Higgs Boson Decays through the Order $O(\alpha\alpha_s)$

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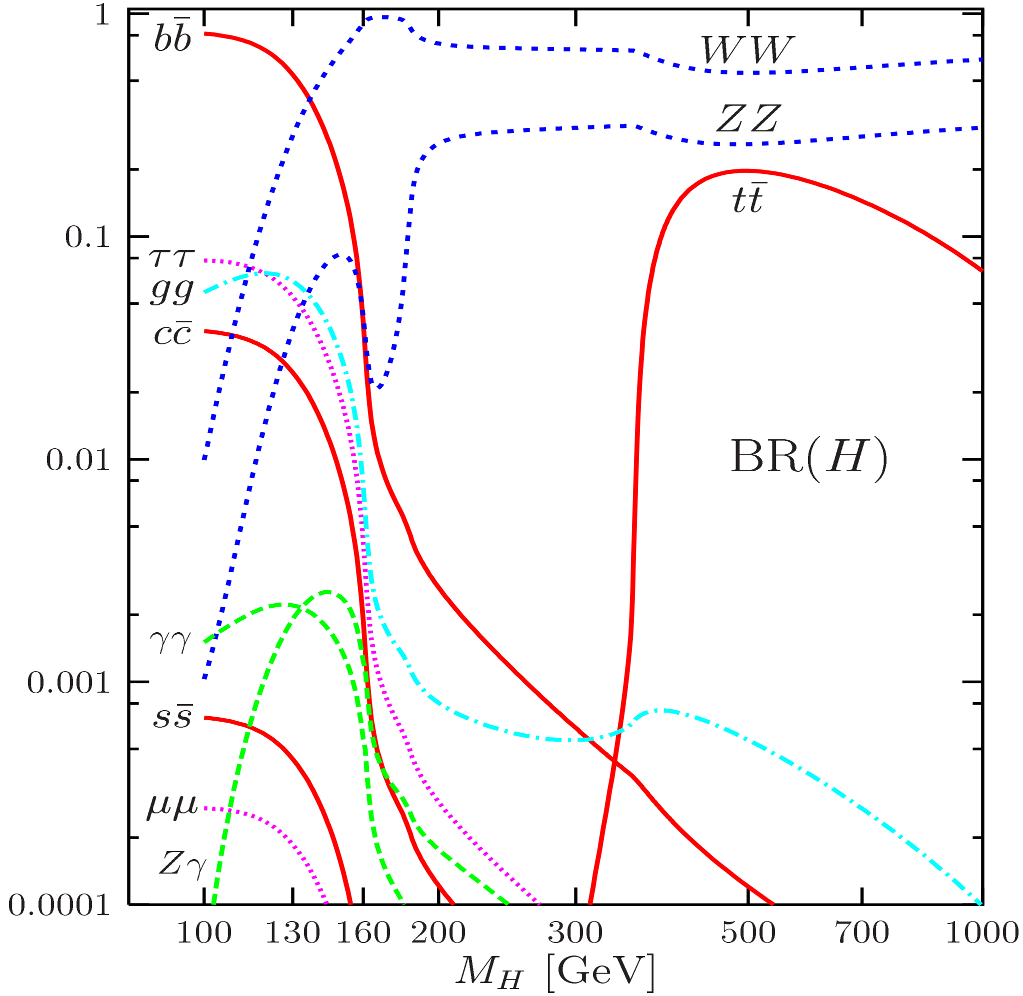
- introduction
- decays into vector bosons
- decays into fermions
- conclusions

branching ratios



decays $H \to WW$, $H \to ZZ$

- lowest order

$$\Gamma_0(H \to VV) = \delta_V \frac{G_F m_H^3}{16\sqrt{2}\pi} \sqrt{1 - \frac{4m_V^2}{m_H^2}} \left(1 - \frac{4m_V^2}{m_H^2} + \frac{12m_V^4}{m_H^4}\right), \qquad \delta_V = \begin{cases} 2 \text{ for } W \\ 1 \text{ for } Z \end{cases}$$

- EW corrections

Fleischer, Jerelehner ‘81, Hioki ’89

Bardin et al ’91, Kniehl ’91

- effective theory
  $\Longrightarrow$ pick up the leading corrections in the limit $m_t \to \infty$
  - $O(\alpha_s G_F m_t^2)$ — Djouadi, Gambino, Kniehl, Sirlin, Spira ’93,’94
  - $O(\alpha_s^2 G_F m_t^2)$ — Kniehl, Steinhauser ’95
  - $O(G_F^2 m_t^4)$ — Djouadi, Gambino, Kniehl ’98
- $O(\alpha\alpha_s)$ correction — this calculation

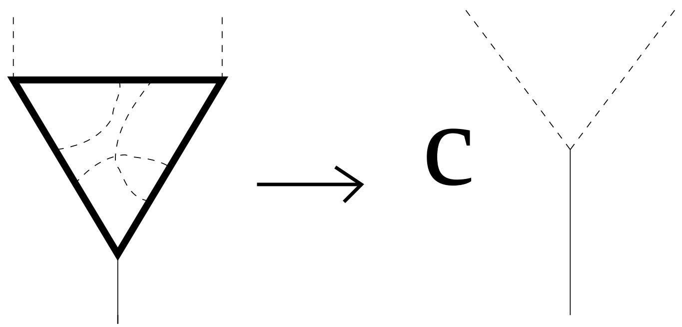
- leading terms $O(\alpha_s^j G_F m_t^2)$ can be computed whithin effective lagrangian approach In the Standard Model

$$L_{\text{Yukawa}} = -\frac{H}{v} m_t \bar{t} t$$

in the limit $m_t \to \infty$ "integrating out" t-quark $\Longrightarrow$

$$L_{\text{eff}} = -\frac{H}{v}\left(+C_g\, G^2_{\mu\nu} + \sum_q (C_q\, m_q \bar{q} q) + C_\gamma\, F^2_{\mu\nu} + \ldots \right)$$

with $C_j = C_j(m_t, \mu_F, \alpha_s)$ – perturbative coefficients



- subleading terms computed with large mass expansion (see later)

decays $H \to VV$. some results

$$\Gamma(H \to VV) = \Gamma^{\mathrm{EW}}_{1-\mathrm{loop}} \left(1 + x_t \frac{\alpha_s}{\pi} N_c C_F \Delta_{VV}\right)$$

$$x_t = G_F m_t^2 / (8\sqrt{2}\pi^2)$$

Using large mass expansion (see later):

$$\Delta_{WW} = \frac{9}{2} - \zeta_2 + \frac{m_W^2}{m_t^2}\Big[-\frac{3}{2} + \frac{m_W^2}{m_H^2}(-\frac{109}{1080} + \frac{\zeta_2}{2})\Big] + \ldots$$

$$\Delta_{ZZ} = \frac{15}{2} - \zeta_2 + \frac{m_W^2}{m_t^2}\Big[\frac{10}{9} - \frac{31}{36\cos^2\theta_W} + \frac{16\sin^2\theta_W}{9} + \frac{m_W^2}{m_H^2}(\frac{37}{135} - \sin^2\theta_W + \frac{4\sin^4\theta_W}{3})\Big] + \ldots$$

and similar for $H \to Z\gamma$.

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Using large mass expansion:

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and similar for $H \to Z\gamma$.

Analytical solutions are possible in cases of:

- gg or $\gamma\gamma$ — Spira, Harlander
- $Z\gamma$ — Spira, Zerwas

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Using large mass expansion:

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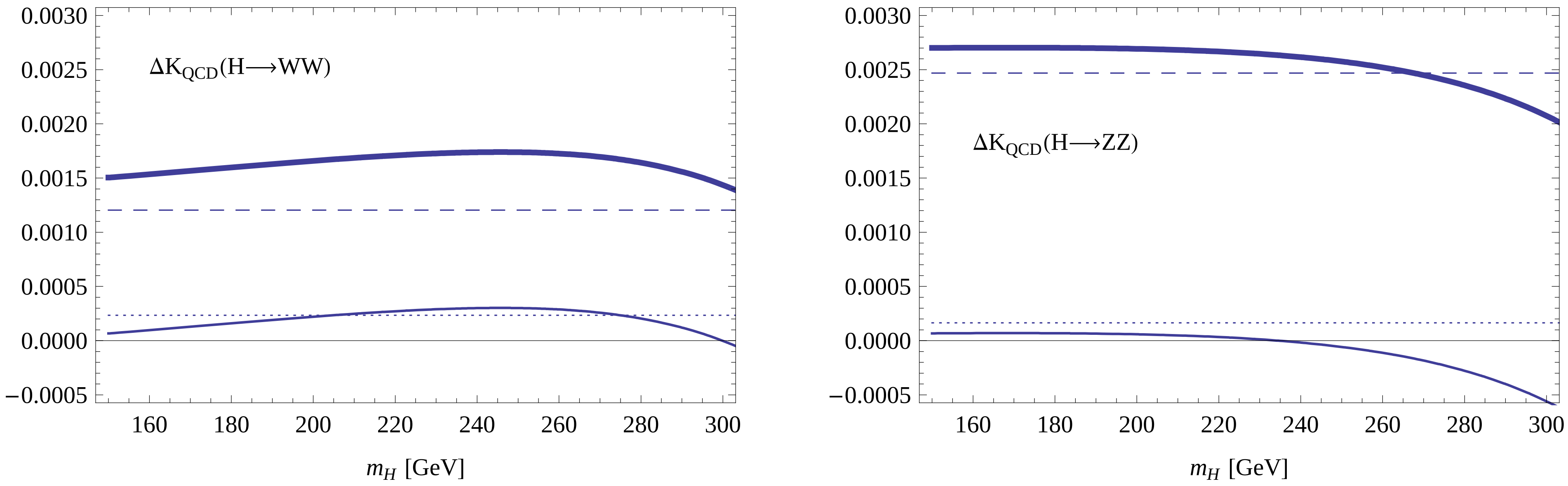
and similar for $H \to Z\gamma$.

e.g. for $m_H = 200$GeV:

$$\Delta_{WW} = 2.855 + 0.629 - 0.072 - 0.030 - 0.010 + \ldots$$
$$\Delta_{ZZ} = 5.855 + 0.238 - 0.058 - 0.028 - 0.010 + \ldots$$

$(H \to VV)$ numerical evaluation

$$\Gamma = \Gamma^{EW}_{1-\mathrm{loop}} \left(1 + \Delta K_{\mathrm{QCD}}\right)$$



- dashed line — $\alpha_s G_F m_t^2$
- solid line — subleading $\alpha_s G_F$ correction (this calculation)
- dotted line — $\alpha_s^2 G_F m_t^2$
- bold line — full QCD correction

decay $H \to \bar{b}b$

$$\Gamma_0(H \to \bar{b}b) = \frac{3G_F m_H}{4\sqrt{2}\pi}\overline{m}_b^2(m_H)(1 + \delta_{\rm EW} + \delta_{\rm QDC} + \delta_{\rm EW/QCD} + \dots)$$

- QCD correction up to $O(\alpha_s^3)$ — Gorishny, Kataev, Larin, Surguladze '91; Larin, van Ritbergen, Vermaseren '95; Chetyrkin, Kwiatkowski '96
- EW corrections — Kniehl, Sirlin '93; Djouadi, Gambino '94
- leading heavy top terms
  $\Longrightarrow$ pick up the leading corrections in the limit $m_t \to \infty$
  – $O(\alpha_s G_F m_t^2)$ — Djouadi, Gambino, Spira '93,'94
  – $O(G_F^2 m_t^4)$ — Butenschön, Fugel, Kniehl '07
- $O(\alpha\alpha_s)$ correction — this calculation

decays $H \to \bar{b}b$

Need to evaluate:

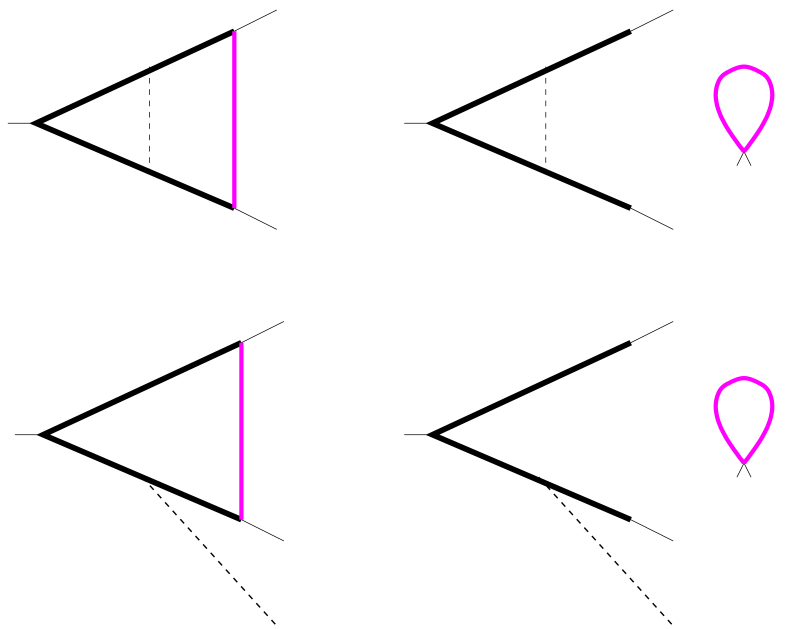
$$H \longrightarrow \bar{b}b, \quad H \longrightarrow g\bar{b}b, \quad H \longrightarrow \gamma\bar{b}b, \quad H \longrightarrow g\gamma\bar{b}b$$

diagrams of 3 types

- with γ-exchange
  $\longrightarrow$ massless diagrams
- with W-exchange (contain top-quark)
  $\longrightarrow$ large mass $(1/m_t)$ expansion
- with Z-exchange
  $\longrightarrow$ differential equations
- "universal" term (e.g. countertems, Δr, pole mass M_b)
  $\longrightarrow$ could be solved without any tricks

heavy top expansion

- consider $m_t^2 \gg m_W^2\,, m_H^2$
- in case of $H \to VV$ the actual parameter is $m_H^2/4m_t^2$
- in case of $H \to \bar{b}b$ the situation is worse
- some improvements are possible (resummations)
- examples:



differential equations

integration by parts identities (IBP):

$$\int d^d k_1 \dots d^d k_L \ \frac{\partial}{\partial k_\mu} \frac{q_\mu}{D_1^{a_1} D_2^{a_2} \dots D_r^{a_r}} = 0$$

$\Longrightarrow$ algebraic relations among integrals with different sets of $(a_1, a_2, \dots, a_r)$ and numerators

Use IBP:

- to reduce tensor integrals to the set of *master integrals*
- to write the differential eqution

differential equations

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$$\int d^dk_1 \ldots d^dk_L \;\; \frac{\partial}{\partial k_\mu} \frac{q_\mu}{D_1^{a_1} D_2^{a_2} \ldots D_r^{a_r}} = 0$$

$\Longrightarrow$ algebraic relations among integrals with different sets of $(a_1, a_2, \ldots, a_r)$ and numerators

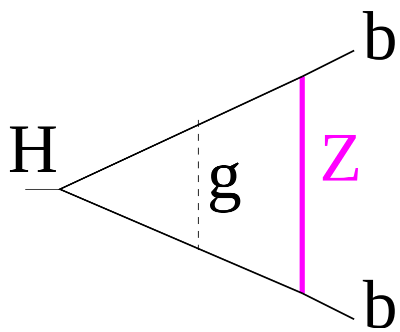
Use IBP:

- to reduce tensor integrals to the set of *master integrals*
- to write the differential eqution

$$H \longrightarrow \bar{b}b$$

diagrams with Z-boson $\longrightarrow$ one parameter

- diff. eq. w.r.t. $z = m_H^2/m_Z^2$
- solution in terms of H-functions



relevant functions

- introduce

$$H_{a,b,\ldots,c}(z) = \int_0^z \frac{dx_1}{x_1 - a} \int_0^{x_1} \frac{dx_2}{x_2 - b} \ldots\ldots \int_0^{x_k} \frac{dx_{k-1}}{x_{k-1} - c}$$

- in case $a, b, \ldots, c = +1, -0, 1$ harmonic polylogarithms

- diagrams with HVV coupling require functions with $a, b, \ldots, c = +1, 0, -1, +e^{i\pi/3}, -e^{i\pi/3}$

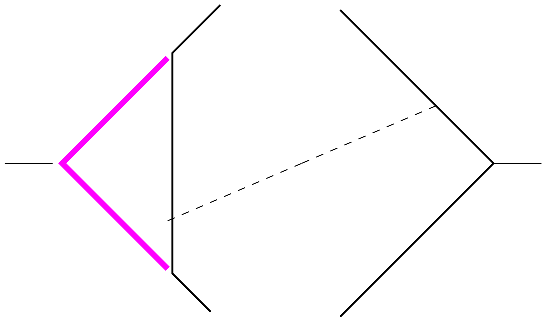
- functions up to weight $4 (= a + b + c + d)$ are required

solutions

- all 2-loop diagrams are the same as in the case of quark formfactor $Z\bar{q}q$

Kühn, Kotikov, OV '03

- some new integrals in three particle cut diagrams



- representation of cut lines

$$\frac{1}{k^2 - m^2} \longleftrightarrow 2i\pi\delta(k^2 - m^2)$$

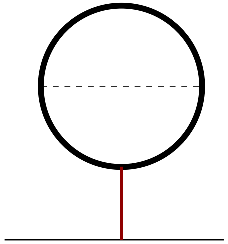
$\longrightarrow$ usual integration by part identities can be applied

the mass of b quark

the mass of b quark

- the are potentially large logarithms $L = \ln(m_H^2/m_b^2)$
- RG resummation with $\overline{\mathrm{MS}}$ mass $\overline{m}_b(\mu)$
- then width has tadpole contributions

$$\sim \frac{\alpha N_c m_t^4}{\pi m_W^2 m_H^2 \sin^2\theta_W}, \qquad \sim \frac{\alpha \alpha_s N_c m_t^4}{\pi^2 m_W^2 m_H^2 \sin^2\theta_W},$$



- if tadpoles are omitted the mass *generally* is gauge dependent
- in the on-shell scheme there are no contributions from tadpoles
- but there are large logs
- mixed schemes?

$\overline{\text{MS}}$ mass of b quark

$$
\begin{aligned}
\frac{\overline{m_b}(\mu)}{M_b} &= 1 + C_F\frac{\alpha_s}{4\pi}\Big(-4+3L_b\Big) + \frac{\alpha}{4\pi}\Big\{N_c\frac{m_t^4}{m_W^2 m_H^2 s_W^2}\Big(1-L_t\Big)\\
&+\frac{m_t^2}{m_W^2 s_W^2}\Big(-\frac{5}{16}+\frac{3}{8}L_t\Big) + \frac{m_t^2 m_W^2}{(m_t^2-m_W^2)^2 s_W^2}\Big(-\frac{3}{8}L_{tw}\Big) + \frac{3}{8}\frac{m_W^2}{(m_t^2-m_W^2)s_W^2}\\
&+a_v^2\frac{m_Z^2}{m_H^2}\Big(-4+12L_z\Big) + \frac{m_H^2}{m_W^2 s_W^2}\Big(-\frac{3}{8}-\frac{3}{8}L_h\Big) + \frac{m_W^2}{m_H^2 s_W^2}\Big(-\frac{1}{2}+\frac{3}{2}L_w\Big)\\
&+Q_b^2\Big(-4+3L_b\Big) + v_b^2\Big(-\frac{5}{2}+3L_z\Big) + a_v^2\Big(-\frac{1}{2}-3L_z\Big)\Big\}\\
&+C_F\frac{\alpha_s}{4\pi}\frac{\alpha}{4\pi}\Big\{N_c\frac{m_t^4}{m_W^2 m_H^2 s_W^2}\Big(-2+16L_t+3L_b-3L_bL_t-6L_t^2\Big)\\
&+\frac{m_t^2}{m_W^2 s_W^2}\Big(-\frac{13}{4}-\frac{3}{2}L_t-\frac{15}{16}L_b+\frac{9}{8}L_bL_t+\frac{9}{8}L_t^2\Big)\\
&\cdots\\
&\cdots\\
&\cdots\\
&+a_b^2\frac{m_Z}{m_H}(1-3L_z)\Big(16-12L_b\Big) + \frac{m_W^2}{m_H^2 s_W^2}(1-3L_w)\Big(2-\frac{3}{2}L_b\Big)\\
&+Q_b^2\Big(\frac{7}{4}-24\zeta_3-60\zeta_2+96\zeta_2\log 2-21L_b+9L_b^2\Big)\\
&+v_b^2\Big(\frac{23}{4}-\frac{15}{2}L_b-9L_z+9L_bL_z\Big) + a_b^2\Big(\frac{55}{4}-\frac{3}{2}L_b-9L_z-9L_bL_z\Big)\Big\}
\end{aligned}
$$

running Fermi constant and Yukawa constant of b quark

Fermi constant:

$$\frac{G_F}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2}(1+\Delta r_0)$$

after $\overline{\text{MS}}$ renormalization we have:

$$\frac{G_F}{\sqrt{2}} = \frac{G_F(\mu)}{\sqrt{2}}(1+\Delta G_F)$$

Relation to Yukawa coupling:

$$\frac{y_b^2(\mu)}{2\sqrt{2}G_F M_b^2} = \frac{\overline{m}_b^2(\mu)}{M_b^2}\,\frac{G_F(\mu)}{G_F}$$

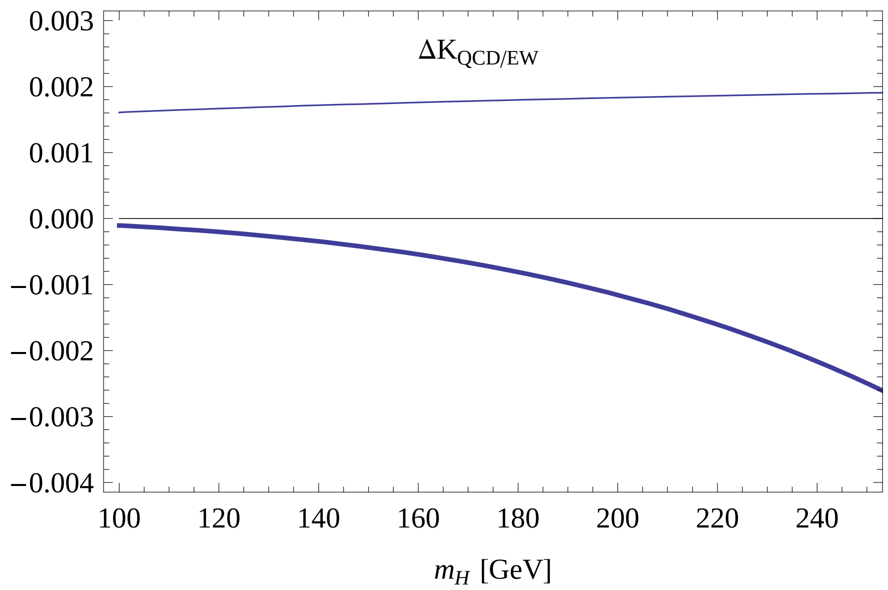
We can introduce another mass:

$$m_{b,\text{Yu}}^2(\mu) = \overline{m}_b^2(\mu)\,\frac{G_F(\mu)}{G_F}$$

$m_{b,\text{Yu}}(\mu)$ is free from large tadpole contributions and gauge invariant!

$(H \to \bar{b}b)$ numerical evaluation

(preliminary, 1-loop corrections are not included)



- solid line — leading
- bold line — subleading

conclusion

- $O(\alpha\alpha_s)$ corrections are evaluated for the main Higgs boson decay modes in the Higgs boson mass range $m_H \sim 100 - 300$ GeV

- for modes $H \to VV$ the subleading $O(\alpha\alpha_s)$ terms are small but at least as large as the 3-loop leading $O(\alpha\alpha_s^2 m_t^2)$ contributions

- for $H \to \bar{b}b$ the subleading $O(\alpha\alpha_s)$ contribution is of the same order as the leading $O(\alpha\alpha_s m_t^2)$ term