Nonlocal PNJL model beyond mean field

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A nonlocal chiral quark model is consistently extended beyond mean field using a strict $1/N_c$ expansion scheme. It is found that the $1/N_c$ corrections lead to a lowering of the temperature of the chiral phase transition in comparison with the mean-field result. On the other hand, near the phase transition the $1/N_c$ expansion breaks down and a non-perturbative scheme for the inclusion of mesonic correlations is needed in order to describe the phase transition region.

1. INTRODUCTION

A quantum field theoretical description of strong interactions in the nonperturbative regime is one of the most interesting and challenging problems of present-day theoretical physics. Quantum chromodynamics is well known only at the *perturbative* level whereas the low-energy region and the most interesting "hadronic" phase in the QCD phase diagram is in the nonpertubative regime. To gain some analytical insights to nonperturbative QCD, continuum approaches, even using effective models, are legitimate tools.

One of the successful models for a description of chiral quark dynamics and the phase diagram is the Nambu–Jona-Lasinio (NJL) model [1] applied to quarks. A generalization of the NJL model has been proposed which includes the coupling of the chiral quark sector to the Polyakov loop being an order parameter of the deconfinement transition (PNJL model [2–10]). Usually, NJL and PNJL models are formulated at the mean-field level. However, there

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are physical problems where the mean-field formulation is not sufficient. One of the most promising beyond mean-field schemes is based on a strict expansion of the inverse number of quark colors $1/N_c$ (see, e.g., Refs. [11–13] and work cited therein). In a nonlocal version of the NJL model the nonlocality leads to an effective regularization which renders the quark (multi-)loop diagrams convergent.

2. NONLOCAL MODEL IN VACUUM

The quark sector of the nonlocal chiral quark model is described by the Lagrangian

$$\mathcal{L}_q \;=\; \bar{q}(x)(i\partial\!\!\!/ - m_c)q(x) + \frac{G}{2}[J_\sigma^2(x) + \mathbf{J}_\pi^2(x)] \;, \tag{1}$$

where m_c is the current quark mass. The nonlocal quark currents are

$$J_{\rm M}(x) \;=\; \int d^4x_1 d^4x_2 \; f(x_1)f(x_2)\bar{q}(x-x_1)\mathbf{\Gamma}_{\rm M}q(x+x_2), \tag{2}$$

where $\mathbf{\Gamma}_\sigma = 1$, $\mathbf{\Gamma}_\pi = i\gamma^5\tau^a$ with $a = 1,2,3$, and $f(x)$ is a form factor. The latter is defined by its Fourier transform in Euclidean space, which we take to be Gaussian, $f^2(p_E^2) = \exp(-p_E^2/\Lambda^2)$. The scalar mean field gives a dynamical contribution to the quark mass

$$\begin{aligned} S(p) \;&=\; \left(p\!\!\!/ - m(p^2)\right)^{-1} = (p\!\!\!/ - m_c - \Sigma(p))^{-1} \,, \\ \Sigma(p) \;&=\; iG\Gamma^\sigma(p,p)\int \frac{d^4k}{(2\pi)^4}\,{\rm Tr}\,[\,\Gamma^\sigma(k,k)\,S(k)\,] \equiv m_d f^2(p^2) \;. \end{aligned} \tag{3}$$

Here the symbol Tr stands for the trace over color-, flavor-, and Dirac-indices, and $\Gamma^{\rm M}(q_1,q_2) = \mathbf{\Gamma}_{\rm M} f(q_1^2)f(q_2^2)$. The amplitude $m_d = -\sigma_{\rm MF}$ is an order parameter for dynamical chiral symmetry breaking. The chiral condensate per flavor is obtained from the non-perturbative part of the quark propagator, $S^{np}(p) = S(p) - S^c(p)$, i.e. after subtracting the perturbative part $S^c(p) = (p\!\!\!/ - m_c)^{-1}$.

Mesons are described as bound state solutions of the quark-antiquark Bethe-Salpeter equation. The meson propagators are given by $\mathrm{D}_p^{\rm M} = (-G^{-1} + \Pi_p^{\rm M})^{-1}$. where $\mathrm{M} = \pi, \sigma$ and $\Pi_p^{\rm M} \equiv \Pi_{\rm M}^{\rm MF}(p^2)$ are the mean field polarization functions .

As usual in the systematic $1/N_c$ expansion, the four-quark coupling constant G is considered to be of the order $1/N_c$. In this case the N_c behavior of pion properties calculated in the model coincides with leading order QCD calculations ($f_\pi \sim \sqrt{N_c}$). As a result any meson propagator line in diagrams has a $1/N_c$ suppression factor.

3. FINITE TEMPERATURE

The model can easily be extended to finite temperature using a Φ-derivable ansatz (see, e.g., Ref. [14]) supplemented by the $1/N_c$ expansion. The central quantity for the analysis is the thermodynamic potential per volume

$$\Omega = i\mathrm{Tr}\ln(\mathbf{S}^{-1}) + i\mathrm{Tr}(\Sigma\mathbf{S}) + \Psi(\mathbf{S}) + U(\Phi,\bar\Phi) - \Omega_0\ , \tag{4}$$

where $\mathbf{S}$ and $\Sigma = (S^c)^{-1} - \mathbf{S}^{-1}$ are the full propagator and the quark selfenergy, respectively, and Tr denotes the trace over all degrees of freedom, internal ones and 4-momenta. At nonzero temperature, we also take into account the Polyakov-loop dynamics, which at $T = 0$ decouples from the quark sector. To that end a constant temporal background gauge field $\phi \equiv \langle A_4\rangle = \langle iA_0\rangle$ is minimally coupled to the quarks and a Polyakov loop potential $U(\Phi,\bar\Phi)$ is added in Eq. (4). Here $\Phi = \frac{1}{N_c}\,\mathrm{Tr}_c\, e^{i\phi/T}$ denotes the Polyakov loop expectation value and $\bar\Phi$ its conjugate. In order to avoid confusion with the potential of the Φ-derivable scheme (“Φ functional”), we denote the latter as Ψ. A subtractive renormalization constant Ω_0 is chosen such that the vacuum ($T = 0$) has vanishing pressure.

The thermodynamic equilibrium corresponds to the (global) minimum of the thermodynamic potential with respect to the full quark propagator and to the Polyakov loop, so that the following necessary conditions (gap equations) must be fulfilled

$$\frac{\partial\Omega}{\partial\mathbf{S}} = 0, \quad \frac{\partial\Omega}{\partial\Phi} = 0\ , \quad \frac{\partial\Omega}{\partial\bar\Phi} = 0\ . \tag{5}$$

We work in Polyakov gauge where the background gauge field is diagonal in color space, i.e. $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$. Following [6], we require $\Phi = \bar\Phi$ to be real with real ϕ_3, ϕ_8. As a consequence $\phi_8 = 0$ and we are left with one variable ϕ_3.

Approximations can be introduced by truncating Ψ at a certain order. In the present paper, we use a “strict $1/N_c$ expansion”, where all contributions beyond the next-to-leading order are discarded. Thus, one gets for the thermodynamic potential

$$\Omega = \Omega^{\mathrm{MF}} + \Omega^{\mathrm{N_c}}, \quad \Omega^{\mathrm{MF}} = \frac{m_d^2}{2G} + U(\Phi,\bar\Phi) - \sum_{i=0,\pm} 4 \int\limits_{k,n} \ln\left[k_{n,i}^2 + M^2(k_{n,i}^2)\right],$$
$$\Omega^{\mathrm{N_c}} = \sum_{M=\pi,\sigma} \frac{d_M}{2} \int\limits_{p,m} \ln\left[1 - G\Pi_M(\mathbf{p},\nu_m)\right] \tag{6}$$

where $k_{n,i}^2 = (\omega_n^i)^2 + \mathbf{k}^2$, the notation $\int\limits_{k,n} \equiv T\sum\limits_n \int d^3k/(2\pi)^3$ has been introduced and summation is over the bosonic Matsubara frequencies. Note that due to the coupling to the

Polyakov loop the fermionic Matsubara frequencies $\omega_n = (2n+1)\pi T$ are partially shifted: $\omega_n^{\pm} = \omega_n \pm \phi_3$, $\omega_n^0 = \omega_n$.

Including the gluon background, we suggest to treat the Polyakov-loop potential as effectively N_c independent[1]. A strict $1/N_c$ expansion of the thermodynamic potential then corresponds to evaluate Eq. (6) for the simultaneous solutions of the gap equations

$$\frac{\partial \Omega^{\rm MF}}{\partial m_d} = 0, \qquad \frac{\partial \Omega}{\partial \phi_3} = 0. \tag{7}$$

Note that ϕ_3 is determined by minimizing the total thermodynamic potential, whereas m_d is obtained from the mean-field part only. Nevertheless, since $\Omega^{\rm MF}$ also depends on ϕ_3, the value of m_d is changed as well compared to the mean-field calculation, due to the modified value of ϕ_3. For the Polyakov loop potential $U(\Phi,\bar{\Phi})$ we adopt the the logarithmic form of [6].

The model predictions for the pressure is shown in Fig. 1. Together with full result the partial contributions to the pressure $p^{\rm MF} = -\Omega^{\rm MF}$ and $p^{\rm N_c} = -\Omega^{\rm N_c}$ are also shown. In Fig. 2 we show the temperature dependence of the quark condensate $\langle \bar{q}q \rangle_T$ and of the Polyakov loop expectation value. In both figures we have also indicated the predictions of lowest-order chiral perturbation theory. At low temperatures, these are well reproduced by the nonlocal model. Note that the $1/N_c$-corrections are absolutely crucial to achieve this agreement.

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[1] In principle the $U(\Phi,\bar{\Phi})$ is proportional to the number of gluons, N_c^2-1 [15, 16]. Its leading contribution to the thermodynamic potential is therefore of the order $O(N_c^2)$, while the quarks only contribute at the order $O(N_c)$ and corrections are of the order $O(N_c^0)$ for both, quarks and gluons. However, since in practice the detailed form of U is not based on a $1/N_c$ expansion, but rather a phenomenological parameterization fitted to quenched lattice data, we believe that it is more appropriate to treat it as N_c independent in the present context.

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1. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
2. P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B **379**, 163 (1996).
3. K. Fukushima, Phys. Lett. B **591**, 277 (2004).
4. E. Megias, E. Ruiz Arriola, and L. L. Salcedo, Phys. Rev. D **74**, 065005 (2006).
5. C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D **73**, 014019 (2006).
6. S. Rößner, C. Ratti, and W. Weise, Phys. Rev. D **75**, 034007 (2007).
7. C. Sasaki, B. Friman, and K. Redlich, Phys. Rev. D **75**, 074013 (2007).
8. H. Hansen *et al.*, Phys. Rev. D **75**, 065004 (2007).
9. D. Blaschke, M. Buballa, A. E. Radzhabov, and M. K. Volkov, Phys. Atom. Nucl. **71**, 1981 (2008).
10. G. A. Contrera, D. Gomez Dumm, and N. N. Scoccola, Phys. Lett. B **661**, 113 (2008).
11. D. Blaschke, M. Buballa, A. E. Radzhabov, and M. K. Volkov, Nucl. Phys. Proc. Suppl. **198**, 51 (2010).
12. D. Blaschke, M. Buballa, A. E. Radzhabov, and M. K. Volkov, Phys. Part. Nucl. **41**, 921 (2010).
13. A. E. Radzhabov, D. Blaschke, M. Buballa, and M. K. Volkov, arXiv: 1012.0664.
14. G. Ripka, *Quarks bound by chiral fields: The quark-structure of the Vacuum and of Light Mesons and Baryons*, Oxford, Clarendon Press, 1997.
15. L. McLerran, K. Redlich, and C. Sasaki, Nucl. Phys. A **824**, 86 (2009).
16. R. D. Pisarski, Phys. Rev. D **62**, 111501 (2000).

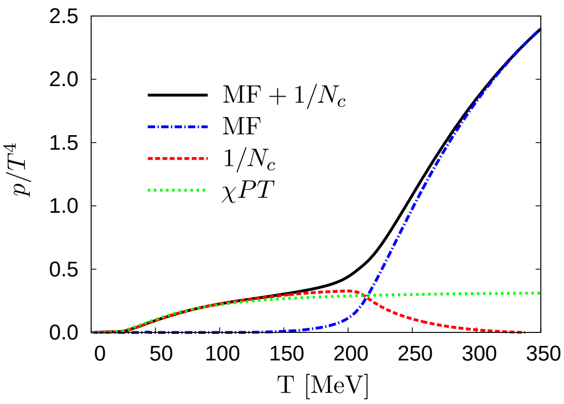

Figure 1. Temperature dependence of the scaled pressure p/T^4: mean field contribution (dash-dotted line), $1/N_c$ contribution (dashed line), mean field + $1/N_c$ contributions (solid line), the lowest order chiral perturbation theory (χPT) result (dotted line).

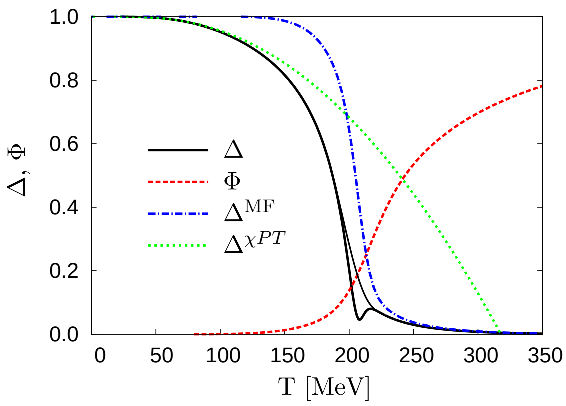

Figure 2. Temperature dependence of the quark condensate normalized to its vacuum value $\Delta = \langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle$ (thick solid line) and the Polyakov loop (dashed line) in the nonlocal PNJL model beyond mean field. Furthermore shown are the mean-field Δ (dash-dotted line), the lowest order chiral perturbation theory (χPT) result (dotted line), a naïve polynomial interpolation in the unstable region of the $1/N_c$ expansion (thin solid line).

FIGURE CAPTIONS

Fig.1: Temperature dependence of the scaled pressure p/T^4: mean field contribution (dash-dotted line), $1/N_c$ contribution (dashed line), mean field + $1/N_c$ contributions (solid line), the lowest order chiral perturbation theory (χPT) result (dotted line).

Fig.2: Temperature dependence of the quark condensate normalized to its vacuum value $\Delta = \langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle$ (thick solid line) and the Polyakov loop (dashed line) in the nonlocal PNJL model beyond mean field. Furthermore shown are the mean-field Δ (dash-dotted line), the lowest order chiral perturbation theory (χPT) result (dotted line), a naïve polynomial interpolation in the unstable region of the $1/N_c$ expansion (thin solid line).