Quark matter conductivity in strong magnetic background

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Applying the ideas and methods of condensed matter physics we calculate the quantum conductivity of quark matter in magnetic field. In strong field quantum conductivity is proportional to the square root of the field.

After a decade of RHIC performance it has become clear that we are encountering an unusual form of matter. To decipher its properties methods traditional to condensed matter physics and hydrodynamics turn out to be effective. A very intriguing effect observed by STAR collaboration at RHIC is the electric current induced in the direction of the magnetic field. This is now famous Chiral Magnetic Effect – see [1, 2] and references therein. The magnetic field created by heavy ion currents at the collision moment is huge, $|eB| \gtrsim m_\pi^2 \sim 10^{19}$ G [2]. Magnetic field of the same order or even higher is expected at LHC. It is therefore clear that the problem of quark matter conductivity in strong magnetic field is an important albeit a complicated one. In what follows we shall propose a solution which may be traced back to the well known results of condensed matter physics [3, 4]. Two remarks are in order before we proceed to the issue. First, is that in the present short contribution details of the calculations are omitted. Second, is that here we shall not try to establish links to the Chiral Magnetic effect.

Our basic assumptions are the following. We postulate the formation of the Fermi surface in dense quark matter and assume that transport coefficients including conductivity are defined by the physical processes occurring in the vicinity of this surface. Next we assume that a certain level of disorder is present giving rise to the scattering time τ (in condensed matter physics this is the electron elastic scattering time on impurities). Following [5] we suppose the quark matter to be in a regime of weak localization [3, 4], i.e. $\tau_\varphi \gg \tau$, $k_F l \sim 1$, where τ_φ is the phase-breaking time, k_F is the Fermi momentum, l is the mean free path

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(at $k_F l < 1$ transition to Anderson localization takes place). Therefore we shall describe conductivity as a quantum process in the background of strong fluctuations [5, 6]. Is so, two physical parameters play the key role phase-breaking time τ_φ and the diffusion coefficient D [3–5]. There is an important difference from the typical picture is condensed matter physics. Namely, quantum contribution to the conductivity is important even in presence of strong magnetic field. This is a subtle point which will be elucidated in detail in the forthcoming publication. We conceive this conclusion to be true in the three-dimensional case considered here. Ultra-relativistic ions resemble two-dimensional discs and in this case quantum conductivity is infrared divergent [3, 4].

The expression for the conductivity reads [3, 4]

$$\sigma_{\alpha\beta} = -\lim_{\omega\to 0} \frac{Q_{\alpha\beta}(\omega)}{i\omega}, \tag{1}$$

where $Q_{\alpha\beta}(\omega)$ is the electromagnetic response operator defined for Matsubara frequencies $\omega_\nu = 2\pi\nu T, \;\; i\omega_\nu \to \omega$. Since we consider the regime of weak localization fan diagrams enter into the response operator on equal footing with ladder diagrams [3, 4]. This means that the response operator has the form

$$Q_{\alpha\beta}(\omega_\nu) = T\sum_{\varepsilon_n}\int \frac{d\mathbf{q}}{(2\pi)^3}\frac{d\mathbf{k}}{(2\pi)^3} j_\alpha G_1 G_2 C(\mathbf{q},\omega_\nu) G_3 G_4 j_\beta, \tag{2}$$

where $G_i (i = 1, 2, 3, 4)$ are relativistic Matsubara propagators at finite T and μ [7] with the following arguments: $G_1 = G(\mathbf{k}, i\tilde{\varepsilon}_n)$, $G_2 = G(\mathbf{k}, i\tilde{\varepsilon}_{n+\nu})$, $G_3 = G(\mathbf{q}-\mathbf{k}, i\tilde{\varepsilon}_{-n-\nu})$, $G_4 = G(\mathbf{q}-\mathbf{k}, i\tilde{\varepsilon}_{-n})$, $\tilde{\varepsilon} = \varepsilon_n + (2\tau)^{-1}\mathrm{sgn}\,\varepsilon_n$, $\varepsilon_n = \pi(2n+1)T$. The quantity $C(\mathbf{q},\omega_\nu)$ is Cooperon [3, 4]

$$C^{-1}(\mathbf{q},\omega_\nu) = 4\pi\nu\tau^2(\omega_\nu + D\mathbf{q}^2 + \tau_\varepsilon^{-1}), \tag{3}$$

where $\nu = \mu k_f/2\pi^2$ is the relativistic density of states at the Fermi surface, $D(v_F^2, T, \tau)$ is the diffusion coefficient [3, 8], the factor 4π is replaced by 2π in nonrelativistic case. Due to Cooperon diffusion like pole appears in the polarization operator. When we impose magnetic field B along z-axis $D\mathbf{q}^2$ is replaced by $(Dq_z^2+\Omega(k+1/2))$, where $\Omega = 4eDB$, e is the absolute value of the quark electric charge, $2D$ replaces the inverse mass in the cyclotron frequency, k enumerates Landau levels. Next one inserts into (2) a complete set of Landau states and the normalization factor counting the number of states per unit area of a full Landau level. Calculation of (2) is a somewhat cumbersome exercise which will be presented elsewhere (for nonrelativistic case see [3, 4]). Here we list our approximations and present the results. Only quark contributions are kept in the propagators G_i (no antiquarks). Integration over $\mathbf{k}$ is replaced by integration around the Fermi surface over $\xi = \sqrt{\mathbf{k}^2 + m^2} - \mu$. Propagators G_i are taken in the τ-approximation (dirty limit [3]). Current operators $j_\alpha = e\gamma_\alpha$ are expressed via the corresponding momenta using Gordon relation. In the static limit the result for quantum contribution to the conductivity reads

$$\sigma = -4e^2 N_f N_c |eB| D \int \frac{dq_z}{2\pi} \sum_k \frac{1}{Dq_z^2 + \Omega(k+1/2) + \frac{1}{\tau_\varphi}}, \tag{4}$$

where we have used the equation for the diffusion coefficient $D = k_F^2 \tau / 3\mu^2$, $\mu = (k_F^2 + m^2)^{1/2}$. This expression is valid in the dirty limit [8]. It comes as not a surprise that we have retrieved the non-relativistic result [3, 4] with only minor changes (we remind that antiquark contribution is omitted). The negative sign in (4) means that quantum effects result in negative magnetoresistence and may drastically suppress the total conductivity.

Next we have to estimate the parameters entering into (4) in order to see what is the value of the magnetic field that kills weak localization. The critical field is

$$|eB_c| \simeq \frac{\pi}{D\tau_\varphi} \gg m_\pi^2, \tag{5}$$

where for the estimate we used the above expression for the diffusion coefficient taken in the chiral limit with $\tau \simeq 1$ fm and took for the phase-breaking time the value $\tau_\varepsilon \simeq 4$ fm. On the other hand the value of the magnetic field at RHIC $|eB| \sim m_\pi^2$ is strong enough to guarantee the smallness of the dimensionless parameter $\delta = (4|eB|D\tau_\varphi)^{-1}$. In this limit expression (4) yields in three-dimensional case the square root dependence on the magnetic field, $\sigma \sim (|eB|)^{1/2}$ [3, 4].

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