Helmholtz International School - Workshop
"Calculations for Modern and Future Colliders"

$Z\gamma\gamma\gamma \to 0$ processes in SANC

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August 1, 2012
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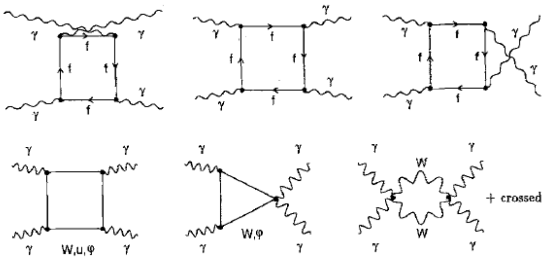
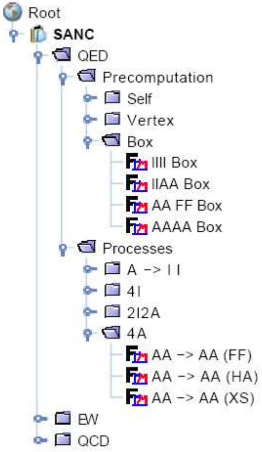
Four bosons physics

Among number of four bosons processes high priority deserve processes with $\gamma\gamma$ and gg (interest for physics at LHC) in the initial state:

$$\gamma\gamma(gg) \to \gamma\gamma(ZZ,\ Z\gamma\ , H\gamma,\ \text{etc.})$$



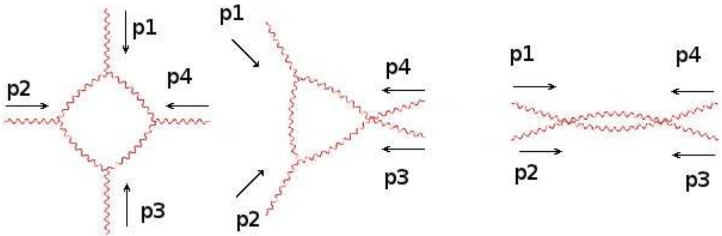
graphs for $\gamma\gamma \to \gamma\gamma$



ArXiv: hep-ph/0611188

Multi-channel approach

$Z\gamma\gamma\gamma \to 0$ processes:



$\gamma\gamma \to \gamma Z$ scattering:
$K_1 = p_1; K_2 = p_2; K_3 = -p_3; K_4 = -p_4;$

$Z \to \gamma\gamma\gamma$ decay:
$K_1 = -p_1; K_2 = -p_2; K_3 = -p_3; K_4 = p_4;$

Precomputation, amplitudes, form factors

Precomputation:
to precompute as many one-loop diagrams and derived quantities (renormalization constants, etc) as possible (to save CPU time)

Covariant Amplitudes (CA) and scalar Form Factors (FF) — $\mathcal{F}_i$

$\mathcal{A} \propto \gamma_\mu \mathcal{F}_1 + \sigma_{\mu\nu} q_\nu \mathcal{F}_2$

Helicity Amplitudes (HA) — $\mathcal{H}_{\{\lambda_i\}}(\mathcal{F}_i)$

Standard approach: $O \propto |\mathcal{A}|^2$
while in terms of HAs: $O \propto \sum_{\{\lambda_i\}} |\mathcal{H}_{\{\lambda_i\}}|^2$

Helicity Amplitudes $\gamma\gamma \to \gamma\gamma$

$$
\begin{aligned}
H^{++++}_{fermion}(s,t,u) &= -1 + \frac{u-t}{s}\left[B_0(u,m_f) - B_0(t,m_f)\right] + \left[\frac{4m_f^2}{s} + 2\left(\frac{tu}{s^2} - \frac{1}{2}\right)\right]\left[uC_0(u,m_f) + tC_0(t,m_f)\right] \\
&\quad -2m_f^2 s\left(\frac{m_f^2}{s} - \frac{1}{2}\right)\left[D_0(s,t,m_f) + D_0(s,u,m_f) + D_0(t,u,m_f)\right] \\
&\quad -tu\left(\frac{4m_f^2}{s} + \frac{tu}{s^2} - \frac{1}{2}\right) D_0(t,u,m_f); \\
H^{++--}_{fermion}(s,t,u) &= 1 - 2m_f^4\left[D_0(s,t,m_f) + D_0(s,u,m_f) + D_0(t,u,m_f)\right]; \\
H^{+++-}_{fermion}(s,t,u) &= 1 - m_f^2\left(s^2 + t^2 + u^2\right)\left[\frac{1}{tu}C_0(s,m_f) + \frac{1}{su}C_0(t,m_f) + \frac{1}{st}C_0(u,m_f)\right] \\
&\quad -m_f^2\left[\left(2m_f^2 + \frac{st}{u}\right)D_0(s,t,m_f) + \left(2m_f^2 + \frac{su}{t}\right)D_0(s,u,m_f) + \left(2m_f^2 + \frac{ut}{s}\right)D_0(u,t,m_f)\right].
\end{aligned}
$$

Comparison with M.Bohm Z.Phys. C63, 219-225 (1994) and G.Jikia arXiv:hep-ph/9312228

$$
\begin{aligned}
H^{++++}_{boson}(s,t,u) &= 1 - \frac{u-t}{s}\left[B_0(u,M_W) - B_0(t,M_W)\right] - \left[\frac{4M_W^2}{s} + 2\left(\frac{tu}{s^2} - \frac{4}{3}\right)\right]\left[uC_0(u,M_W) + tC_0(t,M_W)\right] \\
&\quad + \left[2M_W^2 s\left(\frac{M_W^2}{s} - \frac{4}{3}\right) + \frac{2}{3s^2}\right]\left[D_0(s,t,M_W) + D_0(s,u,M_W) + D_0(t,u,M_W)\right] \\
&\quad + tu\left(\frac{4M_W^2}{s} + \frac{tu}{s^2} - \frac{4}{3}\right) D_0(t,u,M_W).
\end{aligned}
$$

Helicity Amplitudes $\gamma\gamma \to \gamma Z$

$$
\begin{aligned}
H^{++++}_{bosons}(s,t,u) &= 2\Big(\frac{A_1(t,u,s)}{s_1} + \frac{A_2(s,t,u)+A_2(u,t,s)+A_3(u,s,t)}{t} \\
&+ \frac{A_1(u,t,s)}{s_1} + \frac{A_2(s,u,t)+A_2(t,u,s)+A_3(t,s,u)}{u}\Big); \\
H^{++-+}_{bosons}(s,t,u) &= 2\Big(\frac{A_1(s,t,u)-A_1(u,t,s)-A_2(s,t,u)+A_2(u,t,s)}{s_1} + \frac{A_3(s,t,u)}{u} - \frac{uA_3(u,t,s)}{ss_1} \\
&+ \frac{A_1(s,u,t)-A_1(t,u,s)-A_2(s,u,t)+A_2(t,u,s)}{s_1} + \frac{A_3(s,u,t)}{t} - \frac{tA_3(t,u,s)}{ss_1}\Big); \\
H^{+++0}_{bosons}(s,t,u) &= \frac{\sqrt{2}}{M_Z\sqrt{stu}} \times \Big(\frac{su-tM_Z^2}{s_1}A_1(t,u,s) + s(A_2(s,u,t)+A_2(t,u,s)) \\
&+ \frac{suA_3(u,s,t)}{t} - \frac{st-uM_Z^2}{s_1}A_1(u,t,s) - s(A_2(s,t,u)+A_2(u,t,s)) - \frac{stA_3(t,s,u)}{u}\Big);
\end{aligned}
$$

...

Comparison with E.W.N.Glover and A.G.Morgan Z.Phys. C 60 (1993) p.175-180

$$
\begin{aligned}
A_1(s,t,u) &= \frac{1}{4}\Big(\frac{M_Z^2}{M_W^2}-6\Big) \times \Big(\frac{4st}{t-M_Z^2} + \frac{8t}{u}(sB_1(s)-(s-M_Z^2)B_1(t)) - \frac{4M_Z^2(s+2u)t}{(t-M_Z^2)^2}B_1(t) - 8M_W^4 tF \\
&+ \frac{2st(2t+u)}{u^2}E(s,t) + \frac{8M_W^2 t}{u}E(s,t) + \frac{4M_W^2 t}{s}E(t,u) + 4M_W^2(sC(s)+tC(t)+(u-M_Z^2)C_1(u)) \\
&- \frac{8M_W^2(s+2u)t}{t-M_Z^2}C_1(t) - \frac{4M_W^2 st(u+2t)}{u}D(s,t) - 2M_W^2(utD(t,u)+stD(s,t)+usD(u,s))\Big); \\
s_1 &= s - M_Z^2 \ldots
\end{aligned}
$$

SANC Fortran packages

Fortran modules are presented in form of Fortran packages, that provide environment in which they could be tested.

Each fortran package contains

- documentation,
- declaration, initialization and various input files,
- libraries,
- main file,
- **SSFM: files containing subroutines**

SANC Fortran packages download

- 28/10/2008 SANC CC v1.11 package (131 Kb tgz-file) [stable version]

In v1.11 package some bugs in the QCD soft-virtual part are fixed. Details are in the file CHANGES.

This package is intended for calculation of the 1-loop radiative correction to Drell-Yan Charged Current processes at partonic level, see A. Arbuzov et al., Eur. Phys. J. C46 (2006) 407.

SANC 4b package.

- 31/07/2012 SANC 4b v1.00 package (105 Kb tgz-file) [last stable version]
- 19/11/2009 SANC 4A v1.00 package (21 Kb tgz-file) [stable version]

This package is intended for calculation of the 1-loop full EW correction to four bosons processes like light-by-light scattering, see D. Bardin et al., "Light-by-light scattering in SANC", hep-ph/0611188. Current processes: 4A scattering, Z3A decay and scattering.

SANC JAW packages.

- 9/12/2009 SANC JAW butd v1.01 package (6.6 Kb tgz-file) [last stable version]
- 9/12/2009 SANC JAW tbud v1.00 package (6.2 Kb tgz-file) [last stable version]
- 9/12/2009 SANC JAW udtb v1.00 package (5.1 Kb tgz-file) [last stable version]

This packages are intended for calculation of JAW functions arising at the reduction of infrared divergent box diagrams.

SANC Generators.

Drell-Yan Neutral Current processes generator.

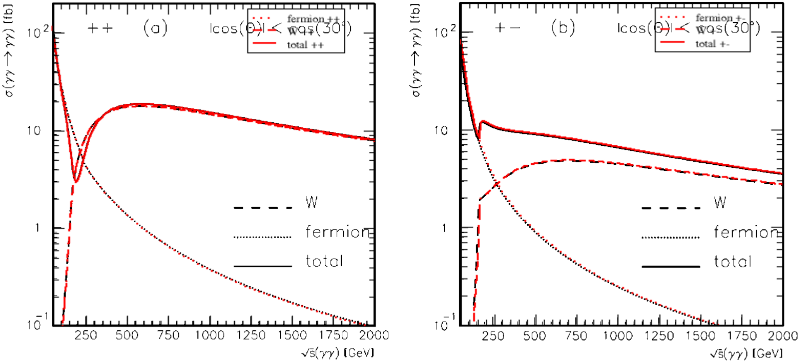
The previous versions of SANC NC generator you can find in SANC Archives.

- 23/05/2008 SANC NC DY FOAM v1.10 package (345 Kb tgz-file) [last stable version]

SANC project web-sites http://sanc.jinr.ru, http://pcphsanc.cern.ch

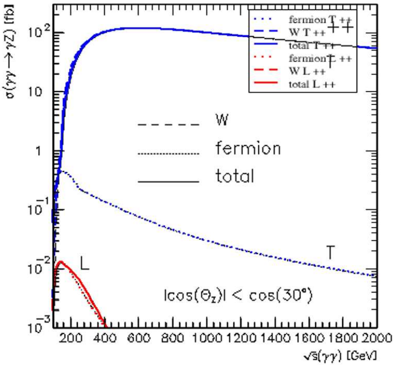
Numeric results

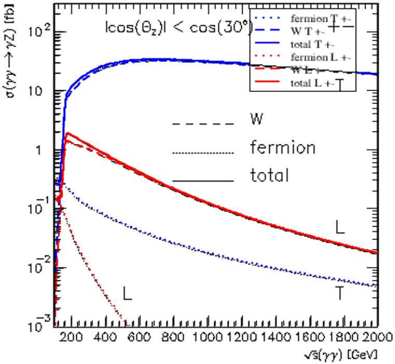
Cross section in monochromatic collisions for $\gamma\gamma \to \gamma\gamma$ process:
(Legend: red line is SANC NLO, black ones - G.Jikia)



Comparison with G.Jikia arXiv: hep-ph/9710459

Cross section in monochromatic collisions for $\gamma\gamma \to \gamma Z$ process:
(Legend: red and blue line is SANC NLO, black ones - G.Jikia)





Comparison with G.Jikia arXiv: hep-ph/9312274;
Th.Diakonidis arXiv: hep-ph/0610085

$Z \to \gamma\gamma\gamma$ decay

$$\Gamma_{fermions} = \frac{\alpha^4}{\sin^2\theta_W \cos^2\theta_W} (\sum e_f V_f) \frac{M_Z}{72\pi^3} X_F = 1.05 \times 10^{-9} GeV;$$

$$\Gamma_{bosons} = \frac{\alpha^4}{\sin^2\theta_W \cos^2\theta_W} \frac{M_Z}{72\pi^3} X_W = 2.03 \times 10^{-11} GeV;$$

$$\Gamma_{total} = 1.35 \times 10^{-9} GeV.$$

Comparison with H.Konig arXiv: hep-ph/9408334;