

# Perspectives in Dynamical Models of Heavy Ion Collisions

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# Thanks to

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- Hannah Petersen (now at Duke U)
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- Marlene Nahrgang
- Dirk Rischke
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# Outline

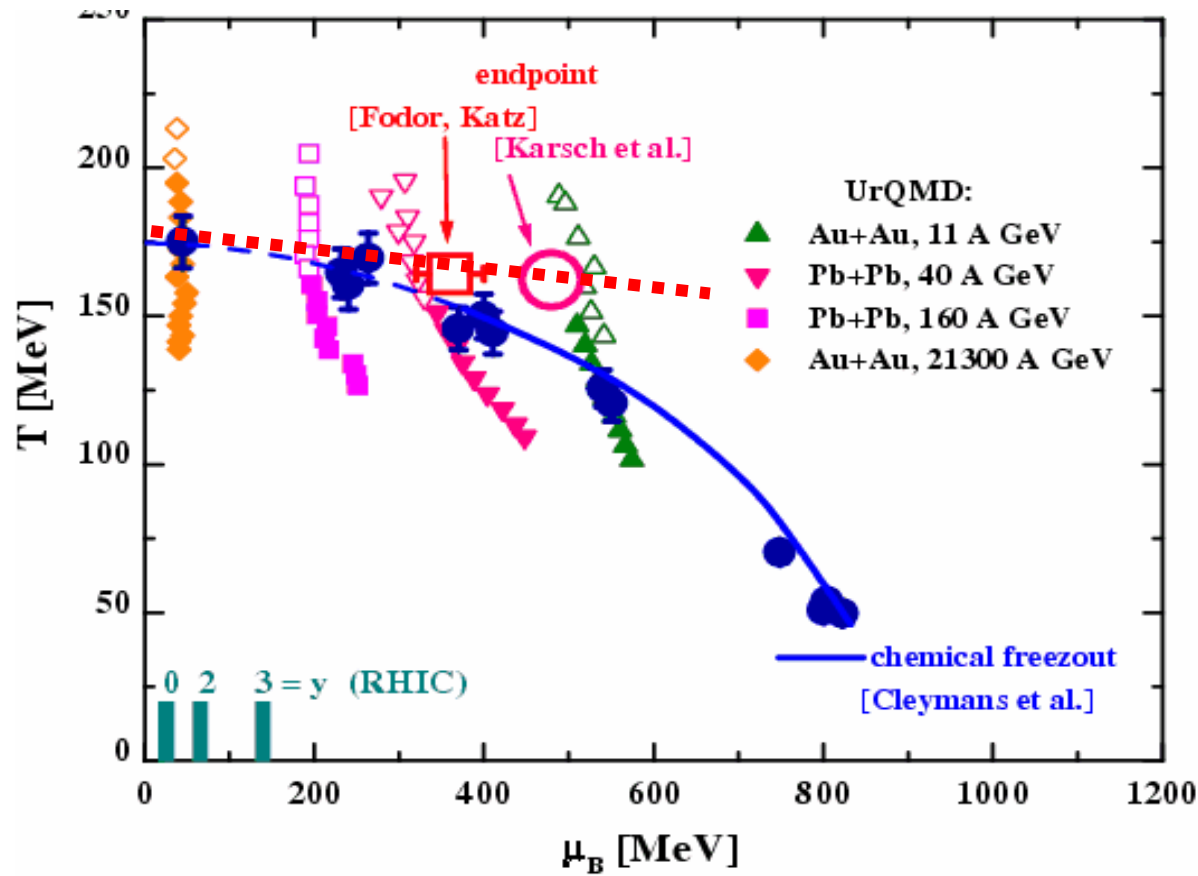
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- Model Description
  - Initial Conditions
  - Equations of State
  - Freeze-out Scenarios
- Multiplicities and Spectra
- HBT Results
- Leptonic Probes
- Open Questions
- Conclusions

(Petersen et al., PRC 78:044901, 2008, arXiv: 0806.1695)

(Petersen et al., arXiv: 0901.3821, PRC in print)

# The QCD Phase Diagram



E. Bratkovskaya, M.B. et al., PRC 2005

In heavy ion collisions heated and compressed nuclear matter is produced under controlled conditions

# Hybrid Approaches (history)

- Hadronic freezeout following a first order hadronization phase transition in ultrarelativistic heavy ion collisions.  
S.A. Bass, A. Dumitru, M. Bleicher, L. Bravina, E. Zabrodin, H. Stoecker, W. Greiner, [Phys.Rev.C60:021902,1999](#)
- Dynamics of hot bulk QCD matter: From the quark gluon plasma to hadronic freezeout.  
S.A. Bass, A. Dumitru, [Phys.Rev.C61:064909,2000](#)
- Flow at the SPS and RHIC as a quark gluon plasma signature.  
D. Teaney, J. Lauret, Edward V. Shuryak, [Phys.Rev.Lett.86:4783-4786,2001](#)
- A Hydrodynamic description of heavy ion collisions at the SPS and RHIC.  
D. Teaney, J. Lauret, E.V. Shuryak, [e-Print: nucl-th/0110037](#)
- Hadronic dissipative effects on elliptic flow in ultrarelativistic heavy-ion collisions.  
T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, [Phys.Lett.B636:299-304,2006](#)
- 3-D hydro + cascade model at RHIC.  
C. Nonaka, S.A. Bass, [Nucl.Phys.A774:873-876,2006](#)
- Results On Transverse Mass Spectra Obtained With Nexspherio  
F. Grassi, T. Kodama, Y. Hama, [J.Phys.G31:S1041-S1044,2005](#)

# Present Approaches

(3+1)dim. hydrodynamics

with nonequilibrium initial conditions (Nexus) and isothermal freeze-out or continuous emission scenario:

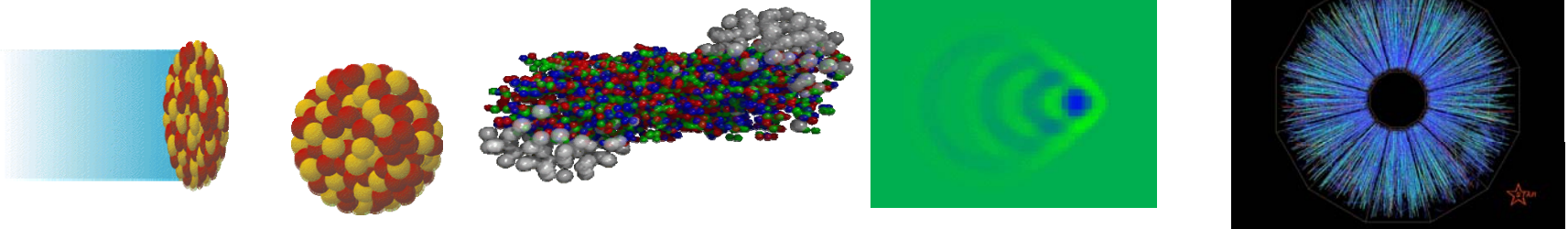
- Results On Transverse Mass Spectra Obtained With Nexspherio  
F. Grassi, T. Kodama, Y. Hama, [J.Phys.G31:S1041-S1044,2005](#)
- K. Werner, M. Bleicher, T. Pierog, Phys. Rev. C (2010)

with Glauber or CGC initial conditions and hadronic afterburner:

- Hadronic dissipative effects on elliptic flow in ultrarelativistic heavy-ion collisions.  
T. Hirano, U. Heinz, D. Kharzeev, R. Lacey, Y. Nara, [Phys.Lett.B636:299-304,2006](#)
- 3-D hydro + cascade model at RHIC.  
C. Nonaka, S.A. Bass, [Nucl.Phys.A774:873-876,2006](#)

# Hybrid Approach

- Essential to draw conclusions from final state particle distributions about initially created medium
- The idea here: Fix the initial state and freeze-out  
→ learn something about the EoS and the effect of viscous dynamics



1) Non-equilibrium  
initial conditions  
via UrQMD

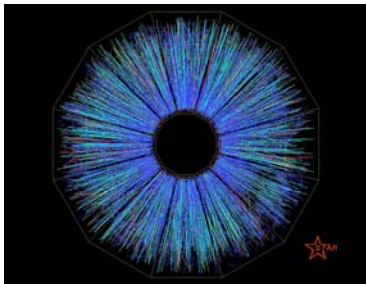
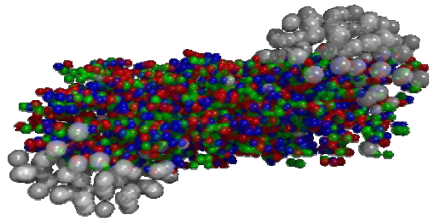
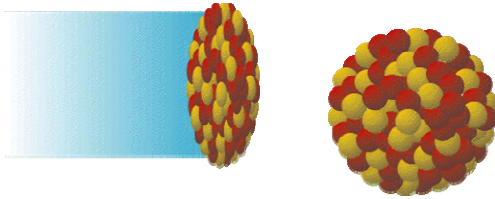
2) Hydrodynamic  
evolution or  
Transport  
calculation

3) Freeze-out via  
hadronic cascade  
(UrQMD)

(Petersen et al., PRC 78:044901, 2008, arXiv: 0806.1695)

# The UrQMD transport approach

UrQMD = Ultra-relativistic Quantum Molecular Dynamics



- Initialisation:

Nucleons are set according to a Woods-Saxon distribution with randomly chosen momenta  $p_i < p_F$

- Propagation and Interaction:

Rel. Boltzmann equation  $(p^\mu \partial_\mu) f = I_{coll}$

Collision criterium

$$d_{\min} \leq d_0 = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- Final state:

all particles with their final positions and momenta

Very successful in describing different observables in a broad energy range  
**But:** modeling of the phase transition and hadronization not yet possible

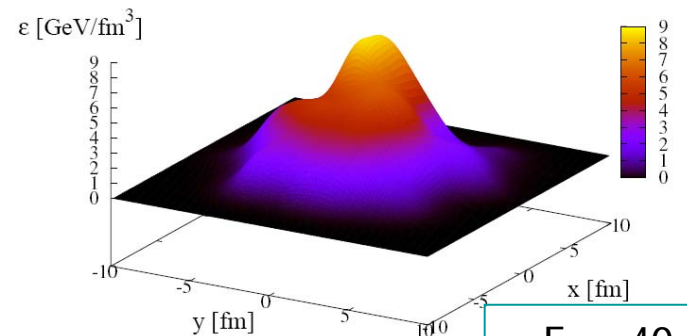
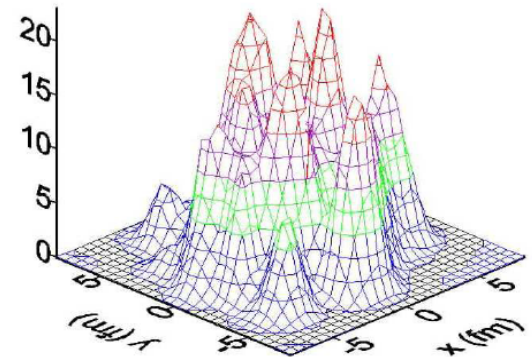
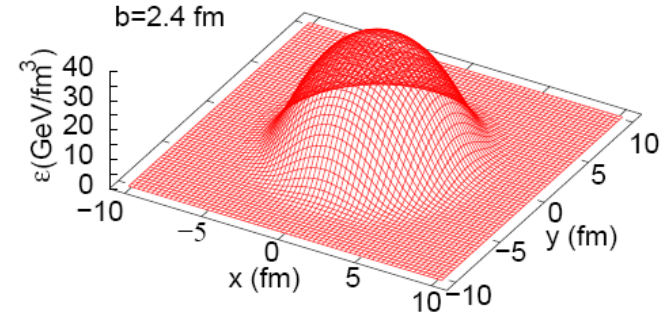


# Initial State

- Contracted nuclei have passed through each other

$$t_{start} = \frac{2R}{\gamma v}$$

- Energy is deposited
- Baryon currents have separated
- Energy-, momentum- and baryon number densities are mapped onto the hydro grid
- Event-by-event fluctuations are taken into account
- Spectators are propagated separately in the cascade



$E_{lab}=40$  AGeV  
 $b=0$  fm

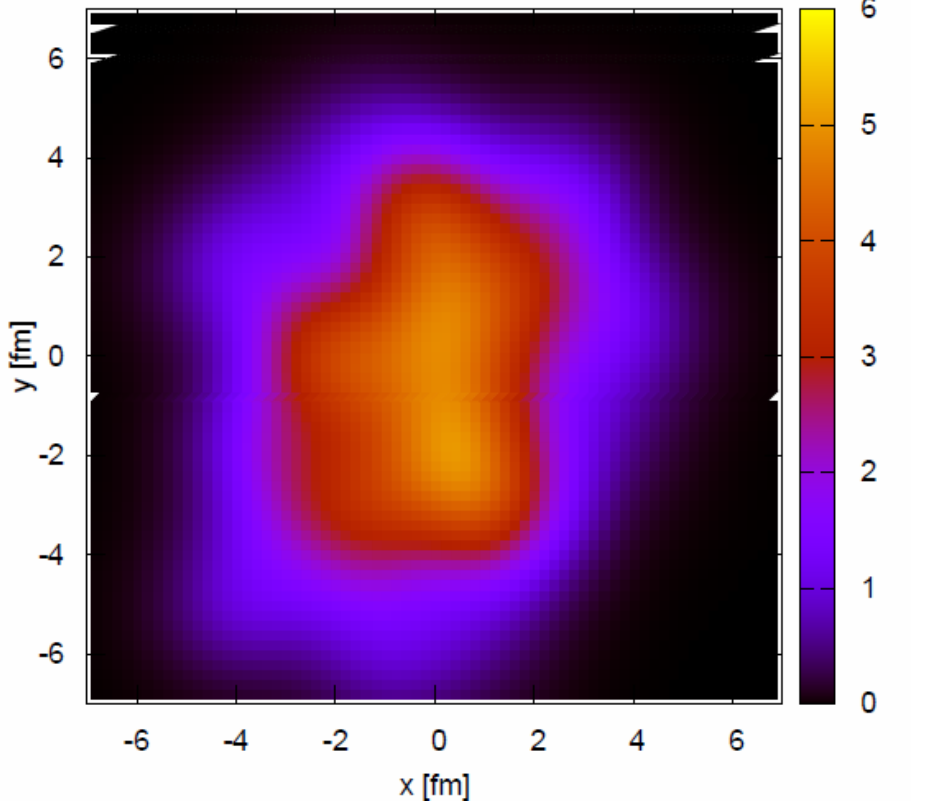
(J.Steinheimer et al., PRC 77,034901,2008)

(nucl-th/0607018, nucl-th/0511021)

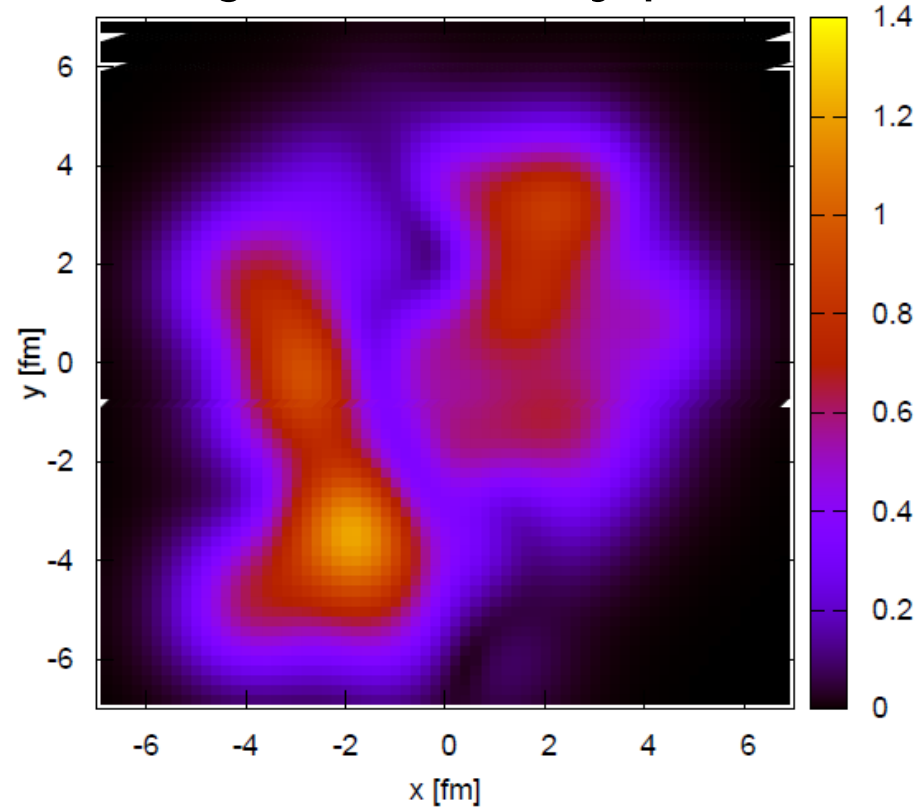
# Initial State for Non-Central Collisions

Pb+Pb at  $E_{\text{lab}}=40$  AGeV with  $b=7$ fm at  $t_{\text{start}}=2.83$  fm

Energy density profile



Weighted velocity profile



→ Event-by-event fluctuations are taken into account

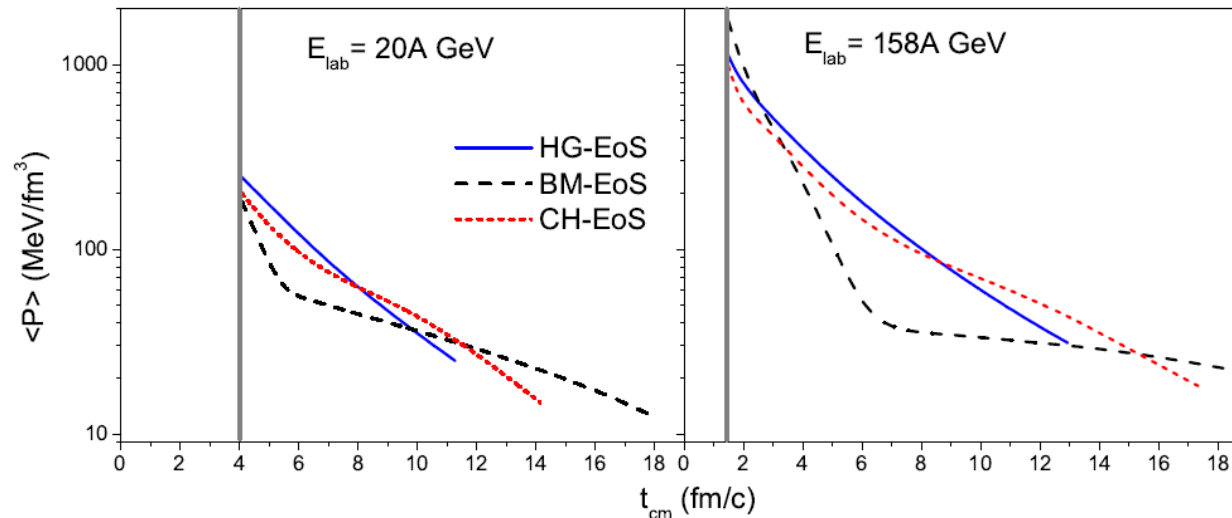
(H.Petersen et.al., arXiv:0901.3821, PRC 2009)

# Equations of State

Ideal relativistic one fluid dynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu (nu^\mu) = 0$$

- HG: Hadron gas including the same degrees of freedom as in UrQMD (all hadrons with masses up to 2.2 GeV)
- CH: Chiral EoS from SU(3) hadronic Lagrangian with first order transition and critical endpoint
- BM: Bag Model EoS with a strong first order phase transition between QGP and hadronic phase



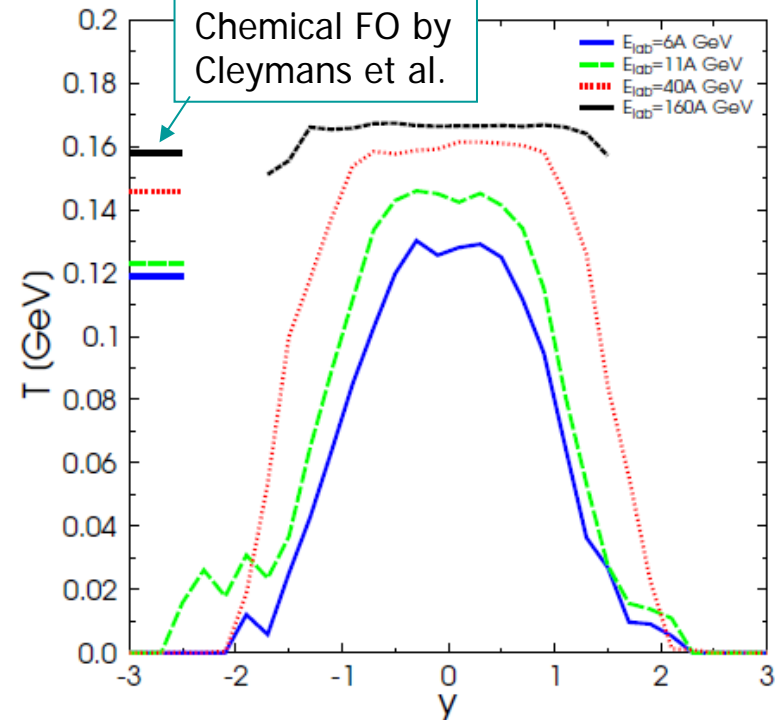
D. Rischke et al.,  
NPA 595, 346, 1995,

D. Zschiesche et al.,  
PLB 547, 7, 2002

Papazoglou et al.,  
PRC 59, 411, 1999

# Freeze-out

- 1) Transition from hydro to transport when  $\varepsilon < 730 \text{ MeV/fm}^3$  ( $\approx 5 * \varepsilon_0$ ) in all cells of one transverse slice (Gradual freeze-out, GF)  
→ iso-eigentime criterion
- 2) Transition when  $\varepsilon < 5 * \varepsilon_0$  in all cells (Isochronuous freeze-out, IF)



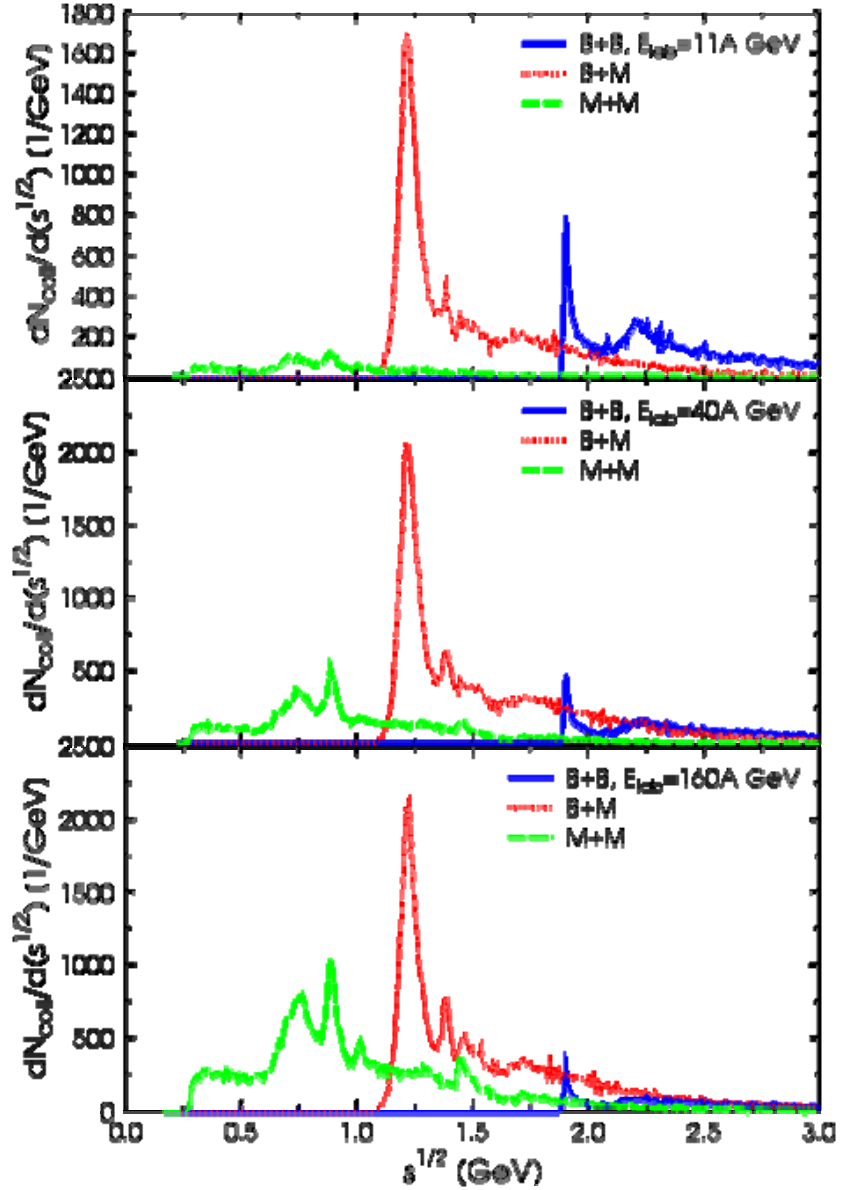
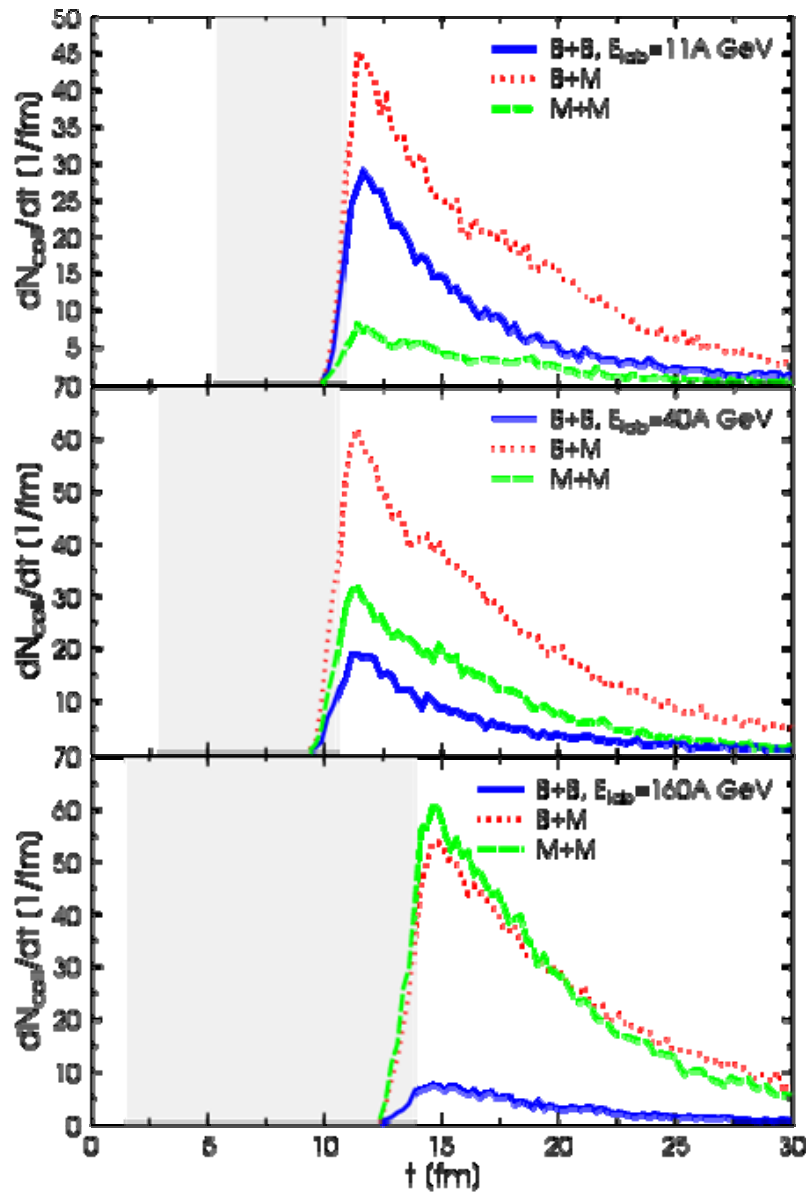
- Particle distributions are generated according to the Cooper-Frye formula

$$E \frac{dN}{d^3p} = \int_{\sigma} f(x, p) p^{\mu} d\sigma_{\mu}$$

with boosted Fermi or Bose distributions  $f(x, p)$  including  $\mu_B$  and  $\mu_S$

- Rescatterings and final decays calculated via **hadronic cascade** (UrQMD)

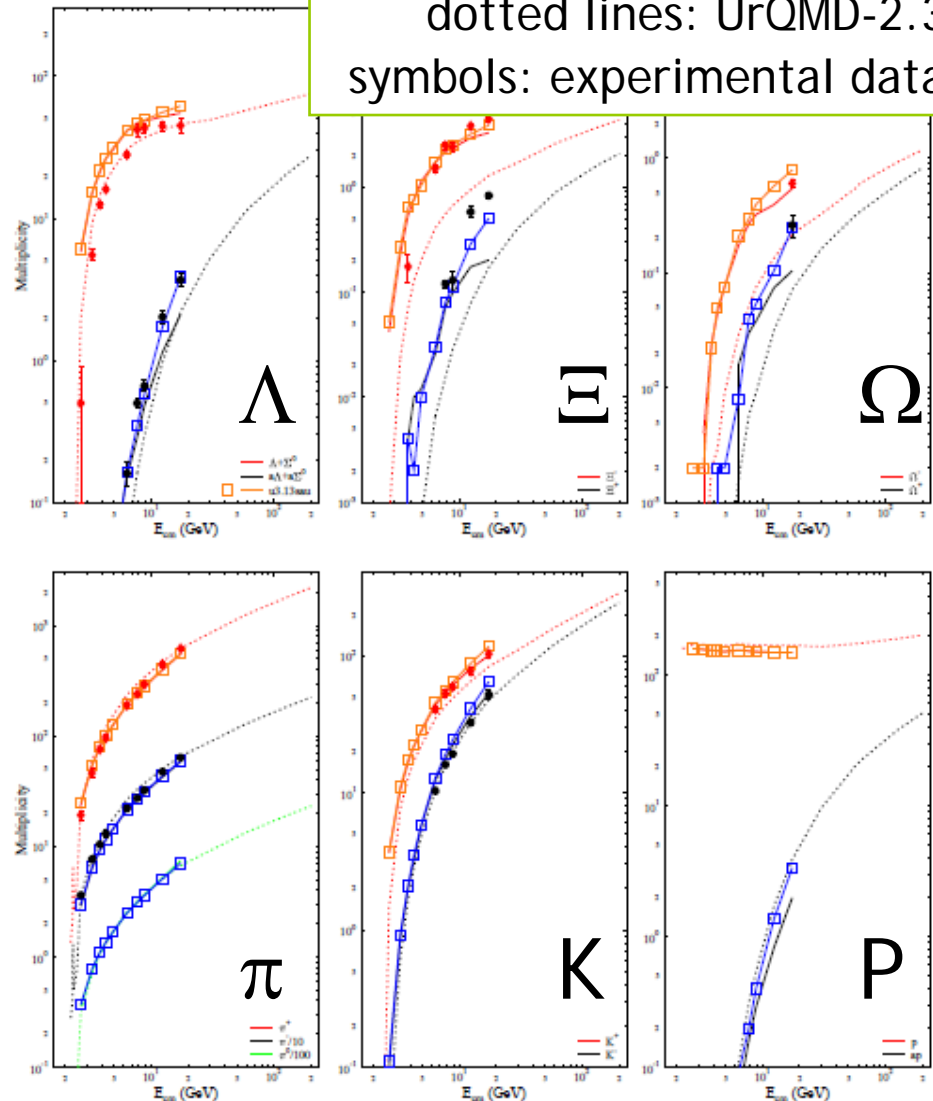
# Final State Interactions (after Hydro)



# Multiplicities vs. Energy

- Both models are purely hadronic without phase transition, but different underlying dynamics

full lines: hybrid model (IF)  
 squares: hybrid model (GF)  
 dotted lines: UrQMD-2.3  
 symbols: experimental data



(Petersen et al., PRC 78:044901, 2008)

→ Results for particle multiplicities from AGS to SPS are surprisingly similar

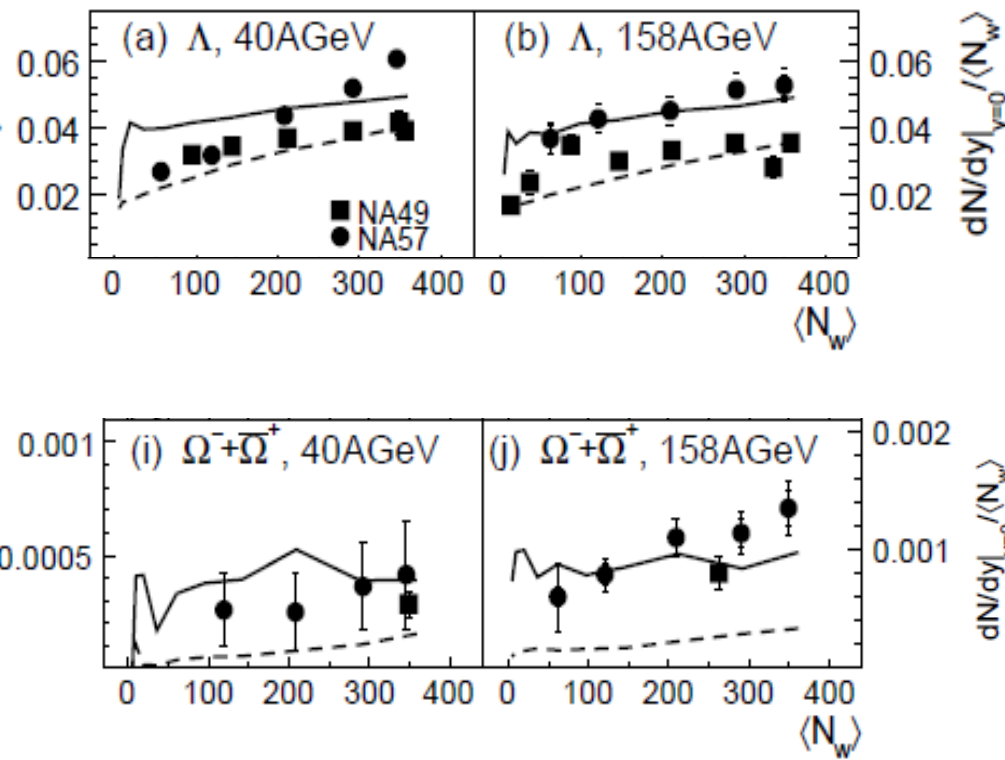
→ Strangeness is enhanced in the hybrid approach due to local equilibration

Central ( $b < 3.4$  fm) Pb+Pb/Au+Au collisions

Data from E895, NA49

Marcus Bleicher, CPOD 2010

# Strangeness Centrality Dependence

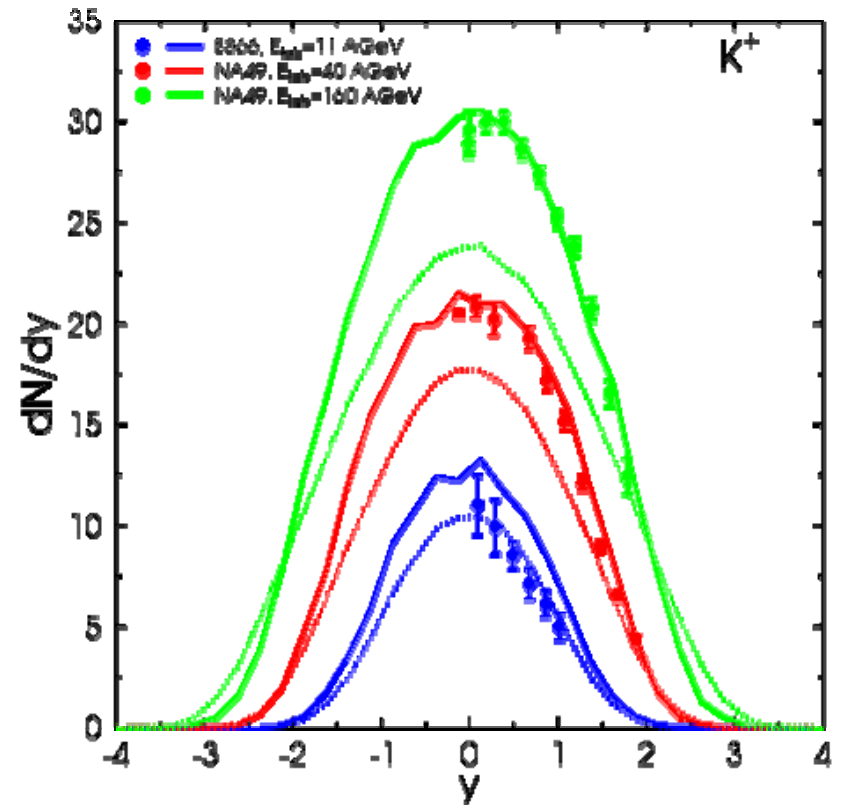
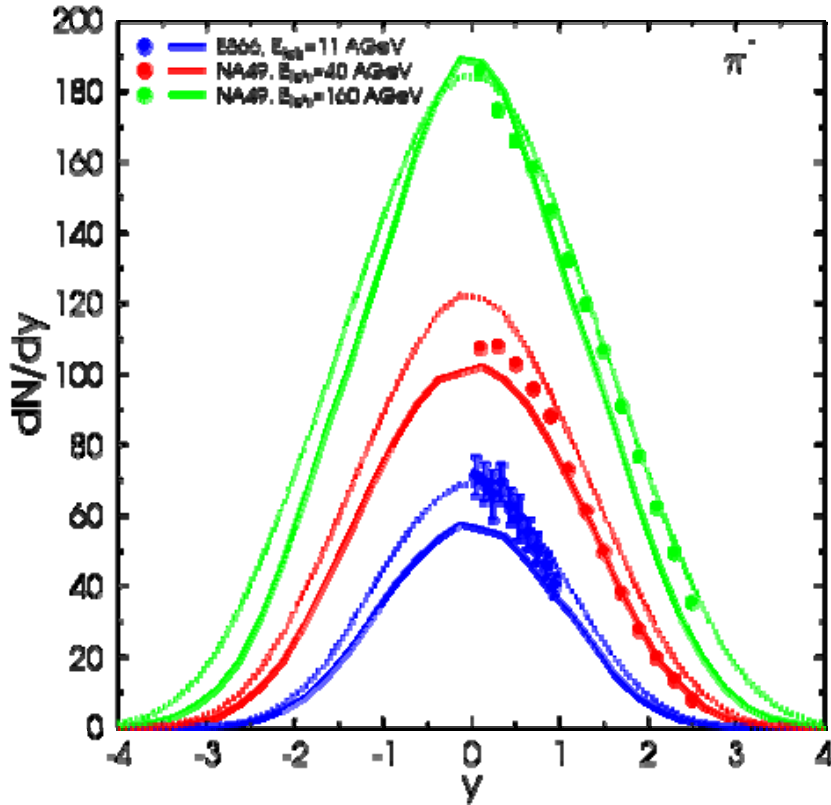


- Thermal production of the particles at transition from hydro to transport
- Centrality dependence of multistrange hyperons is improved

— hybrid model (GF)  
 - - - - UrQMD-2.3

# Rapidity Spectra

full lines: hybrid model  
dotted lines: UrQMD-2.3  
symbols: experimental data

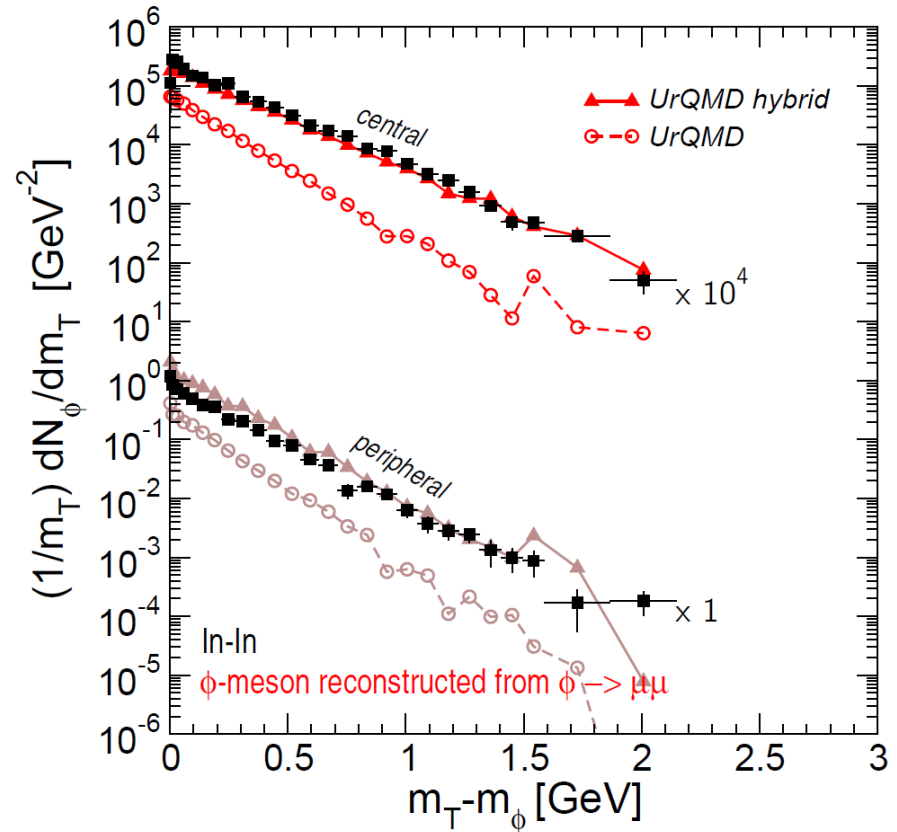


→ Rapidity spectra for pions and kaons have a very similar shape in both calculations



# The Phi

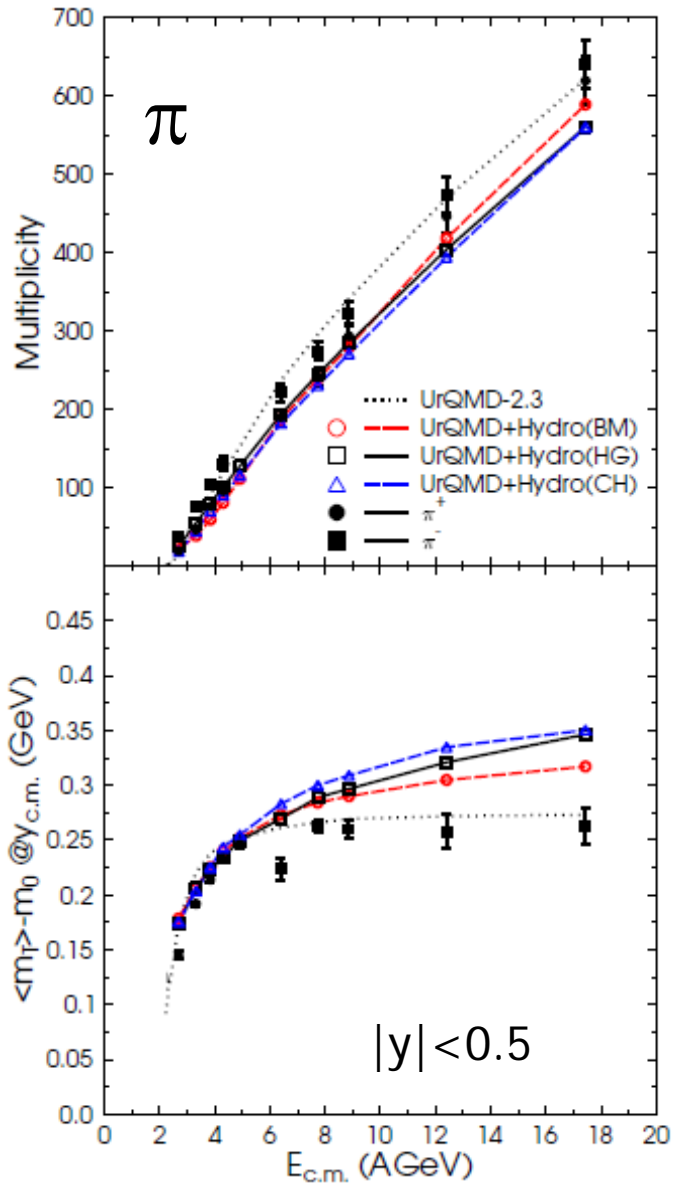
- Comparison between
  - NA60 data
  - UrQMD
  - Hybrid model
- Thermalization is essential to describe phi yield and slope
- Stronger deviation from transport for central reactions



E. Santini, PRC (2010)

Data: NA60

# $\langle m_T \rangle$ Excitation Function

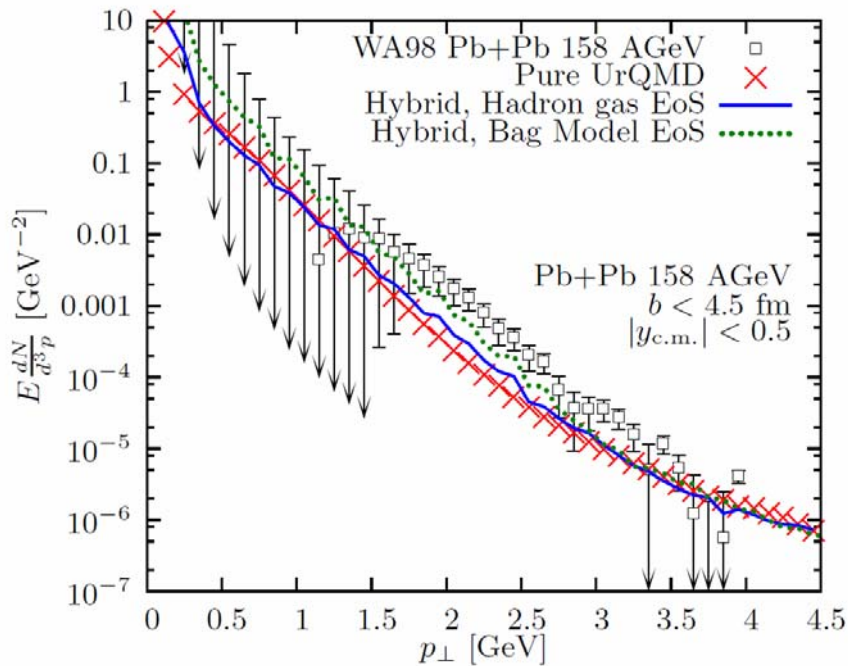


- Resonance excitations and non-equilibrium effects in intermediate energy regime lead to a softening of the EoS in pure UrQMD calculation
- Hybrid calculation with hadronic EoS just rises as a function of beam energy
- Even strong first order phase transition leads only to a small effect

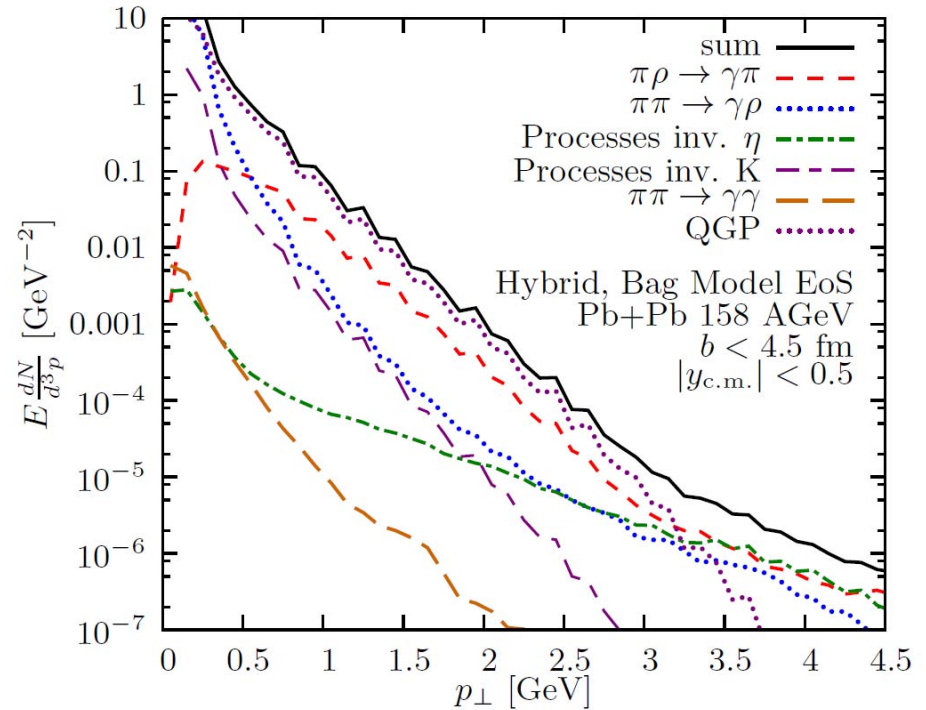
Central ( $b < 3.4$  fm) Au+Au/Pb+Pb collisions,  
Gradual freeze-out for hybrid calculation

(Petersen et al., JPG 36, 055104, 2009)

# Direct Photons: Comparison to data



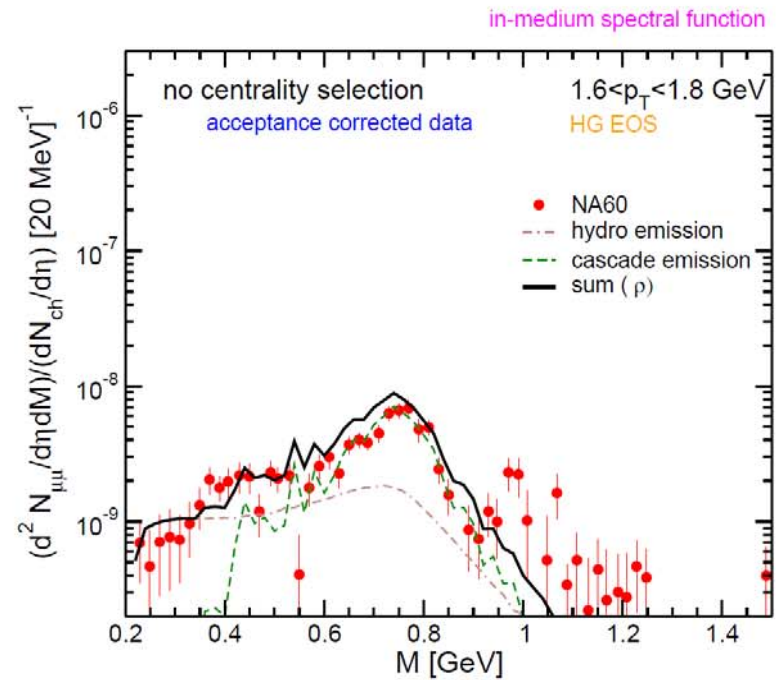
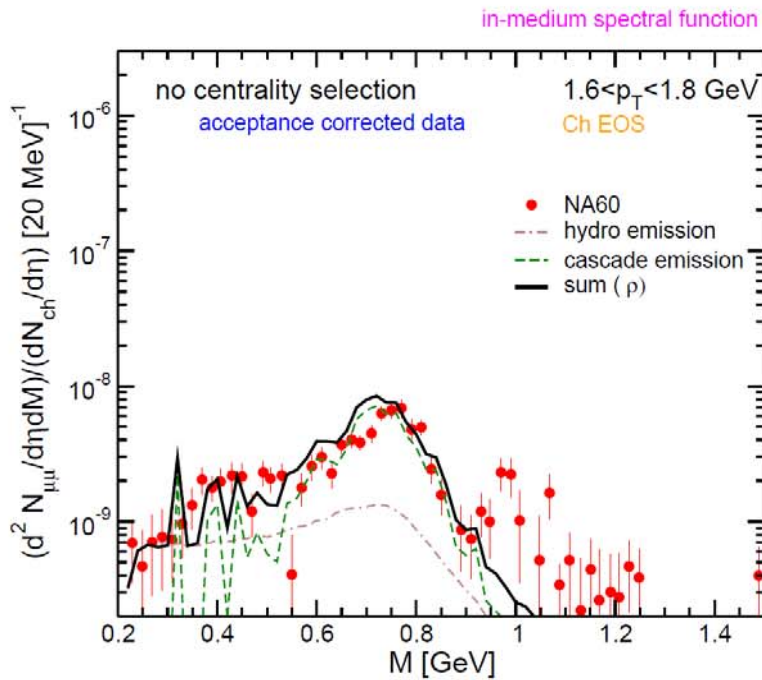
**Comparisons**



**Hybrid, QGP: Channels**

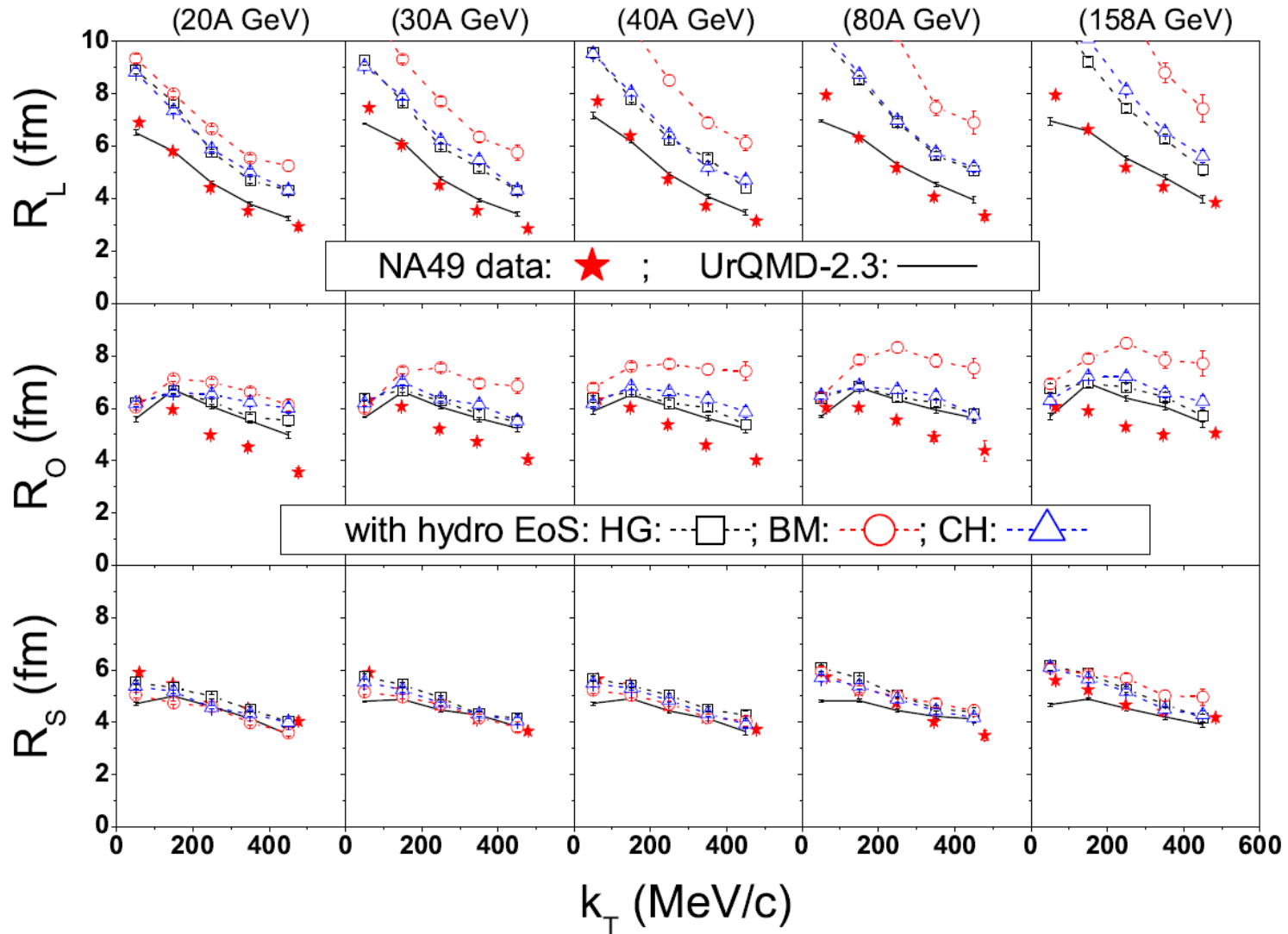
# Hybrid model vs NA60 - DiMuons

intermediate  $p_T$



Santini, Bleicher, 2010

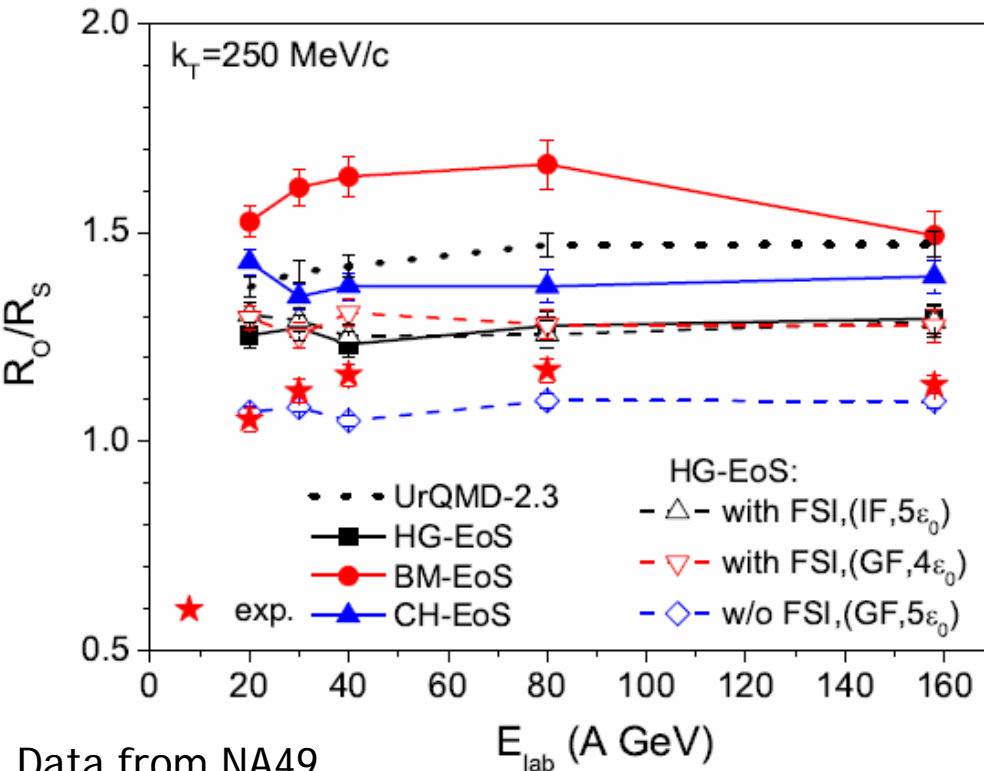
# HBT radii (EoS effects)



(Q. Li et al., arXiv: 0812.0375, PLB in print)

Hydro evolution leads to larger radii, esp. with phase transition

# $R_0/R_s$ Ratio



- Hydro phase leads to smaller ratios
- Hydro to transport transition does not matter, if final rescattering is taken into account
- EoS dependence is visible, but not as strong as previously predicted (factor of 5)

(Q. Li et al., PLB 674, 111, 2009)

# Findings

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- Intermediate hydrodynamics improves description of the data
- (+) Strangeness
- (+) HBT
- (+) Elliptic flow
- (+) Photons / dileptons
- (+) allows direct testing of EoS
- (-) radial flow (viscosities)
- (+/-) no understanding of phase transition
- (+/-) no understanding of thermalisation

# Problems, Challenges, Opportunities

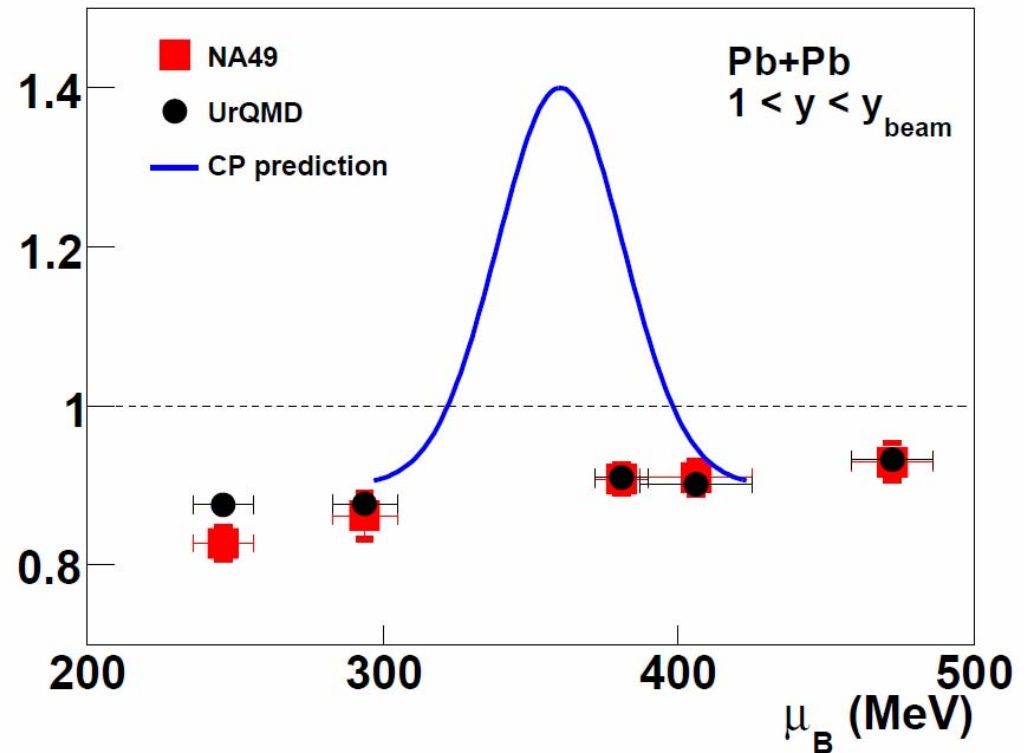
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- Multi-particle interactions
- Hagedorn states
- Hadronization
- Dynamics at the CEP



# CEP: Idea vs Reality

- $\omega = \text{Var}(h^-) / \langle h^- \rangle$
- Prediction: Strong enhancement of fluctuations
- Not observed
- $\rightarrow$  Need better (dynamical) models



B. Lungwitz, M.B., PRC 2007  
M. Stephanov, PRL 1998

# Chiral dynamics

- Effective model (Gell-Mann-Levy):

$$\mathcal{L} = \bar{q} [i\gamma^\mu \partial_\mu - g(\sigma + \gamma_5 \tau \vec{\pi})] q + 1/2 (\partial_\mu \sigma)^2 + 1/2 (\partial_\mu \vec{\pi})^2 - U(\sigma, \vec{\pi})$$

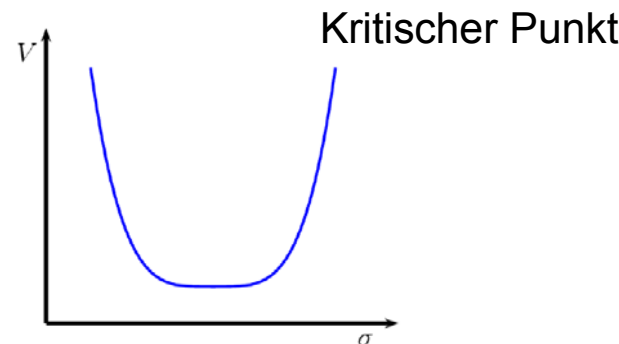
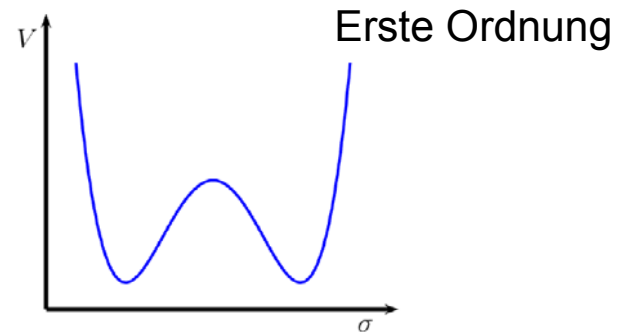
$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 - h_q \sigma - U_0$$

- Potential determines order of phase transition
- At critical point:

$$m_\sigma = \frac{\partial^2 V}{\partial \sigma^2} \rightarrow 0$$

the correl. length becomes

$$\xi = \frac{1}{m_\sigma} \rightarrow \infty$$



# Chiral Hydrodynamics

- Idea: Hydrodynamic simulation with explicit propagation of the fields

- EoM for the fields  $\phi = (\sigma, \vec{\pi})$   
$$\partial_\mu \partial^\mu \phi + \frac{\delta U}{\delta \phi} = -g\rho_\phi \quad \text{with} \quad \rho_\phi = g\phi d_q \int \frac{d^3 p}{(2\pi)^3} \frac{1}{E} f_{FD}(p)$$

- Coupled to hydro

$$\partial_\mu (T_{\text{fluid}}^{\mu\nu} + T_{\text{field}}^{\mu\nu}) = 0 \Rightarrow \partial_\mu T_{\text{fluid}}^{\mu\nu} = g\rho_\phi \partial^\nu \phi$$

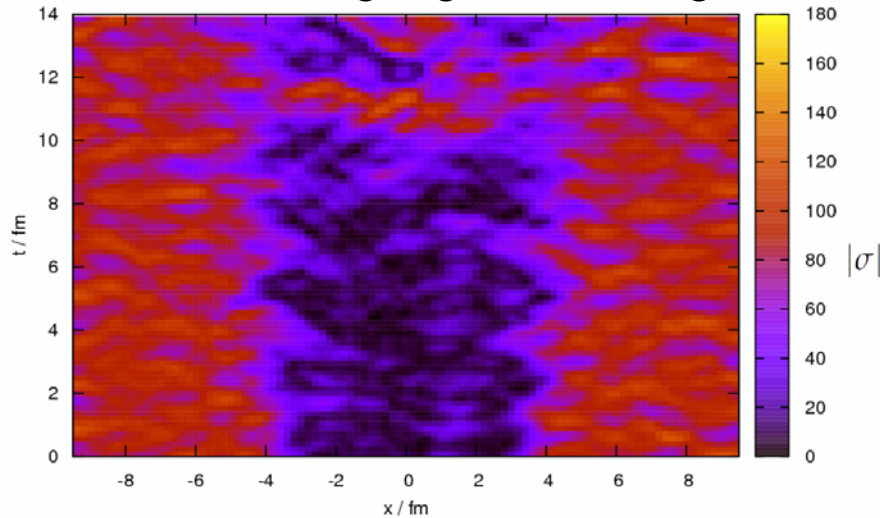
- With ideal E-M-tensor

$$T_{\text{fluid}}^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu}$$

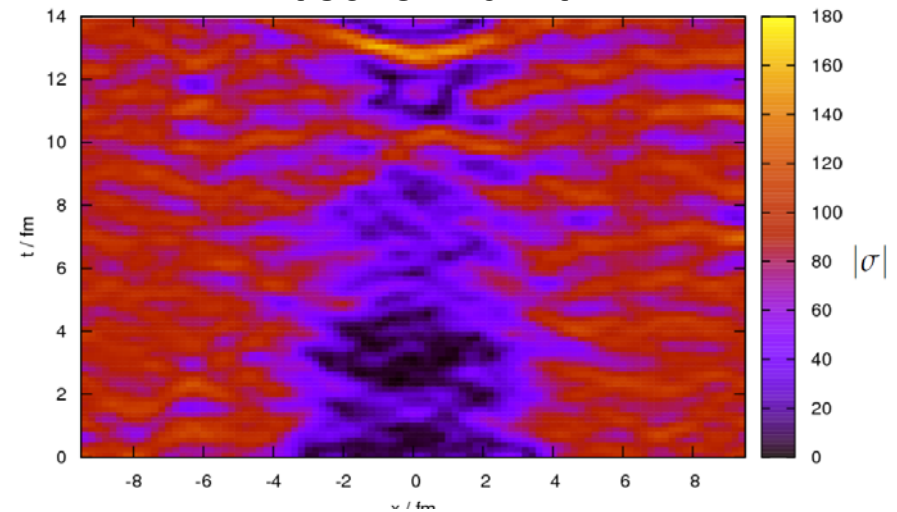
Siehe auch K. Paech, PRC 2004

# Field distribution

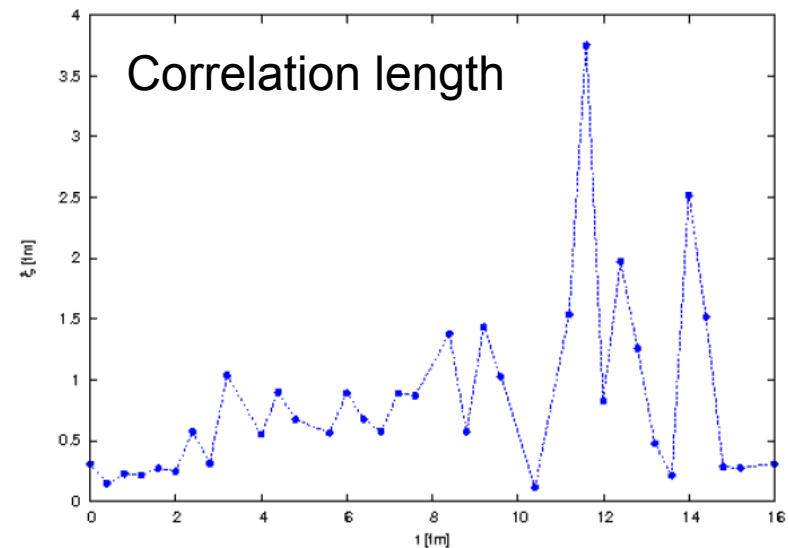
## Phasenübergang 1. Ordnung



## Kritischer Punkt



- Correlation length remains small
- Less fluctuations
- However, model should be improved:
  - Better Lagrangian (pions, P),
  - Langevin term,
  - Particles instead of hydro



# Conclusions

- Hybrid approach combines the advantages of a transport and a hydrodynamic prescription
- Integrated approach with the same initial conditions and freeze-out for different EoS
- Well suited for the FAIR-CBM energy range (but also available for RHIC and LHC)
- Particle multiplicities and spectra are reasonably reproduced, strangeness enhanced
- Open tasks:
  - understand thermalisation
  - multi-particle interactions
  - hagedorn states in dynamical models
  - hadronisation
  - critical phenomena/fluctuations

# Production of Anti-Baryons: Multimesonic channels

$\bar{p}$ -production:  $\bar{p} + N \leftrightarrow n\pi$

R.Rapp and E. Shuryak,  
*Phys.Rev.Lett.* **86** (2001) 2980

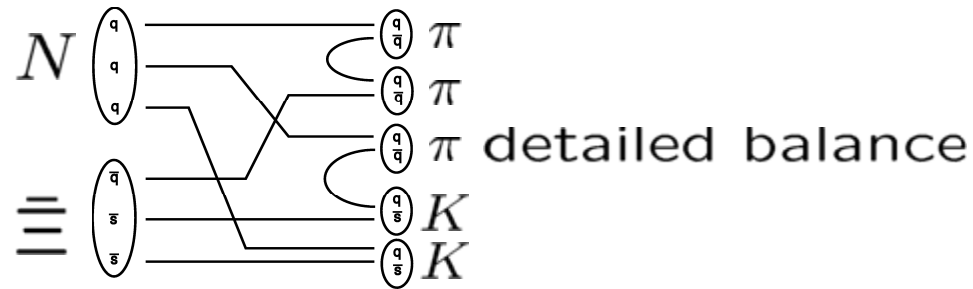
$\bar{Y}$ -production:

C.Greiner and S.Leupold, *J.Phys.* **G27** (2001) L95

$$\bar{\Lambda} + N \leftrightarrow n\pi + K$$

$$\bar{\Xi} + N \leftrightarrow n\pi + 2K$$

$$\bar{\Omega} + N \leftrightarrow n\pi + 3K$$



$$\bar{Y} + N \leftrightarrow n\pi + n_Y K$$

$$\sigma_{N\bar{Y}} \approx \sigma_{N\bar{p}} \approx 50 \text{ mb}$$

annihilation rate  $\Rightarrow$  chemical equilibration rate

$$\tau_{\bar{Y}} := (\Gamma_{\bar{Y}})^{-1} = \frac{1}{\langle\langle \sigma_{N\bar{Y} \rightarrow n\pi + n_Y K} v_{\bar{Y}N} \rangle\rangle \rho_B} \approx 1 - 3 \text{ fm}/c$$

**SPS**

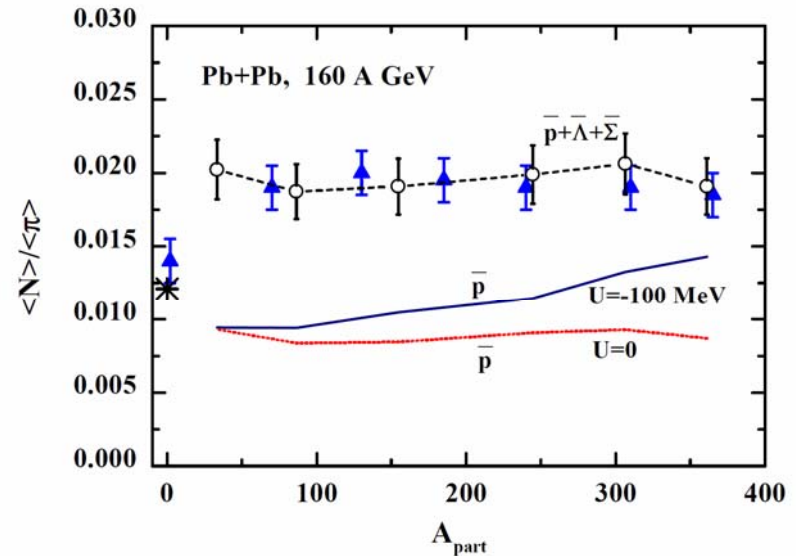
universal behavior:

$$T_{eff} = 155 - 165 \text{ MeV}$$

**But:**  $\xrightarrow{\text{RHIC}}$  **10 [fm/c]**

# Thermalisation: multi-particle interactions

- 10 years ago:  
How can we explain the high yield of anti-protons, anti-hyperons and multi-strange particles?
- $n\pi \rightarrow B\bar{B}$  may be important, MB, Phys.Lett.B485, 2000
- Solution for BBar: Rapp, Shuryak Phys.Rev.Lett.86:2980-2983,2001
- Solution for Ybar: Greiner, Leupold, .Phys.G27:L95-L102,2001
- Solution for thermalization: Wetterich, Stachel, Braun-Munzinger Phys.Lett.B596:61-69,2004



Cassing, NPA 2001

$$\frac{d}{dt} \rho_{\bar{Y}} = - \langle \langle \sigma_{\bar{Y}N} v_{\bar{Y}N} \rangle \rangle \left\{ \rho_{\bar{Y}} \rho_N - \sum_n \mathcal{R}_{(n, n_Y)}(T, \mu_B, \mu_s) (\rho_\pi)^n (\rho_K)^{n_Y} \right\}$$

First explorations: Danielewicz and G. F. Bertsch, NPA533 (1991)712

First implementation: Greiner, Xu, Phys.Rev.C71:064901,2005

# Possible solution by Hagedorn states

- $B\bar{B}$  annihilation at LEAR
  - **statistical** description works well
  - intermediate doorway meson states

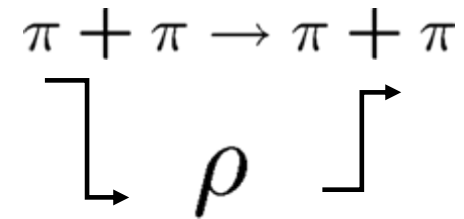
C. Greiner, P. Koch, F. Liu,  
I. Shovkovy, H. Stöcker  
*J.Phys.G31* (2005)

- We propose **Hagedorn States** as intermediate, highly **unstable** states, which decay statistically, for counting and generating **multi-particle** collisions .

$$HS \leftrightarrow n_1 \cdot \pi + n_2 \cdot K + n_3 \cdot \bar{K}$$

$$\leftrightarrow B + \bar{B}$$

$$\leftrightarrow B + \bar{B} + n_1 \cdot \pi + n_2 \cdot K + n_3 \cdot \bar{K}$$



The last **multi-particle** decay will **dominate** over direct  $B\bar{B}$  production.



# Hadronisation

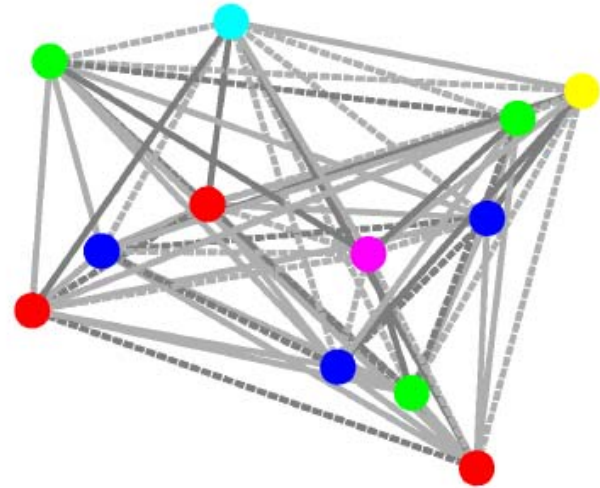
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- How to go from partonic matter to hadronic matter?
  - energy conservation?
  - free quarks in the end?
  - what to do with gluons?
  - decrease in entropy?
  - transition to fragmentation?
- Chromodielectric model
- Quark Molecular Dynamics
- ....

# Quark Molecular Dynamics

Hamiltonian of the model :

$$H = \sum_{i=1}^N \sqrt{\mathbf{p}_i^2 + m_i^2} + \frac{1}{2} \sum_{i \neq j} C_{ij} V(|\mathbf{r}_i - \mathbf{r}_j|)$$



- Potential :

linear potential  $V(r) = \kappa r$

- Color factor  $C_{ij}$  :

can be attractive or repulsive depending on the color of the quarks

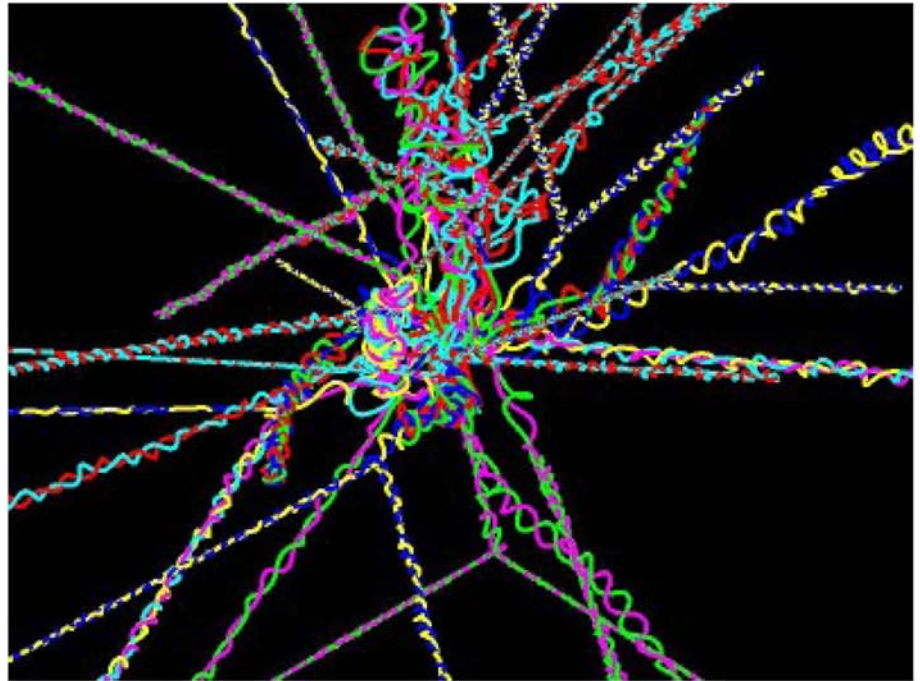
- Quarks :

classical point-particles with light masses  $m_{u,d} = 5 \text{ MeV}$ ,  $m_s = 150 \text{ MeV}$

# Trajectories

## qMD features :

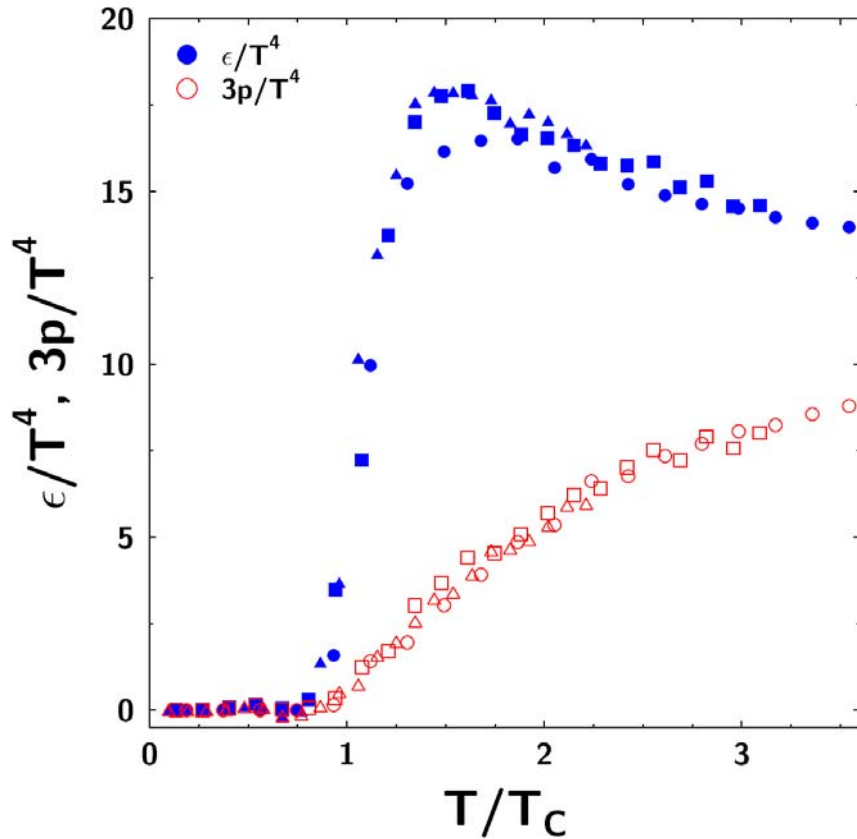
- mesons
- baryons
- confinement
- recombination
- out-of-equilibrium



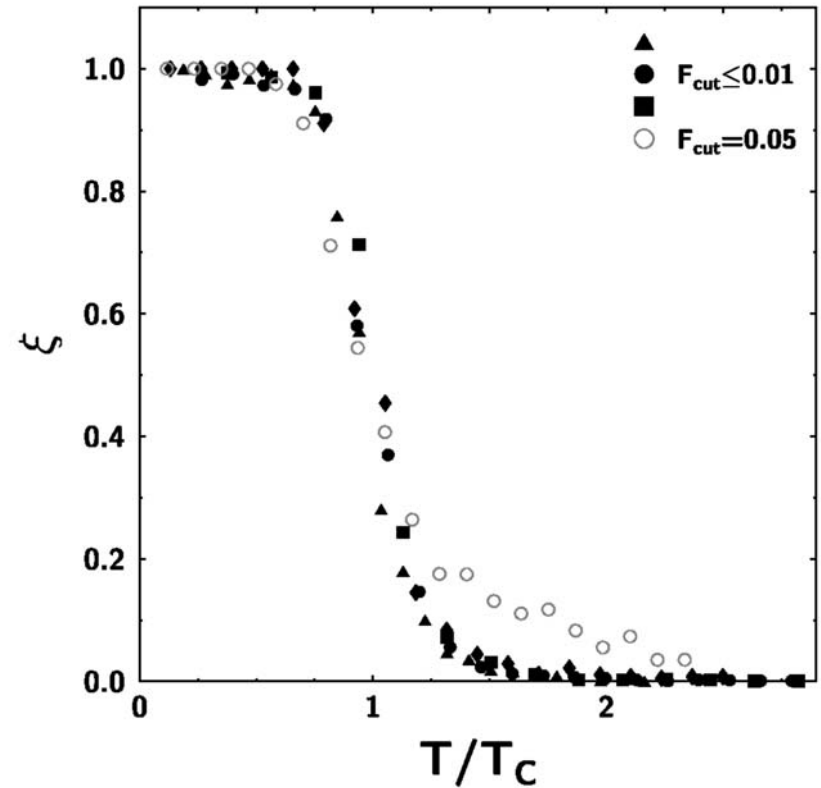
M. Hofmann Ph.D. thesis

Hofmann, Bleicher, Scherer, Neise, Stoecker, Greiner. Phys.Lett.B478:161-171,2000.

# Some properties: equilibrium



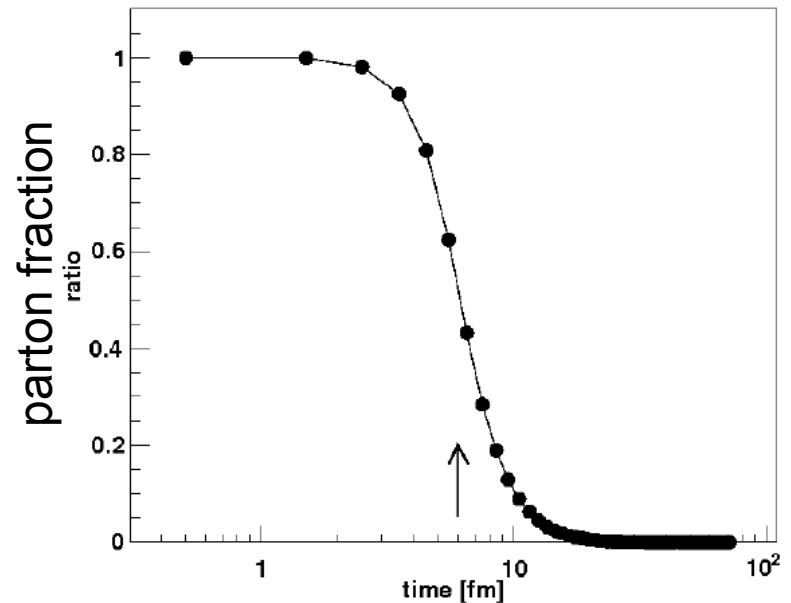
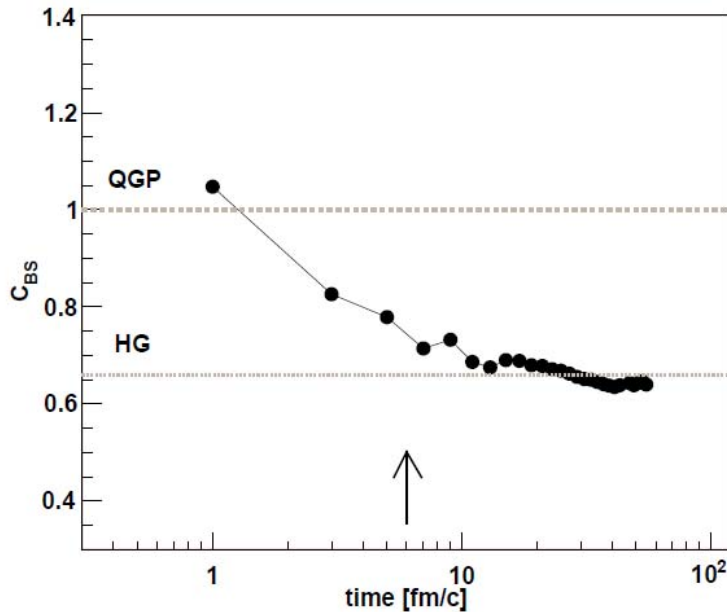
$T_c \sim 140$  MeV



$$\xi = N_{hadrons} / N_{all\ particles}$$

# Time evolution

- Hadronisation blurs the signatures from the QGP phase (here  $C_{BS}$ , also true for others)



# A model for QCD

## Chromodielectric model (CDM)

[G.Martens et.al., Phys.Rev. D70/D73]

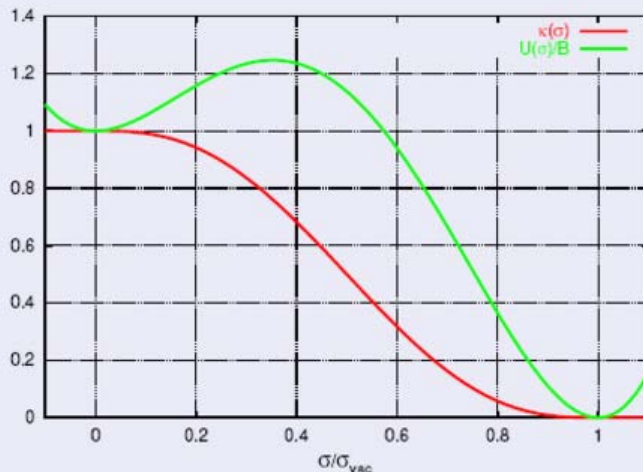
$$\mathcal{L}_{\text{cdm}} = \frac{1}{4} \kappa(\sigma) F_{\mu\nu}^a F^{\mu\nu,a} \quad \mathcal{L}_{\text{glue}}$$

$$-g j_{\mu}^a A^{\mu,a} \quad \mathcal{L}_{q,g}$$

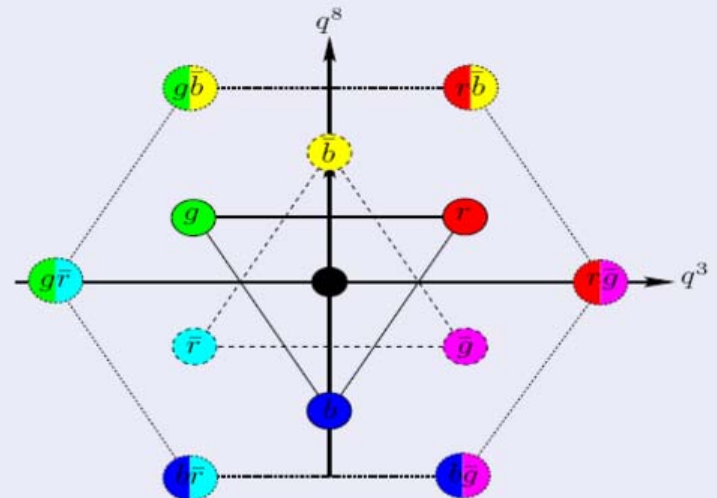
$$+\frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) \quad \mathcal{L}_{\sigma}$$

$$F^{\mu\nu,a} = \partial^{\mu} A^{\nu,a} - \partial^{\nu} A^{\mu,a} \quad a \in \{3, 8\}$$

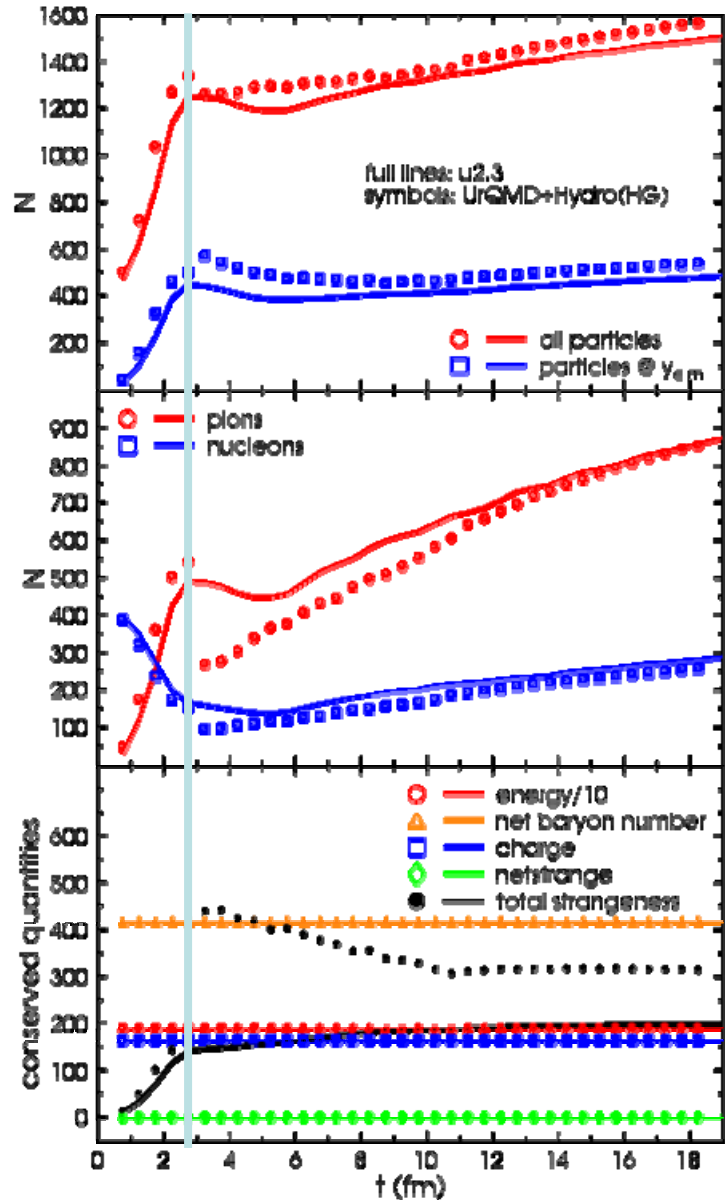
## Self interaction & Dielectric



## Color multipletts



# Time Evolution



Central Pb+Pb collisions at 40A GeV:

- Number of particles decreases in the beginning due to resonance creation

- Qualitative behaviour very similar in both calculations

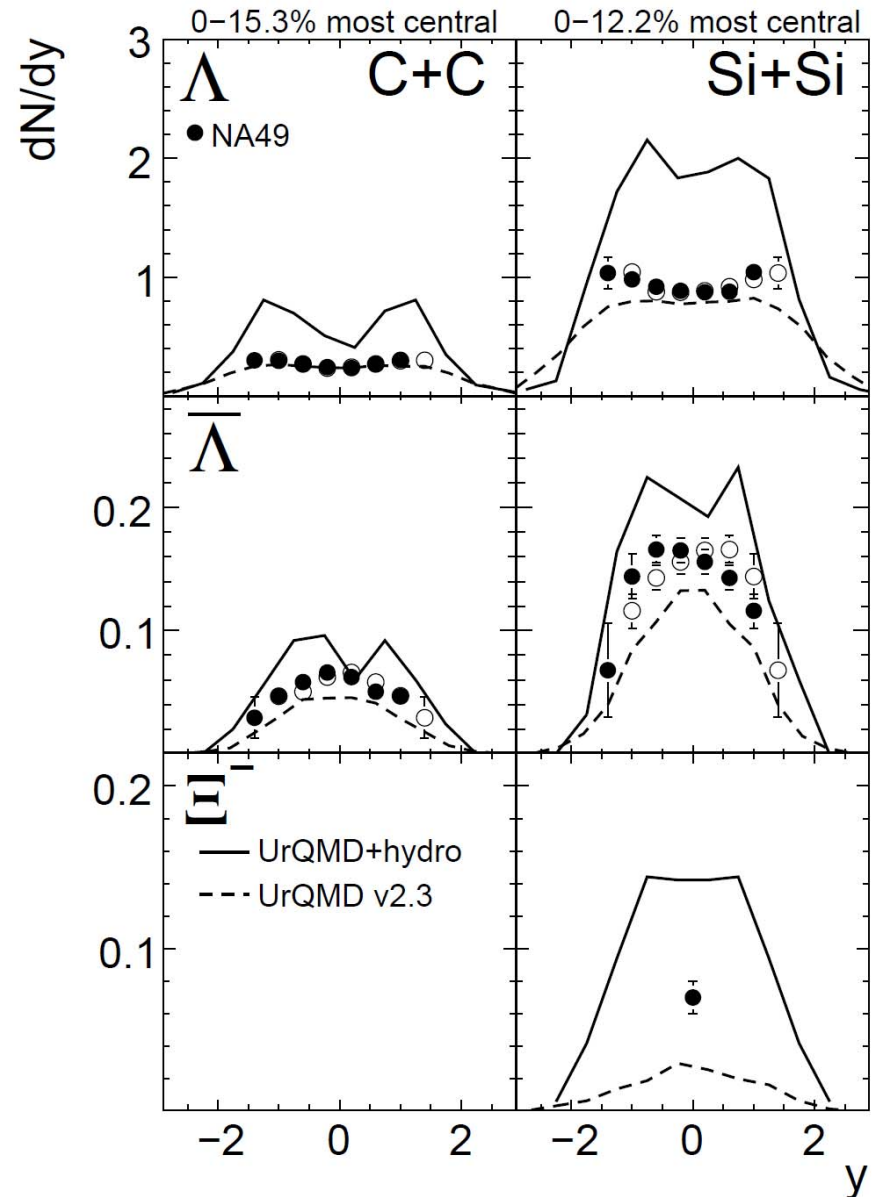
→ UrQMD equilibrates to a rather large degree



# Limitations in small systems

- Small systems lack sufficient thermalisation
- Lambda's etc are still driven by initial state

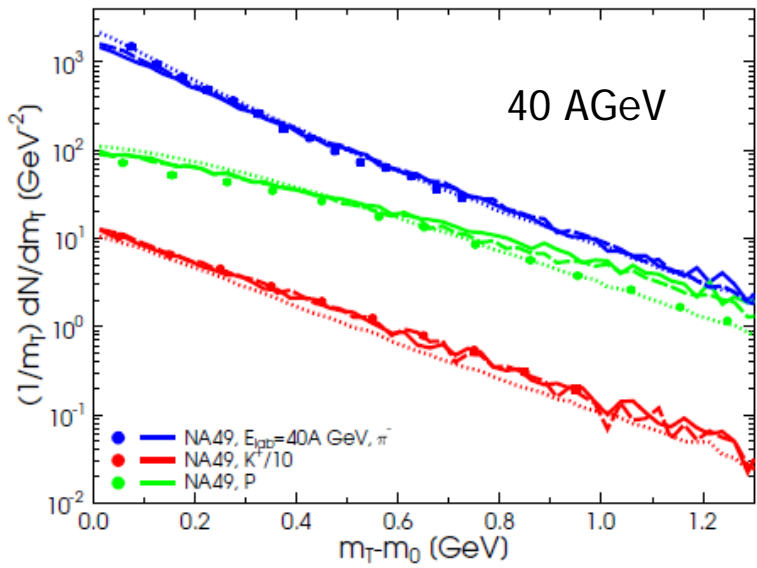
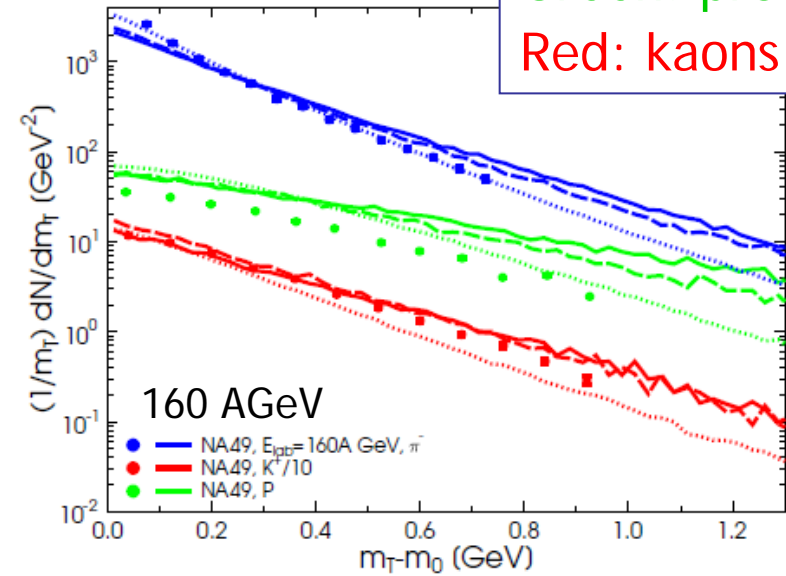
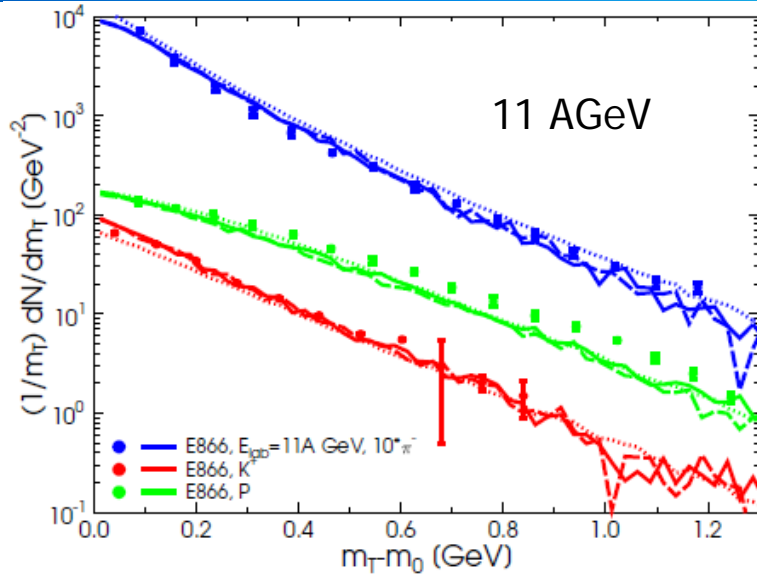
(Petersen et al., arXiv: 0903.0396)





# $m_T$ Spectra

Blue: pions  
Green: protons  
Red: kaons



Full line: hybrid model (IF)  
 Dashed line: hybrid model (GF)  
 Dotted line: UrQMD-2.3

- $m_T$  spectra are very similar at lower energies (11,40 AGeV)
- $\langle m_T \rangle$  is higher in hydro calculation at  $E_{\text{lab}}=160$  AGeV

Central ( $b < 3.4$  fm) Pb+Pb/Au+Au collisions

(Petersen et al., PRC 78:044901, 2008)

# Direct Photons: Hadronic channels

$$\pi + \pi \rightarrow \gamma + \rho, \quad \pi + \rho \rightarrow \gamma + \pi$$

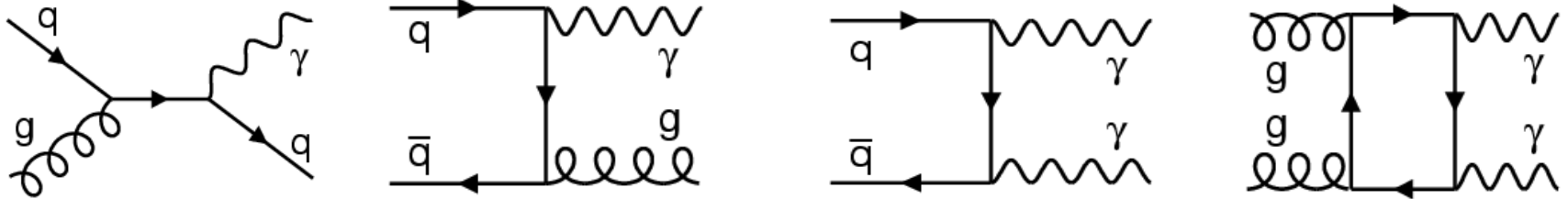
$$\pi + \pi \rightarrow \gamma + \eta, \quad \pi + \eta \rightarrow \gamma + \pi, \quad \pi + \pi \rightarrow \gamma + \gamma$$

$$\pi + K^* \rightarrow \gamma + K, \quad \pi + K \rightarrow \gamma + K^*, \quad \rho + K \rightarrow \gamma + K, \quad K + K^* \rightarrow \gamma + \pi$$

Example for a differential cross section:

$$\frac{d\sigma}{dt} (\pi^\pm \rho^0 \rightarrow \gamma \pi^\pm) = \frac{\alpha g_\rho^2}{12 s p_{\text{c.m.}}^2} \left[ 2 - \frac{s(m_\rho^2 - 4m_\pi^2)}{(s - m_\pi^2)^2} - \frac{(m_\rho^2 - 4m_\pi^2)}{t - m_\pi^2} \left( \frac{s - m_\rho^2 + m_\pi^2}{(s - m_\pi^2)(t - m_\pi^2)} + \frac{m_\pi^2}{(t - m_\pi^2)} \right) \right]$$

# Partonic channels



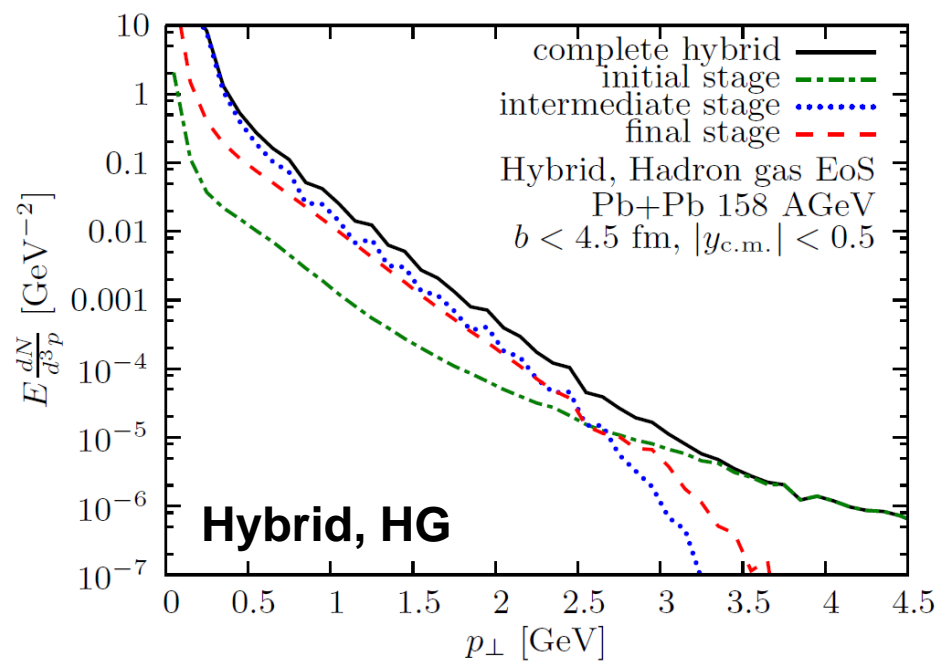
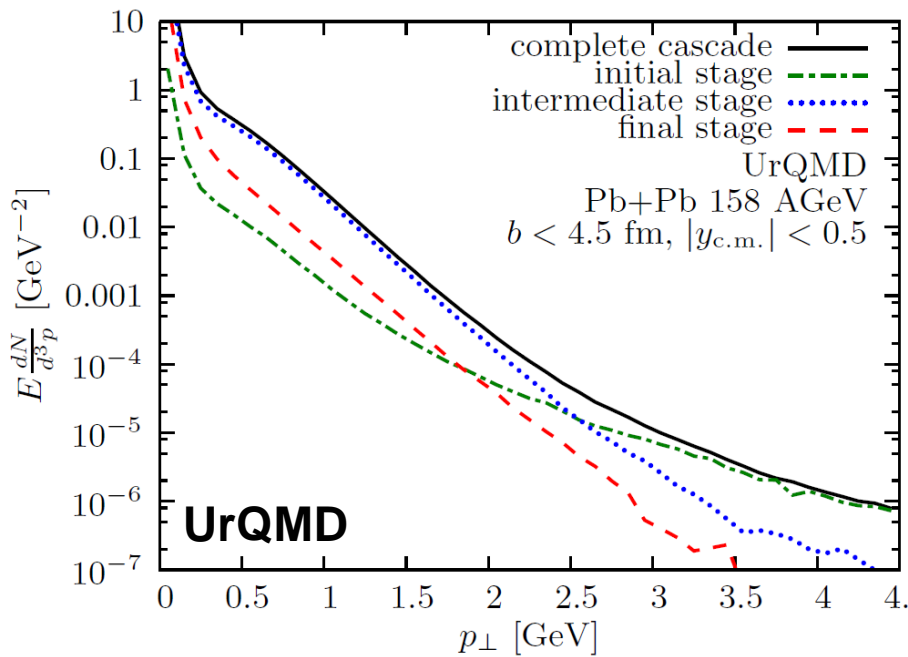
- from QGP: sensitivity to parton density and temperature
- from initial state: sensitivity to PDFs (gluon!)

## Cross section Refs

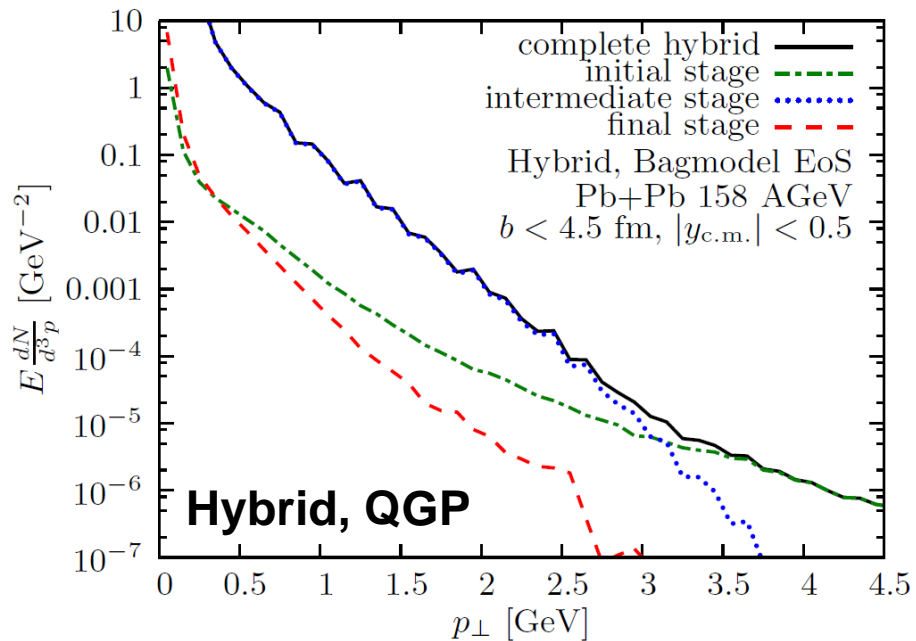
<sup>1</sup>E.g. Aurenche, Fontannaz *et. al*, PRD **73**, 094007 (2006)

<sup>2</sup>Turbide, Rapp and Gale, PRC **69**, 014903 (2004); Turbide, Gale *et al.*, PRC **72**, 014906 (2005); Liu and Werner, arXiv:0712.3612 [hep-ph]; Vitev and Zhang, arXiv:0804.3805 [hep-ph]; Haglin, PRC **50**, 1688 (1994); Haglin, JPG **30**, L27 (2004), Chatterjee *et al.*, Nucl. Phys. A **830** (2009) 503C

<sup>3</sup>Dumitru, Bleicher, Bass, Spieles, Neise, Stöcker and Greiner, PRC **57**, 3271 (1998); Huovinen, Belkacem, Ellis and Kapusta, PRC **66**, 014903 (2002); Li, Brown, Gale and Ko, arXiv:nucl-th/9712048; Bratkovskaya and Cassing, NPA **619**, 413 (1997); Bratkovskaya, Kiselev and Sharkov, arXiv:0806.3465 [nucl.th]



Differential Photon spectra  
 Clear enhancement of  
 Photon production with QGP

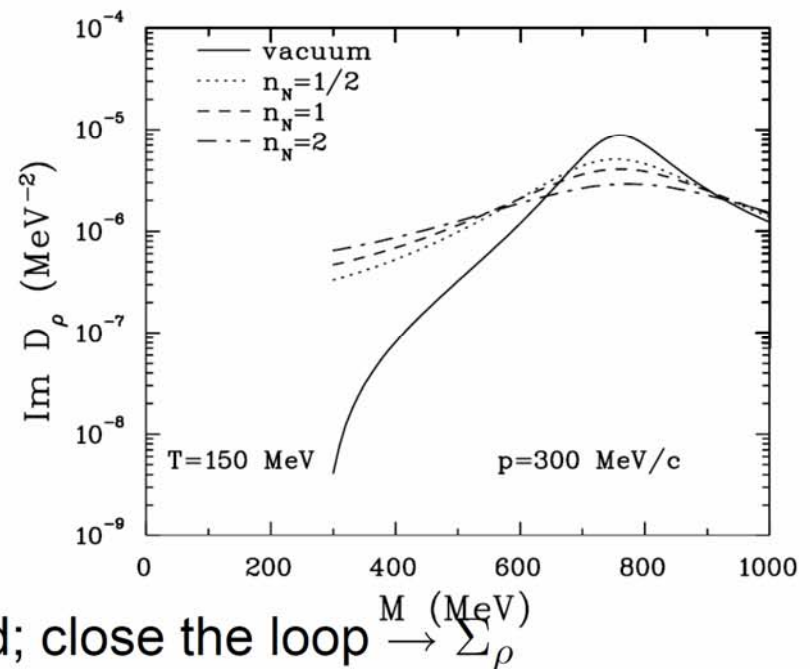


# Di-Leptons from/in the medium

$\rho^* \rightarrow \mu\mu$  production In+In collisions at SPS energies

- ▶ Spectral density for the  $\rho$  meson in a heat bath of  $N$  and  $\pi$  re-derived and labelled

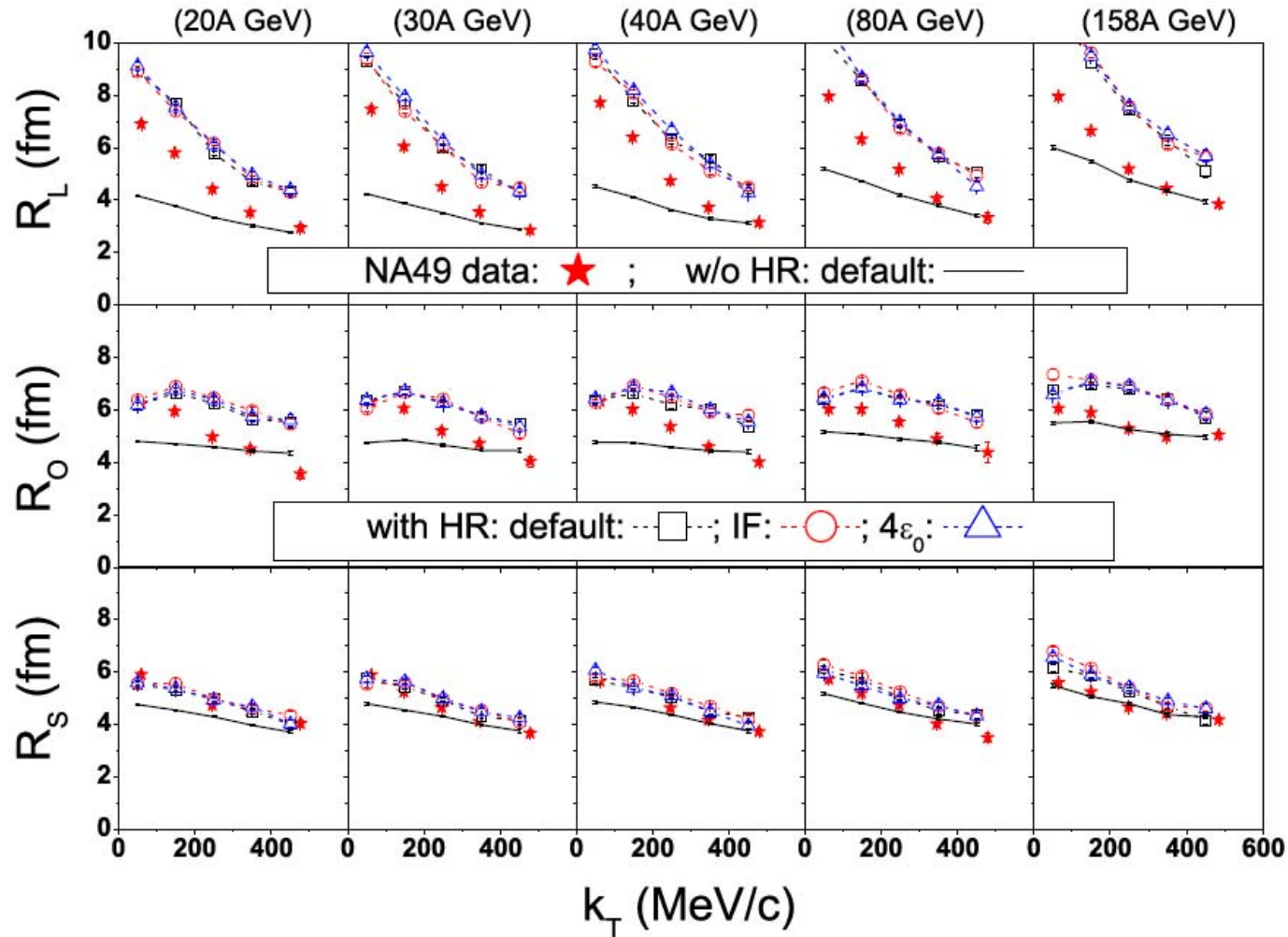
Eletsky, Belkacem, Ellis, Kapusta,  
PRC64 (2001), 035202



Authors give  $f_{\rho a}$  as free to download; close the loop  $\rightarrow \Sigma_\rho$

Santini, Bleicher (2010)

# HBT radii (freeze-out effects)



(Q. Li et al., arXiv: 0812.0375, PLB in print)

Freeze-out effects are small, if hadronic rescattering is included