Viscosity of the QGP from a virial expansion

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In this work we calculate the shear viscosity η in the quark-gluon plasma within a virial expansion approach with particular interest in the ratio of η to the entropy density s, i.e. η/s. We derive a realistic equation of state using a virial expansion approach which allows us to include the interactions between the partons in the deconfined phase. From the interaction we directly extract the effective coupling $\alpha_{\rm V}$ for the determination of η. Our results for η/s show a minimum near to $T_{\rm c}$ very close with the lowest bound and, furthermore, in line with the experimental point from RHIC as well with the lattice calculations.

1. INTRODUCTION

The experimental findings of the heavy ion collisions at the Relativistic Ion Collider (RHIC) led to the announcement about the discovery of the nearly perfect fluidity of the strongly-coupled quark-gluon plasma (sQGP) [1, 2]. Ideal hydrodynamics seems to offer a good description for the experimental data for moderate momenta, in particular of the strong elliptic flow v_2. Nevertheless, ideal hydrodynamics gradually breaks down at larger impact parameter, at lower collision energy and away from midrapidity due of strong viscous effects. The key role of the viscous correction has been also addressed by the very recent v_2 data for PbPb collision at the LHC [3]. Therefore a dynamic calculation of the shear viscosity η within a model describing the strong coupling properties of the QGP is mandatory. Recently, we have developed a generalization of the classical virial expansion approach to calculate the QCD partition function in the partonic phase with an interaction inspired by lattice calculations [4]. We have obtained an Equation of State (EoS) for the QGP that compares well with three-flavor QCD lattice data [5] at nonzero temperature and vanishing quark chemical potential ($\mu_{\rm q} = 0$). Since in the virial expansion approach all thermodynamic

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quantities are based on an explicit parton interaction in form of a potential, this approach is also the ideal starting point for a consistent description of dynamical properties of the sQGP like the shear viscosity η within kinetic theory.

2. η/S AND VIRIAL EXPANSION

In an ultrarelativistic quark-gluon plasma, where the temperature T is much larger than the constituent masses m_i, the ratio of the shear viscosity η and the entropy density s can be estimated, following Ref. [6], as

$$\frac{\eta}{s} \approx \frac{4}{5} \frac{T}{s\sigma_{\mathrm{t}}} \,. \tag{1}$$

The relevant transport cross section is given by

$$\sigma_{\mathrm{t}}(\hat{s}) \equiv \int d\sigma_{\mathrm{el}} \, \sin^2\theta_{\mathrm{cm}} = \sigma_0 \, 4z(1+z) \left[(2z+1)\ln(1+1/z) - 2\right], \tag{2}$$

with the total cross section $\sigma_0(\hat{s}) = 9\pi\alpha_{\mathrm{V}}^2(\hat{s})/2\mu_{\mathrm{scr}}^2$. Here $\alpha_{\mathrm{V}} = \alpha_{\mathrm{V}}(T)$ and μ_{scr} are the effective temperature-dependent coupling constant and the screening mass, respectively, and $z \equiv \mu_{\mathrm{scr}}^2/\hat{s}$. For simplicity, we will assume σ_0 to be energy independent and neglect its weak logarithmic dependence on $\hat{s}$ in the relevant energy range and set $\hat{s} \approx 17T^2$.

In previous works [7] the entropy density and transport cross section have been uncorrelated and strongly simplified, because an ideal EoS and perturbative results for the cross section are used. In [8] a consistent approach based on a virial expansion has been developed. It includes a derivation of the EoS from the interactions between the constituents and the extraction of the effective coupling that enters the transport cross section (2). The virial expansion formalism has been introduced in Ref. [4], where a detailed derivation of the partition function $Z(T,V)$, of all thermodynamic quantities, and of the EoS of the sQGP at vanishing and finite μ_{q} has been presented. We achieve an expansion of $\ln Z$ in powers of the logarithm of the partition function in the Stefan Boltzmann limit $\zeta = \ln Z^{(0)}$ [4]. All quantities can be calculated from the partition function using thermodynamic relations. For the entropy density one obtains

$$s = \frac{\partial P}{\partial T} \qquad \text{with} \quad P = T\ln Z = T\sum_{\nu=1}^{\infty} \frac{b_\nu}{\nu!}\zeta^\nu, \tag{3}$$

where we restrict to the first two coefficients given by

$$b_1 = 1, \qquad\qquad b_2 = \int_V \mathrm{d}^3\mathbf{r} \, \left(\mathrm{e}^{-\beta W_{12}(r)} - 1\right). \tag{4}$$

Following Ref. [4] we use an effective quark-quark potential W_{12} inspired by a phenomenological model which includes non-perturbative effects from dimension two gluon condensates [9]. The effective potential between the quarks explicitly reads

$$W_{12}(r,T) = \left(\frac{\pi}{12}\frac{1}{r} + \frac{\mathcal{C}_2}{2N_\mathrm{c}T}\right) \mathrm{e}^{-M(T)r}, \tag{5}$$

where $\mathcal{C}_2$ is the non-perturbative dimension two condensate and $M(T)$ a Debye mass estimated as

$$M(T) = \sqrt{N_\mathrm{c}/3 + N_\mathrm{f}/6}\; gT = \tilde{g}T. \tag{6}$$

Using a coupling parameter $\tilde{g} = 1.30$ we describe very well the recent three-flavor QCD lattice data [5] for all thermodynamic quantities in the temperature range from 0.8 T_c to 5 T_c. In Fig. 1 the entropy density s (divided by T^3) is shown as a function of the temperature (expressed in units of the critical temperature T_c) from the virial expansion approach using Eq. (1) (solid line) as well as in the SB limit (dashed line). The symbols denote the lattice calculations from Ref. [5]. We show also the entropy density in the confined phase below T_c, where we have calculated within a generalized resonance-gas model (cf. Ref. [4]). Near T_c the deviation of our results from the ideal gas limit are sizeable and huge in the confined phase.

For the evaluation of the transport cross section we have to extract the coupling α_V from the interaction W_{12}. We follow Refs. [8, 10] and define the coupling in the so-called qq-scheme,

$$\alpha_\mathrm{qq}(r,T) \;\equiv\; -\frac{12}{\pi} r^2 \frac{\mathrm{d}W_{12}(r,T)}{\mathrm{d}r}\;. \tag{7}$$

The coupling $\alpha_\mathrm{qq}(r,T)$ then exhibits a maximum for fixed temperature at a certain distance denoted by r_max. By analyzing the size of the maximum at r_max we fix the temperature dependent coupling $\alpha_\mathrm{V}(T)$ as

$$\alpha_\mathrm{V}(T) \;\equiv\; \alpha_\mathrm{qq}(r_\mathrm{max}, T)\;. \tag{8}$$

We only consider the contribution of our dynamical degrees of freedom (quarks and antiquarks) for η/s, since here the gluons are massless and interaction free with respect to each other. Therefore, the gluons do not contribute to the ratio η/s and then the quark specific viscosity is given by

$$\frac{\eta}{s_\mathrm{q}} = \frac{4}{5}\frac{T}{s_\mathrm{q}\sigma_\mathrm{t}}\,, \tag{9}$$

where the quark contribution of the entropy density s_{q} is

$$s_{\mathrm{q}} = s - s_{\mathrm{g}}, \qquad \text{with} \quad s_{\mathrm{g}} = \frac{32\pi^2}{45} T^3, \tag{10}$$

where s_{g} is the gluon contribution to the entropy density in the SB limit. Using the Debye screening mass given in Eq. (6) we show our main results in Fig. 2 in comparison to other estimates. In the deconfined region, $T/T_{\mathrm{c}} \geq 1$, the solid line shows the results for η/s as a function of the temperature (in units of the critical temperature T_{c}) using the EoS (10), the coupling α_{V} (8) derived within the virial expansion approach. Additionally, the experimental point (square) from [11] and the lattice data [12, 13] (triangles and full dots) are shown for comparison. In the confined phase, $T/T_{\mathrm{c}} < 1$, the purple region close to T_{c} shows the estimates for η/s in the resonance-gas model [14]. The dotted line shows the scaling $\eta/s \propto T^{-4}$ from the chiral perturbation theory [15] combined with the requirement that $\eta/s = 1/4\pi$ at T_{c}. Additionally, the range (0.8–1.5) for η/s from perturbative QCD (pQCD) from Ref. [12] is depicted as a blue region and the lowest bound is indicated by the orange area. At T_{c} our result for $\eta/s \approx 0.1$ is very close to the theoretical bound of $1/(4\pi)$. For temperatures $1.5T_{\mathrm{c}} \lesssim T \lesssim 3T_{\mathrm{c}}$ the ratio η/s increases almost linearly until a saturation at high temperatures is achieved. Qualitatively, this increasing behavior of the specific viscosity with the temperature might be confirmed by lattice calculation. However, the large error bars of the lattice data do not allow for a conclusive comparison. In contrast, the "experimental point" is reproduced very well by our result. Additionally, a detailed analysis of the temperature dependence of our results for η/s, using a Taylor expansion around the critical temperature, indicates the existence of a minimum in η/s close to T_{c}.

3. CONCLUSIONS

We have performed an investigation of the specific viscosity η/s in the QGP in a dynamical way within kinetic theory using a generalized virial expansion approach. Our numerical results give a ratio $\eta/s \approx 0.1$ at the critical temperature T_{c}, which is very close to the lower theoretical bound of $1/(4\pi)$. Furthermore, for temperatures $T \leq 1.8T_{\mathrm{c}}$ the ratio η/s is in the range of the present experimental estimates 0.1–0.3 at RHIC. When combining our results for η/s in the deconfined phase with those from chiral perturbation theory or the resonance gas model in the confined phase we observe a pronounced minimum of η/s close to the critical temperature T_{c}.

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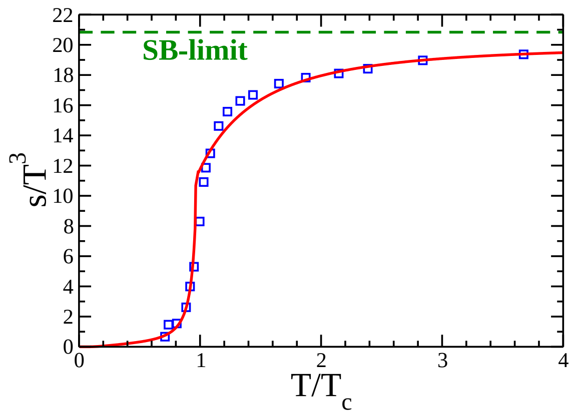

Figure 1. Entropy density s of the QGP as a function of the temperature divided by T^3 from the virial expansion (solid line) using $\tilde{g} = 1.30$. For comparison the corresponding SB limit is displayed by the dashed line. The lattice results (open squares) have been adopted from Ref. [5].

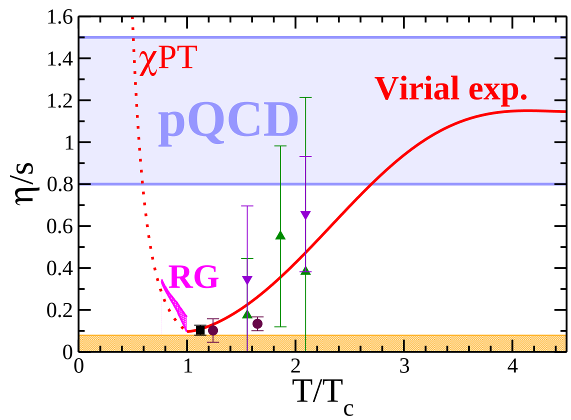

Figure 2. The viscosity/entropy density ratio η/s as a function of the temperature expressed in units of the critical temperature T_c for $T/T_\mathrm{c} < 1$ and $T/T_\mathrm{c} > 1$. The lattice results are from Ref. [12] (triangles) and from Ref. [13] (full dots). The different lines denoted by χPT, RG, and Virial exp. stand for the results from the scaling behavior of the chiral perturbation theory, the hadron resonance gas and the virial expansion approach (for $T/T_\mathrm{c} \geq 1$), respectively (see text).

FIGURE CAPTIONS

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