Energy and system-size dependence of the Chiral Magnetic Effect

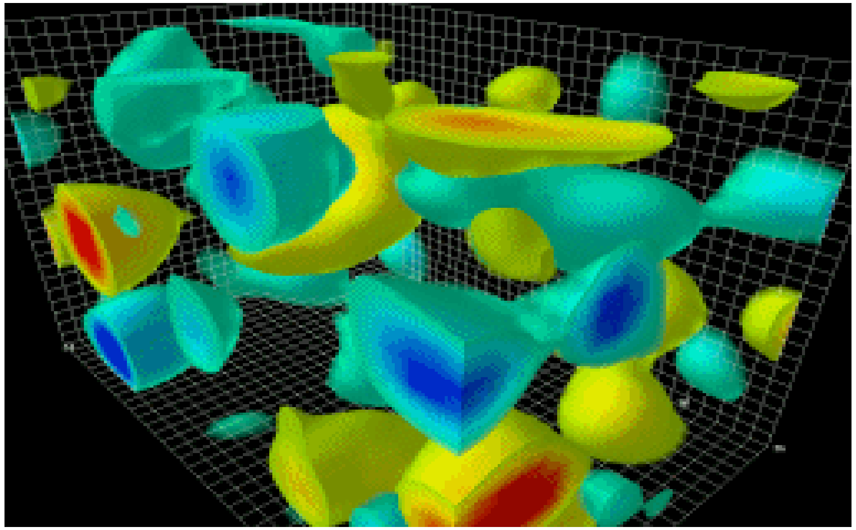
V. Toneev, JINR, Dubna

- *Introductory remarks*
- *A CME estimate (in collab. with D.Kharzeev, V.Skokov)*
- *Kinetics with e.m. fields (in collab. with E.Bratkovskaya, W.Cassing, V.Konchakovski, S.Voloshin, V.Voronyuk)*
- *Conclusions*

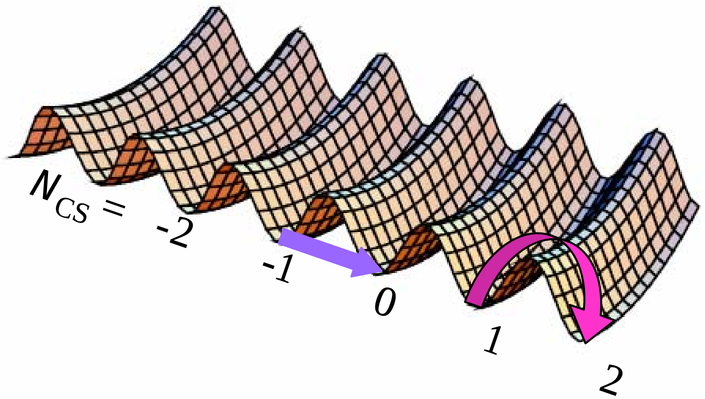
Parity violation in strong interactions

In QCD, chiral symmetry breaking is due to a non-trivial topological effect; among the best evidence of this physics would be event-by-event strong parity violation.

The volume of the box is 2.4 by 2.4 by 3.6 fm.
The topological charge density of 4D gluon field configurations. (Lattice-based animation by *Derek Leinweber)*



Energy of gluonic field is periodic in N_{cs} direction (~ a generalized coordinate)



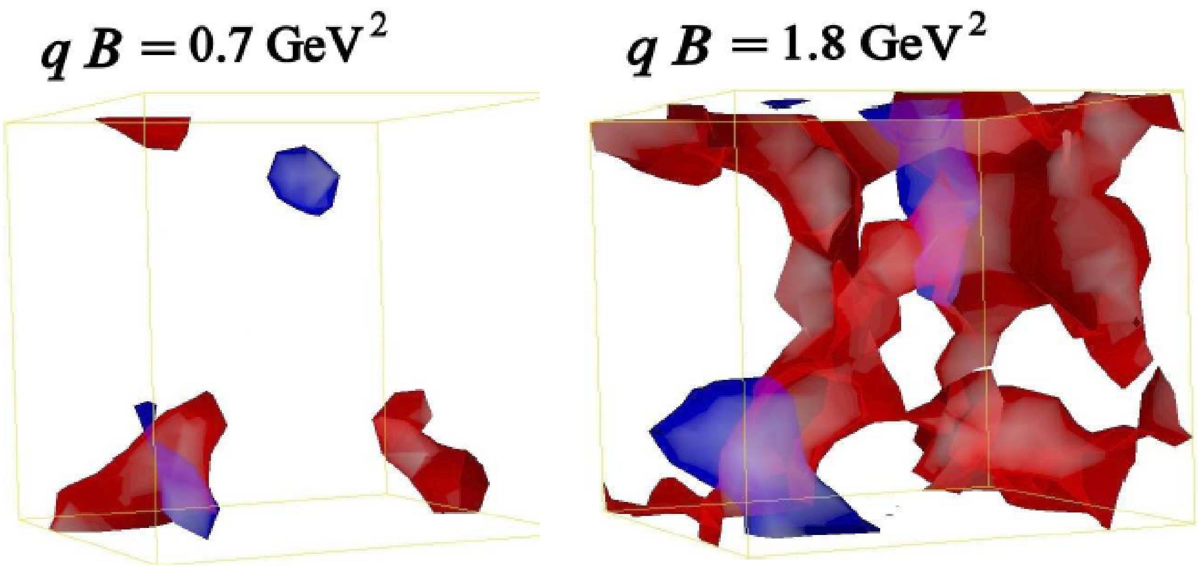
Instantons and sphalerons are localized (in space and time) solutions describing transitions between different vacua via tunneling or go-over-barrier

Dynamics is a random walk between states with different topological charges.

Charge separation: CP violation signal

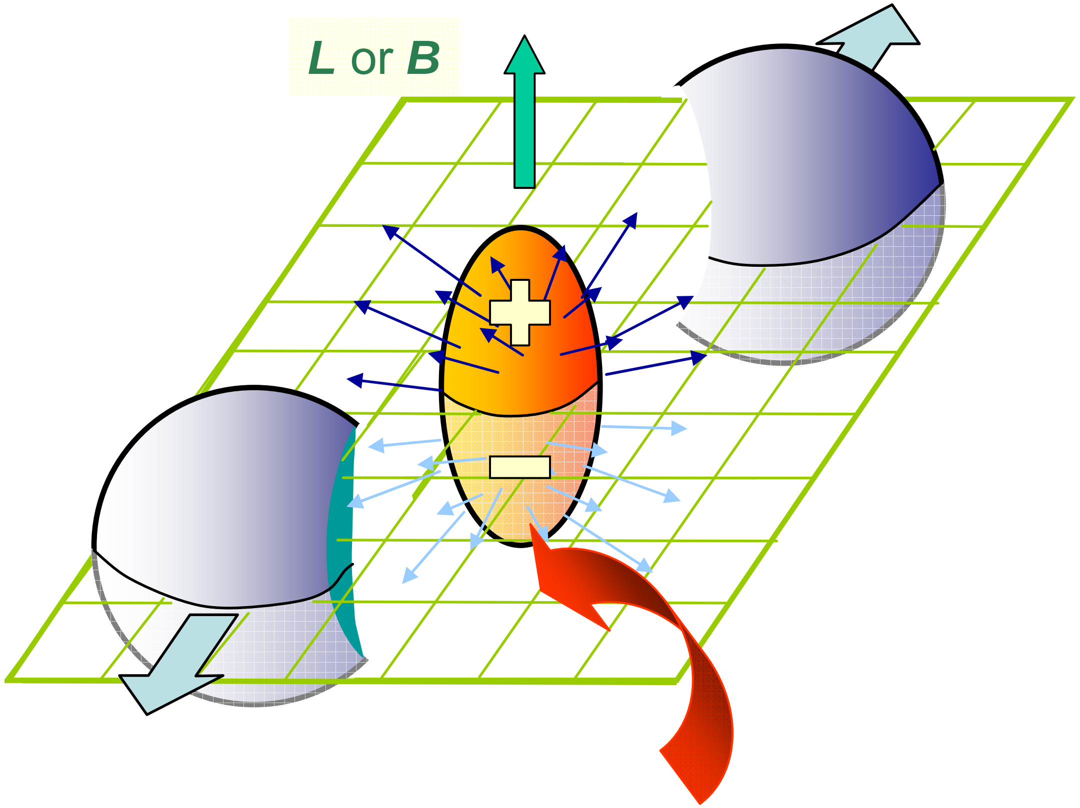
Dynamics is a random walk between states with different topological charges. In this states a balance between left-handed and right-handed quarks is destroyed, $N_R - N_L = Q_T$ → violation of P-, CP- symmetry.
Average total topological charge vanishes $\langle n_w \rangle = 0$ but variance is equal to the total number of transitions $\langle n_w^2 \rangle = N_t$
Fluctuation of topological charges in the presence of magnetic field induces electric current which will separate different charges

Lattice gauge theory



The excess of electric charge density due to the applied magnetic field. Red — positive charges, blue — negative charges. P.V.Buividovich et al., PR **D80**, 054503 (2009)

Charge separation in HIC



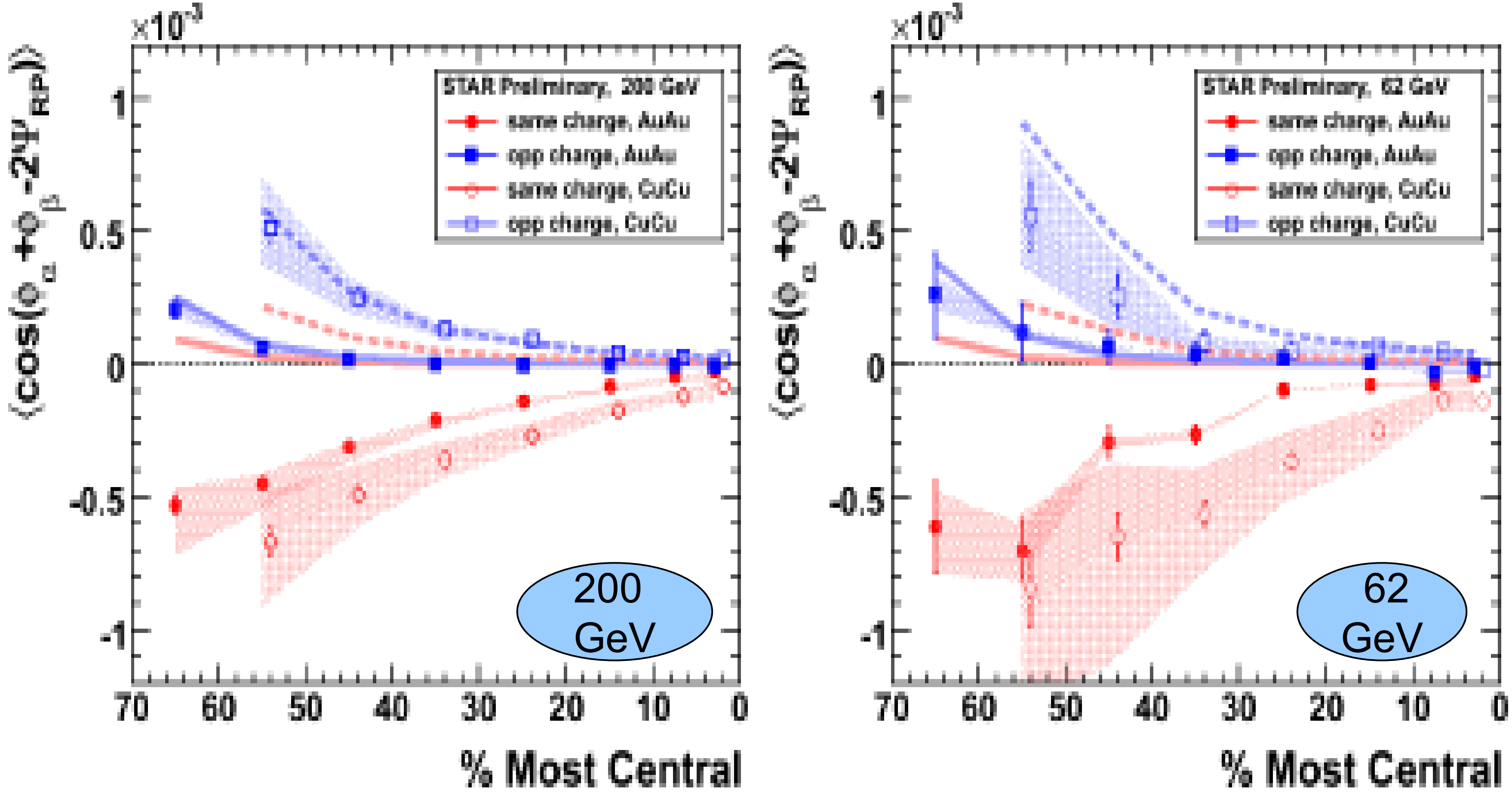
Non-zero angular momentum (or equivalently magnetic field) in heavy-ion collisions make it possible for $\mathcal{P}$- and $\mathcal{CP}$-odd domains to induce charge separation (D.Kharzeev, PL **B 633** (2006) 260).

Electric dipole moment of QCD matter !

Measuring the charge separation with respect to the reaction plane was proposed by S.Voloshin, Phys. Rev. **C 70** (2004) 057901.

Charge separation in RHIC experiments

STAR Collaboration, PRL **103**, 251601 (2009)



Combination of intense B and deconfinement is needed for a spontaneous parity violation signal

Qualitative estimate of the CME

Q_s -- saturation momentum, $\Delta z \simeq \Delta\tau \simeq 1/Q_s, \quad \varepsilon \sim Q_s^4$

$p_t \sim Q_s$

$$\frac{dE_T}{dy} \sim \varepsilon \cdot V = \varepsilon \cdot \Delta z \cdot S$$

$$\sim Q_s^4 \cdot \frac{1}{Q_s} \cdot S \sim Q_s \cdot (Q_s^2 S) \sim Q_s \cdot \frac{dN_{hadron}}{dy}$$

The generated topological charge $n_w \equiv \sqrt{Q_s^2} = \sqrt{\Gamma_s \cdot V \cdot \tau_B}$

$\Gamma_s \sim \lambda^2 T^4$ (SUSY Y-M)

$$n_w \sim \sqrt{\frac{dN_{hadron}}{dy}} \cdot \sqrt{Q_s \tau_B}$$

Sphaleron transition occurs only in the deconfined phase, the lifetime is

$$\tau_B = min\{\tilde{\tau}_B, \tau_\varepsilon\}$$

Analysis strategy

$$\langle \cos(\psi_\alpha + \psi_\beta - 2\Psi_{RP}) \rangle =$$
$$= \langle \cos(\psi_\alpha + \psi_\beta - 2\psi_c) \rangle / v_{2,c} = v_{1,\alpha} v_{1,\beta} - a_\alpha a_\beta$$

Average correlators are related to the topological charge
(D .Kharzeev, Phys. Lett. B **633** (2006) 260)

$$a \sim \frac{n_w}{dN_{hadron}/dy} \sim \frac{\sqrt{Q_s \tau_B}}{\sqrt{dN_{hadron}/dy}}$$

For numerical estimates $Q_s^2 \sim s_{NN}^{1/8} \sim dN_{hadron}/dy$

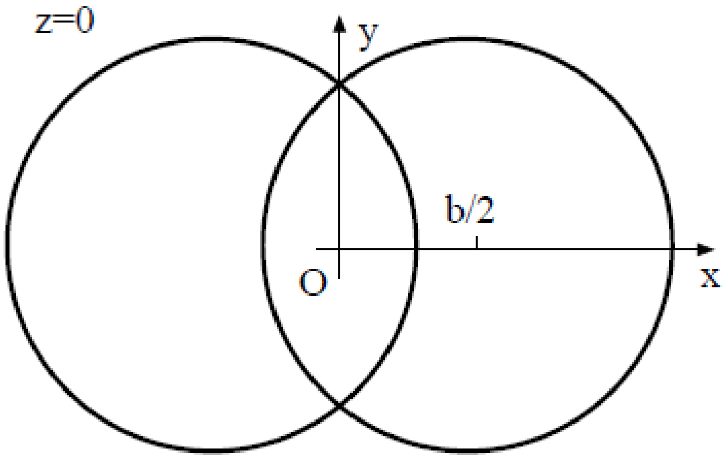
$$a^2 \sim \frac{\tau_B}{Q_s} \sim (\sqrt{s_{NN}})^{-1/8} \cdot \tau_B = K\ (\sqrt{s_{NN}})^{-1/8} \cdot \tau_B$$

At the fixing point $K = \frac{a_{exp}^2 \cdot (200)^{1/8}}{\tau_B(200)}$

Magnetic field calculation

The Lienart-Wiechard potential is applied to the time evolution of heavy-ion collisions within the UrQMD model

$$e\vec{B}(t,\vec{x}_0) = \alpha_{\rm EM} \sum_n Z_n \frac{1-v_n^2}{\left(R_n - \vec{R}_n \vec{v}_n\right)^3} \left[\vec{v}_n \times \vec{R}_n\right],$$



$$\vec{R}_n = \vec{x}_n - \vec{x}_0$$

with the retardation condition

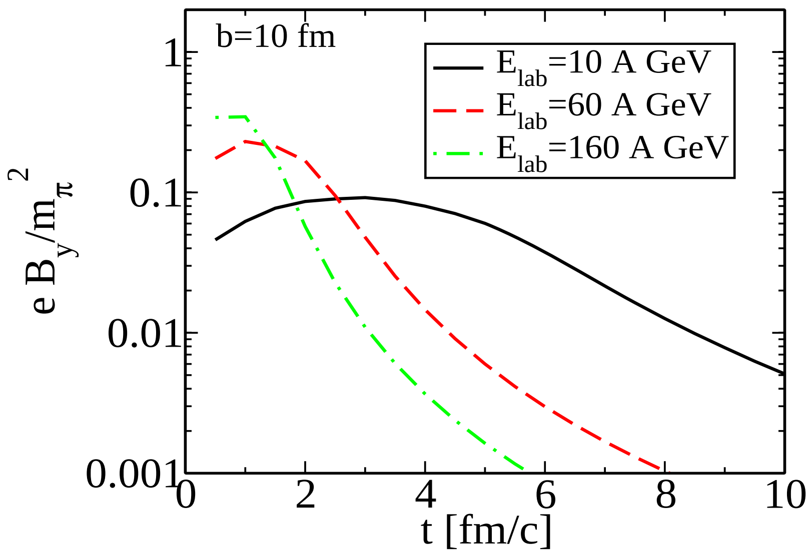
$$|\vec{x}_0 - \vec{x}_n(t')| + t' = t.$$

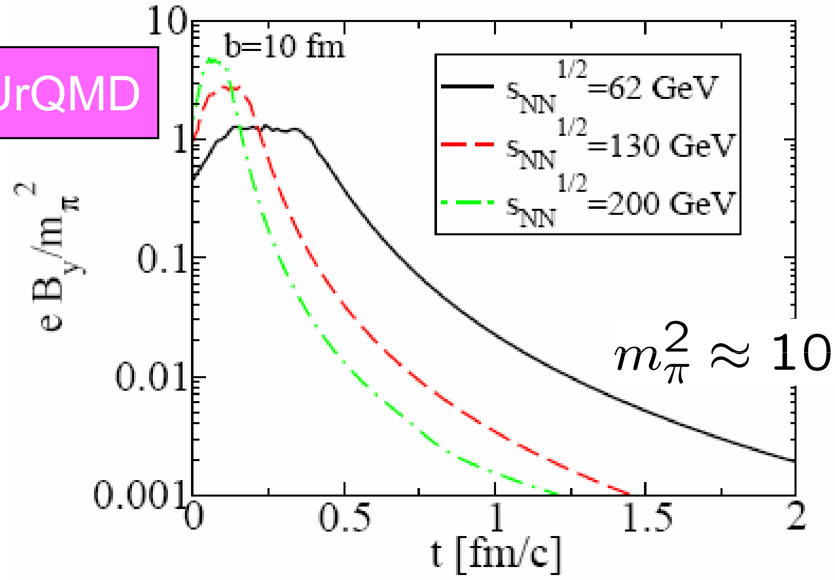
- Field will have only B_y nonzero component
- Field will be negligible for low bombarding energies
- For ultrarelativstic energies the magnetic field is felt by particles close to the transverse plane
- For symmetry reasons the magnetic field is negligible for small b

V.Skokov, A.Illarionov, V.T, IJMP **A24**, 5925 (2009)

Magnetic field and energy density evolution in Au+Au collisions at b=10 fm

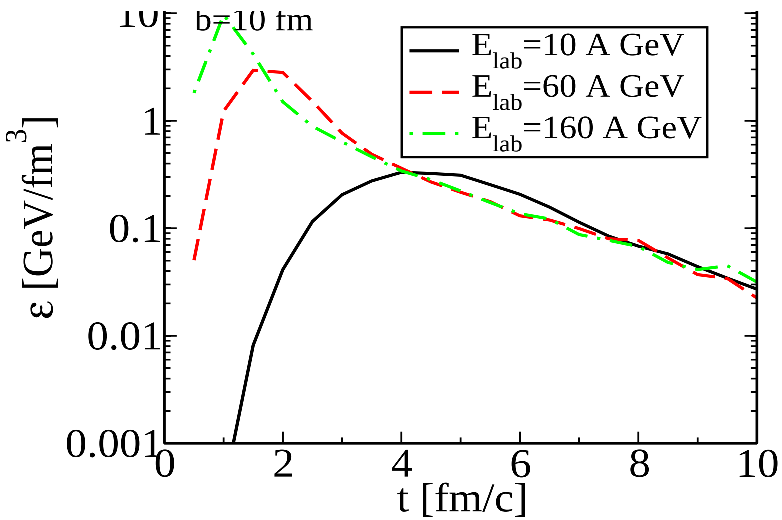
UrQMD

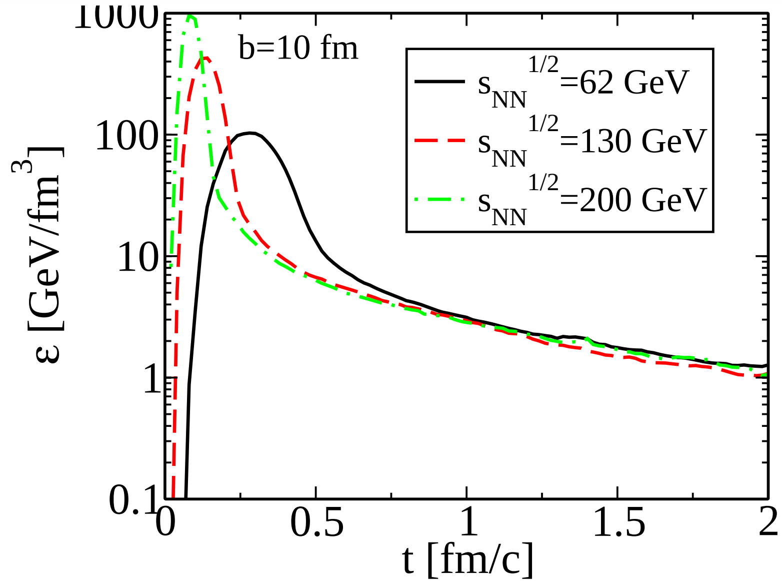




eB_y

$m_\pi^2 \approx 10^{18}$Gauss





ε

[~2π/S_d] $B_{crit} \approx (10. — 0.2)\ m_\pi^2$ [~$(\alpha_S T)^2$] and $\varepsilon_{crit} \approx 1$ GeV/fm^3

Characteristic parameters for the CME

$\sqrt{s_{NN}}$ GeV	$s_{NN}^{1/16}$	$\tilde{\tau}_B$, fm/c	τ_ϵ, fm/c	a^2
$4.5 \cdot 10^3$	2.86	0.018	>1	$0.016 \cdot 10^{-4}$
130	1.84	0.33	$\sim$2.3	$0.45 \cdot 10^{-3}$
62	1.68	0.62	$\sim$2.2	
17.9	1.43	1.41	$\sim$2.	$2.48 \cdot 10^{-3}$
11.	1.35	1.66	$\sim$ 1.9	$3.10 \cdot 10^{-3}$
4.7	1.21	0.	0.	0.

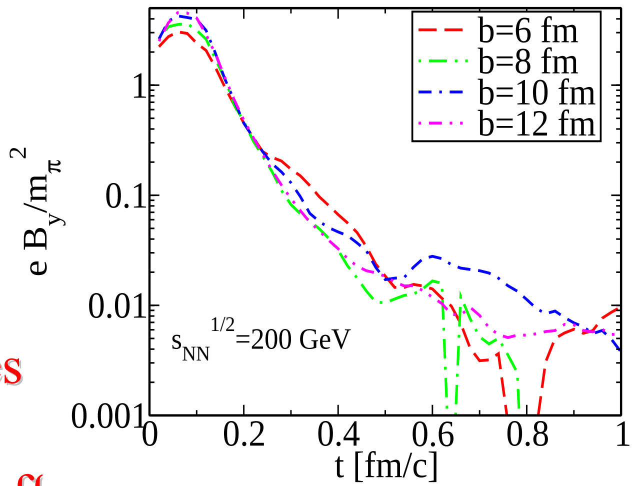
The lifetimes are estimated at $eB_{crit}=0.2m_\pi^2$ and $\varepsilon_{crit}=1$ GeV/fm^3 for Au+Au collisions with b=10 fm ($K_{Au}=2.52\ 10^{-2}$)

- For all energies of interest $\tau_B < \tau_\varepsilon$
- **The CME increases with energy decrease till the top SPS/NICA energy**
- If compare $\sqrt{s_{NN}}$ = 200 and 62 GeV, the increase is too strong !

Ways to remove the discrepancy

The correlator ratio at two measured energies for b=10 fm

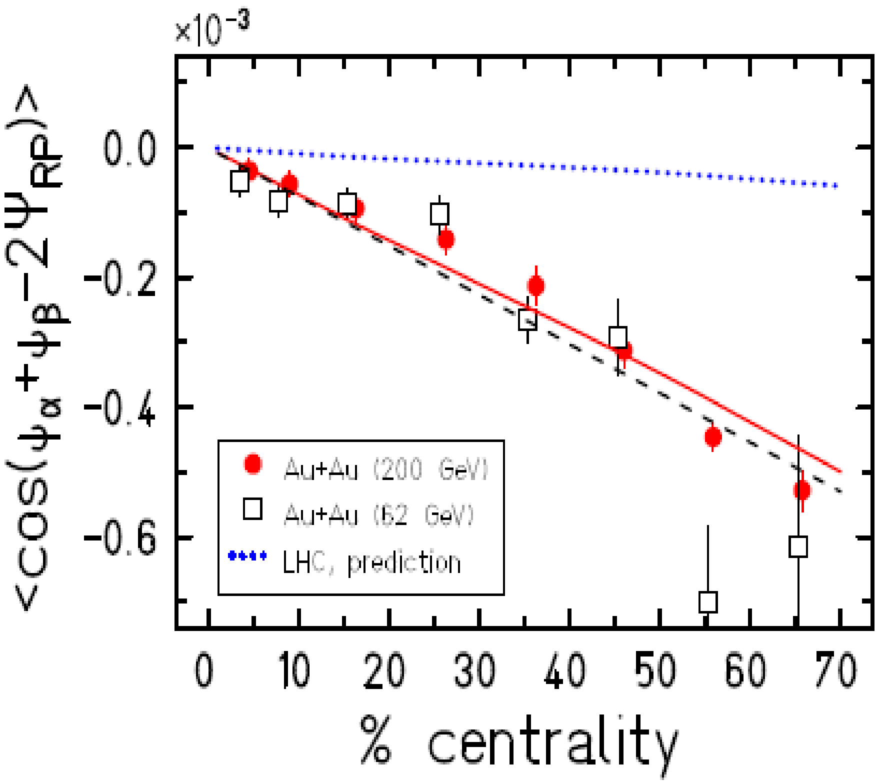
$$\frac{a^2(200)}{a^2(62)} = \frac{\tau_B(200)}{\tau_B(62)}\left(\frac{62}{200}\right)^{1/8}$$
$$= 0.387\ (0.31)^{\beta} \approx 0.72. \text{(exp)}$$



- Uncertainity in $\sqrt{s}_{NN}$ dependence does not help; $\beta<0$?!
- Should be $\tau_B(62) = 1.2\ \tau_B(200)$ (instead of ~3); lifetimes
- Uncertainty in impact parameter; not essential
- Inclusion of participant contribution to eB; very small eff
- To decrease eB_{crit} till $0.01 m_\pi^2$ to reach regime $\tau_B = \tau_\varepsilon$; <62
- If eB_{crit} increases the lifetime ratio is correct for $eB_{crit} \approx 1.05\ m_\pi^2$ very close to the maximal $eB_{crit} = 1.2\ m_\pi^2$; questionable, no CME for Cu
- To introduce the initial time when equilibrium of quark-gluon matter is achieved, $t_{i,\varepsilon} > 0$, associated with a maximum in ε-distribution, $\tau_B(62) / \tau_B(200) \approx (0.62-0.32)/(0.24-0.08) \approx 2.0$; not enough
- To combine the last two scenarios; success !

The calculated CME for Au+Au collisions

Calculated correlators for Au+Au (b=10 fm) collisions at $\sqrt{s_{NN}}$=200 and 62 GeV agree with experimental values for $eB_{crit} \approx 0.7\ m_\pi^2$, K=6.05 10^{-2}. No effect for the top SPS energy!
In a first approximation, the CME may be considered as linear in b/R (D.Kharzeev et al., Nucl. Phys. **A803**, 203 (2008))

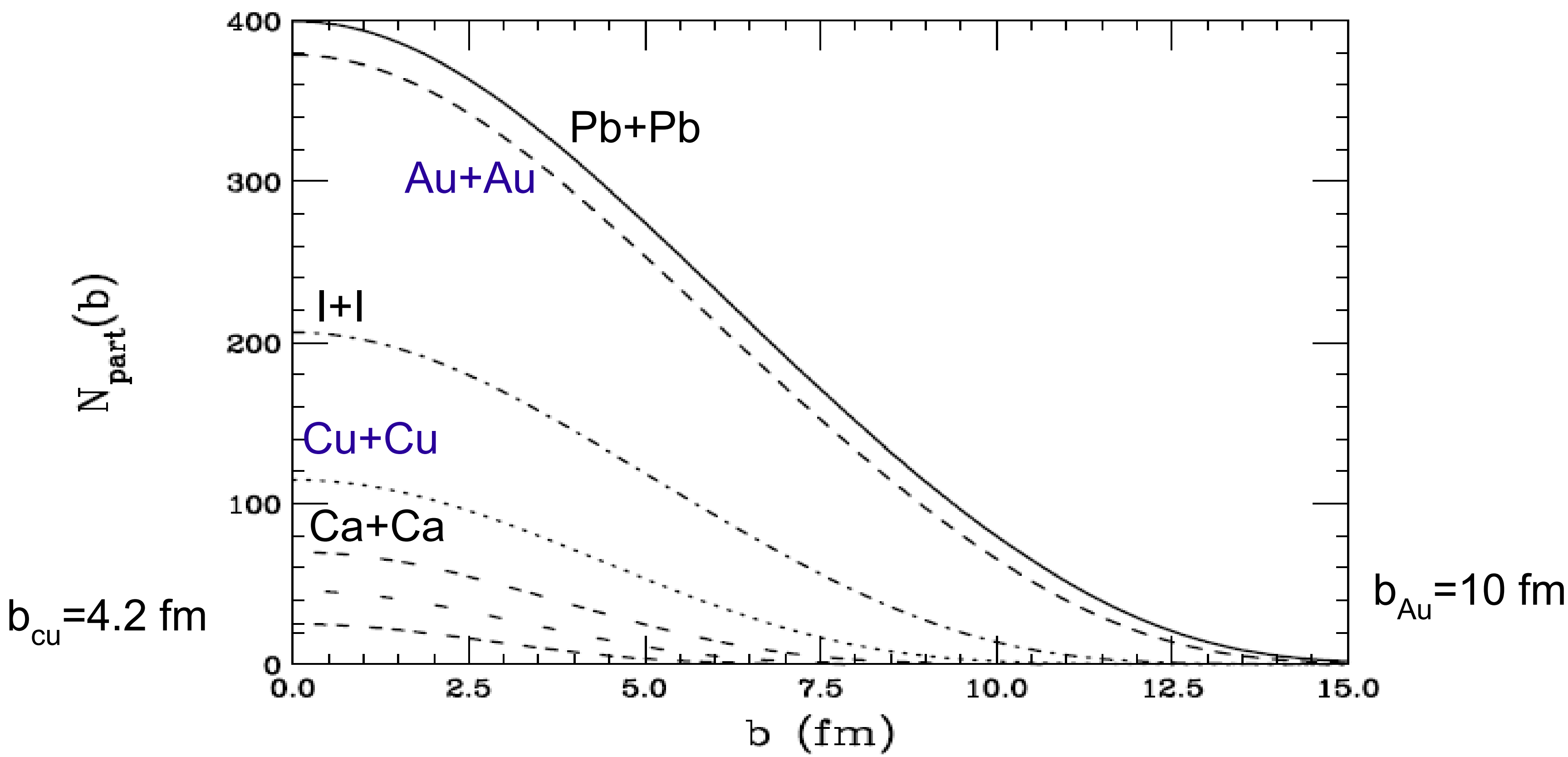


Normalized at b=10 fm (centrality 0.4-0.5) for Au+Au collisions

System-size dependence

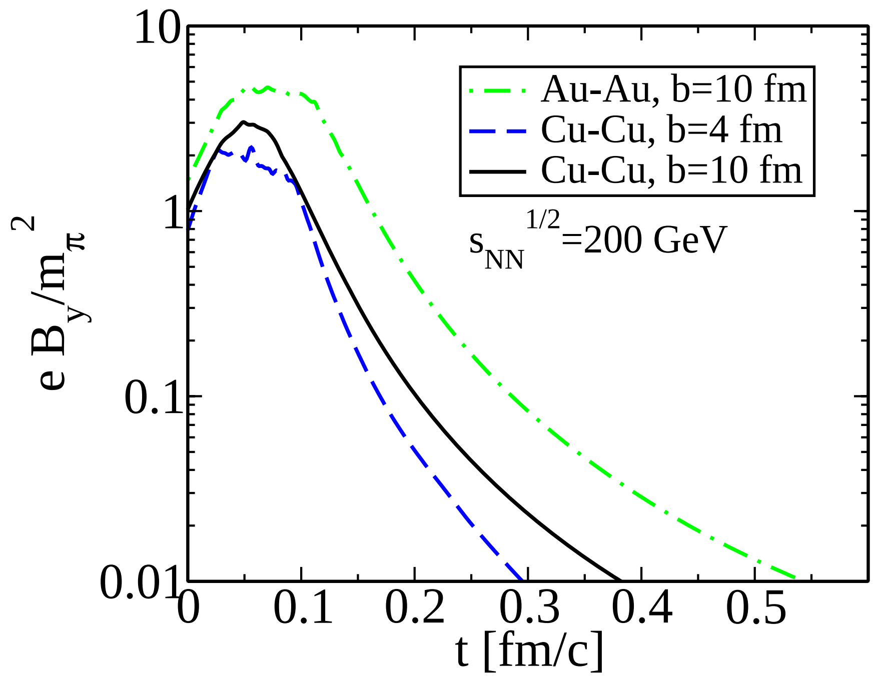
The CME should be proportional to the nuclear overlap area $S \equiv S_A(b)$

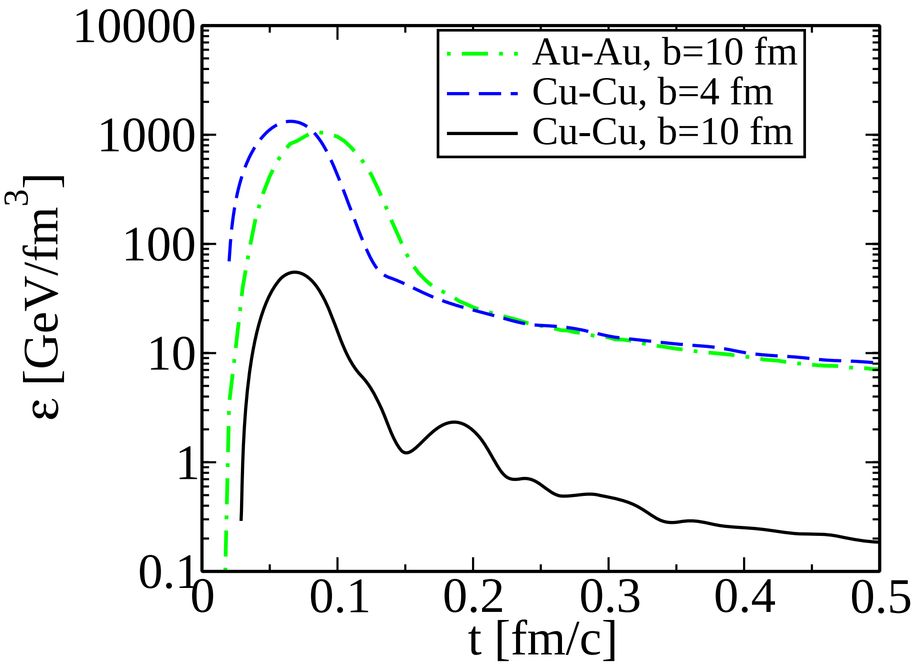
Centrality ⇨ $e_0 N_{part}$



Correlation between centrality and impact parameter

Comparison of eB_y and ε evolution for Au+Au and Cu+Cu collisions

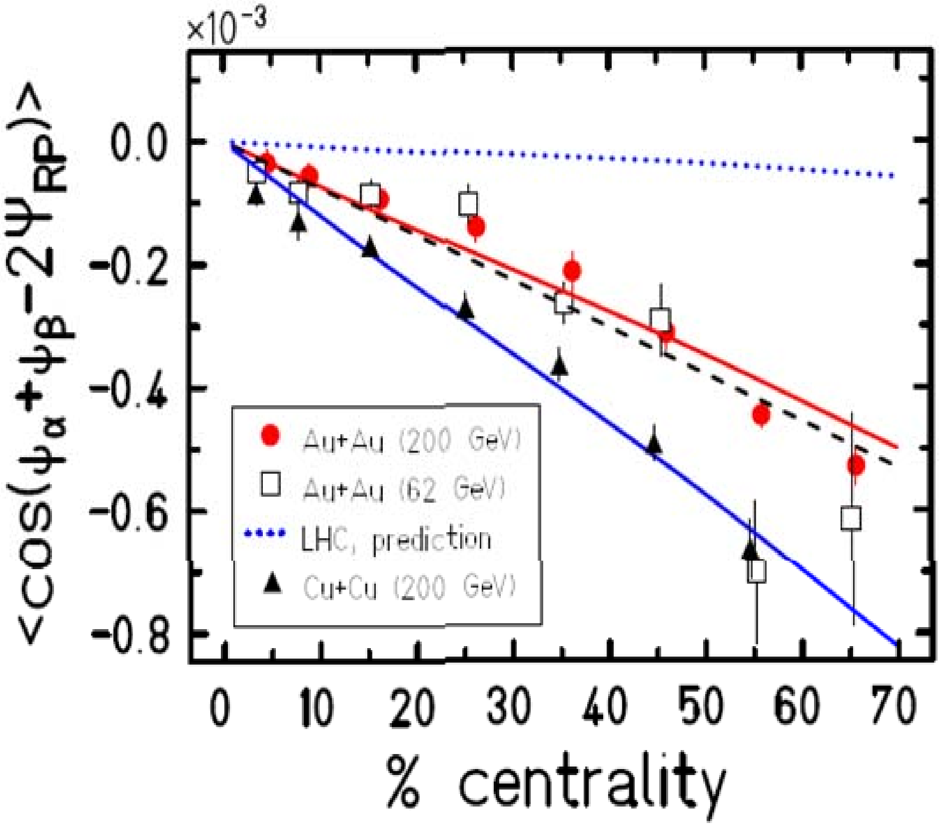




Only lifetime ratio is relevant !

The CME for Cu+Cu collisions

A rough approximation for the «almond» area :
$S_A(b)=(R_A^2 - b^2/4)^{1/2} (R_A - b/2)$,
so $S_{cu}(b=4.2)/S_{Au}(b=10) \approx 1.5$.
More accurate estimate: 1.65

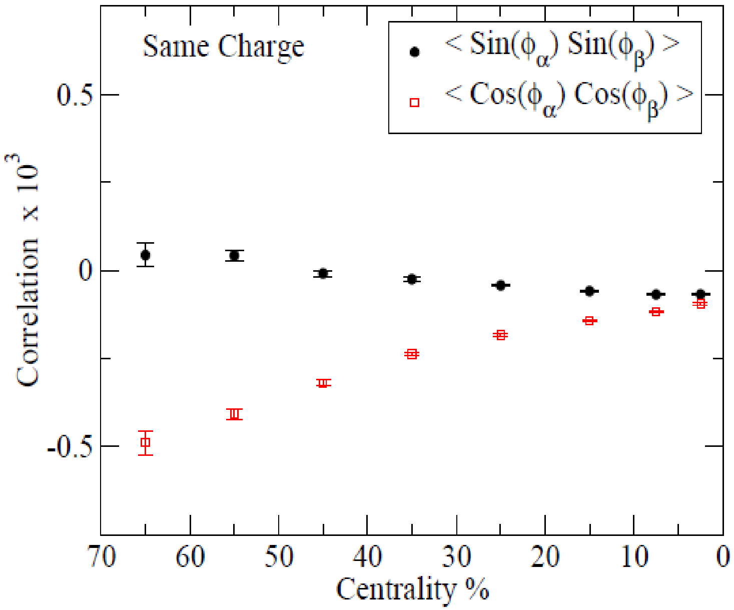


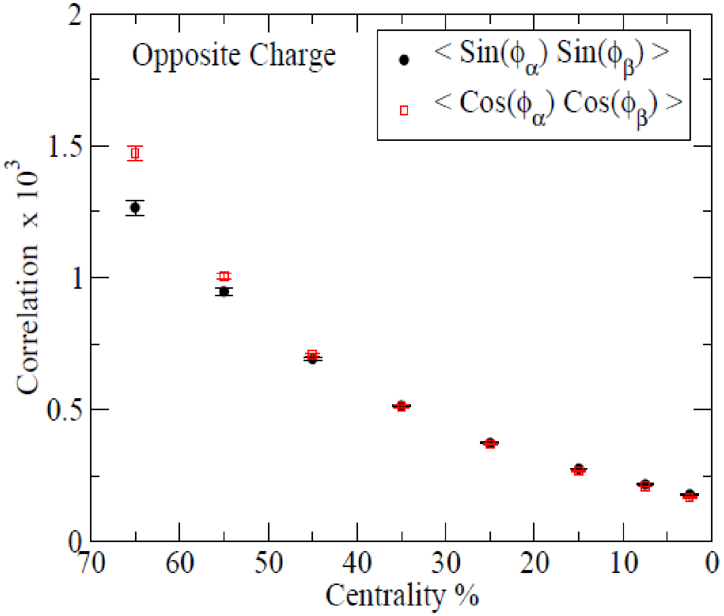
Only the coefficient K should be renormalized:
$K_{cu}=K_{Au} \cdot S_{Cu}(b=4.2)/S_{Au}(b=10)$
$=1.65\ K_{Au}$

This works for $\sqrt{s_{NN}}$ =200 GeV but NOT for 62 GeV
(due to different conditions for eB_{crit})

The system-size dependence is not only a geometrical effect

Worrying remarks





$$< cos(\phi_a + \phi_b) >=$$

$$< cos(\phi_a)\ cos(\phi_b) > - < sin(\phi_a)\ sin(\phi_a) >$$

⇧ In-plane Out-of-plane ⇧

The same charge pairs are mainly in-plane and not out-of-plane.

If there is a parity violating component it is large and, surprisingly, of the same magnitude as the background.

A.Bzdak, V.Koch, J.Liao,
Phys. Rev. **C81**, 034910 (2010)

Transport model with e.m. field

The Boltzmann equation is the basis of QMD like models:

$$\{\frac{\partial}{\partial t} + \dot{\vec{r}} \cdot \vec{\nabla}_r + \dot{\vec{p}} \cdot \vec{\nabla}_p\} \, f(\vec{r},\vec{p},t) = I_{coll}(f, f_1, ... f_N)$$

Generalized on-shell transport equations in the presence of electromagnetic fields can be obtained formally by the substitution:

$$\dot{\vec{r}} \rightarrow \frac{\vec{p}}{p_0} + \vec{\nabla}_p U \ ,$$

$$\dot{\vec{p}} \rightarrow -\vec{\nabla}_r U + e\vec{v} \times \vec{B}$$

$$U \sim Re(\Sigma^{ret})/2p_0$$

$$\{\frac{\partial}{\partial t} + \left(\frac{\vec{p}}{p_0} + \vec{\nabla}_{\vec{p}}\, U\right) \vec{\nabla}_{\vec{r}} - \left(\vec{\nabla}_{\vec{r}}\, U - e\vec{v} \times (\vec{\nabla} \times \vec{A})\right) \vec{\nabla}_{\vec{p}} \} \, f(\vec{r},\vec{p},t)$$

$$= I_{coll}(f, f_1, ... f_N) + S_B(\vec{r},\vec{p},t)$$

A general solution of the wave equations

$$\begin{cases} \vec{B} = \vec{\nabla} \times \vec{A} \\ \vec{E} = -\vec{\nabla}\Phi - \dfrac{\partial \vec{A}}{\partial t} \end{cases}$$

is as follows

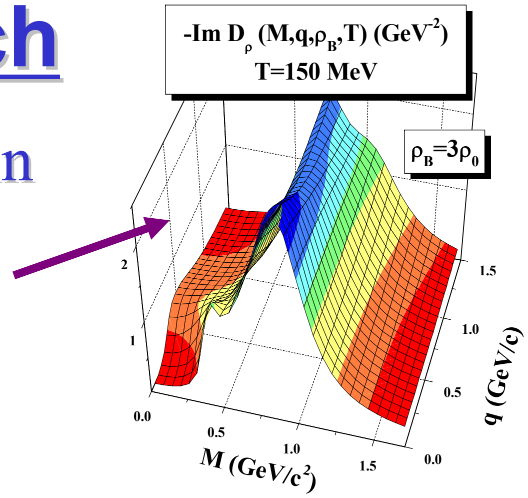
$$\vec{A}(\vec{r},t) = \frac{1}{4\pi} \int \frac{\vec{j}(\vec{r'},t')\ \delta(t - t' - |\vec{r} - \vec{r'}|/c)}{|\vec{r} - \vec{r'}|} \, d^3r' dt'$$

$$\Phi(\vec{r},t) = \frac{1}{4\pi} \int \frac{\rho(\vec{r'},t')\ \delta(t - t' - |\vec{r} - \vec{r'}|/c)}{|\vec{r} - \vec{r'}|} \, d^3r' dt'$$

For point-like particles $\rho(\vec{r},t) = e\ \delta(\vec{r} - \vec{r}(t));\quad \vec{j}(\vec{r},t) = e\ \vec{v}(t)\ \delta(\vec{r} - \vec{r}(t))$ $\quad \vec{\nabla} \times \vec{A} \rightarrow LW eq.$

HSD off-shell transport approach

Models predict changes of the particle properties in the hot and dense medium, e.g. broadening of the spectral function



→ **Accounting for in-medium effects requires off-shell transport models!**

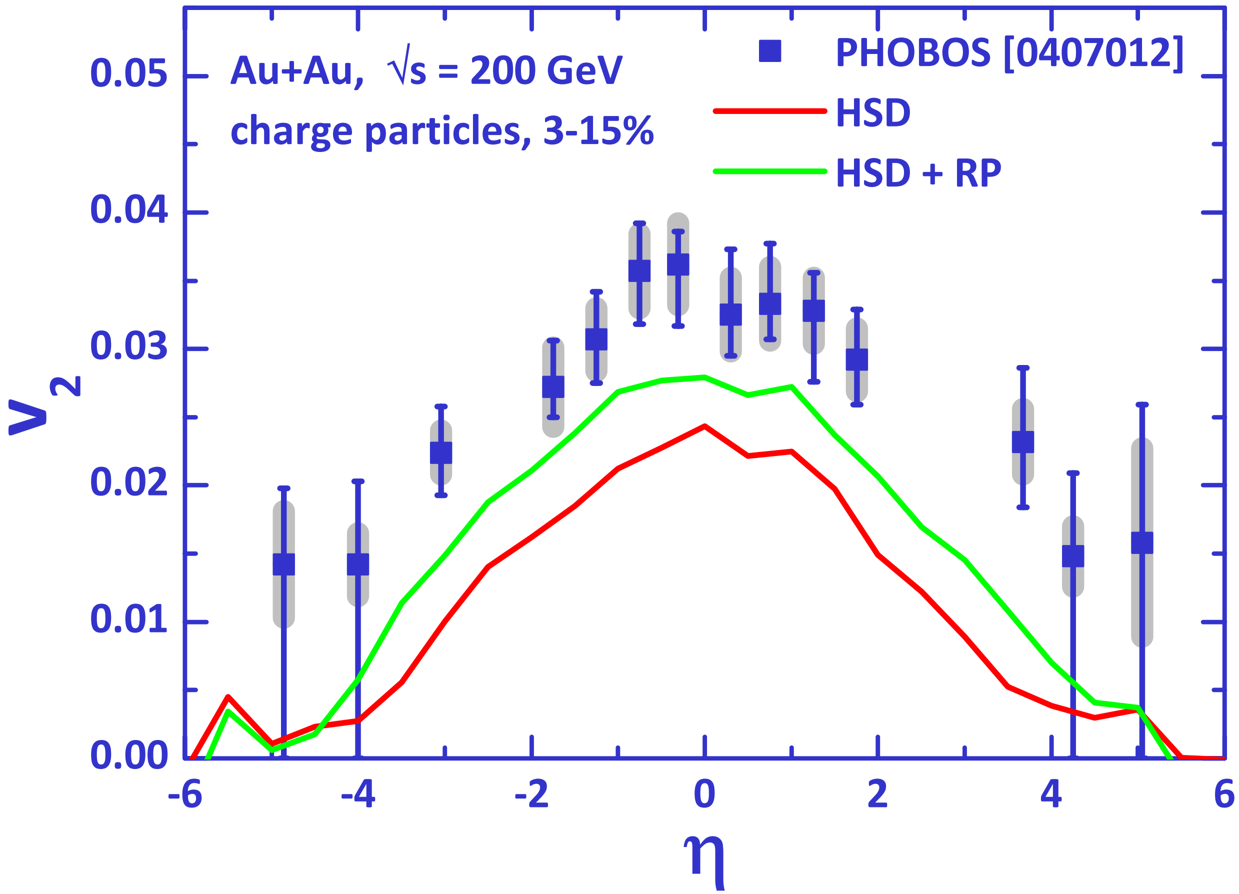
Generalized transport equations on the basis of the Kadanoff-Baym equations for Greens functions - accounting for the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations beyond the quasiparticle approximation (i.e. beyond standard on-shell models) – are incorporated in HSD and PHSD

W. Cassing et al., NPA **665** (2000) 377; **672** (2000) 417; **677** (2000) 445

→ The off-shell spectral functions change their properties dynamically by propagation through the medium and become on-shell in the vacuum

E. Bratkovskaya, NPA **686** (2001), E. Bratkovskaya & W. Cassing, NPA **807** (2008) 214

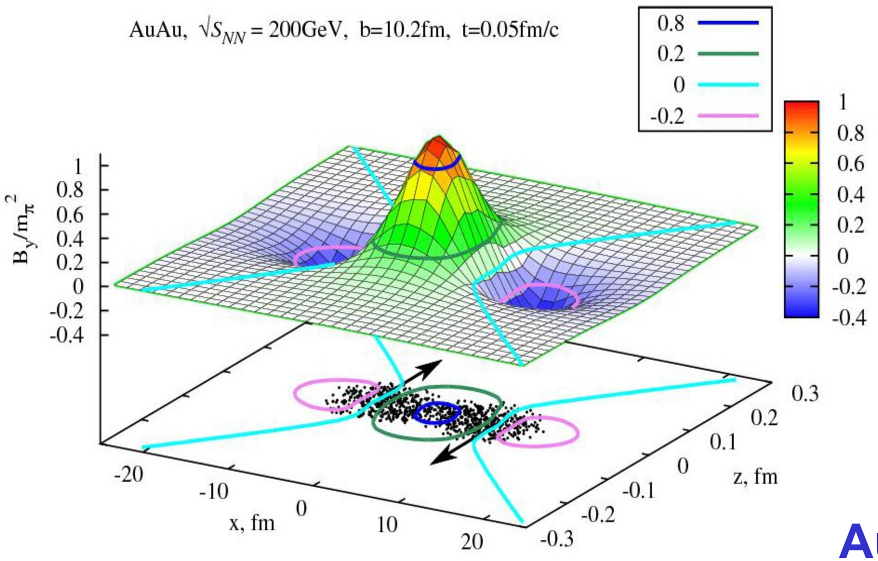
Elliptic flow v_2 in the HSD model

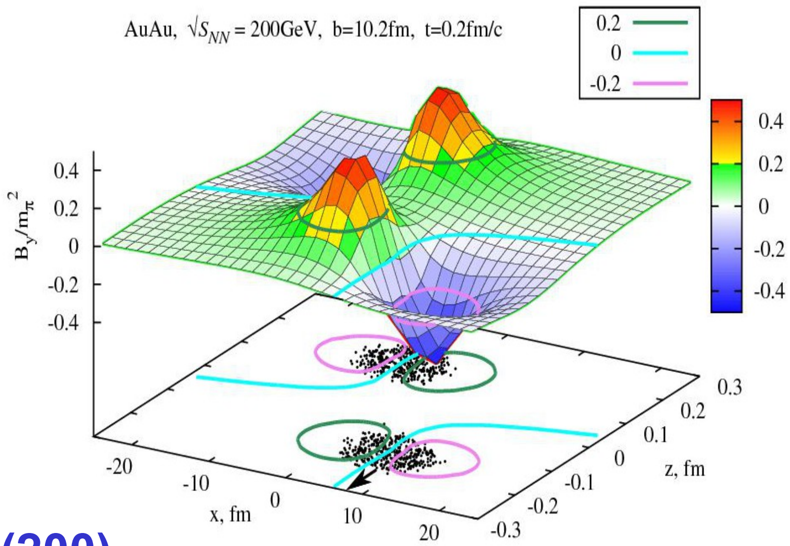


PHOBOS Collaboration,
PR **C72**, 0510901 (2005)

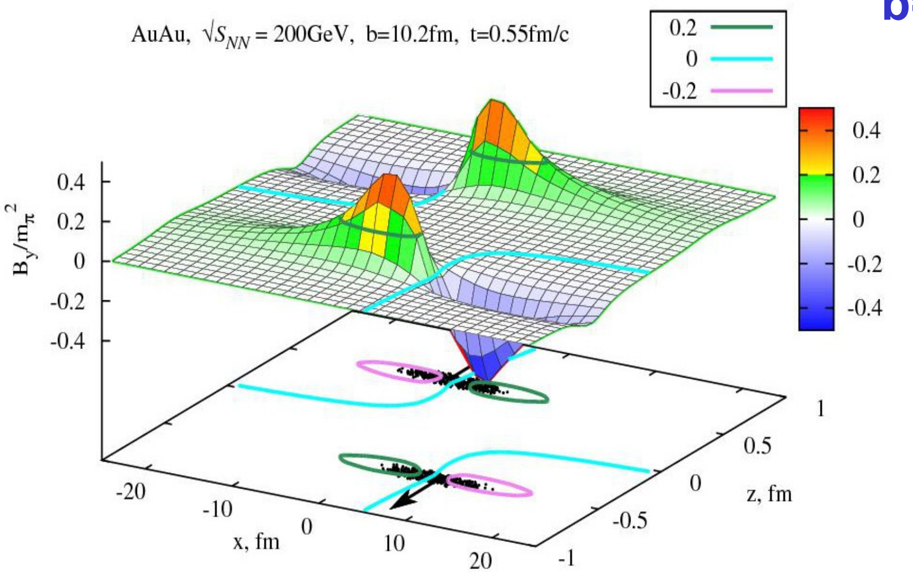
Note: method to define the reaction plane is important!

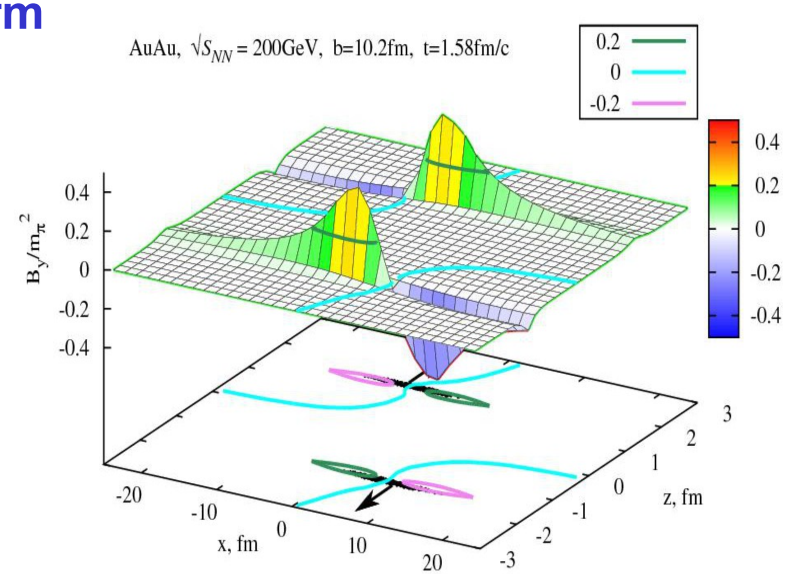
Magnetic field evolution



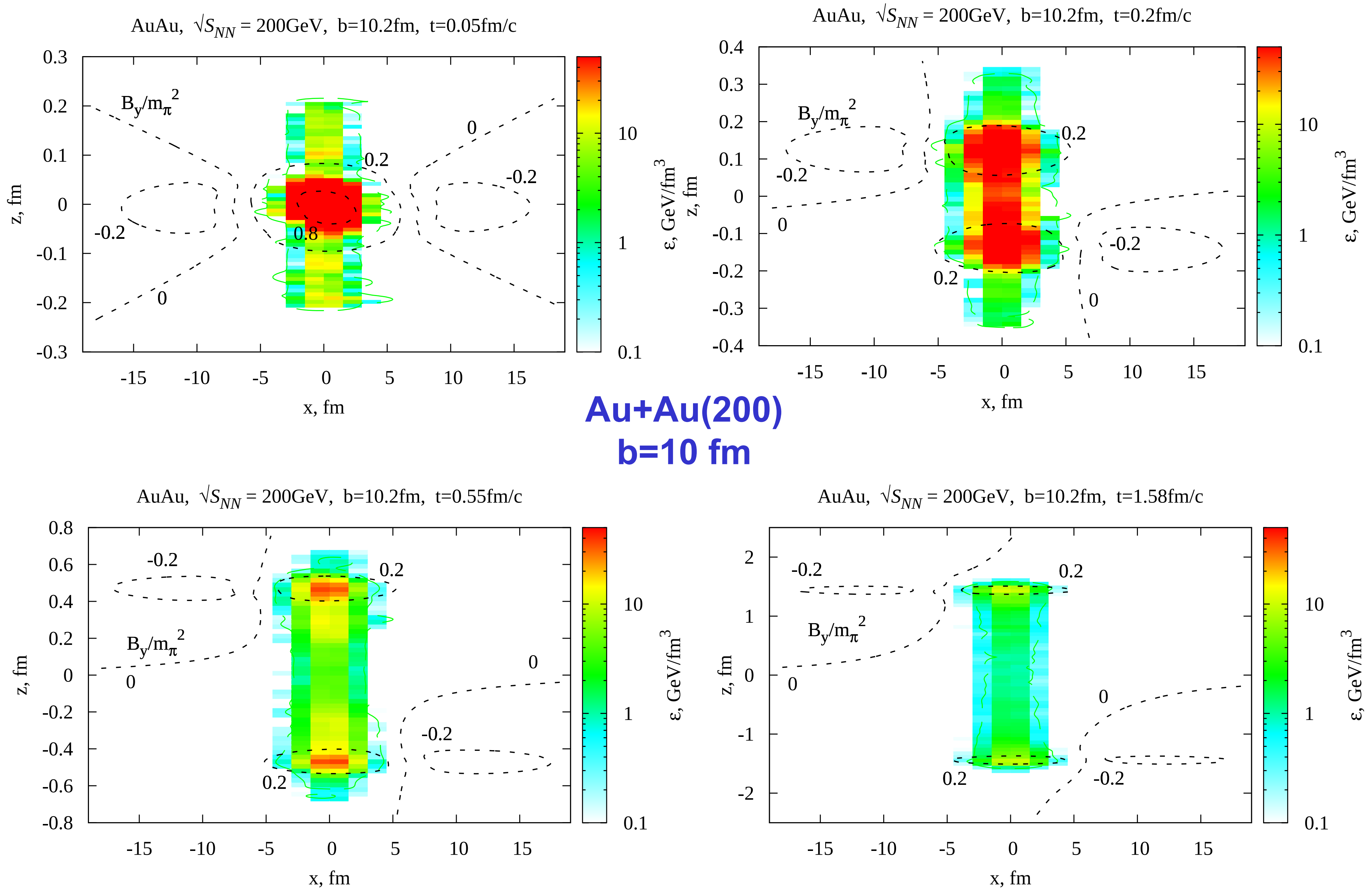


Au+Au(200)
b=10 fm

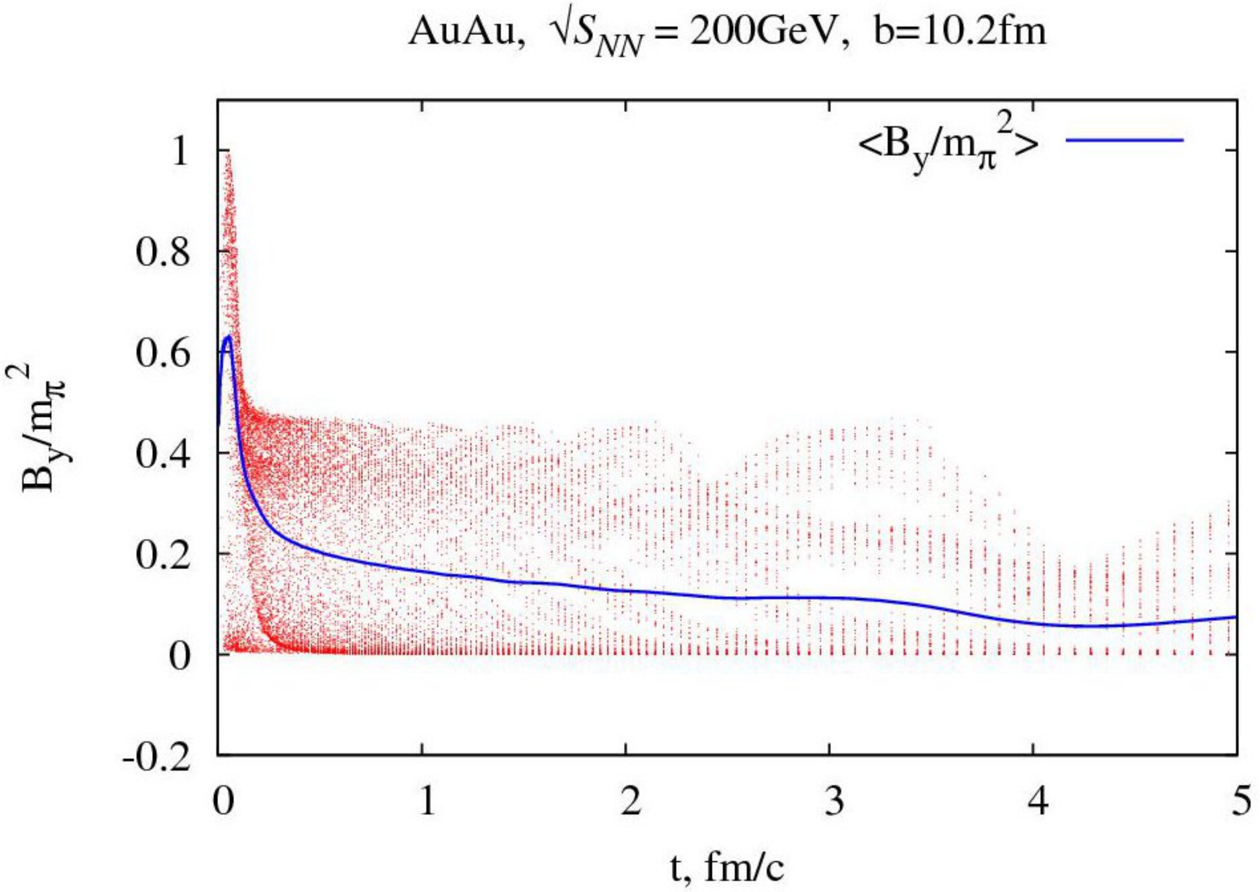




Magnetic field and energy density evolution



Magnetic field acting on charged pions



Effect is stongest at the very beginning of a collision (partonic phase ?)

Conclusions

The magnetic field and energy density of the deconfined matter reach very high values in HIC for $\sqrt{s_{NN}} \geq 11$ GeV satisfying necessary conditions for a manifestation of the CME.

Our consideration predicts $a^2 \sim (s_{NN})^{-1/8}$ which nevertheless is too strong to describe the observable energy behavior of the CME in the RHIC range. The model energy dependence can be reconciled with experiment by a detailed treatment of the lifetime taking into account both magnetic field and energy density evolution.

For the chosen parameters we are able to describe data on charge separation at two available energies. We predict that the CME will be much smaller at LHC energies and disappears at energies below top SPS energies.

Experiments on the CME planned at RHIC by the low-energy scan program are of great interest since they hopefully will allow to infer the critical magnetic field eB_{crit} governing the spontaneous local CP violation. Other possible mechanisms of CP violation and explanation of the observed charge separation ?

Further development of the HSD/PHSD transport model with respect to retarded electromagnetic fields is needed.