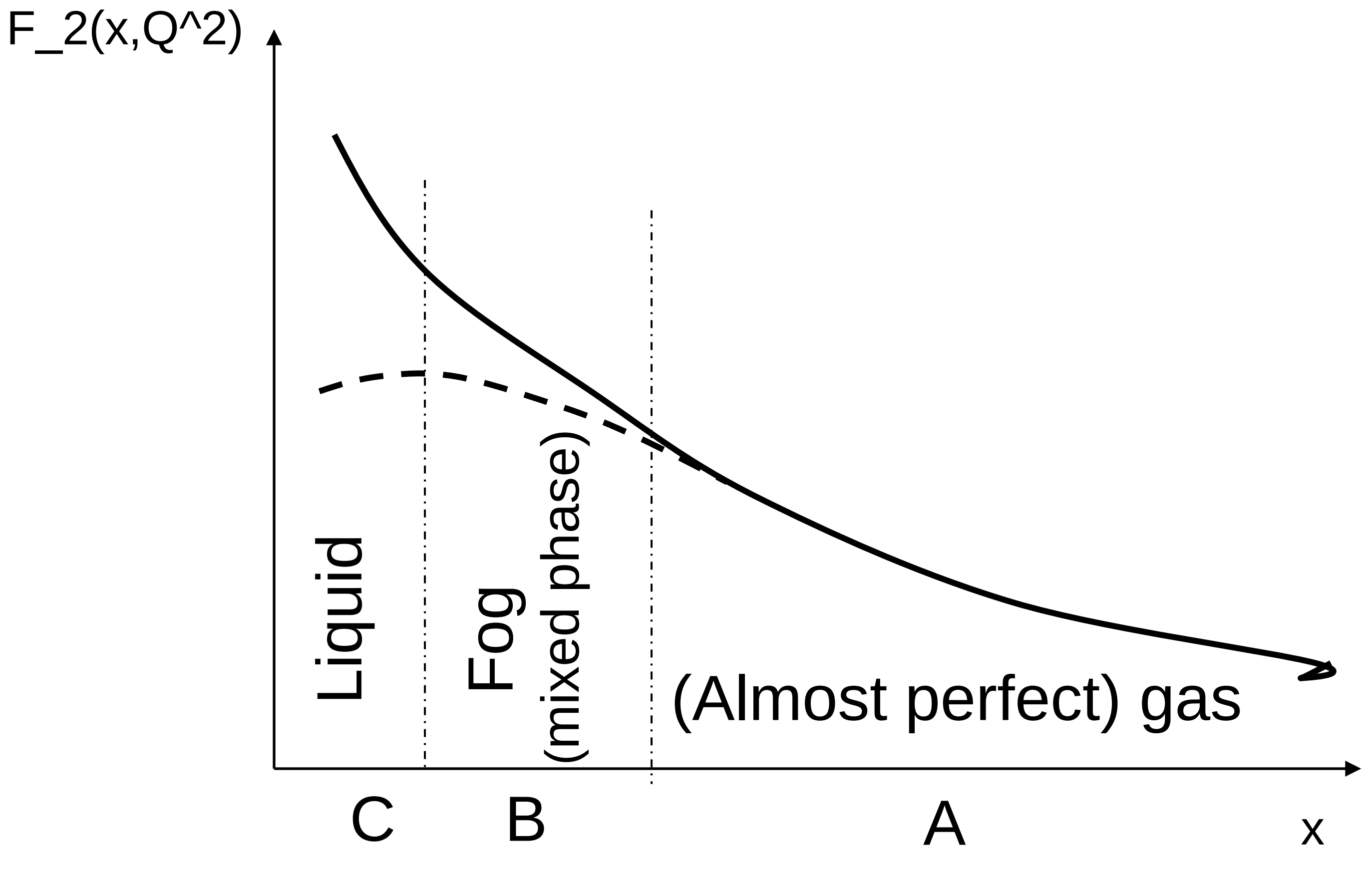
where $n = N/V$ is the particle number density, a is the strength of the mean-field attraction, and b governs the short-range repulsion.



Pressure-to-number density dependence from a Van der Waasl EoS

Figure 2




F_2(x,Q^2)
Liquid
Fog
(mixed phase)
(Almost perfect) gas
C
B
A
x

Conclusions and Outlooks

- Bjorken scaling in DIS, observed at moderate x and Q^2 (with weak, logarithmic Q^2-dependence) of the SF, corresponds to a proton filled with a nearly free gas of partons – valence and sea quarks and gluons;
- With decreasing x (increasing energy) and increasing Q^2, the partons in the nucleon start overlapping, gradually filling (saturating) the available space in the nucleon $\sim R(s)^3 \sim \ln^3 s$. According to the observed violent increase of the SF towards small x, the space occupied by the partonic gas increases faster than the volume of the nucleon, thus leading to its saturation and, consequently to the condensation (coalescence) of the partonic gas into a partonic liquid (phase transition?). This phenomenon can be described by the methods of statisctical physics, similar to the case of hadronic or nuclear collisions.

Thank you!