CPOD2010 Dubna

QUARK MATTER RESPONSE TO A MAGNETIC FIELD

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The status of the work:

"Now this is not the end. It is not even the beginning of the end. But it is, perhaps, the end of the beginning"

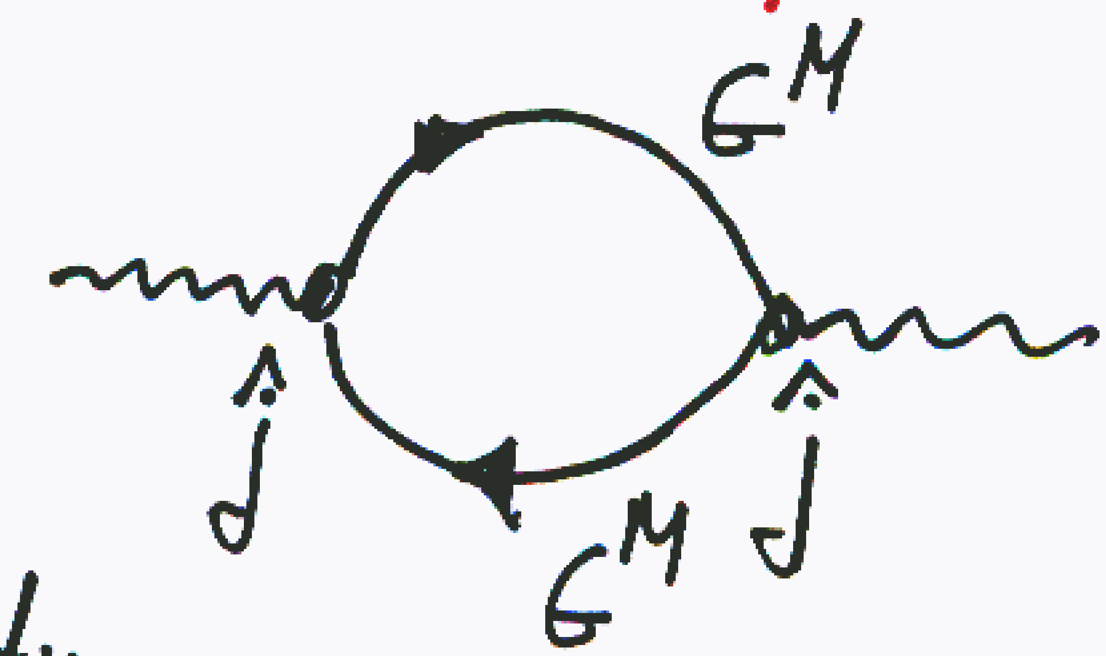
Sir Winston Churchill

Only after a decade of RHIC operation attention was drawn to the fact that colliding heavy ion beams create the largest B known in Nature

$$eB \sim 10^4 \, MeV^2 \sim 10^{18} \, G$$

This B is at the core of the (controversial) Chiral Magnetic Effect – CME – – see S. Voloshin and other talks

- RHIC partisans claim that a new form of matter has been created
- Important insight into the nature of this form of matter may be gained by decoding the properties of the transport coefficients
- Kubo teaches us that transport coefficients (diffusion, viscosity, conductivity) are defined by polarization operator (susceptibility), e.g.,

$\sigma \sim$ 

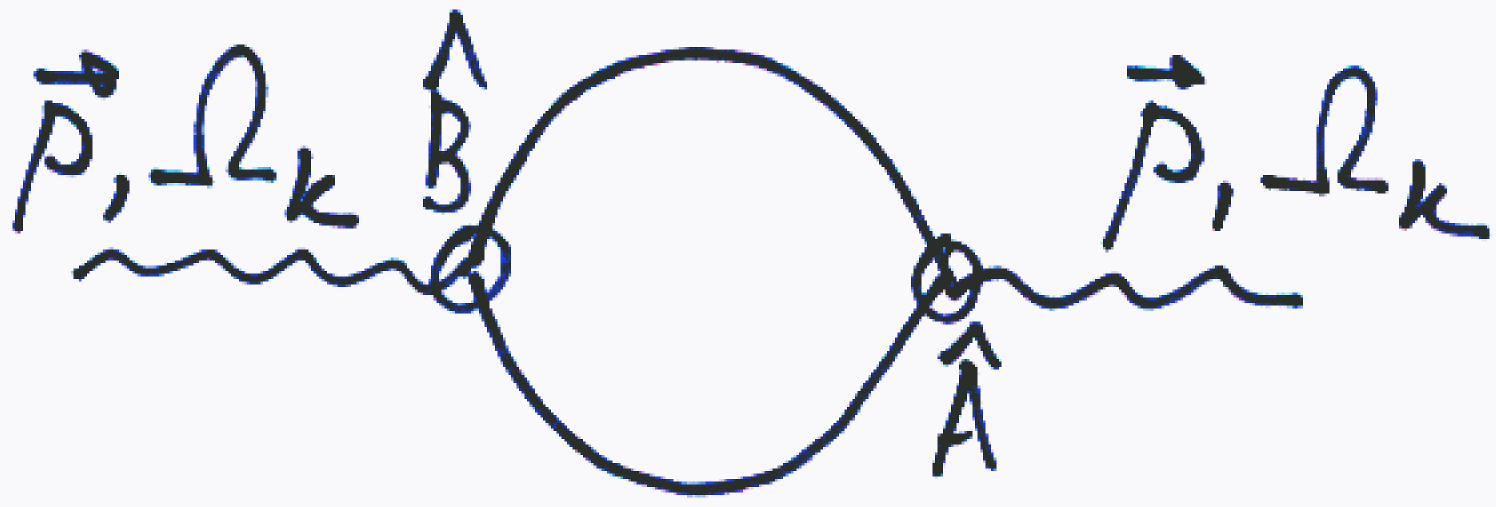
$\uparrow$
conductivity

$G^M(\vec{k}, i\varepsilon_n)$ – Matsubara Green's function

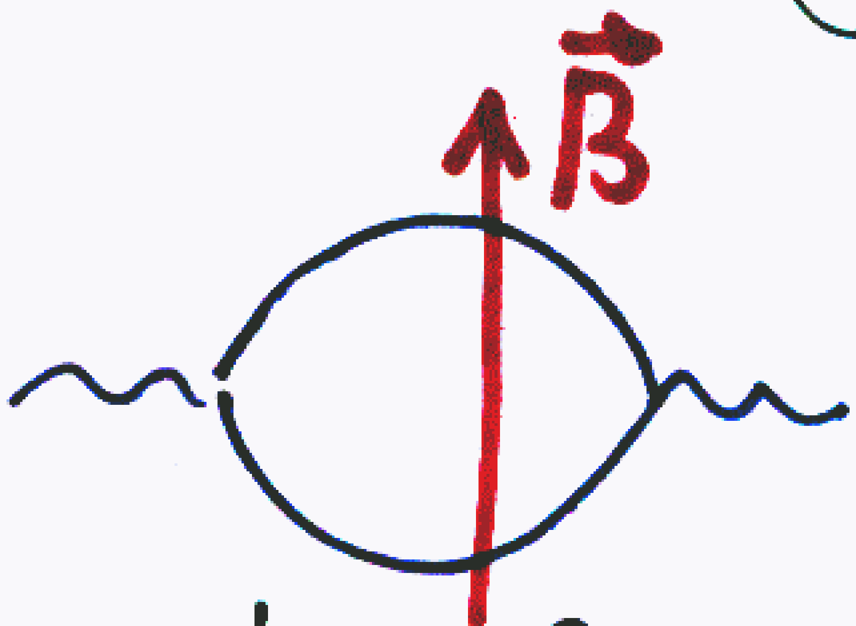
Susceptibility of $\hat{A}$ with respect to $\hat{B}$

$$\chi^M_{AB}(i\Omega_k, \vec{p}) = \frac{1}{\beta}\sum_{\varepsilon_n}\int\frac{d\vec{k}}{(2\pi)^3}\, G^M(\vec{k}, i\varepsilon_n)\,\hat{B}\,G^M(\vec{p}-\vec{k}, i(\Omega_k-\varepsilon_n))\,\hat{A}$$

$\beta = \frac{1}{T}$, $\varepsilon_n = \frac{\pi}{\beta}(2n+1)$ for fermions



It is commonly accepted that at $\vec{B} \neq 0$



$\propto |\vec{B}|$

e.g. Fukushima
Kharzeev
Warringa

why ?

$$\int\frac{d^4p}{(2\pi)^4} \underset{T\neq 0}{\Longrightarrow} \sum_{\varepsilon_n}\frac{1}{\beta}\int\frac{d\vec{p}}{(2\pi)^3} \underset{\vec{B}\neq 0}{\Longrightarrow} \frac{|eB|}{2\pi}\sum_{k=0}^{\infty}\sum_{\varepsilon_n}\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi}$$

$k=0$: Landau levels

$\uparrow^2 \uparrow \vec{B}$

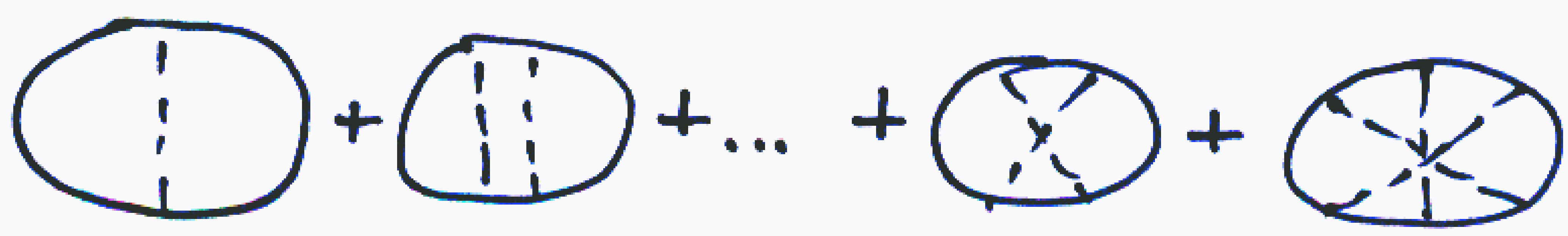
In vacuum there is no any dimensionful parameter to alter the B dependence.
This is not the case in dense medium.

- What is the nature of the matter created at RHIC? Most ideal liquid in Nature? Strongly interacting quark-gluon plasma? Color Superconductor?
- We need only rather weak assumptions:

(i) The system has a Fermi surface

$$\int \frac{d\vec{k}}{(2\pi)^3} \longrightarrow \frac{\mu k_F}{2\pi^2}\int \frac{d\Omega}{4\pi}\, d\xi\,, \quad \xi = \sqrt{\vec{k}^2+m^2}-\mu$$

- There is a quantum disorder



ladder diagrams $\qquad$ fan diagrams

$\frac{1}{\tau}$ – the frequency of elastic collisions

New important dimensionful parameter is the **diffusion constant**

$$D \simeq \begin{cases} \frac{1}{3} v_F^2 \tau, & T\tau \ll 1, \text{ dirty limit} \\ v_F^2/6T, & T\tau \gg 1. \end{cases}$$

Another parameter – phase-breaking time τ_φ – weak localization – – a precursor to Anderson localization

Critical magnetic field B_c which kills weak localization

$$|eB_c| \simeq \frac{\pi}{D\tau_\varphi} \sim |eB(\text{RHIC})|_{max}$$

A glimpse of technical details

$$G(q) = \frac{1}{\gamma_0(q_0+\mu) - \vec{\gamma}\vec{q} - m} \Rightarrow G^M(\tilde{\varepsilon}_n, \vec{q})$$

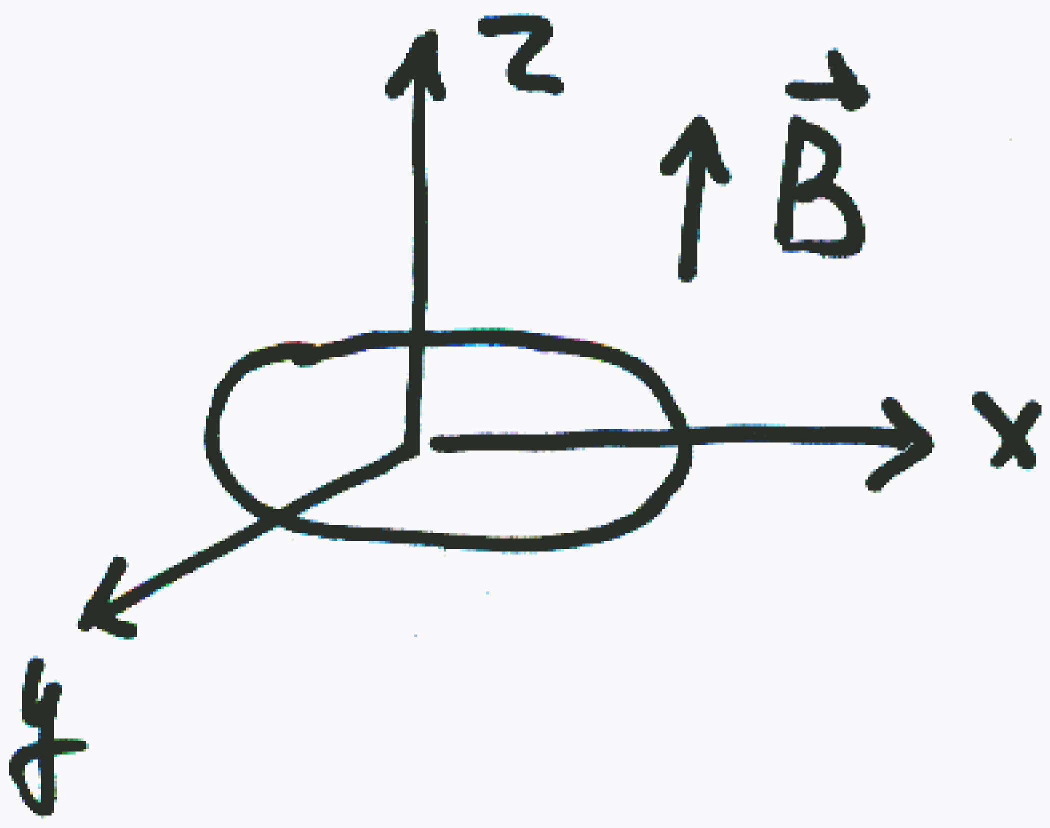
$$q_0 \Rightarrow i\tilde{\varepsilon}_n = i\left\{\frac{\pi}{\beta}(2n+1) + \frac{1}{2\tau}\,\mathrm{sgn}\,\varepsilon_n\right\}$$

$$\Rightarrow \frac{1}{-i\Omega_k + D\vec{p}^{\,2} + \frac{1}{\tau_\varphi}}$$

diffusion, or soft mode

Imposing $\vec{B}$: $D\vec{p}^{\,2} \Rightarrow Dp_z^2 + \omega_c^*(k+1/2)$

$\omega_c^* = 4DeB$, $\quad k = 0, 1, 2, \ldots$ – Landau levels



spins

$$dN = \frac{eBSL_z}{4\pi^2}\, 2\, dp_z$$

number of states at the Landau level

The observable physical quantity – – conductivity σ

$$B = 0 \Rightarrow \delta\sigma = -\frac{2e^2 D}{\pi}\int \frac{d\vec{p}}{(2\pi)^3}\frac{1}{-i\Omega + D\vec{p}^{\,2}}$$

$$p_{max} \sim 1/\ell$$

$B \neq 0$

$$\delta\sigma = -\frac{e^2(eB)}{\pi^2}\int\frac{dp_z}{2\pi}\sum_k \frac{1}{-i\Omega + Dp_z^2 + \omega_c^*(k+1/2) + \frac{1}{\tau_\varphi}} \Rightarrow$$

$$\Rightarrow \Omega = 0 \text{ (static conductivity)} \Rightarrow k_{max} \simeq \frac{D}{\omega_c^* \ell^2} \Rightarrow$$

$$\Rightarrow \delta\sigma = \frac{e^2(eB)\sqrt{D}}{\pi^2}\sum_{k=0}^{k_{max}} \frac{1}{\sqrt{n + 1/2 + \frac{1}{4eBD\tau_\varphi}}}$$

Strong Field (RHIC) $eB \gg 1/4D\tau_\varphi$

$$\rightarrow \delta\sigma(B) - \delta\sigma(0) \simeq \underline{e^2\sqrt{eB}}$$