Confining but chirally symmetric dense and cold matter

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The possibility for existence of cold, dense chirally symmetric matter with confinement is reviewed. The answer to this question crucially depends on the mechanism of mass generation in QCD and interconnection of confinement and chiral symmetry breaking. This question can be clarified from spectroscopy of hadrons and their axial properties. Almost systematical parity doubling of highly excited hadrons suggests that their mass is not related to chiral symmetry breaking in the vacuum and is approximately chirally symmetric. Then there is a possibility for existence of confining but chirally symmetric matter. We clarify a possible mechanism underlying such a phase at low temperatures and large density. Namely, at large density the Pauli blocking prevents the gap equation to generate a solution with broken chiral symmetry. However, the chirally symmetric part of the quark Green function as well as all color non-singlet quantities are still infrared divergent, meaning that the system is with confinement. A possible phase transition to such a matter is most probably of the first order. This is because there are no chiral partners to the lowest lying hadrons.

1. INTRODUCTION

A key question to QCD at high temperatures and densities is whether and how deconfinement and chiral restoration transitions (crossovers) are connected to each other. In order to answer this question we need understanding of hadron mass generation in QCD, how both confinement and chiral symmetry breaking influence the origin of mass. We know from the trace anomaly in QCD that the hadron mass (we discuss here only the light quark sector) almost entirely consists of the energy of quantized gluonic field. However, this tells us nothing about the effect of chiral symmetry breaking on hadron mass. The chiral symmetry is

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dynamically broken in the QCD vacuum and this phenomenon is crucially important for the mass origin of the lowest lying hadrons, such as pion, nucleon or rho-meson. Phenomenologically it follows from the well established (pseudo) Nambu-Goldstone nature of pion as well as from the absence of chiral partners to the lowest lying hadrons. Their mass is determined to large degree by the quark condensate of the vacuum, which can be seen from the SVZ sum rules [1, 2], as well as from many different microscopical models.

At the same time almost systematical parity doubling in both highly excited baryons [3] and mesons [4] suggests that chiral symmetry breaking in the vacuum is almost irrelevant to the mass generation of these hadrons, i.e. the chiral (and $U(1)_A$) symmetry gets effectively restored, for a review see [5]. This conjecture is strongly supported by the pattern of strong decays of excited hadrons [6]. Experimental discovery of the still missing chiral partners to some high lying states is required, however, for the unambiguous conclusion [7, 8].

If effective chiral restoration in excited hadrons is correct, then there is a possibility in QCD for a phase with confinement (i.e. elementary excitations are of the color-singlet hadronic type) and where at the same time chiral symmetry is restored.

2. QUARKYONIC MATTER

In the large N_c world with quarks in the fundamental representation there are neither dynamical quark-antiquark nor quark-quark hole loops. Consequently, at low temperatures there is no Debye screening of the confining gluon propagator and gluodynamics is the same as in vacuum. Confinement persists up to arbitrary large density. In such case it is possible to define quarkyonic matter [9]. In short, it is a strongly interacting matter with confinement and with a well defined Fermi sea of baryons or quarks. At smaller densities it should be a Fermi sea of nucleons (so it matches with standard nuclear matter), while at higher densities, when nucleons are in a strong overlap, a quark Fermi surface should be formed. While a quark Fermi sea is formed, the system is still with confinement and excitation modes are of the color-singlet hadronic type. Obviously, the question of existence or nonexistence of the quarkyonic phase in the real world $N_c = 3$ cannot be formulated and studied with models that lack explicit confinement of quarks.

Note that nothing can be concluded from this simple argument about existence or nonexistence of the chiral restoration phase transition, i.e., whether there is or not a "subphase" with restored chiral symmetry within the quarkyonic matter (quite often, unfortunately, a quarkyonic matter is confused with confining but chirally symmetric phase.)

In the large N_c 't Hooft limit such a matter persists at low temperatures up to arbitrary large densities. At which densities in the real $N_c = 3$ world will we have a deconfining transition (which could be a very smooth crossover) to a quark matter with uncorrelated single quark excitations? Lattice results for $N_c = 2$ suggest that such a transition could occur at densities of the order 100×nuclear matter density [10]. If correct, then it would imply that at all densities relevant to future experiments and astrophysics we will have a dense quarkyonic (baryonic) matter with confinement.

The most interesting question concerns the fate of chiral symmetry breaking in this dense, cold quarkyonic matter with confinement. Indeed, in a dense matter one expects that all lowest lying quark levels (that are required for the very existence of the quark condensate) are occupied by valence quarks and Pauli blocking prevents dynamical breaking of chiral symmetry. Consequently, one would obtain a confining but chirally symmetric phase within the quarkyonic matter [11]. To this end one needs a quark that is confined but at the same time its Green function is chirally symmetric. Is it possible?

There is no way to answer today this question within QCD itself. What can be done is to clarify the issue whether it is possible or not in principle. If possible, a key question is about the physical mechanism that could be behind such a phase. Then to address this question one needs a model that is manifestly confining, chirally symmetric and provides dynamical breaking of chiral symmetry. Such a model does exist. The answer to the question above is "yes", at least within the model.

3. CONFINING AND CHIRALLY SYMMETRIC MODEL

We will use the simplest possible model that is manifestly confining, chirally symmetric and guarantees dynamical breaking of chiral symmetry in a vacuum [12, 13]. It is assumed within the model that the only gluonic interaction between quarks is a linear instantaneous potential of the Coulomb type. (This model can be considered as a 3+1 dim generalization of the 't Hooft model [17]. In the 't Hooft model, that is the large N_c QCD in 1+1 dimensions, the only interaction between quarks is a linear confining potential of the Coulomb type.) Such potential in 3+1 dimensions is a main ingredient of the Gribov-Zwanziger scenario in

Coulomb gauge [14] and is indeed observed in Coulomb gauge variational calculations [15] as well as in Coulomb gauge lattice simulations [16].

A key point is that the quark Green function (that is a solution of the gap equation in a vacuum) contains not only the chiral symmetry breaking part A_p, but also the manifestly chirally symmetric part B_p:

$$\Sigma(\mathbf{p}) = A_p + (\boldsymbol{\gamma} \cdot \hat{\mathbf{p}})[B_p - p]. \tag{1}$$

The linear potential requires the infrared regularization. Otherwise all loop integrals are infrared divergent. All observable color-singlet quantities are finite and well defined in the infrared limit (i.e., when the infrared cutoff approaches zero). These are hadron masses, the quark condensate, etc. In contrast, all color-nonsinglet quantities are divergent. E.g., single quarks have infinite energy and consequently are removed from the spectrum. This is a manifestation of confinement within this simple model.

Given a quark Green function obtained from the gap equation, one is able to solve the Bethe-Salpeter equation for mesons. A very important aspect of this model is that it exhibits the effective chiral restoration in hadrons with large J [5, 18, 19]. This is because chiral symmetry breaking is important only at small momenta of quarks. But at large J the centrifugal repulsion cuts off the low-momenta components in hadrons and consequently the hadron wave function and its mass are insensitive to the chiral symmetry breaking in the vacuum. The chiral symmetry breaking in the vacuum represents only a tiny perturbation effect: Practically the whole hadron mass comes from the chiral invariant dynamics. This explicitly demonstrates that it is possible to construct hadrons in such a way that their mass origin is not the quark condensate. If so, it is clear apriori that there are good chances to obtain a confining but chirally symmetric matter within this model.

Now we want to see what will happen with confinement and chiral symmetry at zero temperature and large density. There are no quark loops within this model and consequently Debye screening of the confining potential is absent. Confinement persists up to arbitrary large density. Will it be possible to restore chiral symmetry at some density?

This 3+1 dim model is complicated enough and it is not possible to solve it exactly for a dense baryonic matter. What can be done is a kind of a mean-field solution. To obtain such a solution we need an additional assumption. Namely, we assume that both rotational and translational invariances are not broken in a medium. This implies that we assume a liquid phase.

This assumption is rather important and it is worth to discuss its relevance. We know that within the $N_c = 3$ QCD both translational and rotational invariances are not broken in the one nucleon system. We also know that the nuclear matter is a liquid, i.e. the translational and rotational invariances are not broken.

This real life situation is drastically different from exactly solvable 1+1 dimensional theories (the 't Hooft model and the Gross-Neveu model) [20]. In the latter cases the translational invariance is broken both for a one-nucleon and baryonic matter solutions and one obtains a chiral spiral with inhomogeneous chiral condensate. The baryonic matter is in a crystal phase. In contrast, we assume that in the real life a dense baryonic matter is a liquid, like standard nuclear matter. Such assumption is also supported by the fact that at $N_c = 3$ and very high densities the relevant phase is a color-superconducting matter, where the system is also a (ideal) liquid. It is difficult to imagine a phase diagram, where at $T = 0$ a liquid-crystal-liquid sequence of phases with increasing density would exist. Hence we assume that at zero temperature we always have a liquid, all the way up to very high density. Obviously, properties of this liquid should be quite different at different densities. Such assumption implies that both the rotational and translational invariances are not broken in a dense matter.

Given unbroken translational and rotational invariances and assuming that confinement survives up to rather large density we can now try to answer the question about possible existence of confining but chirally symmetric phase [11].

Imagine that we have a dense baryon (quarkyonic) matter consisting of overlapping baryons with a well defined quark Fermi sea and the quark Fermi momentum is P_f. At the same time the interquark linear potential is not yet screened (in the large N_c limit it is not screened at any density). In order to understand what happens with chiral symmetry we have to solve a gap equation for a probe quark with momentum above P_f. All intermediate quark levels below P_f are Pauli blocked and do not contribute to the gap equation. Consequently, at sufficiently large P_f a chiral restoration phase transition happens, see Fig. 1. Chiral symmetry is restored like in the NJL model, because there is not available phase space in the gap equation to create a nontrivial solution with broken chiral symmetry. This required phase space is removed by the Pauli blocking of levels with positive energy. The standard quark-antiquark condensate of the vacuum vanishes.

Above the chiral restoration phase transition the chiral symmetry breaking part of the quark self-energy identically vanishes, $A_p = 0$. What crucially distinguishes this confining model from the nonconfining models like NJL, is that the quark Green function contains also a chirally symmetric part, B_p. This chirally symmetric quark self-energy, B_p, does not vanish both below and above the chiral restoration phase transition and is infrared-divergent, like in vacuum. This means that even in the chirally restored regime the single quark energy is infinite and a single quark is removed from the spectrum. This infrared divergence is exactly canceled, however, in any color-singlet excitation of baryonic or mesonic type. Consequently, a spectrum of excitations consists of a complete set of all possible chiral multiplets. Energies of these excitations are finite and well defined quantities. A mass of this confining matter is chirally symmetric and comes from the chiral invariant dynamics.

Such confined phase with restored chiral symmetry can be viewed as a system of chirally symmetric baryons that are in a strong overlap. Confining gluonic fields are not screened, but quarks can move not only within each individual baryon, but also within the matter by hoping from one baryon to another. So one cannot say to which specific baryon a given quark belongs. However, it would be a mistake to consider these quarks as free particles. They are colored and are subject to strong gluonic confining fields. Their dispersion law is a complicated one (it contains the infrared divergent term), by far not as for free particles. This picture is different from the naive perlocation picture, where one thinks that due to perlocation of baryons the quarks are free.

An interesting question is what happens near the Fermi surface of such dense confining matter with vanishing quark-antiquark condensate. Could be there some surface phenomena like chiral density waves [21]? These chiral density waves have been derived so far as an instability of the quark Fermi sea (with free unconfined quarks) due to a gluon exchange force between quarks and quark holes with large momenta near the Fermi surface. However, in the confining baryonic mode the relevant degrees of freedom near the Fermi surface are color singlet baryons. In such case all lowest excitations are of the baryon-baryon hole type (i.e., of three quarks-three quark holes type), because it costs no energy to excite a baryon from the Fermi surface to the next not occupied level. Here the gluon-exchange force in the quark-quark hole pairs is simply absent and the chiral density waves cannot be formed. The quark-quark hole excitations with the color-exchange force between the quark and the quark hole would correspond to intrinsic excitations of baryons near the Fermi surface which costs a lot of energy.

4. WHAT ORDER IS THE CHIRAL RESTORATION PHASE TRANSITION?

A very interesting question is about the order of the phase transition from the confining matter with broken chiral symmetry to the confining matter with restored chiral symmetry at low temperatures. Within the above mentioned model this question can be answered by microscopical calculations. However, such model answer to this question is of little relevance to real QCD with $N_c = 3$.

The point is that in vacuum the hadron spectrum within the model above can be considered as a set of linear chiral multiplets and the chiral partners within each multiplet are split by the chiral symmetry breaking effects [11]. Hence, there is a one-to-one correspondence between the chiral partners. Consequently, across the phase transition there is a continuous transformation of hadrons from the broken chiral symmetry phase to the phase with restored chiral symmetry and the phase transition is of the second order.

However, in real QCD it is by far not so. Chiral symmetry is strongly broken in the lowest lying hadrons, so there are not chiral partners to hadrons like nucleon or ρ-meson. (It would be possible to assign approximate chiral partners only if the chiral symmetry breaking effect were weak). This can be directly seen from the lowest lying spectroscopy in vacuum: there is not a one-to-one correspondence of positive and negative parity hadrons in the low lying part of the spectrum. Hence, it is not possible to arrange all low lying hadrons into approximate linear chiral multiplets. Only for the high lying hadrons such a correspondence can be seen.

This question can be also investigated on the lattice. The chiral content of the ρ-meson has been studied in dynamical simulations [22]. It turned out that the physical ρ-meson is a strong mixture of two chiral representations and it is not possible to assign the lowest a_1 and the h_1 mesons as rho's chiral partners. For the nucleon the chiral symmetry breaking effect is probably even stronger and the nucleon is a strong mixture of several different chiral representations.

In the chirally symmetric phase with confinement *all* hadrons must be arranged into chiral multiplets. Then it is not possible to connect continuously the lowest lying hadrons in the Nambu-Goldstone mode, that have no chiral partners, with hadrons in the Wigner-Weyl mode, which are all arranged into exact chiral multiplets. The phase transition is then necessarily discontinuous, i.e., of the first order.

A possible schematic phase diagram is shown on Fig. 2.

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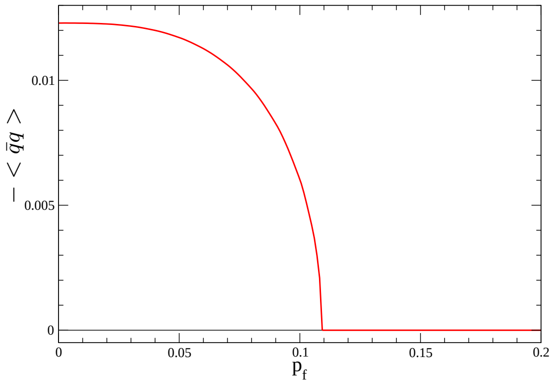

Figure 1. Chiral restoration phase transition in a dense quarkyonic matter with unbroken translational and rotational symmetries.

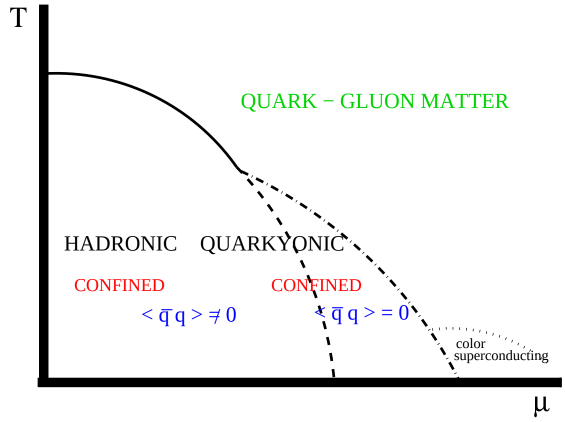

Figure 2. A schematic phase diagram.

FIGURE CAPTIONS

Fig.1: Chiral restoration phase transition in a dense quarkyonic matter with unbroken translational and rotational symmetries.

Fig.2: A schematic phase diagram.