Anomalies and Asymmetries in Quark-Gluon Matter

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The manifestations of axial anomaly and related effects in heavy ion collisions are considered. Special role is played by various asymmetries. The azimuthal correlational asymmetries of neutron pairs at NICA/FAIR energy range may probe the global rotation of strongly interacting matter. The conductivity is related to the angular asymmetries of dilepton pairs. The strong magnetic field generated in heavy ion collisions leads to the excess of soft dileptons flying predominantly in the scattering plane.

1. INTRODUCTION

The axial anomaly is known to be on of the most subtle effect in quantum field theory. Its appearance in heavy ion collisions is due to renowned Chiral Magnetic Effect (CME) which may interpreted as the local violation [1] of discrete symmetries in strongly interacting QCD matter. There is an interesting counterpart of this effect, Chiral Vortaic Effect (CVE) [2] due to coupling to P-odd medium vorticity. In its original form [2] this effect leads to the appearance of the same electromagnetic current as CME and it was recently [3] straightforwardly generalized of CVE resulting in generation of all conserved-charge currents. In particular, we address the case of the *baryonic* charge and the corresponding asymmetries of baryons, especially neutrons (not affected by CME), which can be measured by the Multi-Purpose Detector (MPD) [4] at the Nuclotron-based Ion Collider fAcility (NICA) [5] at the Joint Institute for Nuclear Research (JINR).

As soon as the suggested observables are bilinear in the induced current, one may address another probe of two current correlators, represented by angular asymmetries of dilepton pairs.

Another related issue is the manifestation of anomaly in medium. One is usually expect-

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ing the appearance of massless pole due to t'Hooft consistency condition. At the same time the experience from the studies of anomaly for virtual photons [6] manifested in the meson transition form factors [7] tells that in that case anomaly is related to the whole meson spectrum. One may expect that temperature and/or chemical potential play the role analogous to photon mass resulting in the integral relation rather than massless pole.

2. CHIRAL VORTAIC EFFECT AND NEUTRON ASYMMETRIES AT NICA

The basic point in the emergence of CME is the coupling of the topological QCD field θ to the electromagnetic field A_α controlled by the triangle axial-anomaly diagram. Similar interaction of θ with the velocity field V_α exists in relativistic hydrodynamics due to the new coupling

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha \tag{1}$$

involving the chemical potentials μ_j (for various flavours j) and the current J^α. It provides also the complementary description [8] of the recently found contribution of fluid vorticity to the anomalous non-conserved current [9]. Note that the similarity between the effects of the magnetic field and the rotation mentioned in [2] is very natural as the rotation is related by the Equivalence Principle to the so called *gravitomagnetic* field (see e.g. [10] and references therein).

CVE leads to similar (to CME) contribution to the electromagnetic current:

$$J_e^\gamma = \frac{N_c}{4\pi^2 N_f} \varepsilon^{\gamma\beta\alpha\rho} \partial_\alpha V_\rho \partial_\beta (\theta \sum_j e_j \mu_j) \ , \tag{2}$$

where N_c and N_f are the numbers of colours and flavours, respectively. If variation of the chemical potential is neglected, the charge induced by CVE for a given flavour can be obtained from that due to CME by substitution of the magnetic field with the curl of the velocity: $e_j \mathbf{H} \to \mu_j \mathbf{\nabla} \times \mathbf{V}$.

In order to estimate the vorticity one may appeal [2] to the Larmor theorem relating the magnetic field to the angular velocity of the rotating body, which in turn is proportional to the vorticity. As a result, for $\mu \sim 500$ MeV (in the NICA energy range) the order of magnitude of CVE should be the same as that of CME.

On one hand, CVE provides another source for the observed consequences of CME, relating with both light and strange [11] quarks (regarded as the heavy ones [12]). On the other hand (this is the basis of our following discussion), CVE leads also to the separation of charges different from the electric one. This becomes obvious if the current is calculated from the triangle diagram, where quark flavours j carry various charges $g_{i(j)}$ (see Fig. 1). The calculation may also be performed following [2] by variation of the effective Lagrangian with respect to the external vector field. In that case this vector field can be not only the electromagnetic potential [2] (entering the Lagrangian describing the interaction with the real electromagnetic field) but also an arbitrary (auxiliary) field coupled to any conserved charge.

If variation of the chemical potential in Eq. (2) is neglected, the current of that charge g_i selecting the specific linear combination of quark triangle diagrams is related to electromagnetic one as follows (see Fig. 1):

$$J_i^\nu = \frac{\sum_j g_{i(j)}\mu_j}{\sum_j e_j\mu_j}\, J_e^\nu \ . \tag{3}$$

In another extreme case of dominance of chemical potential gradients (assumed to be collinear) one gets the relation

$$|J_i^0| = \frac{|\boldsymbol{\nabla} \sum_j g_{i(j)}\mu_j|}{|\boldsymbol{\nabla} \sum_j e_j\mu_j|}\, |J_e^0| \tag{4}$$

which might be useful, e.g. for the mixed phase [13] description.

In particular, the large baryonic chemical potential (actually the largest one which is achievable in accelerator experiments [14]), appearing in the collisions at comparatively low energies at the FAIR and NICA (and possibly SPS and RHIC at low energy scan mode) facilities, may result in the separation of the baryonic charge. Of special interest are manifestations of this separation in *neutron* asymmetries with respect to the production plane, as soon as the neutrons, from the theoretical side, are not affected by CME and, from the experimental side, there is a unique opportunity to study neutron production and asymmetries by MPD at NICA. Besides that, the noticeable strange chemical potential at the NICA energy range (see, e.g. [15] and references therein) might result in the strangeness separation.

The numerical smallness of such expected vortaic effect makes it highly improbable to search it on an event-by-event basis. To collect statistics from different events one needs to construct a quadratic variable which does not depend on the varying sign of topological field fluctuations.

This problem was solved in the experimental studies of CME [16–19] by consideration of the angular asymmetries of *pairs* of particles with the same and opposite charges with respect to the reaction plane. Moreover, one can use three-particle correlations as well in order to avoid the necessity of fixing the reaction plane. We used [3] the similar correlations for baryonic charge.

Note that the comparison of above-mentioned correlations for various particles could be very useful [3]. Note also that as was recently shown, the baryonic charge separation due to CME is suppressed [20] if three quark flavours are taken into account. At the same time, the strange quark mass effects [11] may lead to non-complete cancellation with light quarks contribution.

For the studies of CVE we suggested [3] the collider NICA[1] which is expected to operate with average luminosity $L \sim 10^{27}$ cm^{-2}s^{-1} for AuAu collisions in the energy range $\sqrt{s_{NN}} =$ 4–11 AGeV (for Au^{79+}). For the estimation of CVE we could explore the same three-particle correlator of azimuthal angles $\langle \cos(\phi_\alpha + \phi_\beta - 2\phi_c) \rangle$ which was used for the detection of CME [17–19].

The possible magnitude of the statistical errors for the three-particle correlator with this number of collected events from the UrQMD model [21] collisions of AuAu at $\sqrt{s_{NN}} = 9$ GeV is shown in Fig. 2. The number of neutrons in each event within the mid-rapidity range is much smaller than the number of charged particles. Hence, in order to determine CVE with the same value of precision as in the CME case at RHIC [19], we need a much larger number of events. Precisely while $\sim$ 15M of events were sufficient at RHIC for targeted precision in the CME case, at NICA we need $\sim$ 1000M of events for the same precision in CVE measurements, which could be accumulated in a few months of NICA/MPD running time.

3. BILINEAR CURRENT CORRELATORS IN MEDIUM AND DILEPTON ASYMMETRIES

The two current correlators in medium reveal themselves also via the dilepton production rate [22] and angular asymmetry (anisotropy) [23] related to the tensor polarization of virtual

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[1] The value of CME at NICA is under intensive discussion [13].

photon.

The two-current correlators may be studies on the lattice [24], revealing the effects of magnetic field. They are especially pronounced for soft dileptons, when the corresponding conductivity is non-zero in the direction of magnetic field only. In that case the angular distribution of (each) lepton with respect to magnetic field direction is [2]

$$d\sigma \sim \sin^2\theta \,. \tag{5}$$

It means that the excessive soft dileptons are emitted predominantly in the reaction plane and this fact may explain why there are no obvious experimental indications of their appearance.

Note that there is another source of the in-plane photons and (soft) dileptons due to magnetic field which just the synchrotron radiation. As soon as the magnetic field is of order of the typical hadronic scale it should lead to the fast loss of energy by the fast quarks from the colliding beams and the emission of multiple in-plane photons with the energies $\sim B/m$.

One may continue such an analysis [25] by considering the finite mass photons. The existence of different spectral functions in the directions parallel and orthogonal to to the magnetic field leads to the appearance of effective spectral function dependent on the polar angle

$$\rho(\omega,\theta) = \rho_{zz}(\omega)\sin^2\theta + 2\rho_{xx}(\omega)\,. \tag{6}$$

The angular averaged distribution is

$$\bar{\rho}(\omega) = \rho_{zz}(\omega)/2 + 2\rho_{xx}(\omega)\,. \tag{7}$$

One may also introduce the effective angular dependent meson mass

$$m(\theta) = \frac{\int d\omega\omega\rho(\omega,\theta)}{\int d\omega\rho(\omega,\theta)}\,. \tag{8}$$

4. DOES T'HOOFT CONSISTENCY PRINCIPLE ALWAYS IMPLY THE MASSLESS POLE?

The t'Hooft consistency principle plays a decisive role in matching the fundamental and effective theories. It is usually formulated in terms of massless modes reproducing the massless pole (say, dues to massless quarks) in fundamental theory.

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[2] Note the obvious misprint in [24].

This situation is however not a general one and changes in the presence of some mass scale. The particular example is VVA triangle diagram with one of the photons being virtual. In that case one may derive the exact (due to t'Hooft principle) "Anomaly Sum Rule" [6] which implies the existence of massless pole in the real photon limit only.

The hadronic content of this sum rule (Fig. 3) was explored recently [7] in connection with the experimental data of BaBar collaboration which could be interpreted as a violation of QCD factorization.

One can see that pion contribution is rapidly decreasing with growing photon virtuality making the anomaly the collective effect of the whole spectrum.

One may expect the similar behaviour in the case of finite temperature and/or density, when anomaly effect should be also attributed to the whole meson spectrum. One may recall, that the interaction in hadronic phase is emerging due to the collective hadron spectrum.

Such manifestation of anomaly would not require the existence of massless excitations. This may, in particular, allow the confined phase with unbroken chiral symmetry and affect the analysis of possible quarkionic phases.

5. CONCLUSIONS AND OUTLOOK

Here the various manifestations of axial anomalies and related effects in heavy-ion collicions are discussed. The Chiral Vortaic Effect is of special interest. as it is related to the fundamental properties of medium to manifest vortaic flow.

As soon as both CME and CVE are studied due to quadratic effects, the complementary probes of bilinear current correlators in medium by dilepton asymmetries are quite important.

Also, the anomaly in medium may be a collective effect do not necessarily related to existence of massless pole.

As an outlook, let us first mention that the non-perturbative (in particular, lattice QCD [26]) studies of vorticity effects are very important. Let us also note that the large chemical potential might result in meson decays forbidden in the vacuum, like C-violating $\rho \to 2\gamma$ [27, 28] or recently considered CP-violating $\eta \to 3\pi$ [29].

Vorticity is related to the global rotation of hadronic matter, an interesting observable by itself. Its calculations in the framework of various models are very desirable, as well as studies of its possible relations with other collective effects due to non-centrality of heavy ion collisions, like directed (v_1) and elliptic (v_2) flows.

Another interesting problem is the possible manifestation of vorticity in the polarization of Λ particles was suggested some time ago in [30] although the experimental tests at RHIC [17] did not show any significant effect. One may think that such a polarization can emerge due to the anomalous coupling of vorticity to the (strange) quark axial current via the respective chemical potential, being very small at RHIC but substantial at FAIR and NICA energies. In that case the Λ polarization at NICA [13] due to triangle anomaly can be considered together with other probes of vorticity [31] and recently suggested signals [32] of hydrodynamical anomaly.

One can expect that the polarization is proportional to the anomalously induced axial current [9]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\,\mu\, n}{3\,(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho}\, V_\nu\, \partial_\lambda V_\rho\,, \tag{9}$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ-dependence of the polarization has to be more strong than that of CVE leading to the effect rapidly increasing with decreasing energy. This option may be explored in the framework of the program of polarization studies at NICA [13] performed in the both collision points as well as at the low-energy scan program at RHIC.

To collect the polarization data from different events one need to supplement the production plane with a sort of orientation. For this purpose one might use the left-right asymmetry of *forward* neutrons as it was done at RHIC [17, 18] or another observable, interesting by itself. The last comment regards handedness [33], namely, the P-odd multiparticle momenta correlation. Its exploration in heavy ion collisions provides a way of orienting the event plane and collecting data for Λ polarization and other P-odd observables.

Finally, let us mention that the chiral charge density (zeroth component of axial current (9) is proportional to fluid helicity $\mathbf{v} \cdot \mathbf{v}$ which may, in particular emerge from chaotic ABC flows (see, e.g. [34] and Ref. therein) manifesting themselves in the turbulent dynamo problem. Such relation between anomaly and chaos seems rather intriguing, although its specific manifestations remain to be studied [35].

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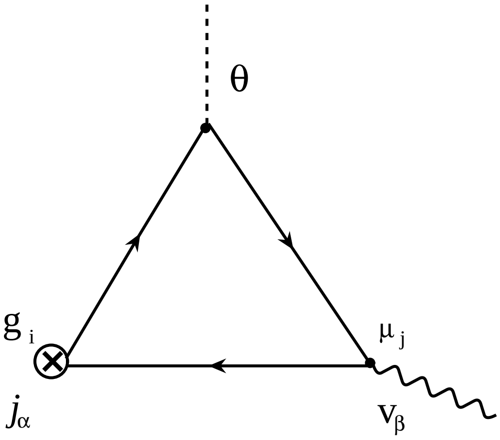

Figure 1. The generation of the current of the conserved charge g_i by the chemical potential μ_j.

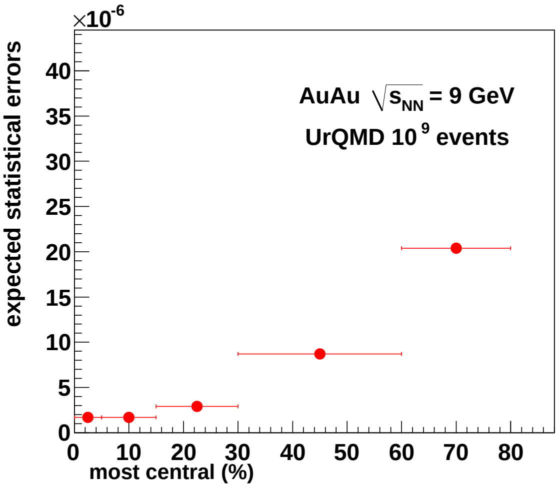

Figure 2. Estimation of statistical errors.

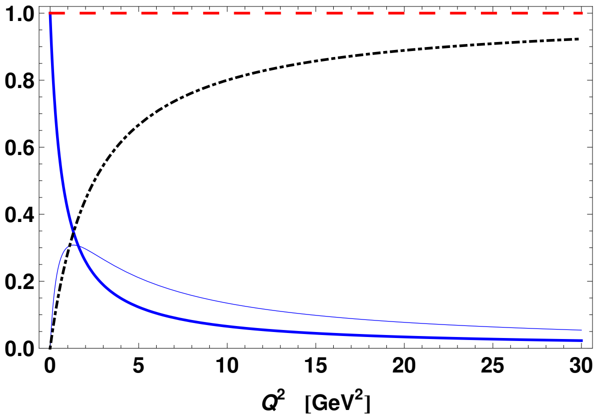

Figure 3. Contributions of pion (thick solid line), a_1 (thin solid line), and continuum (dot-dashed line) to the Anomaly Sum Rule (dashed line).

FIGURE CAPTIONS

Fig.1: The generation of the current of the conserved charge g_i by the chemical potential μ_j.

Fig.2: Estimation of statistical errors.

Fig.3: Contributions of pion (thick solid line), a_1 (thin solid line), and continuum (dot-dashed line) to the Anomaly Sum Rule (dashed line).