How to extract physics from $\nu_{\rm dyn}$

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In this paper we summarize the properties of $\nu_{\rm dyn}$ and discuss briefly how the results can be interpreted in terms of a simple pair production model. The ideas are then illustrated in detail with a PYTHIA simulation study of forward-backward correlations in pp collisions.

1. INTRODUCTION

In physics one often encounters observables that are positively correlated because of a common origin rather than direct dynamical effects. An example of this is the charged particle multiplicities at forward, $N_{\rm fwd}$, and backward, $N_{\rm bck}$, rapidities in high energy heavy ion collisions which depends primarily on the number of interacting (wounded) nucleons.

To study the fluctuations of dynamic origin the first challenge is to separate out the trivial fluctuations due to the variation in the number of interacting nucleons. In these conditions it is advantageous to use $\nu_{\rm dyn}$ which was proposed in [1, 2], since with this observable the trivial fluctuations cancels out.

The second challenge is to interpret the physics of the dynamical correlations. In a recent paper we have shown how one can interpret $\nu_{\rm dyn}$ in term of a simple pair production model [3].

Here, in Section 2 we summarize these and previous results before we in Section 3 applies it to the study of simulated forward-backward correlations in pp.

2. THE BASIC PROPERTIES OF $\nu_{\rm DYN}$

Derivations of the results presented here and other details can be found in [3].

For each event we measure two numbers m and n. If we consider the expression:

$$\frac{m}{\langle m\rangle} - \frac{n}{\langle n\rangle}, \tag{1}$$

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it is easy to see that it has the average value 0.

We define ν as the variance of Eq. (1) so that:

$$\nu = \mathrm{V}\left[\frac{m}{\langle m\rangle} - \frac{n}{\langle n\rangle}\right] = \left\langle\left(\frac{m}{\langle m\rangle} - \frac{n}{\langle n\rangle}\right)^2\right\rangle \tag{2}$$

$$= \frac{\langle m^2\rangle}{\langle m\rangle^2} - 2\frac{\langle mn\rangle}{\langle m\rangle\langle n\rangle} + \frac{\langle n^2\rangle}{\langle n\rangle^2}. \tag{3}$$

For purely statistical fluctuations this expression is reduced to

$$\nu_{\mathrm{stat}} = \frac{1}{\langle m\rangle} + \frac{1}{\langle n\rangle}, \tag{4}$$

and thus we have

$$\nu_{\mathrm{dyn}} = \nu - \nu_{\mathrm{stat}} = \frac{\langle m(m-1)\rangle}{\langle m\rangle^2} - 2\frac{\langle mn\rangle}{\langle m\rangle\langle n\rangle} + \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}. \tag{5}$$

The important properties of ν_{dyn} are:

- Easy statistical behavior (ν_{stat}). No need for event mixing.
- Independent of detection efficiency (NB! the interpretation is not).
- The three terms *together* cancels trivial correlations such as e.g. centrality variation.

If we consider single particle probabilities $p, q = 1-p$ and pair probabilities $P_M(m,m)$, $P_M(m,n)$ [1], $P_M(n,n)$ then we can rewrite ν_{dyn} as:

$$\nu_{\mathrm{dyn}} = \frac{\langle M(M-1)\rangle}{\langle M\rangle^2}\left(\frac{P_M(m,m)}{p^2} - \frac{P_M(m,n)}{pq} + \frac{P_M(n,n)}{q^2}\right), \tag{6}$$

where $M = m + n$.

For uncorrelated pairs we have that $P_M(m,m) = p^2$, $P_M(m,n) = 2pq$, and $P_M(n,n) = q^2$, respectively. Inserting in Eq. (6) we find that in this case $\nu_{\mathrm{dyn}} = 0$. This shows that ν_{dyn} measures the correlated part of the pair probabilities.

Now to see what we mean with correlated pair probabilities we consider a particular model of pair production. We assume that events consists of $M/2$ correlated pairs, where each *pair* (2 particles) have the production probabilities: $P_2(m,m) = p^2 + \varepsilon$, $P_2(m,n) = 2pq - 2\varepsilon$, and $P_2(n,n) = q^2 + \varepsilon$ [2]. It turns out that ν_{dyn} is then [3]:

[1] $P_M(m,n)$ is the probability to pick, independent of order, 1 m and 1 n from a sample of M particles.

[2] This is the only parameterization that ensures that the one particle probabilities come out correctly. Note that $P_2(m,m)$ is for two particles while $P_M(m,m)$ is for $\langle M\rangle$ particles. In the presentation at CPOD2010 there was a mistake about this in the more intuitive derivation of Eq. (7).

$$\nu_{\rm dyn} = \frac{1}{\langle M \rangle} \left(\frac{\varepsilon}{p^2 q^2} \right) . \tag{7}$$

This is a very simple model for $\nu_{\rm dyn}$, but as we shall see in the next section quite powerful for understanding data. Note that when $\varepsilon < 0$ ($\varepsilon > 0$) identical pairs are less (more) likely to be produced and $\nu_{\rm dyn} < 0$ ($\nu_{\rm dyn} > 0$), i.e. the variance of Eq. (1) is smaller (larger) than for pure statistical variation. Finally we see that it is natural for the interpretation of $\nu_{\rm dyn}$ to study it as a function of $\langle M \rangle = \langle m + n \rangle$.

Another important result for interpreting $\nu_{\rm dyn}$ is the sum rule which we shall return to in the next section.

3. EXAMPLE: FORWARD-BACKWARD CORRELATIONS IN PP

The correlation between forward and backward multiplicity correlations have been studied in both *pp* and AA collisions by, e.g. UA5 and STAR [4, 5], but $\nu_{\rm dyn}$ has to our knowledge not been used before. The idea is to relate for each event the multiplicity measured in one pseudorapidity interval to the multiplicity in another pseudorapidity interval to study the correlation. Such correlations are an indication of collective effects in *pp* collisions supposedly due to multi parton interactions. With $\nu_{\rm dyn}$ we will be insensitive to the direct correlation but instead study whether or not the fluctuations *between* the multiplicities are statistical or not, i.e are there in addition to the variation in the number of interaction partons also dynamical effects that drives the correlations.

To show an example of what one can expect to learn from real data using $\nu_{\rm dyn}$ we use the *pp* Monte Carlo simulation program PYTHIA [6, 7]. We have chosen to study collisions at $\sqrt{s} = 200$ GeV, where experimental data from STAR is available, but the arguments are relevant also for other energies and heavy ion collisions. To have a large average multiplicity in each *pp* collision we use broad pseudo-rapidity intervals (1 unit wide).

Fig. 1 shows the dynamic fluctuations $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$ as a functions of $N_{\rm tot} = N_{\rm fwd} + N_{\rm bck}$, where $N_{\rm fwd}$ ($N_{\rm bck}$) is the charged particle multiplicity in the pseudorapidity intervals $0 < \eta < 1$ ($0 > \eta > -1$). As we use charged particles it is clear we have $N_{\rm fwd} = N^+_{\rm fwd} + N^-_{\rm fwd}$ and $N_{\rm bck} = N^+_{\rm bck} + N^-_{\rm bck}$ so that we in fact have $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck}) = \nu_{\rm dyn}(N^+_{\rm fwd} + N^-_{\rm fwd}, N^-_{\rm bck} + N^+_{\rm bck})$. In [3] we have derived a sum rule which relates $\nu_{\rm dyn}(N^+_{\rm fwd} + N^-_{\rm fwd}, N^-_{\rm bck} + N^+_{\rm bck})$ to the 6 "underlying terms" one can construct by calculating $\nu_{\rm dyn}$ for two subterms, e.g.,

$\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm bck})$. Because we have forward-backward symmetry and almost charge symmetry we expect that $\nu_{\rm dyn}(N^+_{\rm fwd}, N^+_{\rm bck}) \sim \nu_{\rm dyn}(N^-_{\rm fwd}, N^-_{\rm bck})$, $\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm bck}) \sim \nu_{\rm dyn}(N^-_{\rm fwd}, N^+_{\rm bck})$, and $\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm fwd}) \sim \nu_{\rm dyn}(N^+_{\rm bck}, N^-_{\rm bck})$. Therefore it is enough to measure three terms in this case, e.g., the terms before the $\sim$ sign. These three terms are also shown in Fig. 1. Note that the last term is the dynamical fluctuations of charged particles in the same pseudorapidity interval ("auto-fluctuations") which plays a crucial role in understanding the results.

Now we will try to use our pair production model to understand the physics in Fig. 1. By symmetry of the forward and backward pseudo-rapidity intervals we have $p = q = 0.5$. So that:

$$\nu_{\rm dyn} = \frac{16\varepsilon}{\langle M\rangle}, \tag{8}$$

where $\langle M\rangle = N_{\rm tot}$ for $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$, and $\langle M\rangle \sim N_{\rm tot}/2$ for the underlying terms.

The minimum is when $\varepsilon = -p^2 = -1/4$, where $\nu_{\rm dyn} = -4/\langle M\rangle$. This curve is also shown in Fig. 1. We observe that $\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm bck})$, and $\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm fwd})$ seem to follow a similar trend which is easy to understand since this must be due to charge conservation, i.e, the number of positive and negative charges are both for neighboring pseudorapidity intervals ($\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm bck})$) and in the same pseudorapidity interval ($\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm fwd})$) nearly the same. Note that actually the conservation is weaker than it seems because in Fig. 1 all $\nu_{\rm dyn}$ are shown as a function of $N_{\rm tot}$ for later comparison. This is a factor 2 difference in $\langle M\rangle$ as previously mentioned, so minimum $\nu_{\rm dyn}$ for the *underlying terms* in Fig. 1 is really $\nu_{\rm dyn} \sim -8/(\langle N_{\rm tot}\rangle)$.

One also observes that $\nu_{\rm dyn}(N^+_{\rm fwd}, N^+_{\rm bck}) \sim 0$ which is maybe not so surprising since if we in general produces pairs of 1 positive and 1 negative particles, this term goes as dynamical fluctuations of pairs of pairs. However, given that the other two underlying terms are dominated by charge conservation, this term is where one would expect to be more sensitive to interesting dynamical effects for real data.

The final strange observation in Fig. 1 is that $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$ is positive while all the underlying distributions are negative. To understand this we use the sum rule (Eq. (12) in [3]) to write up $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$ as a sum of the 3 independent terms:

$$\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck}) = \frac{1}{2}[\nu_{\rm dyn}(N^+_{\rm fwd}, N^+_{\rm bck}) + \nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm bck}) - \nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm fwd})]. \tag{9}$$

So we see that the negative "auto-fluctuations" gives a *positive* contribution to $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$. At first this might look surprising, but if we recall that $\nu_{\rm dyn}$ is related to pair probabilities Eq. (6) it should be quite clear why that is so.

So in summary without looking into the simulation code we have already a quite good understanding – qualitative and quantitative – for the dynamical fluctuations we observe in Fig. 1.

Fig. 2 shows what happens to $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$ and the underlying distributions when we move the pseudorapidity intervals apart. At first it is surprising that $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$ increases, but from the previous discussion it should be clear what happens. The "auto-fluctuations" are essentially the same, but now both the other terms in Eq. (9) are close to 0 because the real dynamic long range fluctuations are small and so as a very rough approximation $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck}) \sim -1/2\nu_{\rm dyn}(N^+_{\rm fwd}, N^-_{\rm fwd})$.

We hope that the results in this section has demonstrated the potential of $\nu_{\rm dyn}$ and also the caveats. Note that we have shown in detail in the paper [3], how if one makes a very simple model for $\pi - K$ fluctuations requiring that pions and kaons are only produced in pairs, $\pi^+\pi^-$ or K^+K^-, one obtains from auto-fluctuations alone [3]:

$$\nu_{\rm dyn}(N(K), N(\pi)) = \frac{1}{\langle N(\pi)\rangle} + \frac{1}{\langle N(K)\rangle}, \tag{10}$$

where $N(\pi) = N(\pi^+) + N(\pi^-)$ and $N(K) = N(K^+) + N(K^-)$.

4. CONCLUSIONS

In this paper we have used simulated forward-backward multiplicity fluctuations to illustrate how $\nu_{\rm dyn}$ works. It is essential for understanding the physics content of $\nu_{\rm dyn}$ to use some model, e.g., pair production and be aware that strong dynamical fluctuations are introduced by e.g. charge, strangeness, and baryon number conservation. One will have similar problems for other fluctuation observables because of the quadratic terms (e.g. $\langle m^2\rangle$) as they are sensitive to the pair probabilities. For $\nu_{\rm dyn}$ the sum rule can help separate out these trivial fluctuations from fluctuations related to phase transitions.

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[3] This can be derived using the sum rule, but is actually also covered by our pair production model with $\varepsilon = pq$ ($P_2(m, n) = 0$) and Eq. (7) can be directly used.

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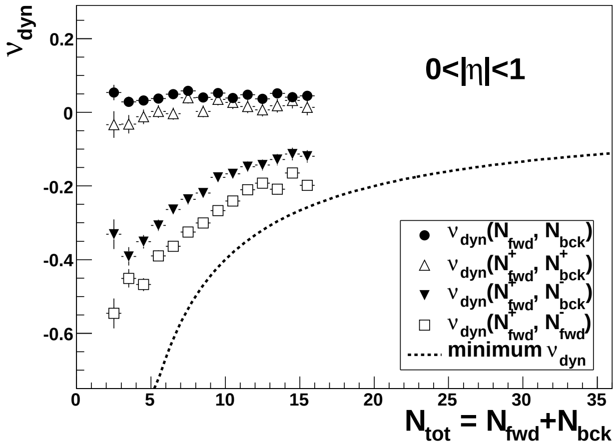

Figure 1. Dynamical fluctuations, $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$, as a function of $N_{\rm tot} = N_{\rm fwd} + N_{\rm bck}$ for $|\eta| < 1$, and the underlying contributions (see text). The dashed line shows the minimum value of $\nu_{\rm dyn}$ corresponding to $N_{\rm fwd} = N_{\rm bck}$ for all events.

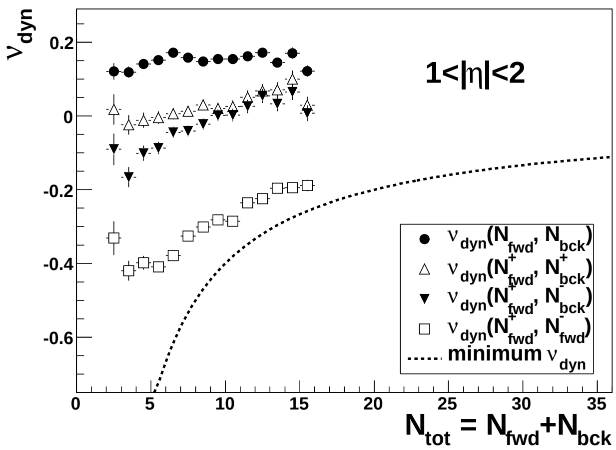

Figure 2. Same as Fig. 1 but for dynamical fluctuations in the pseudorapidity intervals $1 < \eta < 2$ and $-1 > \eta > -2$.

FIGURE CAPTIONS

Fig. 1: Dynamical fluctuations, $\nu_{\rm dyn}(N_{\rm fwd}, N_{\rm bck})$, as a function of $N_{\rm tot} = N_{\rm fwd} + N_{\rm bck}$ for $|\eta| < 1$, and the underlying contributions (see text). The dashed line shows the minimum value of $\nu_{\rm dyn}$ corresponding to $N_{\rm fwd} = N_{\rm bck}$ for all events.

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