Quantum simulations of strongly coupled electromagnetic and quark-gluon plasmas.

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OUTLINE

- Phase diagram of strongly coupled quantum Coulomb systems
- Basic assumptions of semi-classical theory for non-Abelian plasma and limits of applicability
- Simulation of thermodynamics of quantum many-particle systems by Feynman path integral Monte Carlo method
- Wigner approach to simulations of quantum dynamics
- Applications to the semi-classical models of quark-gluon plasma
- Applications to the strongly coupled electromagnetic plasma

Interaction and quantum effects in strongly coupled Coulomb systems with different masses of particles.

Classical one-component plasma - COCP

Quantum one-component plasma - QOCP

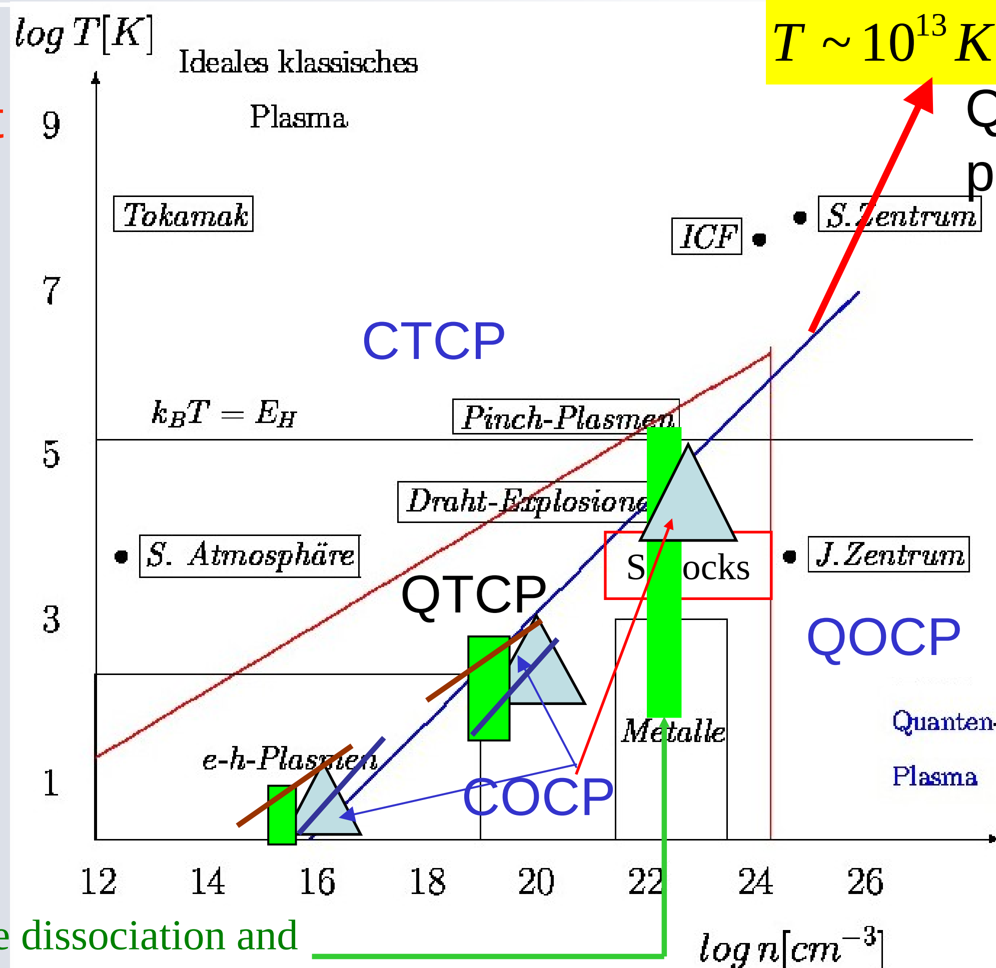
Classical two-component plasma - CTCP

Quantum two-component plasma model - QTCP

Coulomb interaction: $U_{ab}(r) = e_a e_b / r$

$T \sim 10^{13} K, n \sim 10^{45} cm^{-3}$

Quark-gluon plasma- QGP



— Nonideality boundary:

$$< U_{Coul} > = < E_{Kin} >$$

Inside: Strong Coulomb interaction, Many-body effects atomes, molecules, clusters

Degeneracy boundary

$$\lambda_e = \bar{r}$$

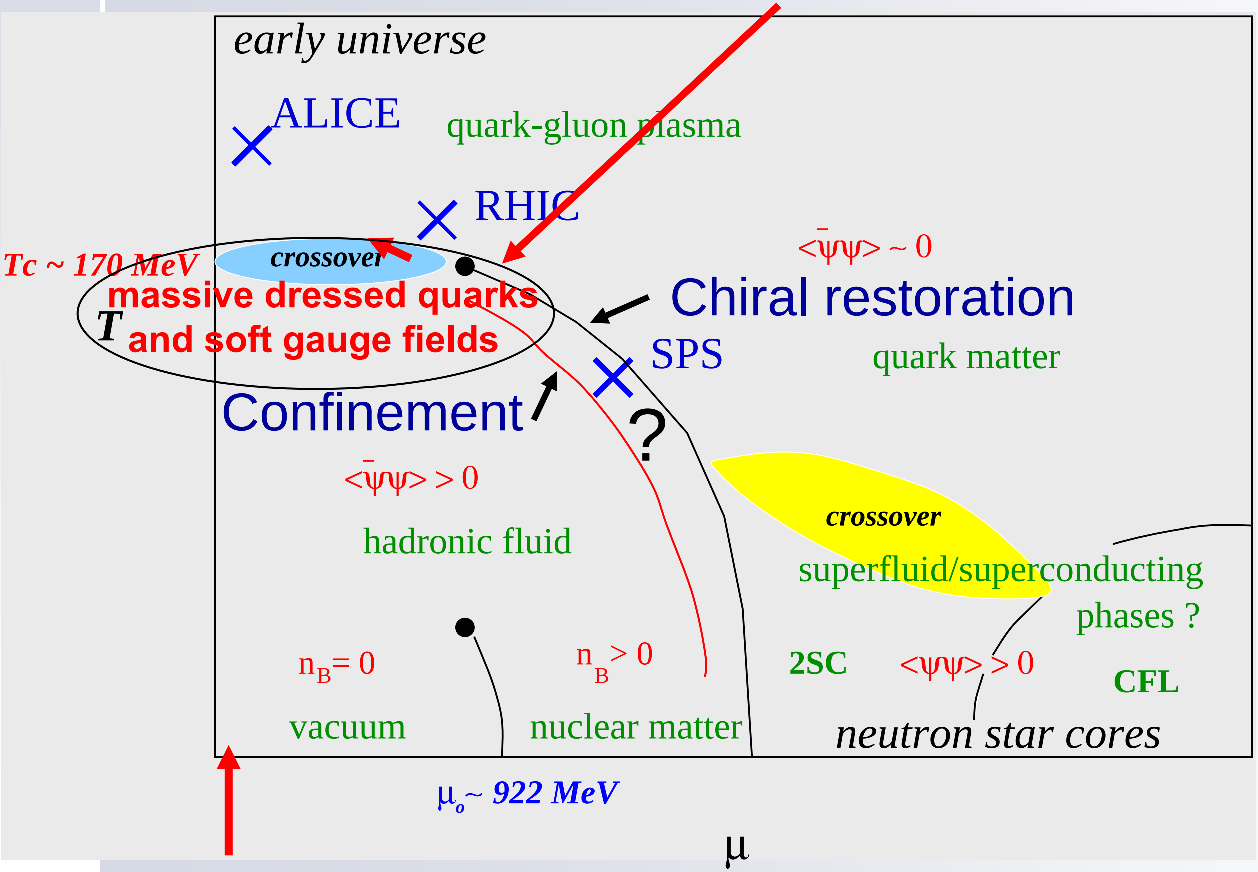
Below: overlapping electron Wave functions, Quantum and spin effects

Pressure dissociation and ionization, Mott effect

Semi-classical theory for non-Abelian system of color Coulomb quasi-particles

is **based on resummation technique and lattice simulations** allowing for consideration of quark-gluon plasma as system of **dressed** quark, antiquark and gluon presented by **color Coulomb quasiparticles** with T-dependent dispersion curves and width

Litim, Manuel, Stoecker,Bleicher,Feinberg, Richardson, Bonasera,Maruyama, Hatsuda,Shuryak, Fukushima,….



Phase diagram

(F.Karsch)

Basic asumptions of the semi-classical quasiparticle model of quark – gluon plasma

is based on resummation technique and lattice simulations allowing for consideration of quark-gluon plasma as system of dressed quark, antiquark and gluon presented by color Coulomb quasiparticles with T-dependent dispersion curves and width.

(Phys.Lett.B478,161(2000), Phys. Rev. C, **74**, 044909, (2006))

- All color quasiparticles are massive (m > T) and move non-relastivistically
- Interparticle interaction is domonated by a color Coulomb potential with distance dependent coupling constant.
- The color operators are substituted by their average values
  - classical color vectors in SU(3) (8D vectors with 2 Casimirs condit.).

The model input requires :

- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.

All the input quantiries should be deduced
from lattice QCD calculations
and substitued in quantum Hamiltonian.

Thermodynamics of quark - gluon plasma in grand canonical ensemble within Feynman formulation of quantum mechanics

$$H_\beta = K_\beta + U_C = \sum_a \sqrt{p_a^2 + m_a^2(\beta)} + U_C \approx$$

$$\approx \sum_a (N_a m_a(\beta) + \frac{p_a^2}{2m_a(\beta)}) + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}, m_a >> T$$

Grand canonical partition function

$$\Omega\left(\mu, \mu_g = 0, V, \beta\right) =$$

$$= \sum_{N_q, N_{\underline{q}}, N_g} \exp(\beta\mu(N_q - N_{\underline{q}}))Q\left(N_q, N_{\underline{q}} N_g, \beta\right) / N_q! N_{\underline{q}}! N_g!$$

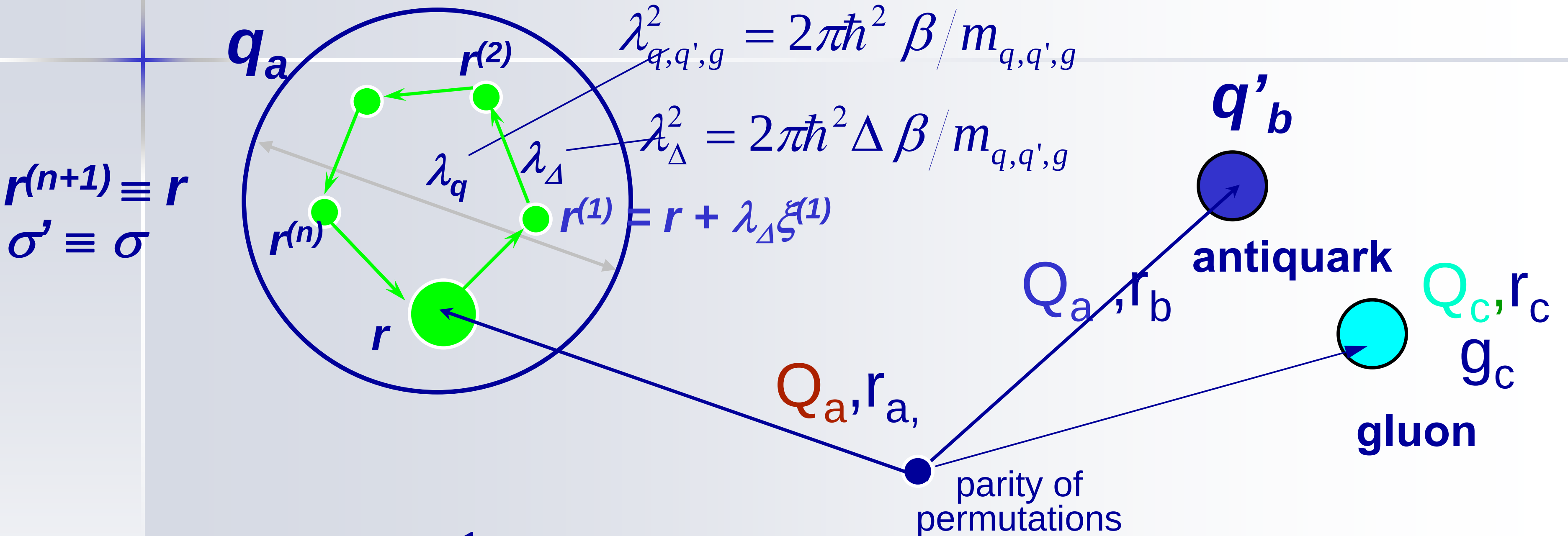
$$Q\left(N_q, N_{\underline{q}}, N_g, \beta\right) = \sum_\sigma \int_V drd\mathbf{Q}\rho\left(r, \mathbf{Q}, \sigma; \beta\right)$$

$$\rho = \exp(-\beta H(\beta)) = \underbrace{\exp(-\Delta\beta H(\beta)) \times \ldots \times \exp(-\Delta\beta H(\beta))}_{n+1}$$

$$\beta = 1/kT \qquad \Delta\beta = \beta/(n+1)$$

PATH INTEGRALS
MONTE-CARLO METHOD

quark, antiquark, gluon



$$\rho(r,Q,\sigma;\beta) = \frac{1}{\lambda_{\Delta q}^{3N_q}\lambda_{\Delta q'}^{3N_{q'}}\lambda_{\Delta g}^{3N_g}} \sum_{P=P_q,P_{q'},P_g} (\pm 1)^{\kappa_P} \int_V dr^{(1)}\ldots dr^{(n)} dQ^{(1)}\ldots dQ^{(n)} \times$$

$$\rho(r,Q;r^{(1)},Q^{(1)};\Delta\beta)\ldots\rho(r^{(n)},Q^{(n)};\hat{P}r^{(n+1)},\hat{P}Q^{(n+1)};\Delta\beta)\, S(\sigma,\hat{P}\sigma')$$

spin matrix

$$\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l+1)}) \approx \delta(Q^{(l)}-Q^{(l+1)})\,\rho(r^{(l)},Q^{(l)};r^{(l+1)},Q^{(l)})$$

Density matrix

$$\sum_\sigma \rho(r,Q,\sigma;\beta) = \frac{1}{\lambda_\Delta^{3N_q}\lambda_\Delta^{3N_{q'}}\lambda_\Delta^{3N_g}} \sum_{s=0}^{N_q}\sum_{s'=0}^{N_{q'}}\sum_{s''=0}^{N_g} \rho_{ss's''}([rQ],\beta)$$

$$\rho_{ss's''}([rQ],\beta) = \frac{C_{N_q}^{s}}{2^{N_q}}\frac{C_{N_{q'}}^{s'}}{2^{N_{q'}}}\frac{C_{N_g}^{s''}}{2^{N_g}} \exp\{-\beta U([rQ],\beta)\}\times$$

$$\times \prod_{l=1}^{n}\prod_{p=1}^{N_e}\phi_{pp}^{l}\,\det\left|\psi_{ab}^{n,1}\right|_s \prod_{p=1}^{N_i}\tilde{\phi}_{pp}^{l}\,\det\left|\tilde{\psi}_{ab}^{n,1}\right|_{s'} \prod_{p=1}^{N_i}\tilde{\tilde{\phi}}_{pp}^{l}\,\mathrm{per}\left|\tilde{\tilde{\psi}}_{ab}^{n,1}\right|_{s''}$$

$$U([rQ],\beta) = \sum_{l=0}^{n}\frac{U_l^{qq'g}([r^{(l)}Q],\beta)}{n+1}$$

Pairwise sum of Kelbg potentials for each l=0,…,n

Exchange matrix

$$\left\|\psi_{ab}^{n,1}\right\|_s \equiv \left\|\exp\left\{-\frac{\pi}{\lambda_\Delta^2}\left|(r_a-r_b)+y_a^n\right|^2\right\}\right\|_s$$

Color Kelbg potential

Richardson, Gelman, Shuryak, Zahed, Harmann, Donko, Leval, Kalman (r=0 ?)

$$x_{ab} = |\mathbf{r}_{ab}| / \tilde{\lambda}_{ab} \qquad \tilde{\lambda}_{ab} = \hbar^2 \varepsilon / 2\mu_{ab}$$

$$\Phi^{ab}(x_{ab}, \varepsilon) = \frac{<\vec{Q}_a | \vec{Q}_b > C_{ab} g^2}{4\pi \tilde{\lambda}_{ab} x_{ab}} \left\{ 1 - e^{-x_{ab}^2} + \sqrt{\pi} x_{ab} \left[1 - erf(x_{ab}) \right] \right\}$$

$$|\mathbf{r}_{ab}| \to 0$$

$$\sim \frac{< Q_a | Q_b > g^2 \sqrt{\pi}}{4\pi \tilde{\lambda}_{ab}}$$

$$|\mathbf{r}_{ab}| >> \tilde{\lambda}_{ab}$$

$$\frac{< Q_a | Q_b > g^2}{4\pi \tilde{\lambda}_{ab} |x_{ab}|}$$

Objects Q are color coordinates of quarks and gluons
There is no divergence at small interparticle distances and it has a true asymptotics (T, x_{ab})

Ha -> $k_B T_c$, T_c =175 Mev,
$T_c < T$, $m_a \sim 5 k_B T_c / c^2$,
$L_o \sim hc / k_B T_c$, $r_s = <r>/L_o < 0.1$,
$L_o \sim 7\ 10^{-15}$ m, $x_{ab} \sim 1$

First studies and testing method within simplified quasiparticle model of quark – gluon plasma.

- All color quasiparticles are massive (m > T) and move non-relastivistically
- All quasiparticle masses are the same.
- We do not distiguish between quark flavors.
- Interparticle interaction is domonated by a color Coulomb potential with interparticle distance dependent coupling constant.
- The color operators are substituted by their average values
  – classical color vectors in SU(2) (3D vec.with 1Cas.) instead of SU(3).

Canonical ensemble instead of grand canonical ensemble.

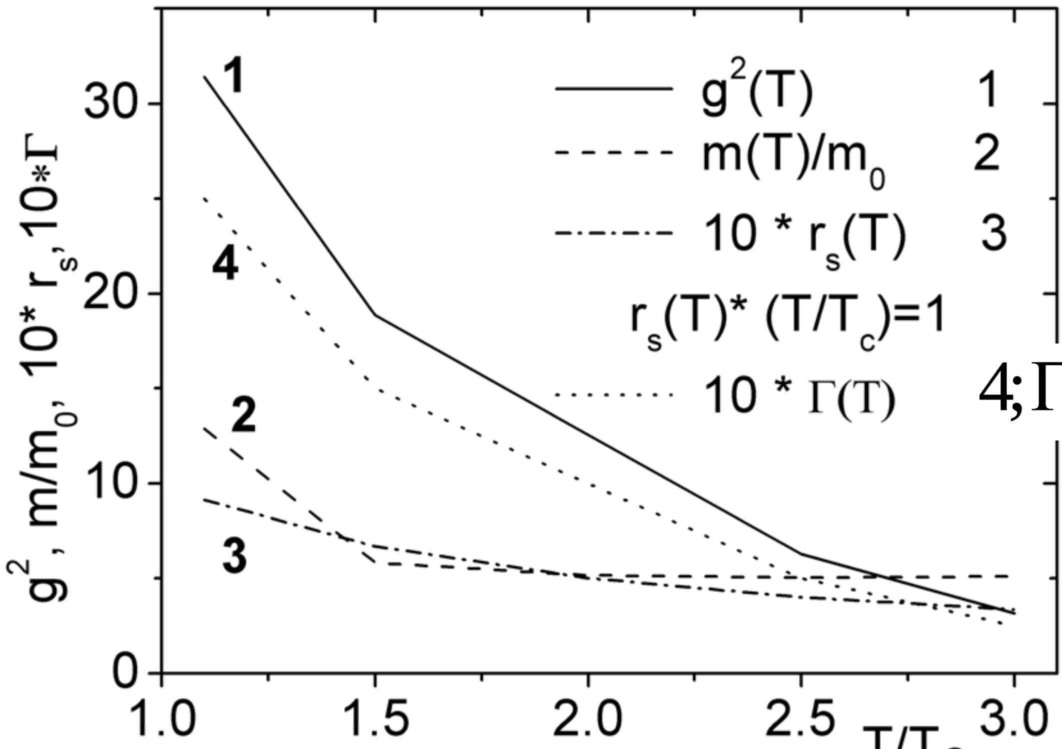
- Numbers of quarks, antiquarks and gluons are equal.

The model input requires :

- The temperature dependence of the quasiparticle mass.
- The temperature dependence of the coupling constant.
- The temperature dependence of the quasiparticle density.

All the input quantiries should be deduced from lattice QCD calculations.

Input quantities from lattice calculations in simplified version



Coupling constant

$$\alpha(T) = g^2(T)/4\pi \sim 1$$

Ratio of potential to kinetic energy per quasiparticle

$$4;\Gamma(T) \sim U/K \sim 1$$

$$< Q_a | Q_b > g^2$$

SU(3) SU(2)

Quasiparticle masses: $m(T)/T_c \approx \frac{0.9}{(T/T_c - 1)} + 3.45 + 0.4T/T_c$

Density: $n\sigma^3 \approx 0.24(T/T_c)^3$ $4\pi r_s^3 n\sigma^3/3 = 1$ $\sigma \approx 1.1 fm$ $r_s(T) = < r > /\sigma \approx 1/(T/T_c)$

[Phys. Rev. C, **74**, 044909, (2006), Phys. Rev. D, 73, 014509, (2006)]

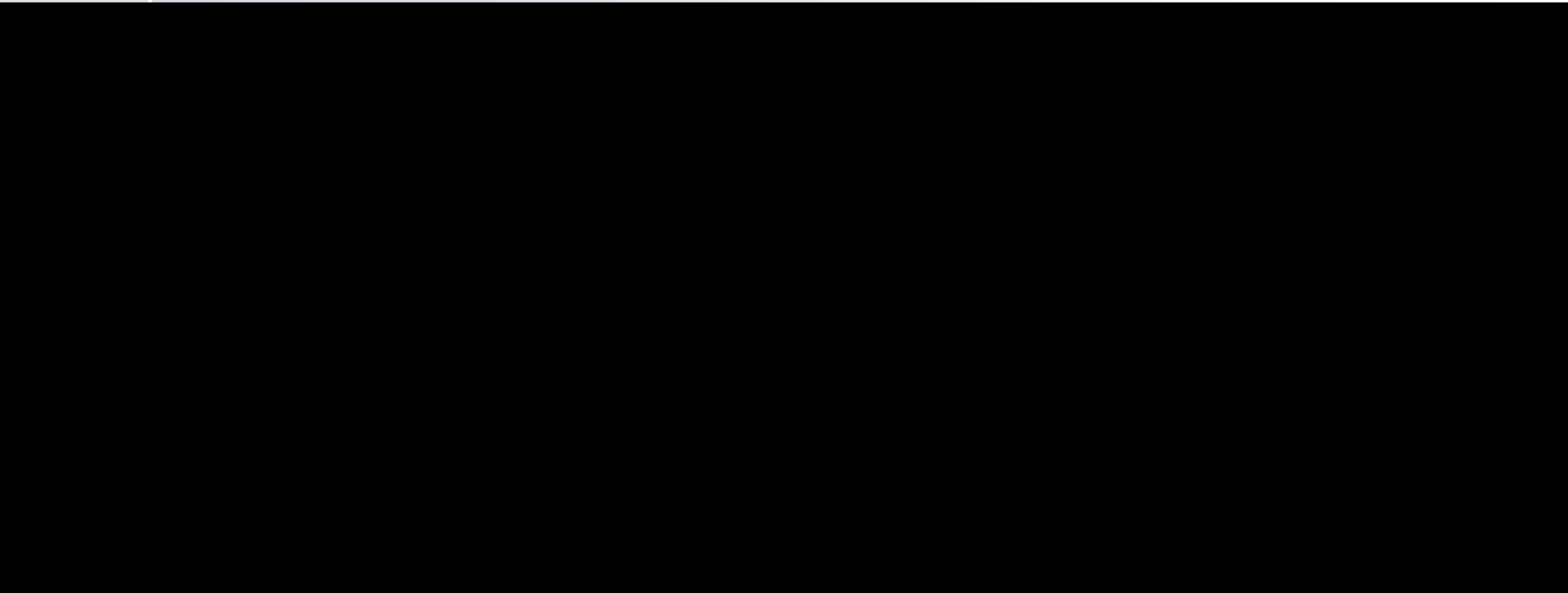
Snapshots of typical configurations

$T=1.1T_0$

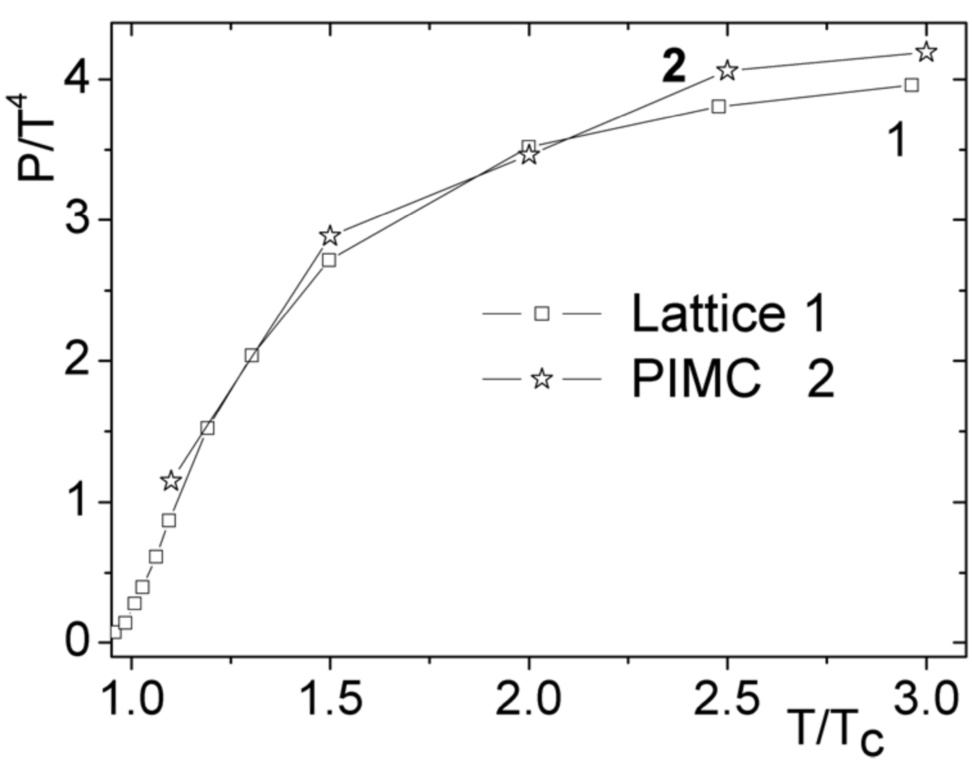
Gas-like rarefied system of 3-4 quasiparticle clusters

$T=3T_0$

Liquid-like dense system of individual quasiparticles



Equation of State.
Comparison path integral results with lattice (2+1) QCD



Pair distribution functions in canonical emsemble
Color correlation functions

$$H_\beta \approx \sum_a (N_a m_a(\beta) + \frac{p_a^2}{2m_a(\beta)}) + \sum_{a,b} \frac{g^2(|r_a - r_b|, \beta) C_{ab} < \vec{Q}_a | \vec{Q}_b >}{4\pi |r_a - r_b|}$$

$$Z(N_q, N_{q'} N_g, V, \beta) = Q(N_q, N_{q'} N_g, \beta) / N_q! N_{q'}! N_g!$$

$$Q(N_q, N_{q'}, N_g, \beta) = \sum_\sigma \int_V drdQ \rho(r, Q, \sigma; \beta)$$

$$g_{ab}(|R_1 - R_2|) = g_{ab}(R_1, R_2) = \frac{1}{Q(N_q, N_{q'}, N_g)} \times$$

$$\sum_\sigma \int_V drd\, Q\, \delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) \rho(r, Q, \sigma; \beta),$$

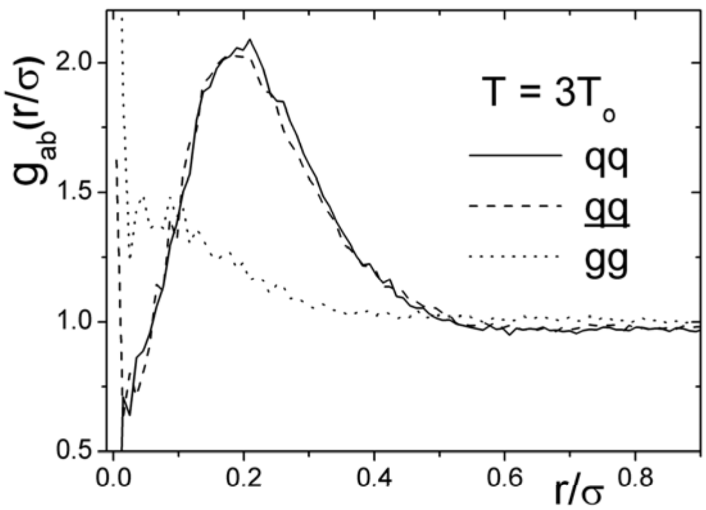
$$c_{ab}(R_1 - R_2)_{Def} = \frac{1}{Q(N_q, N_{q'}, N_g)} \sum_\sigma \int_V drd\, Q \times$$

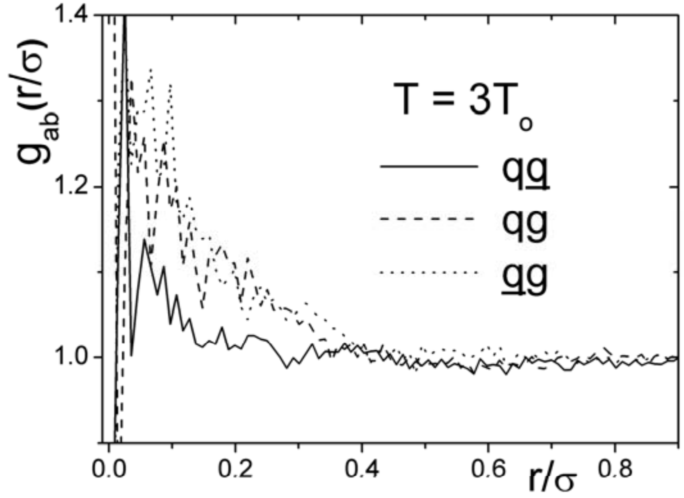
$$\delta(R_1 - r^a{}_1) \delta(R_2 - r^b{}_2) < Q^1{}_a | Q^2{}_b > \rho(r, Q, \sigma; \beta)$$

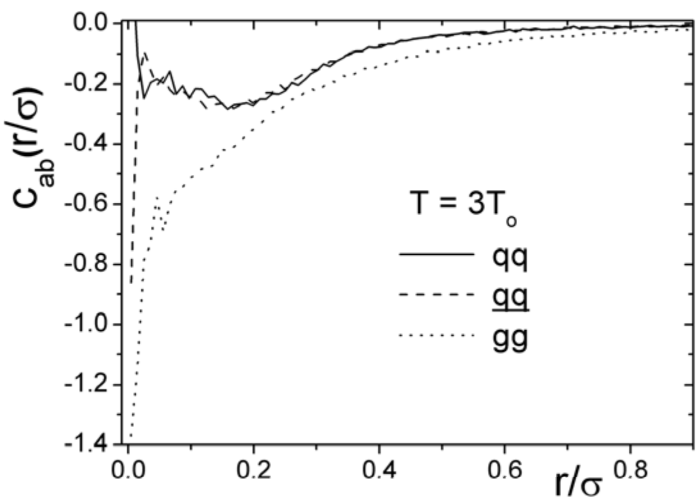
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

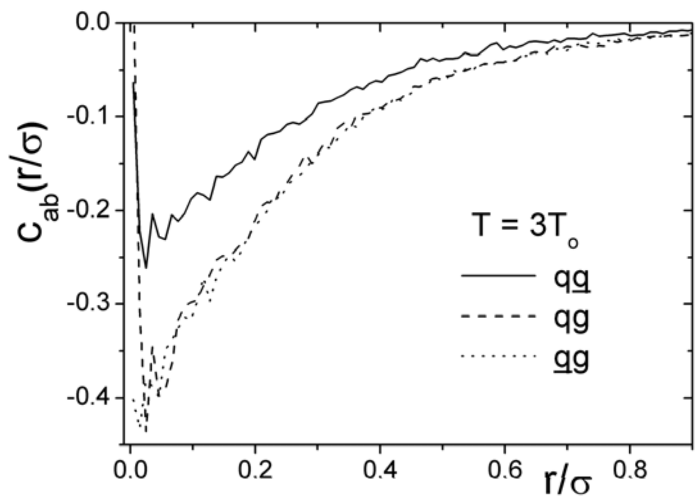
Similar quasiparticles

Different quasiparticles





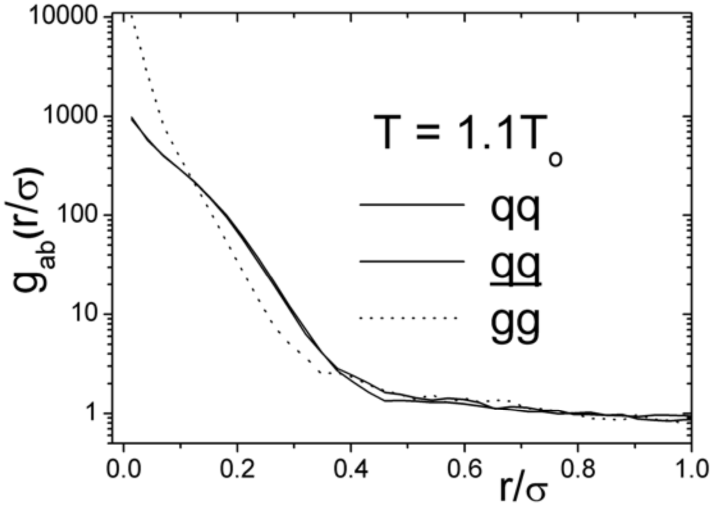


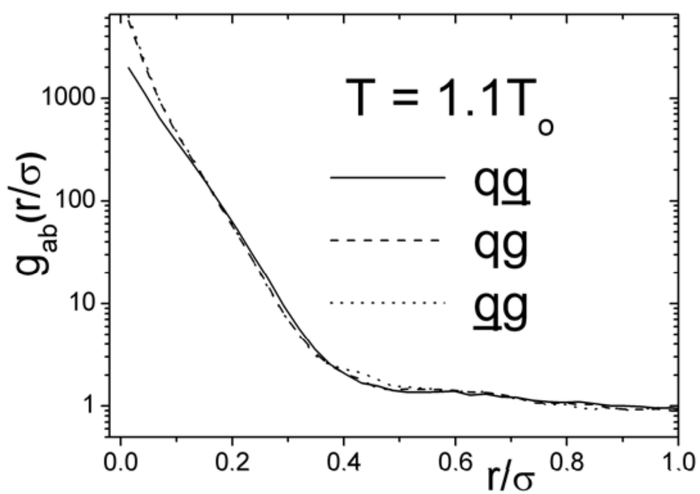


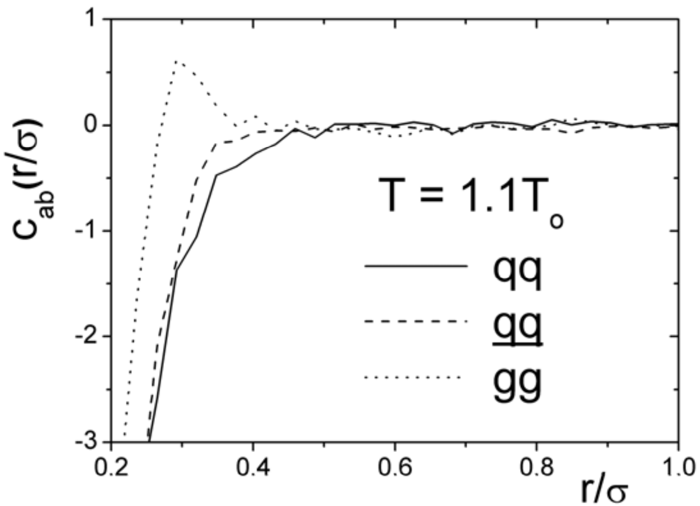
PAIR DISTRIBUTION AND COLOR CORRELATION FUNCTIONS

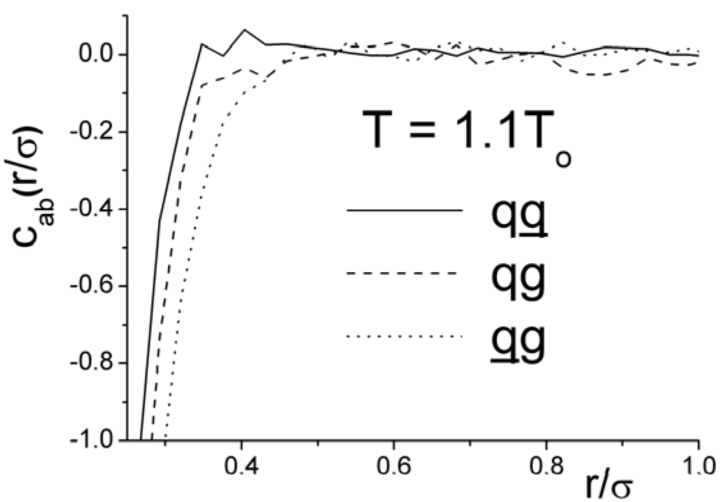
Similar quasiparticles

Different quasiparticles









Estimation of the quasiparticle bound states

The product $r^2 g_{ab}(r)$ has the physical meaning of a probability to find an two quasipartices at a distance |r| from each other. On the other hand, the corresponding quantum mechanical probability is the product of r^2 and the two-particle Slater sum $\sum_{ab}$

$$\sum_{ab} = 8\pi^{3/2}\lambda_{ab}^3 \sum_{E_\alpha}^{\infty} |\Psi_\alpha(r)|^2 \exp(-\beta E_\alpha) = \sum_{ab}{}^d + \sum_{ab}{}^c$$

$$\sum_{ab}{}^c >> \sum_{ab}{}^d \Rightarrow r^2 * \sum_{ab}{}^c \sim r^2$$

$$\sum_{ab}{}^d = 8\pi^{3/2}\lambda_{ab}^3 \sum_{E_\alpha}^{E'} |\Psi_\alpha(r)|^2 \exp(-\beta E_\alpha)$$

$$\sum_{ab}{}^c << \sum_{ab}{}^d \Rightarrow r^2 * \sum_{ab}{}^d >> r^2$$

$$r^2 g(r) \sim r^2 (\sum_{ab}{}^d + \sum_{ab}{}^c)$$

$$r^2 g(r) \sim r^2 \sum_{ab}{}^d > r^2, r < a_b$$

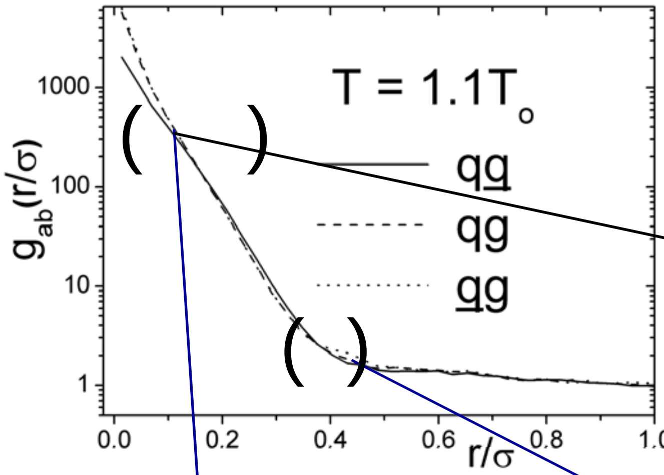
$$r^2 g(r) \sim r^2 \sum_{ab}{}^c \sim r^2, r > a_b$$

Peak related to bound states at interparticle distances of order one Bohr radius exists if discrete bound states in electron-hole or hydrogen plasma are well populated (low temperatures and small densities)

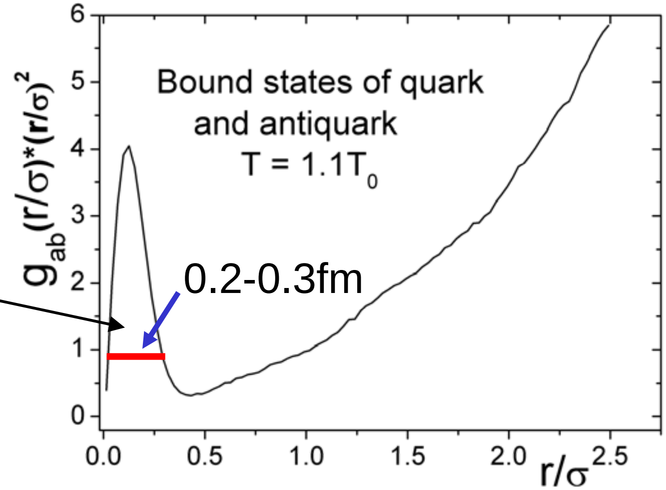
For low densities it is reasonable to choose $E' > -1/\beta$ while for high densities is appropriate $E' = -Ry/r_s$ since the quasiparticle in states with energy $E_\alpha > E'$ can be considered as free particles.

Color bound states and mean force potential ($T = 1.1\,T_c$)

Distribution functions

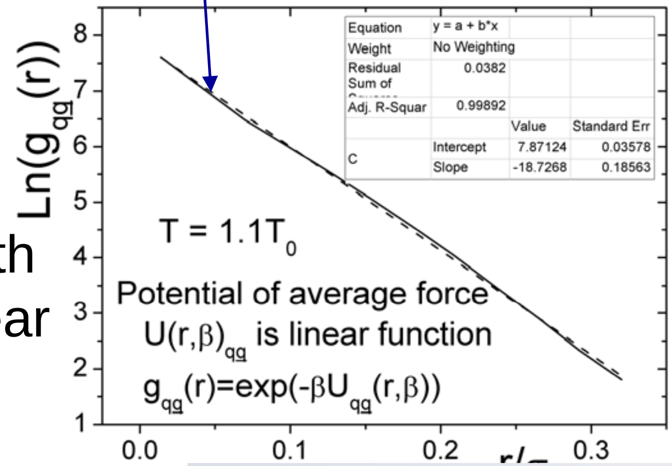


r^2 * Distribution function

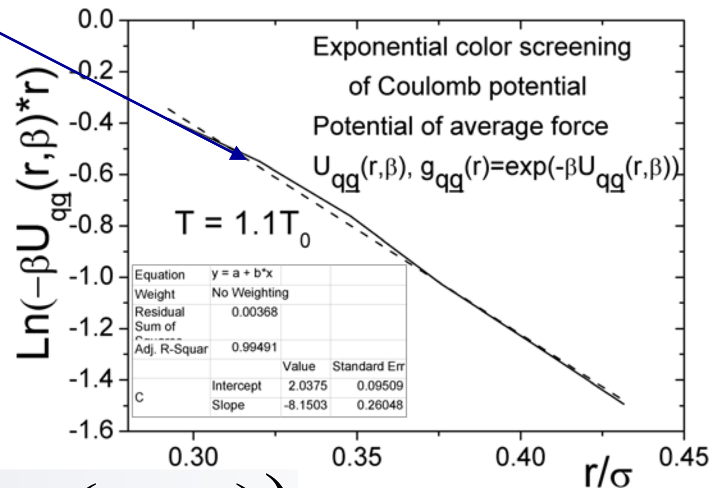


Linear part of the mean force potential

Depth U > 1 GeV agree with lattice near T_c



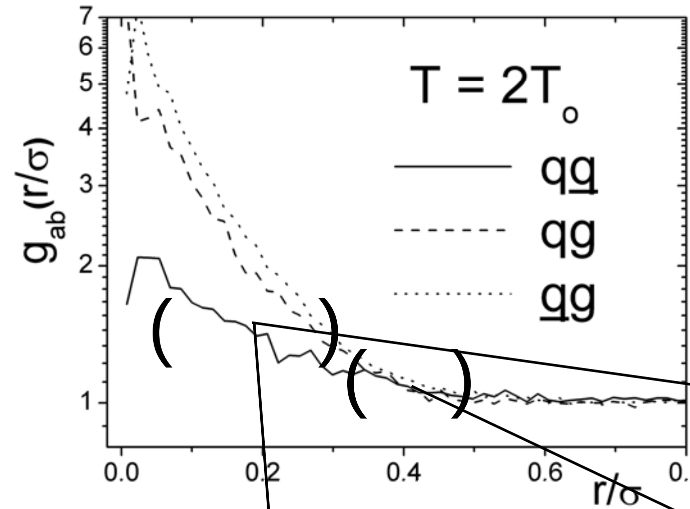
Color screening part of mean force potential



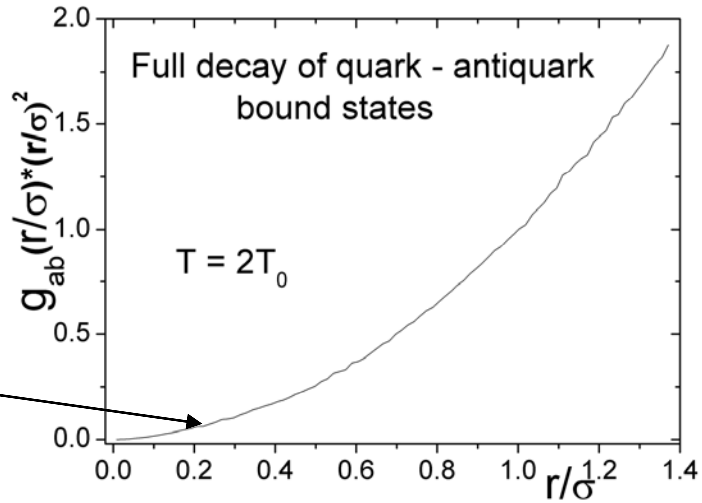
$$g_{q\bar{q}}(r) = \exp\left(-\beta U_{q\bar{q}}(r,\beta)\right)$$

Decay of color bound states and mean force potential ($T = 2T_c$)

Distribution functions

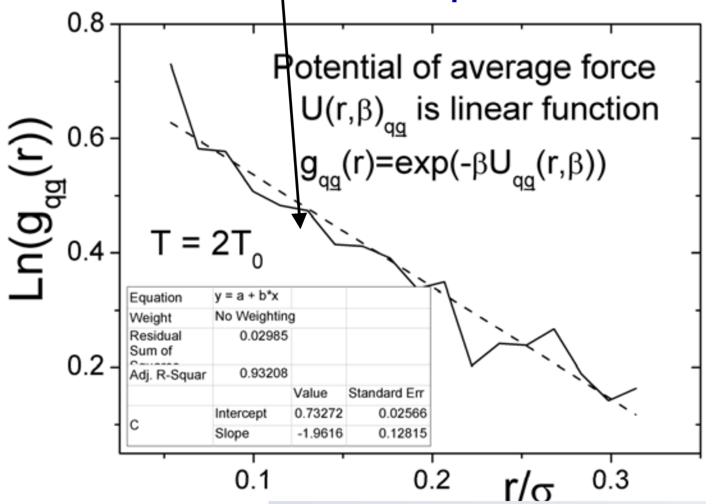


r^2 * Distribution function

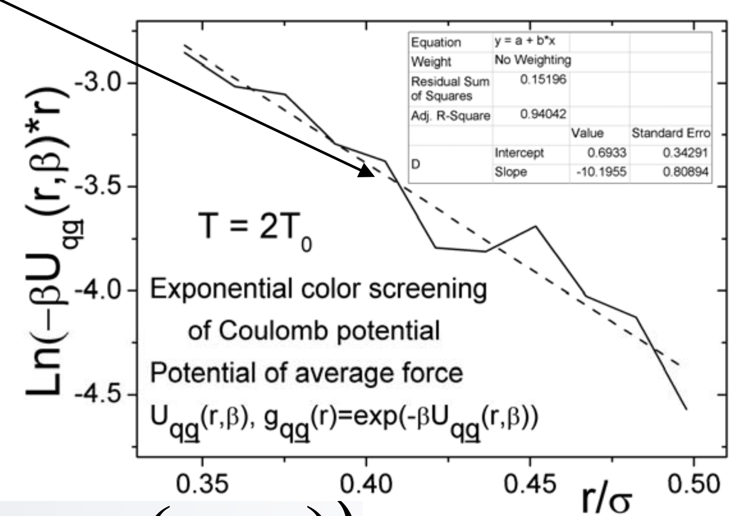


Linear part of the mean force potential

Depth~ 175 MeV



Color screening part of mean force potential



$$g_{q\bar{q}}(r) = \exp\left(-\beta U_{q\bar{q}}(r,\beta)\right)$$

Kinetic properties of quark – gluon plasma in canonical ensemble

$$C_{FA}(t) = Z^{-1} Tr\{F \exp(i\frac{Ht_c}{h}) A \exp(-i\frac{Ht_c}{h})\};$$

$$H = K + V(qQ), t_c = t - i\frac{\beta h}{2}, \beta = \frac{1}{kT},$$

$$Z = Tr\{\exp(-\beta H)\}$$

$$C_{FA}(t) = \frac{1}{(2\pi h)^{2\nu}} \iint dQ_1 dp_1 dq_1 dp_2 dq_2 F(p_1, q_1) A(p_2, q_2) \times$$

$$W(p_1, q_1, Q_1; p_2, q_2, Q_1; t; i\beta h),$$

In this model we use approximation

$$\delta(Q_1 - Q_1^{'})\delta(Q_2 - Q_2^{'})\delta(Q_1 - Q_2)$$

$$A(p,q) = \iint d\xi \exp(-i\frac{p\xi}{h}) < q - \frac{\xi}{2} | A | q + \frac{\xi}{2} >$$

Weil symbols of operators

$$W(p_1, q_1, Q_1; p_2, q_2, Q_1; t; i\beta h) = Z^{-1} \iint d\xi_1 d\xi_2 \exp(i\frac{p_1\xi_1}{h}) \exp(i\frac{p_2\xi_2}{h}) \times$$

$$< q_1 + \frac{\xi_1}{2} | \exp(i\frac{Ht_c}{h}) | q_2 - \frac{\xi_2}{2} >< q_2 + \frac{\xi_2}{2} | \exp(-i\frac{Ht_c}{h}) | q_1 - \frac{\xi_1}{2} >$$

Integral equation

$$W(p_1,q_1,Q_1;p_2,q_2,Q_1;t;i\beta h)=\bar{W}(p^0{}_1,q^0{}_1,Q_1^0;p^0{}_2,q^0{}_2,Q_2^0;0;i\beta h)+$$

$$+\int_0^t d\tau\iint ds\iint d\eta W(p_1^\tau-s,q_1^\tau,Q_1^\tau;p_2^\tau-\eta,q_2^\tau,Q_2^\tau;\tau;i\beta h)\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau),$$

$$\gamma(s,q_1^\tau,Q_1^\tau;\eta,q_2^\tau,Q_2^\tau)=\frac{1}{2}\{\omega(s,q_1^\tau,Q_1^\tau)\delta(\eta)-\omega(\eta,q_2^\tau,Q_2^\tau)\delta(s)\},F(q,Q)=-\nabla_q V(q,Q)$$

$$\omega(\eta,q,Q)=\frac{4}{(2\pi h)^\nu h}\iint dq'V(q-q',Q)Sin(\frac{2sq'}{h})+F(q,Q)\bullet\frac{d\delta(s)}{ds}$$

Positive time direction

$$\frac{dq_1^t}{dt}=\frac{1}{2m}p_1^t,\frac{dp_1^t}{dt}=\frac{1}{2}F(q_1^t,Q_1^t),$$

$$\frac{dQ_{1,i}^{t,a}}{dt}=\frac{1}{2}\sum_{b,c}f^{abc}Q_{1,i}^b\nabla_{Q_{1,i}^c}V(q_1^t,Q_1^t),$$

$$p_1^t(t,p_1,q_1,Q_1)=p_1,q_1^t(t,p_1,q_1,Q_1)=q_1,Q_1^t(t,p_1,q_1,Q_1)=Q_1$$

$$\frac{dq_2^t}{dt}=-\frac{1}{2m}p_2^t,\frac{dp_2^t}{dt}=-\frac{1}{2}F(q_2^t,Q_2^t),$$

$$\frac{dQ_{2,i}^{t,a}}{dt}=-\frac{1}{2}\sum_{b,c}f^{abc}Q_{2,i}^b\nabla_{Q_{2,i}^c}V(q_2^t,Q_2^t),$$

$$p_2^t(t,p_2,q_2,Q_1)=p_2,q_2^t(t,p_2,q_2,Q_1)=q_2,Q_2^t(t,p_2,q_2,Q_1)=Q_1$$

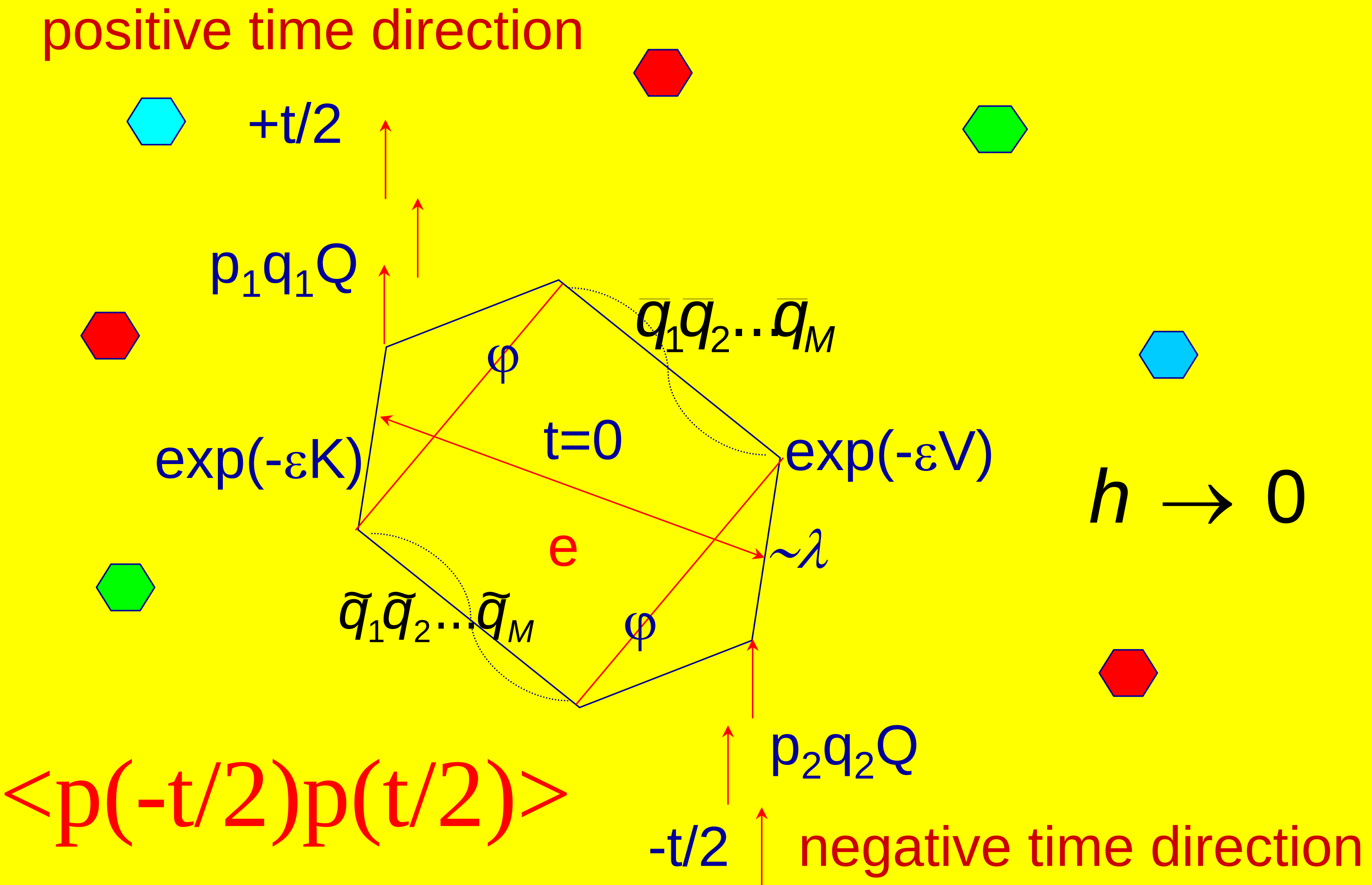
Color dynamics in SU(2) or SU(3)

Initial conditions

Hamiltonian eqations

Negative time direction

Schematic snapshot for color phase space dynamics

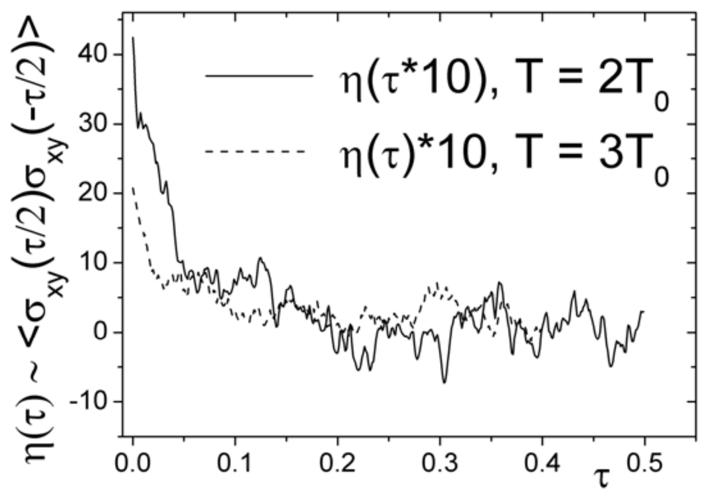


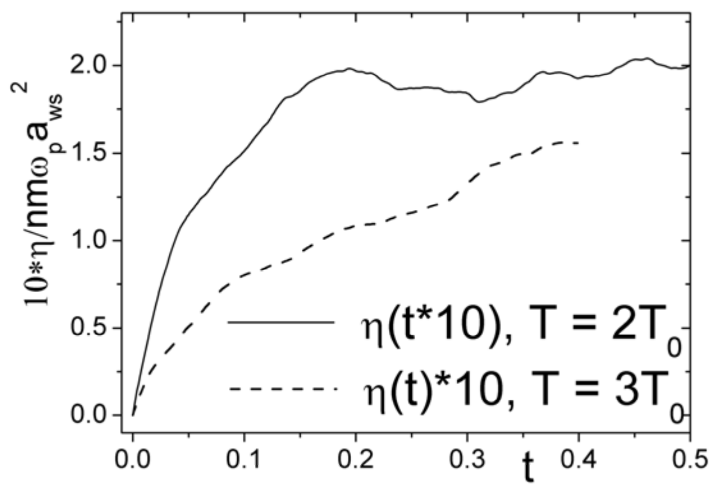
Time autocorrelation function of the stress energy tensor and shear viscosity of quark –gluon plasma

$$\eta(\tau) = \frac{n}{3k_B T}\left\langle \sum_{X<Y} \sigma_{XY}(\tau/2)\sigma_{XY}(-\tau/2) \right\rangle$$

$$\sigma_{XY}(\tau) = \frac{1}{N}\left(\sum_{i=1}^{N} m_i v_{ix} v_{iy} + \frac{1}{2}\sum_{i \neq j} r_{ij,x} F_{ij,y} \right)$$

$$\eta = \lim_{t\to\infty} \eta(t) = \lim_{t\to\infty} \int_0^t d\tau \eta(\tau)$$



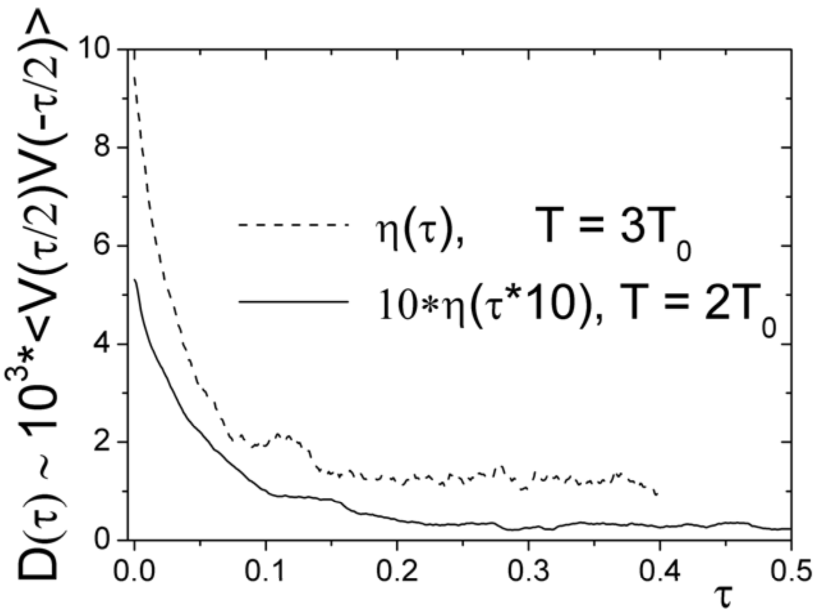


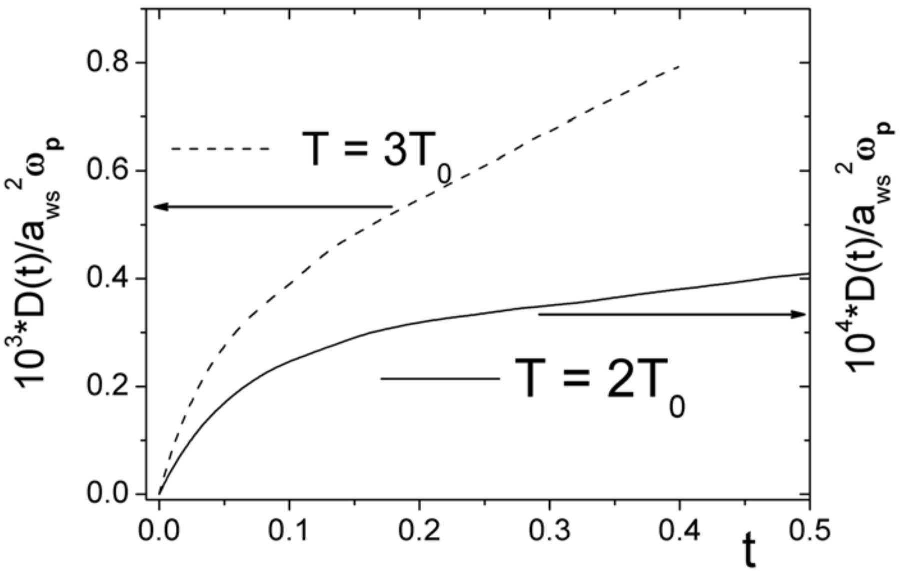
Velocity autocorrelation function and diffusion constant QGP

$$D(\tau) = < v(\tau/2) v(-\tau/2) > =$$

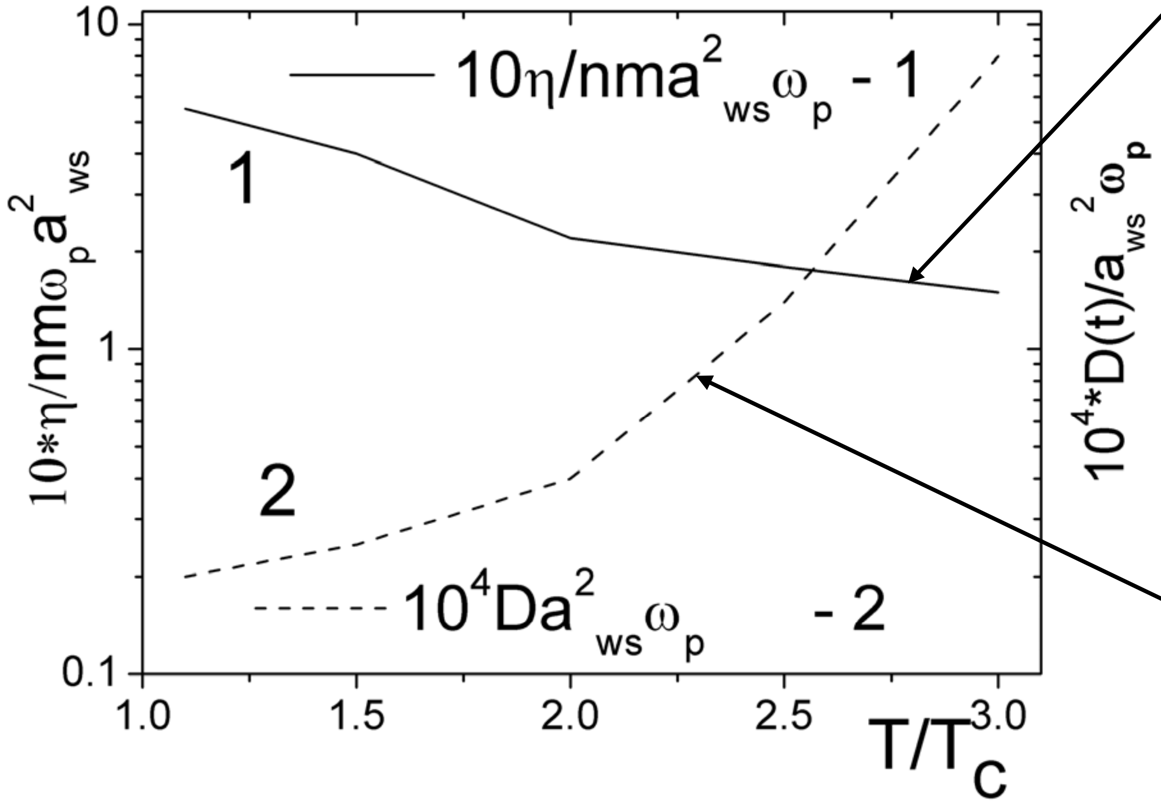
$$= \frac{1}{3N} < \sum_{i=1}^{N} \vec{v}_i(\tau/2) \bullet \vec{v}_i(-\tau/2) >$$

$$D = \lim_{t\to\infty} D(t) = \lim_{t\to\infty} \int_0^t d\tau D(\tau)$$





Diffusion coefficient and shear viscosity



Shear viscosity agrees with Gelman et al., 2006

Diffusion coefficient is ~10^3 lower in comparison with Gelman et al., 2006

Electromagnetic plasma
Crystallization of protons

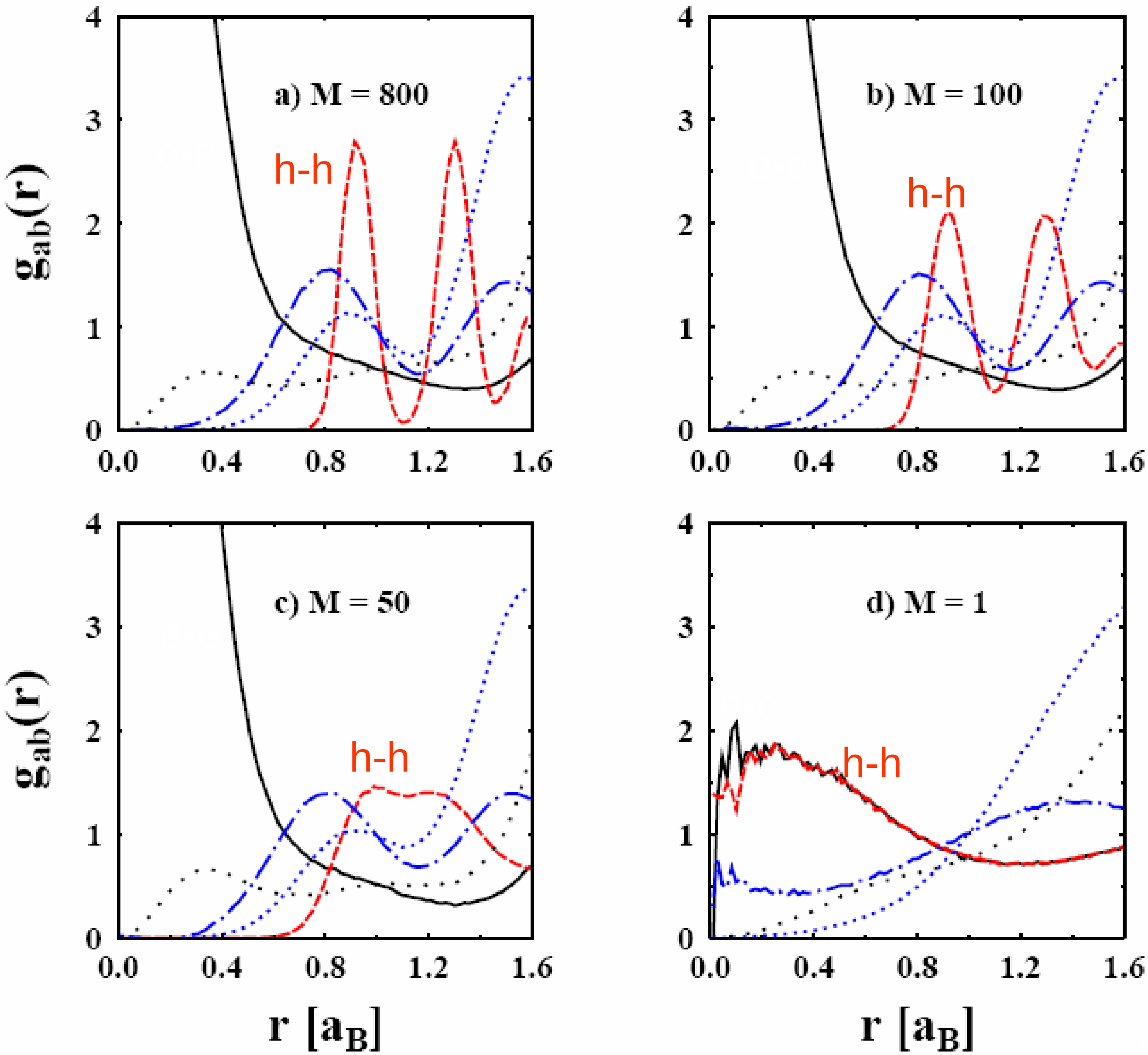
HYDROGEN, PIMC-SIMULATION,
n = 10^{25} cm^{-3}, T=10 000 K°
Bloch oscillation of electron density in periodic potential

HOLE CRYSTALLIZATION AND QUANTUM MELTING

$\langle r \rangle / a_B = 0.63$

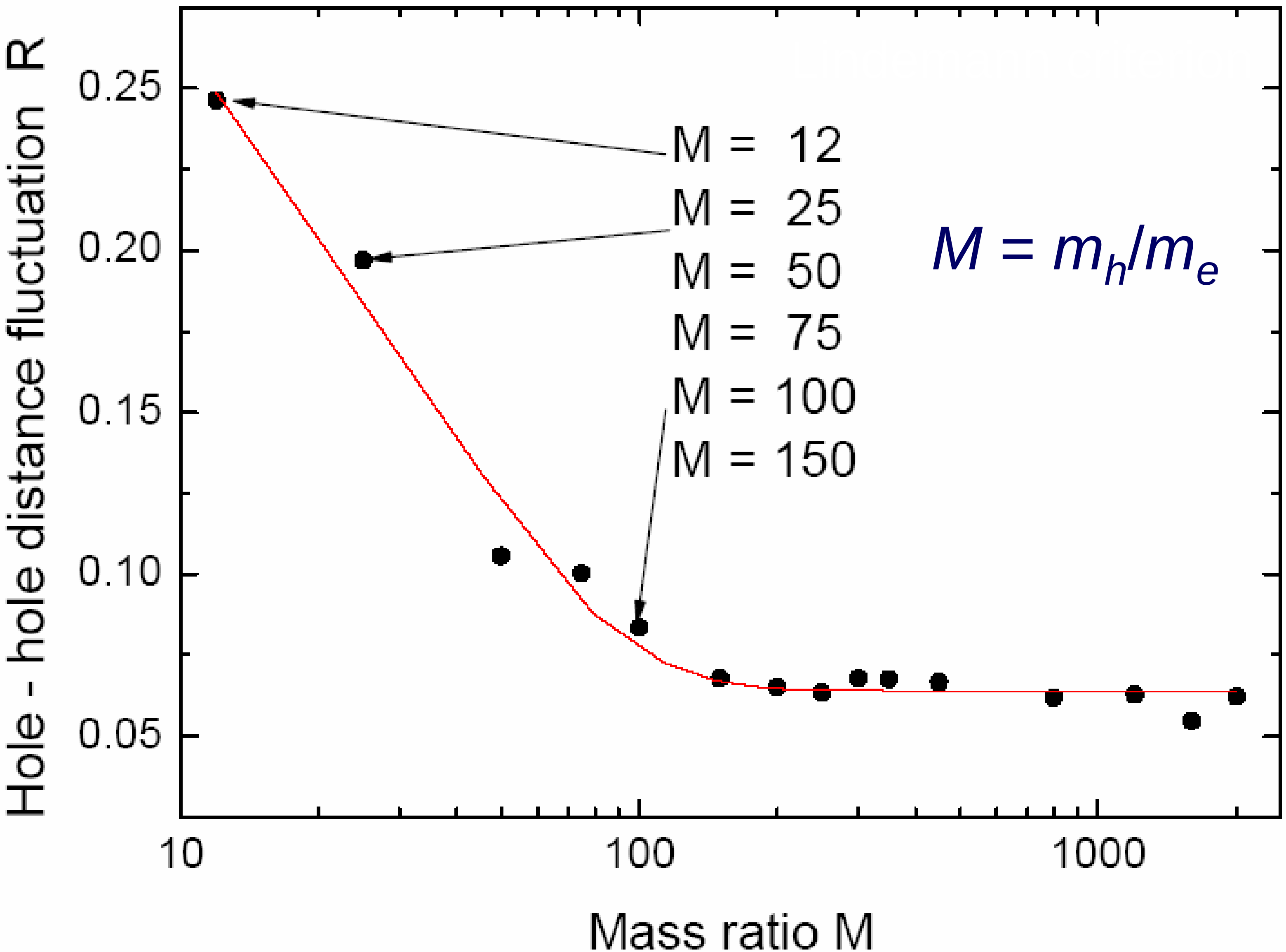
$T = 0.064 E_b$

$M = m_h / m_e$



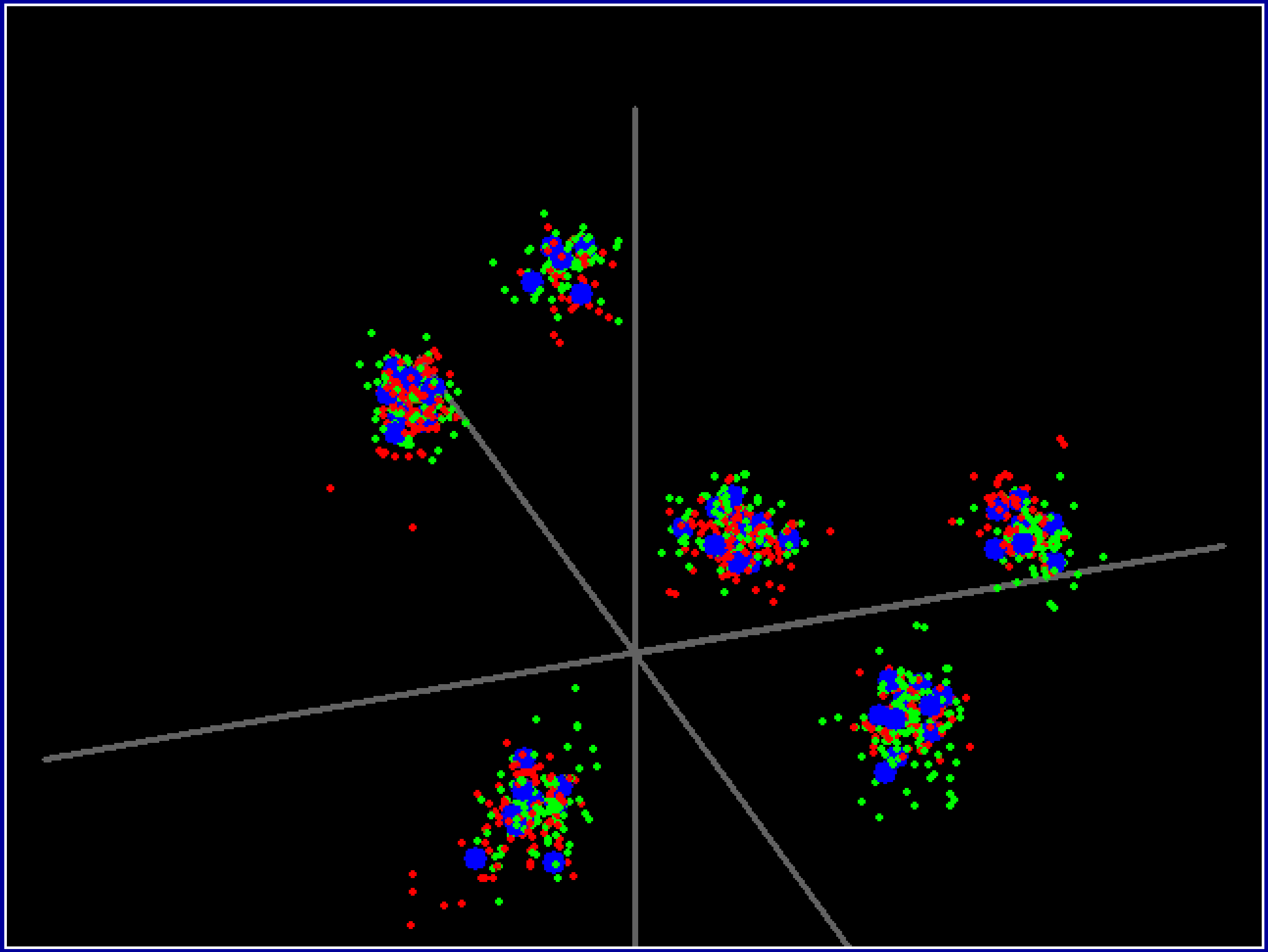
e-e
h-h
e-h
(e-e) · r^2
(e-h) · r^2

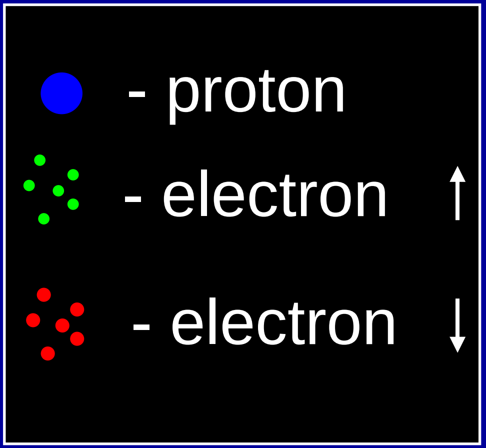
HOLE-HOLE DISTANCE FLUCTUATIONS



Phase transition to metallic state
Metallic drops and many particle clusters in hydrogen plasma

3D quantum two-component plasma.





T = 10000 K, n = 10^{22} cm^{-3}, ρ = 0.0167 g/cm^{3}

CONCLUSIONS

• Path integral Monte Carlo is a reliable and very fast method of simulation thermodynamic properties in a wide range of plasma parameters

•Quantum dynamics can be constructed on the basis of Feynman and Wigner formulation of quantum mechanics

•The developed numerical approach can be applied to consideration of EM and QG plasmas.

•Results of simulations agree with available theoretical and experimental data.

Thank you for attention.

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