Statistical Models of Hadron Production

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1. Statistical Ensembles
2. Particle Number Fluctuations
3. New Concept of Statistical Ensembles
4. MCE/sVF:
   a) KNO-scaling;
   b) power-law in **p_T**

GCE and CE

$$Z_{gce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} = \exp(2z)$$

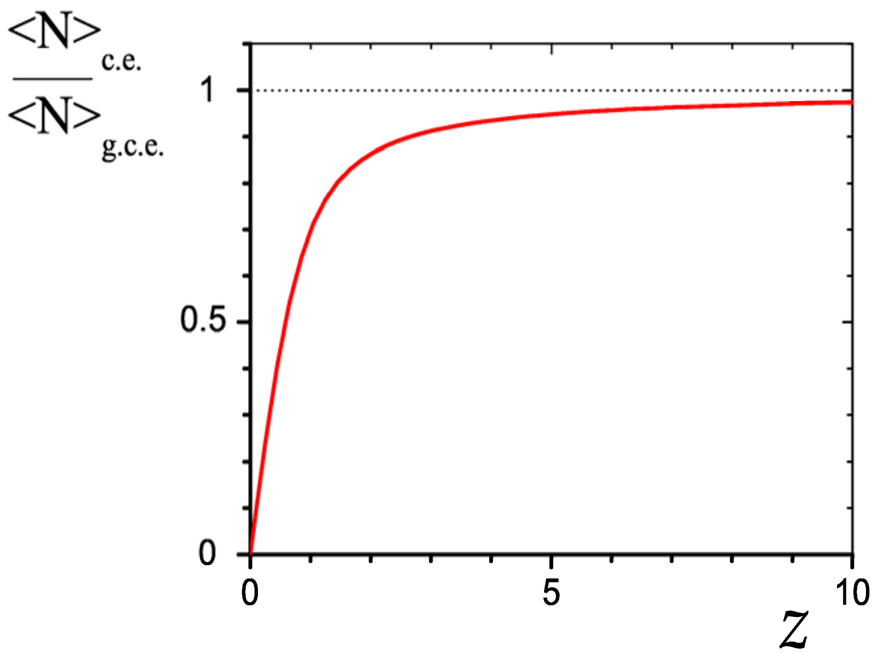
$$z = \frac{V}{2\pi^2} Tm^2 K_2(m/T)$$

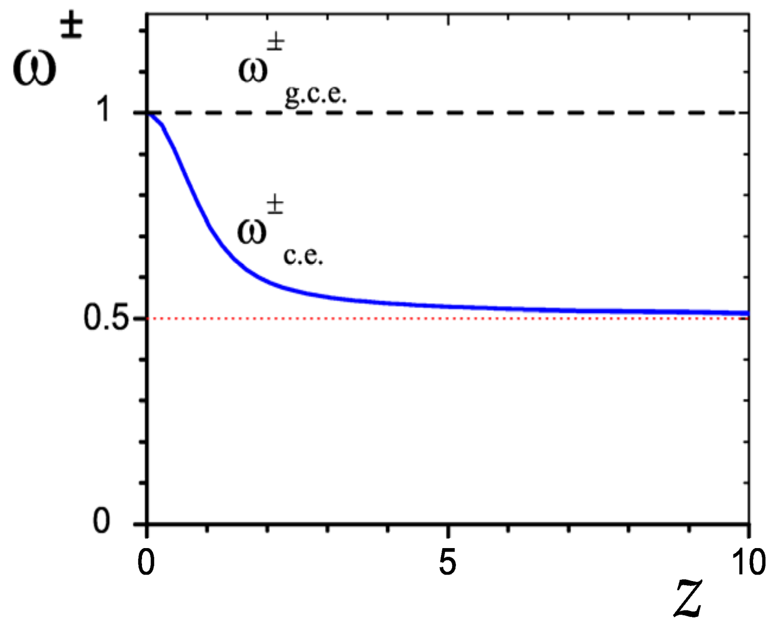
$$Z_{ce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} \delta(N_+ - N_-) = I_0(2z)$$

$$\omega^- = \frac{\langle N_-^2\rangle - \langle N_-\rangle^2}{\langle N_-\rangle}, \qquad \langle N_-\rangle_{gce} = z, \qquad \omega^-_{gce} = 1$$

$$\langle N_-\rangle_{ce} = z \frac{I_1(2z)}{I_0(2z)}, \qquad \omega^-_{ce} = 1 - z \left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)}\right]$$

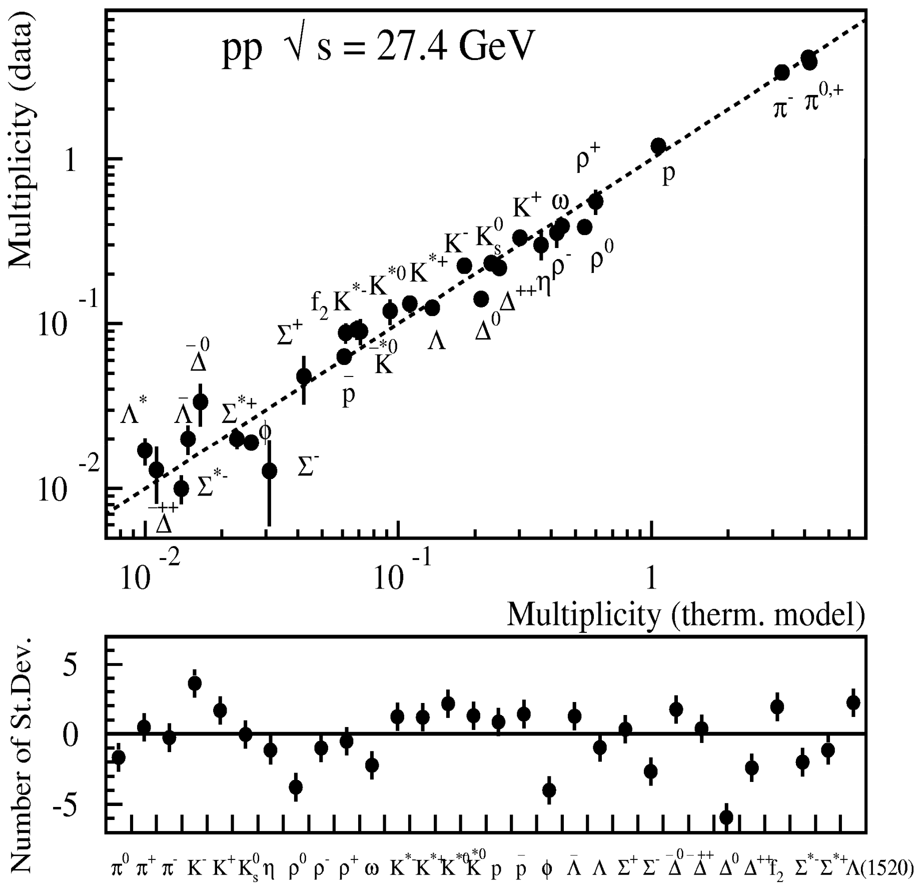
GCE and CE

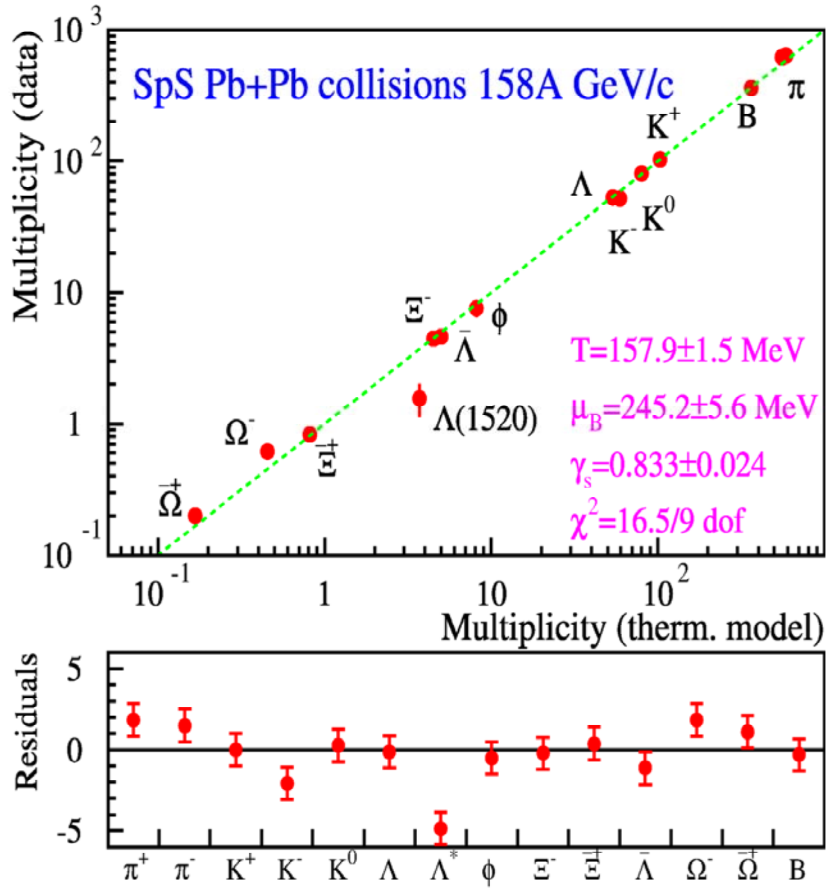




Begun, Gazdzicki, M.I.G., Zozulya
Phys. Rev. C (2004)

Mean Multiplicities



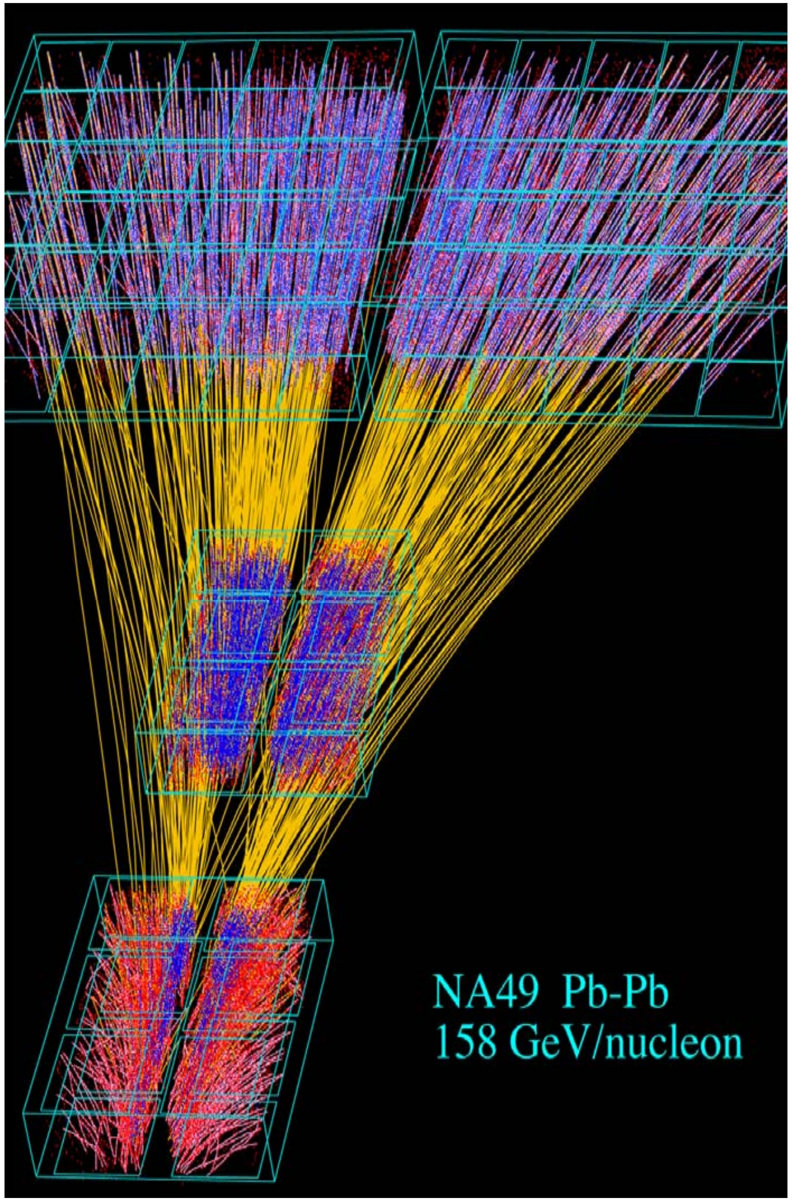


Becattini, Heinz, Z. Phys. (1997) CE in p+p

Becattini, Manninen, Gazdzicki, Phys. Rev. C (2006) GCE in A+A

M.I.G., Gazdzicki, Greiner, Phys. Lett. B (2000)
CE for antibaryons in p+A

M.I.G., Kostyuk, Stoecker, Greiner, Phys. Lett. B (2001)
CE for charmed hadrons



$$N = 10^2 \div 10^4$$

$$P(N)\ , \quad \langle N^k \rangle = \sum_N N^k \, P(N)$$

$$Var(N) = \langle N^2 \rangle - \langle N \rangle^2$$
$$= \langle (N - \langle N \rangle)^2 \rangle = \langle (\Delta N)^2 \rangle$$

$$\omega = \frac{Var(N)}{\langle N \rangle}$$

Scaled Variances are not equal to each other in different SE

Statistical Ensembles **E, V, Q**

$E \Longleftrightarrow T$

$V \Longleftrightarrow p$

$Q \Longleftrightarrow \mu_Q$

$2^3 = 8$

E, V, Q **MCE**

T, V, Q **CE**

T, V, μ_Q **GCE**

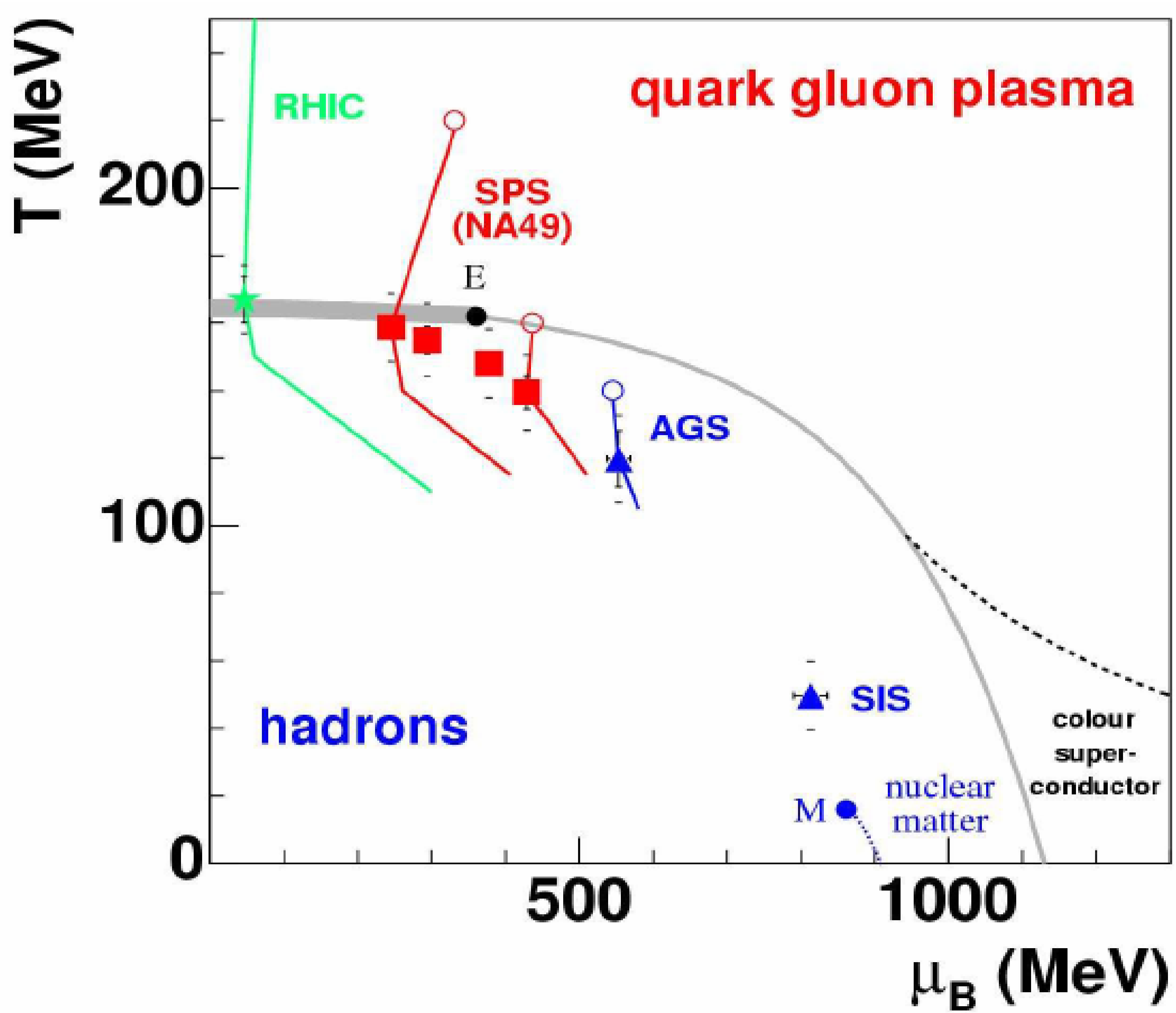
E, V, μ_Q **MGCE**

E, p, Q $\quad$ T, p, Q

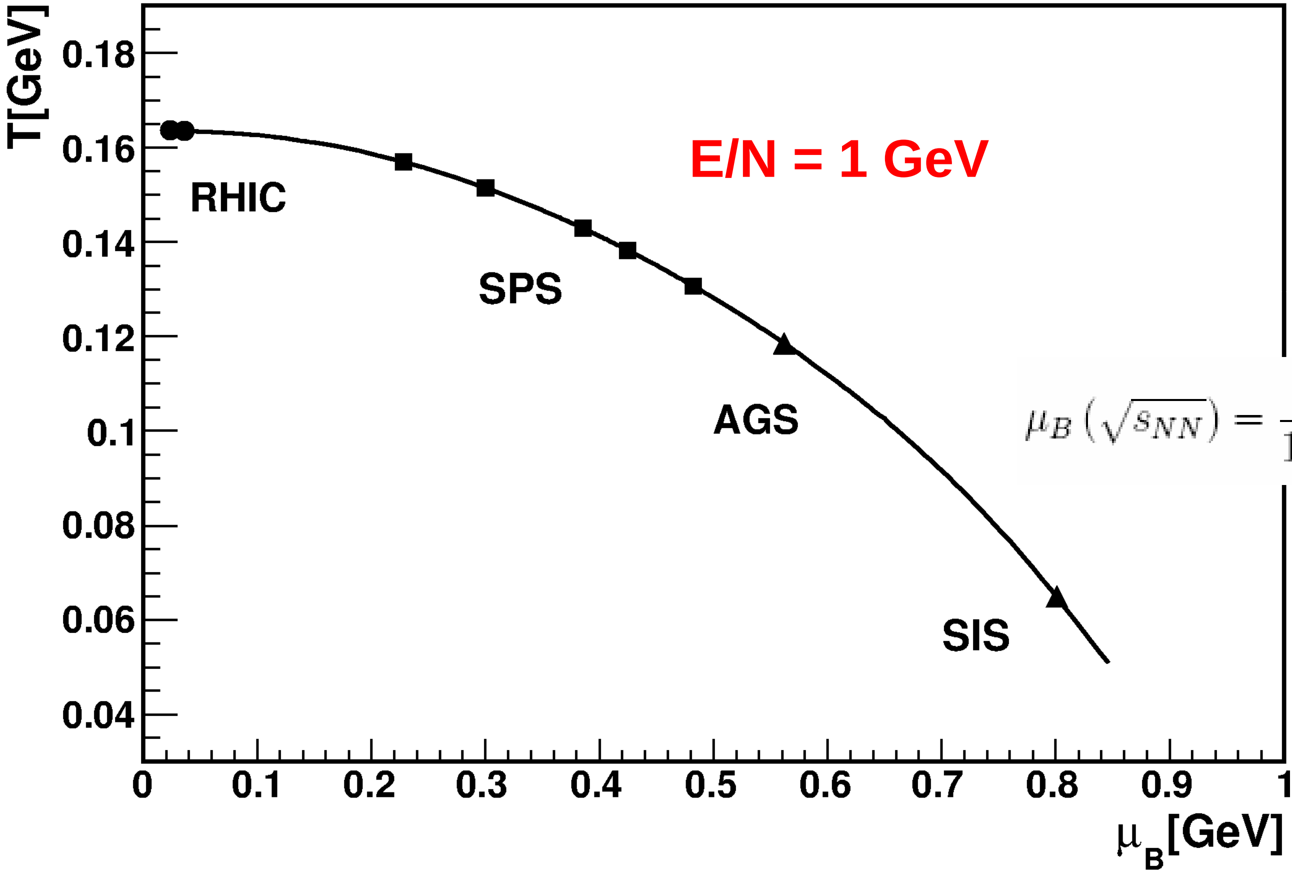
T, p, μ_Q $\quad$ E, p, μ_Q

} **Pressure Ensembles**

Gorenstein, J. Phys. G (2008)


T (MeV)
200
100
0
500
1000
μB (MeV)
quark gluon plasma
RHIC
SPS
(NA49)
E
AGS
SIS
hadrons
colour
super-
conductor
nuclear
matter
M

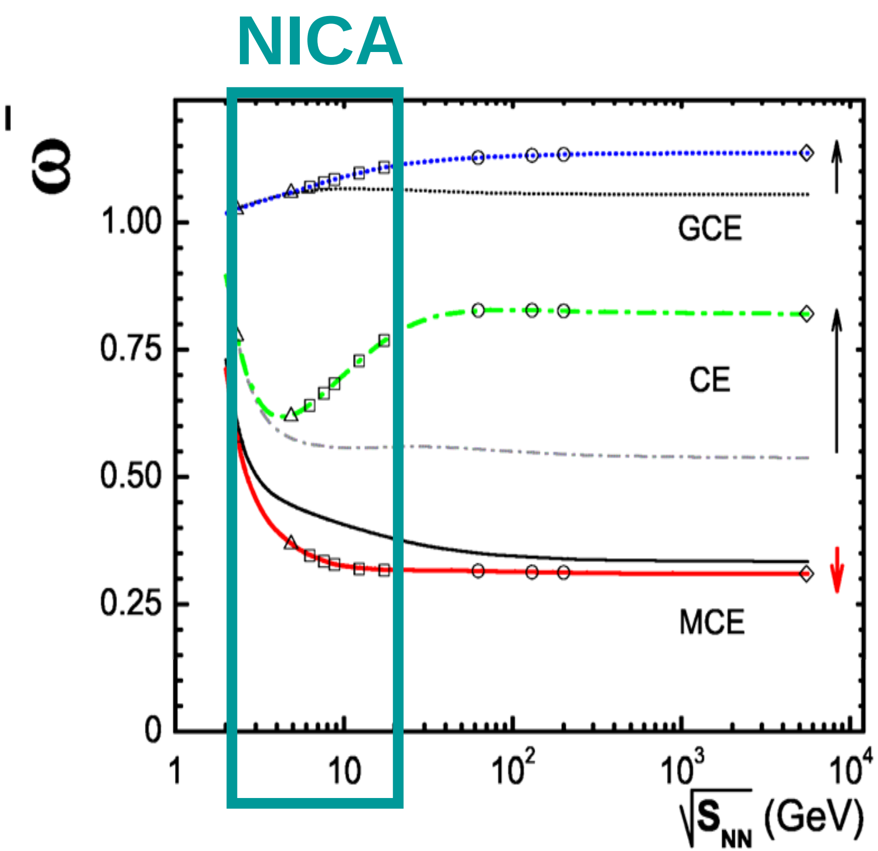
Line of the chemical freeze-out

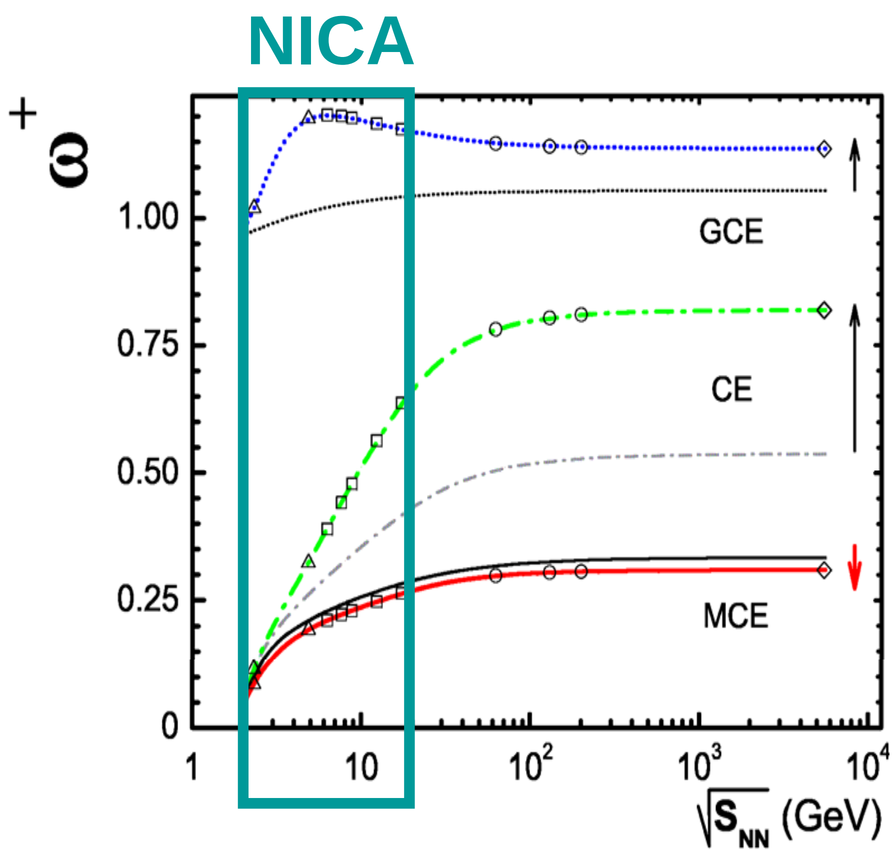


$$\mu_B\left(\sqrt{s_{NN}}\right) = \frac{1.308\ \mathrm{GeV}}{1 + 0.273\ \sqrt{s_{NN}}} ,$$

Cleymans and Redlich, Phys. Rev. Lett. (1998)

The prediction of hadron gas model

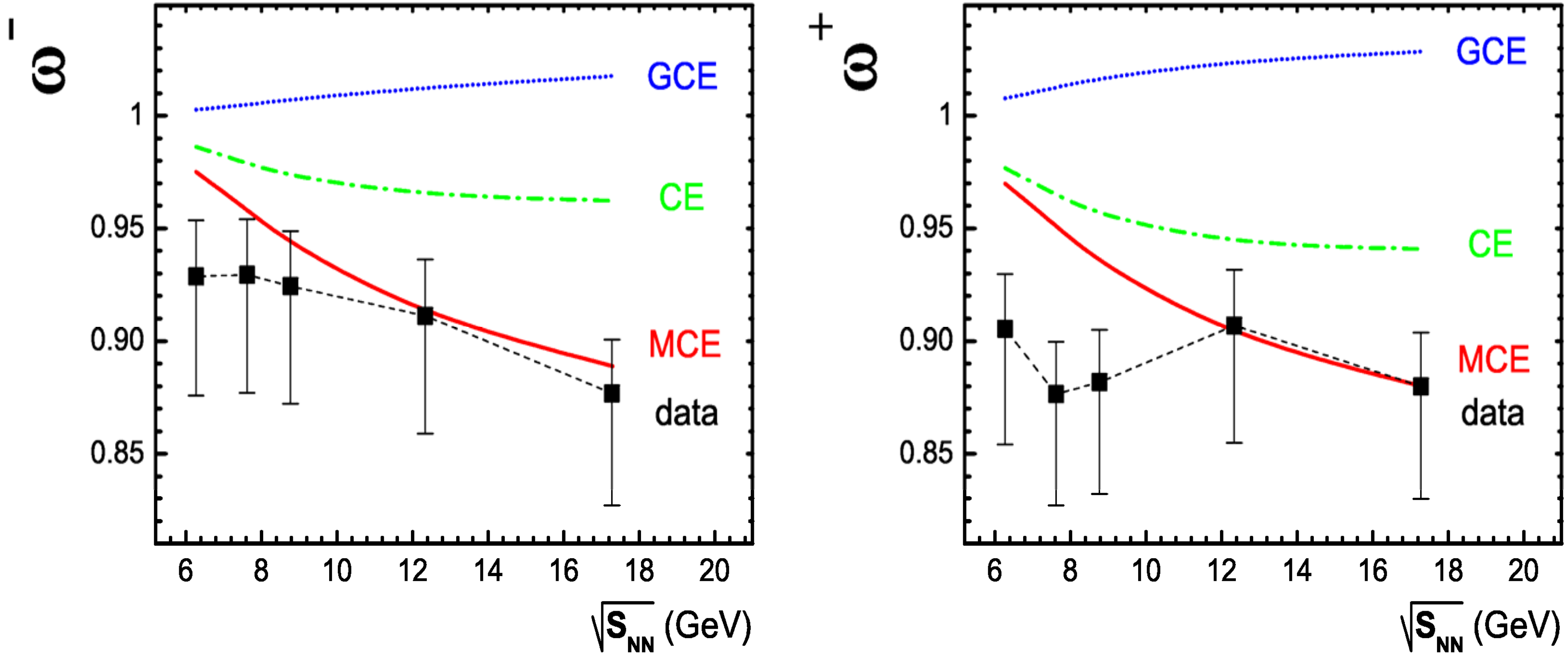




$$\omega^{\pm}_{4\pi} \equiv \frac{\langle (\Delta N_{\pm})^2 \rangle}{\langle N_{\pm} \rangle} \qquad \omega^{\pm}_{acc} = 1 - q + q\,\omega^{\pm}_{4\pi}$$

Begun, Gazdzicki, M.I.G. Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2007)

Comparison with the NA49 data



$$q = 0.038,\ 0.063,\ 0.085,\ 0.131,\ 0.163$$

Begun, Gazdzicki, M.I.G. Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2007)

$$\vec{A} \; = \; (E, V, Q_1, \ldots, Q_k)$$

Alpha-Enesmble

$$P_\alpha(X) \; = \; \int \; d\vec{A} \; P_\alpha(\vec{A}) \; P_{mce}(X; \vec{A})$$

M.I.G. and Hauer, Phys. Rev. C (2008)

Problems of the statistical approach

I. Multiplicity distribution in $e^+e^-,\ pp,\ p\bar{p}$

II. Power law at high p_t and high m

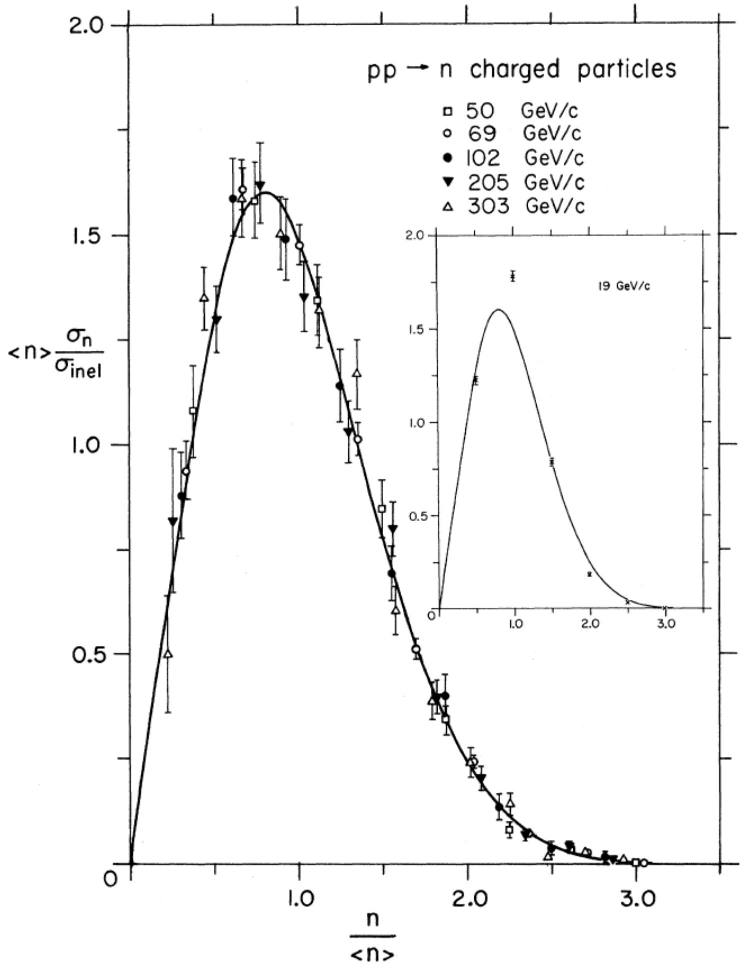
$$\frac{d^3N_i}{dp^3} \sim C\ p_T^{-K_p} \qquad p_T \gg m_i$$

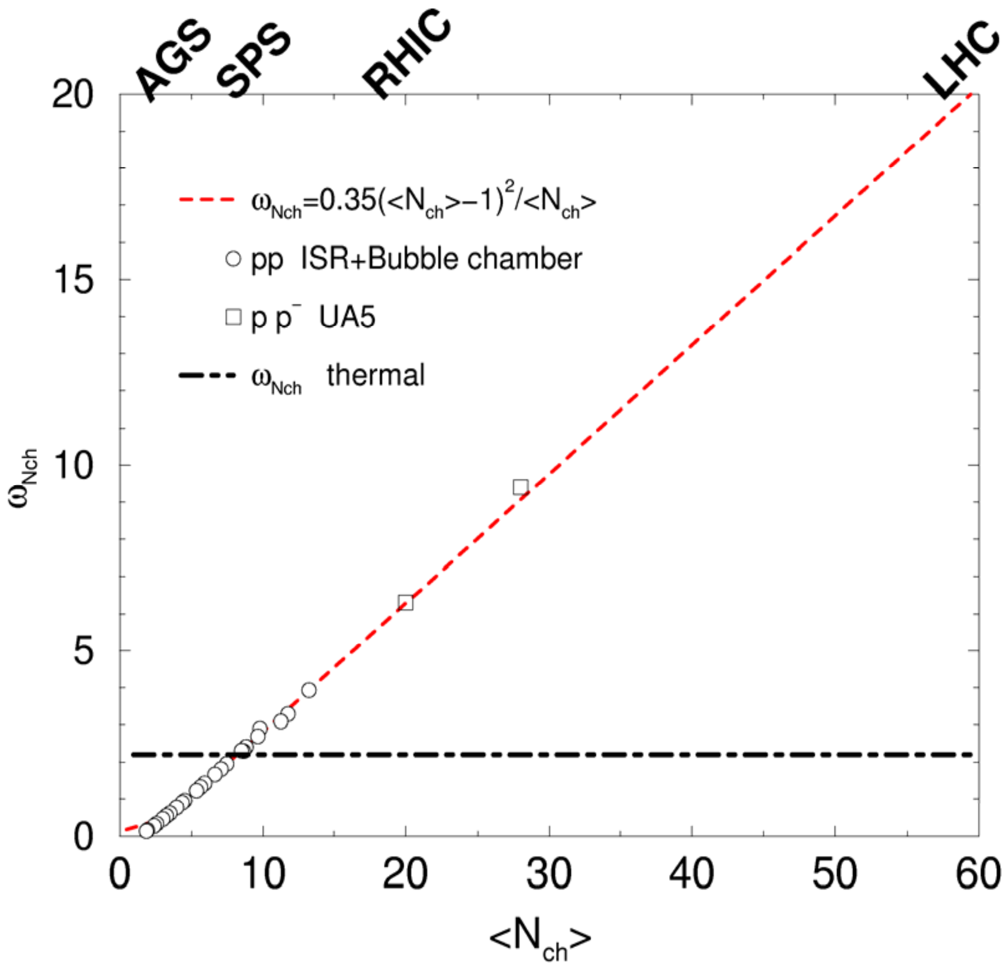
$$\langle N_i \rangle \sim C\ m_i^{-K_m}$$

$$K_p \approx 8$$

$$K_m \approx K_p - 3$$

KNO scaling & Large fluctuations





Slattery, Phys. Rev. Lett. (1972); Heiselberg, Phys. Rept. (2001)

Momentum Spectra

Becattini, Passaleva,

Eur. Phys. J. (2002),

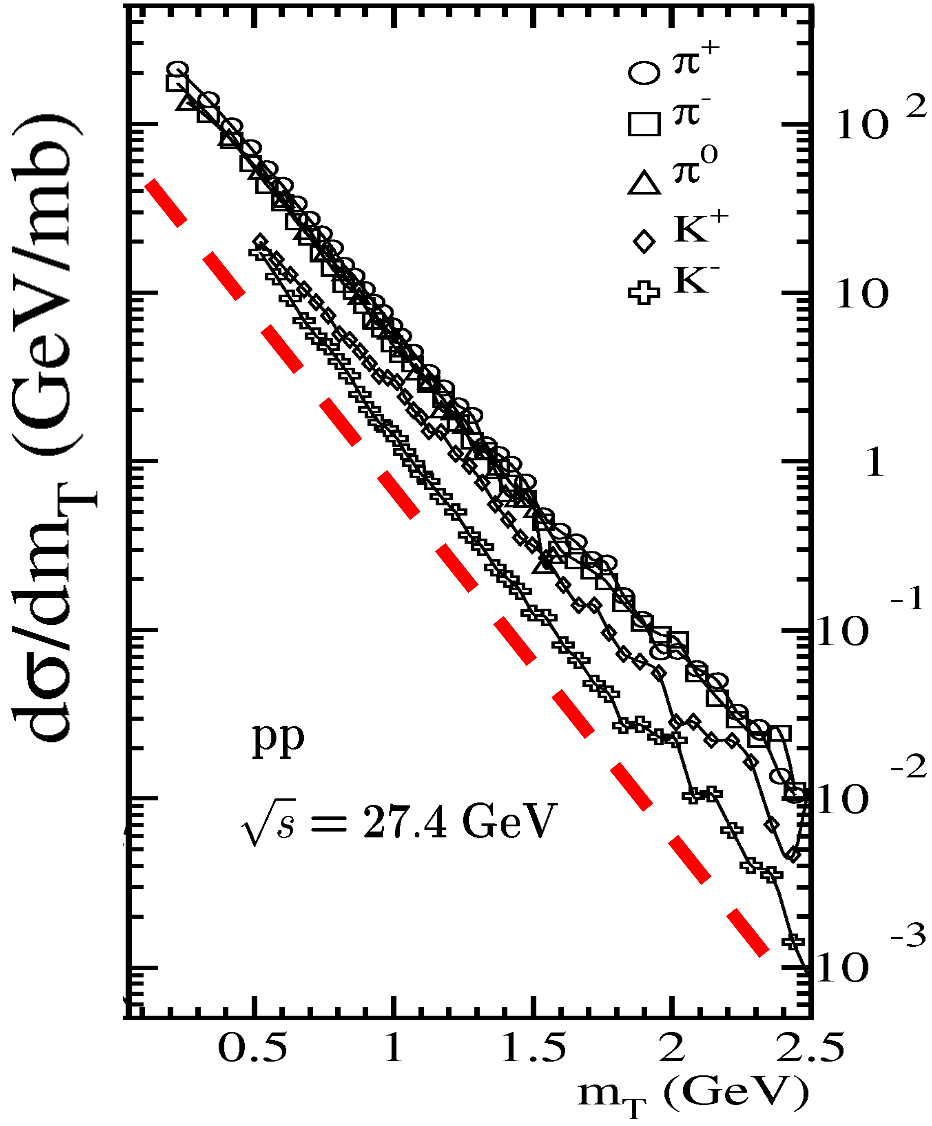
data from

Aguilar-Benitez et al.,

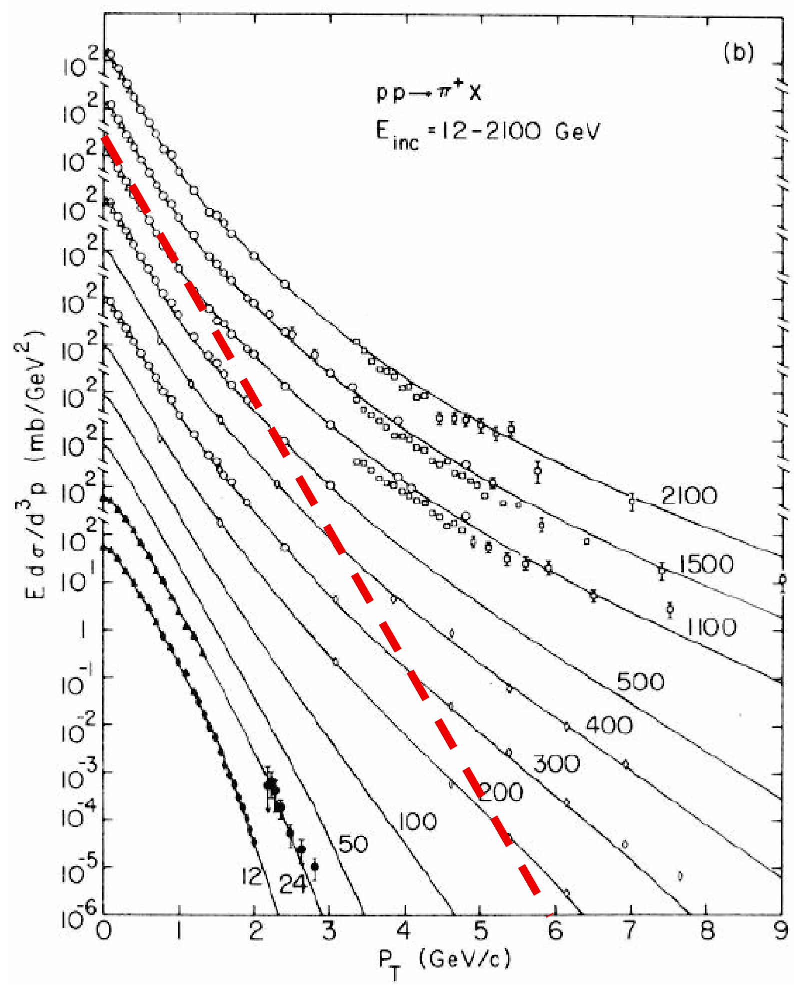
Z. Phys. C (1990)

$$\exp\left(-\frac{\mathrm{m_T}}{\mathrm{T}}\right)$$

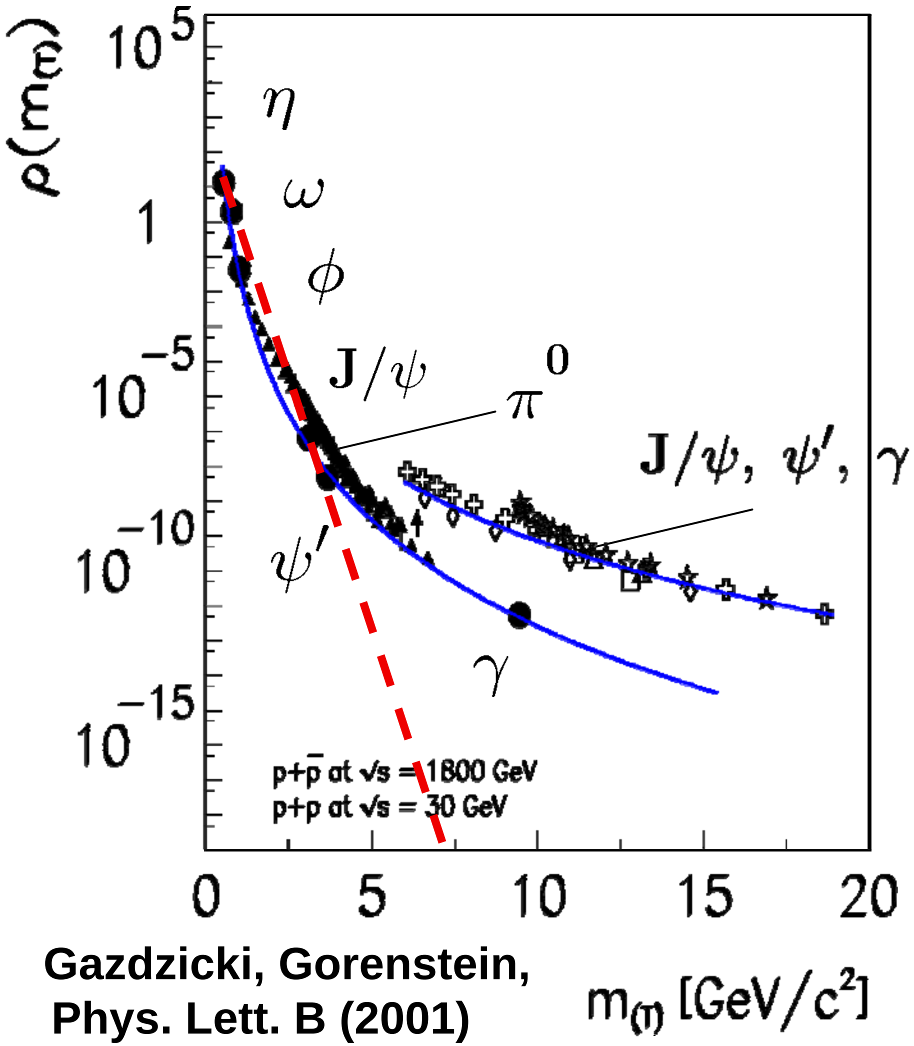
$$\mathrm{m_T} = \sqrt{p_T^2 + m^2}$$



Power law at high p_t and high m



Beier *et al.*, Phys. Rev. D (1978)



Gazdzicki, Gorenstein, Phys. Lett. B (2001)

I. Multiplicity distribution
KNO scaling & Large fluctuations

Data:

$$P(N) = \frac{1}{\langle N \rangle} \Psi_\alpha \left(\frac{N}{\langle N \rangle} \right) ,$$

Koba, Nielsen, Olesen, Nucl. Phys. B (1972)

$$\omega \equiv \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} \propto \langle N \rangle$$

Statistical Models:

$$P(N) \cong \frac{1}{\sqrt{2\pi\,\omega\,\langle N \rangle}} \exp\left[-\frac{(N - \langle N \rangle)^2}{2\,\omega\,\langle N \rangle} \right] ,$$

$$\omega \approx \mathrm{const} \approx 1$$

Micro Canonical Ensemble with scaling Volume Fluctuations (MCE/sVF)

$$P_\alpha(X;E) \;=\; \int_0^\infty dV \; P_\alpha(V) \; P_{mce}(X;E,V)$$

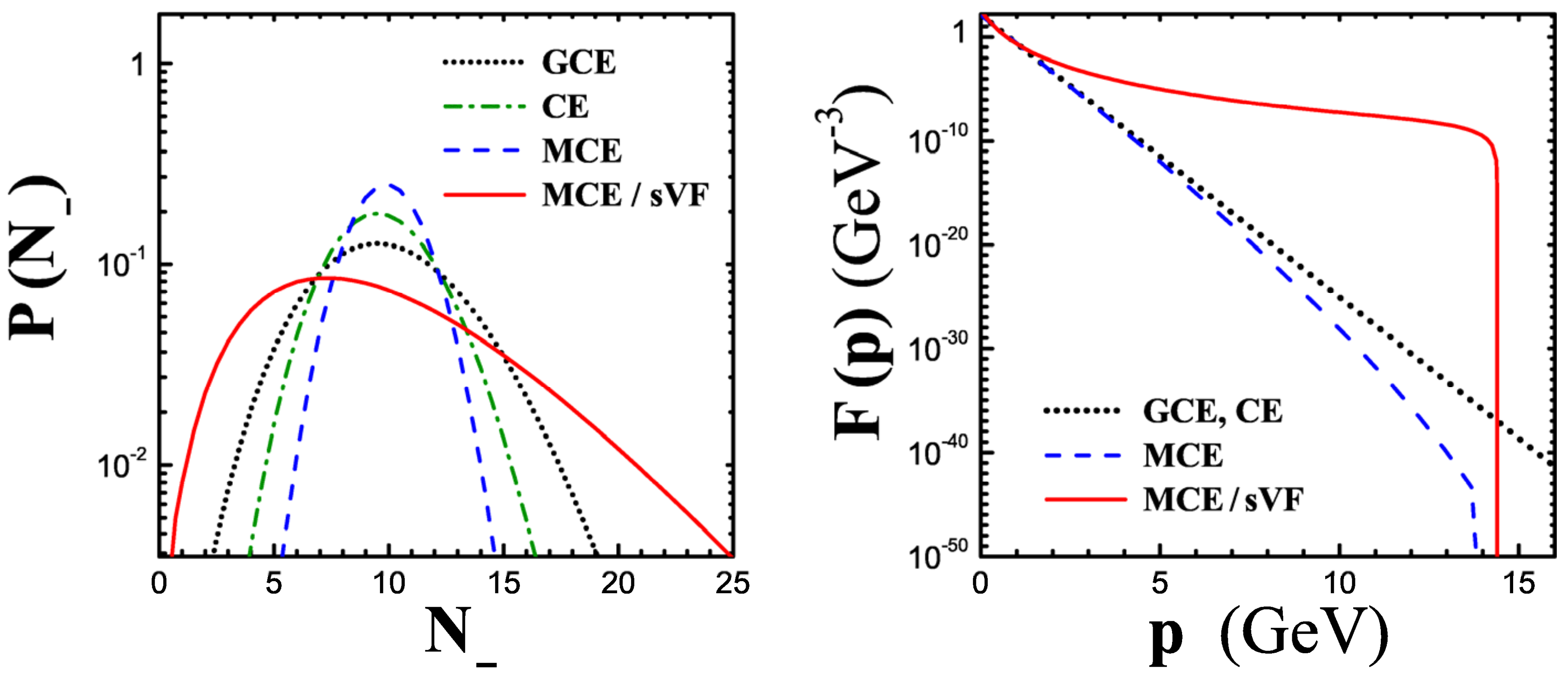
$$X = N, \; p$$

Begun, Gazdzicki, Gorenstein, Phys. Rev. C (2008)

$$P_\alpha(V) \;=\; \frac{1}{\overline{V}}\,\Phi_\alpha(V/\overline{V})$$

Scaling volume fluctuations selected to fit experimental multiplicity distribution

Particle Number Distributions and Spectra

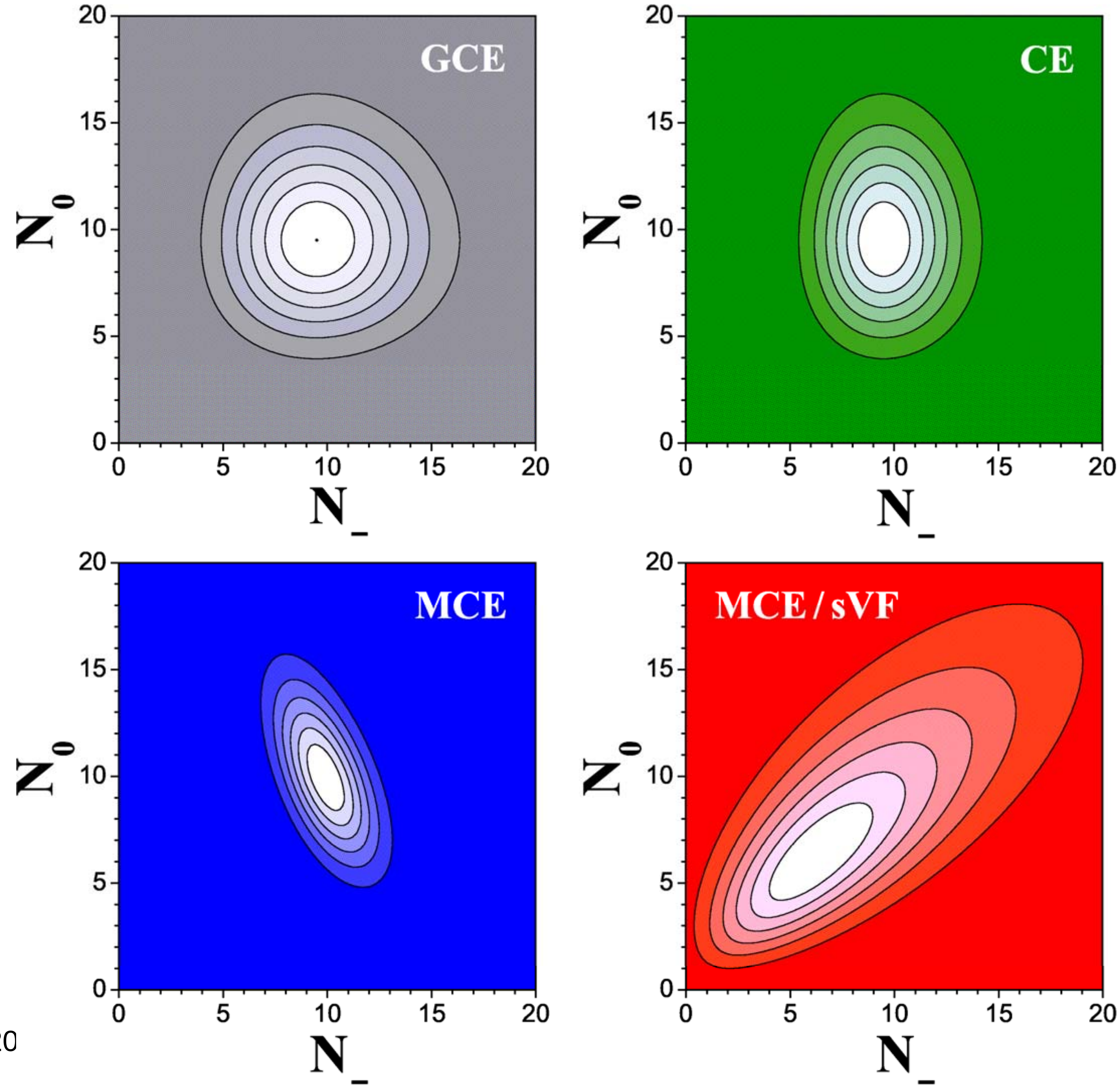


$$P_\alpha(N) = \frac{1}{\overline{\overline{N}}}\,\Psi_\alpha(N/\overline{N})$$

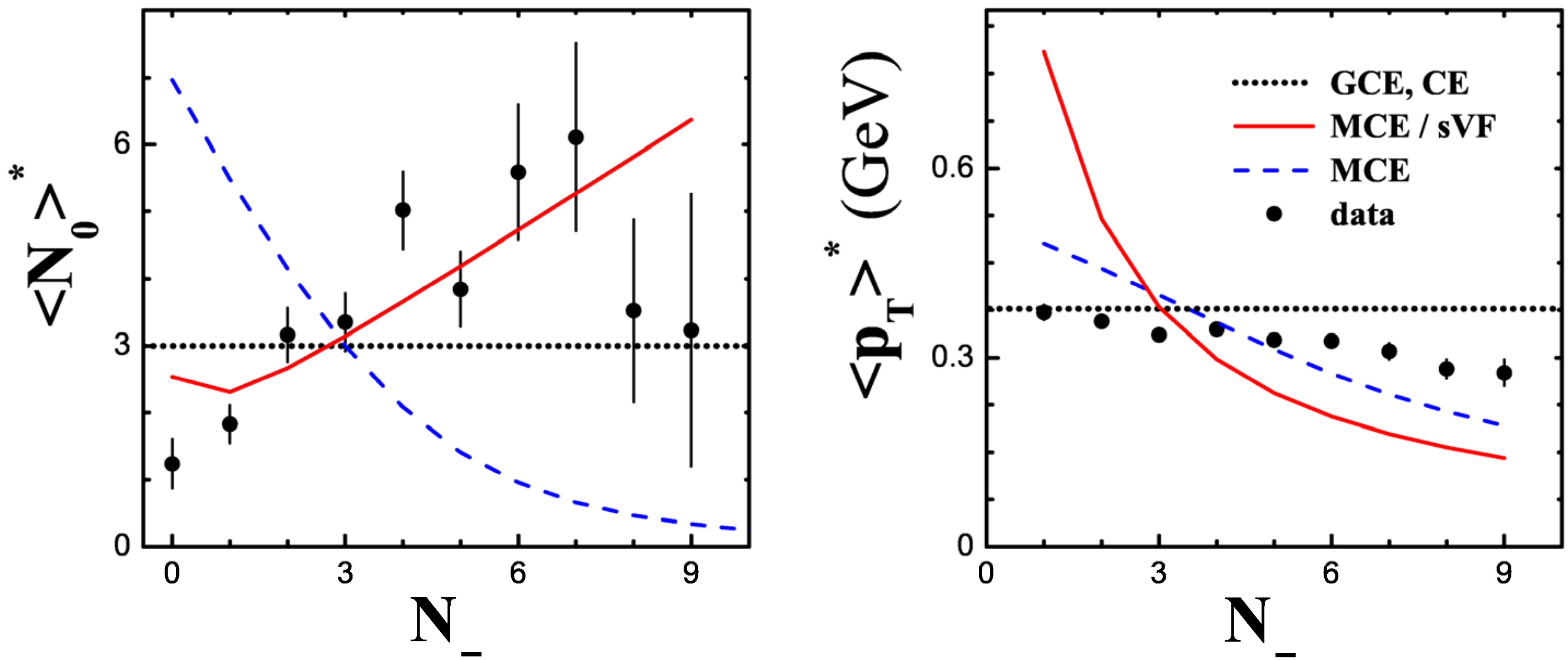
$$\Psi_\alpha(y) = \frac{k^k}{3!}\, y^{k-1}\ \exp(-k\,y)$$

$$F_\alpha(p) \cong \frac{k^k\Gamma(k+4)}{2\Gamma(k)} T^{k+1}\,(p + kT)^{-k-4}$$

$$\cong 11.27\ \mathrm{GeV}^5\,(p + 4T)^{-8}$$


GCE
CE
MCE
MCE / sVF
N_0
N_-
0
5
10
15
20

Semi-Inclusive Observables



Begun, Gazdzicki, M.I.G, Phys. Rev. (2009)

Summary

1. Statistical Ensembles with
   Fluctuating Extensive Quantities

2. MCE/sVF
   a) Large Particle Number Fluctuations
   b) Power Law at Large Transverse Momenta

3. Semi-Inclusive Observables in
   Statistical Mechanics

References

- ***Statistical Ensembles with Fluctuating Extensive Quantities***
  Gorenstein and Hauer, Phys. Rev. C78, 041902 (2008)

- ***Statistical Ensembles with Volume fluctuations***
  Gorenstein, J. Phys. G 25, 125102 (2008)

- ***Power Law in Micro-Canonical Ensemble with scaling Volume Fluctuations***
  Begun, Gazdzicki, Gorenstein, Phys. Rev. C78, 024904 (2008)

- ***Semi-Inclusive Observables in Statistical Models***
  Begun, Gazdzicki, Gorenstein, Phys. Rev. C (2009)