Precise theoretical predictions for Drell-Yan processes at hadron colliders: lecture 2

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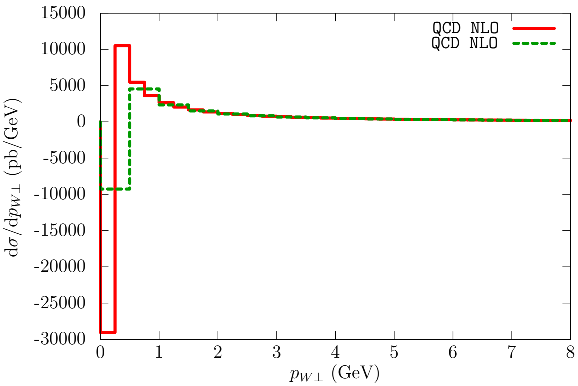
Example: W/Z transverse momentum



- NLO calculation totally unpredictive in the region of small $p_{\perp}^{V}$
- actually the tree-level prediction is zero $\Longrightarrow$
  - in the large $p_{\perp}$ region predictions are LO
  - extremely sensitive to extra radiation, in particular to multigluon radiation

resummed predictions for $Q_\perp$ (I)

review by S. Marzani, arXiv:1207.4279[hep-ph]

- $Q_\perp \sim M_Z \Longrightarrow$ perturbative prediction reliable
- $\Lambda_{\rm QCD} \ll Q_\perp \ll M_Z \Longrightarrow$ p.t. still reliable provided $\ln\left(\frac{Q_\perp}{M_Z}\right)$ are summed up to all orders
- $Q_\perp \sim \Lambda_{\rm QCD}$ non perturbative effects become dominant
- integrating the real correction to the LO cross section

$$\frac{d\sigma}{dQ_\perp^2} = \frac{\alpha_s}{2\pi}\left(\frac{A}{Q_\perp^2}\ln\frac{M^2}{Q_\perp^2} + \frac{B}{Q_\perp^2} + C(Q_\perp^2)\right)$$

- the logarithmic structure holds to all orders of p.t.

$$\frac{d\sigma}{dQ_\perp^2} = \frac{\sigma_0}{Q_\perp^2}\left[\frac{\alpha_s}{2\pi}A_1\ln\frac{M^2}{Q_\perp^2} + \ldots + \left(\frac{\alpha_s}{2\pi}\right)^n A_n\ln^{2n-1}\frac{M^2}{Q_\perp^2} + \ldots\right]$$

resummed predictions for $Q_\perp$ (II)

- the coefficients A_i can be calculated in the soft/collinear limit
- resummed solution

$$\frac{d\sigma}{dQ_\perp^2} = \sigma_0 \frac{d}{dQ_\perp^2} \exp\left(-\frac{\alpha_s}{2\pi} C_F \ln^2 \frac{M^2}{Q_\perp^2}\right)$$

- in order to improve the double log accuracy also subleading effects like momentum conservation in the transverse plane have to be introduced. To this aim it is convenient to use
  - Mellin transforms (in order to transform PDF convolution in ordinary products)
  - impact-parameter representation of the δ function

$$\delta^{(2)}\left(\sum_{i=1}^{n} \bar{k}_{\perp i} + \bar{Q}_\perp\right) = \frac{1}{4\pi^2} \int d^2\bar{b} e^{i\bar{b}\cdot\bar{Q}_\perp} \prod_{i=1,n}^{n} e^{i\bar{b}\cdot\bar{k}_{\perp i}}$$

resummed predictions for $Q_\perp$ (III)

- solution with Mellin transformation $\phi(n) = \int_0^\infty dx\, x^{n-1} \phi(x)$

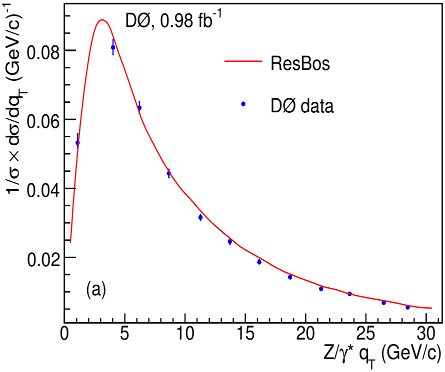
$$\frac{d\tilde{\sigma}}{dQ_\perp^2}(N) = \tilde{\sigma}_0(N) \frac{1}{4\pi^2} \int d^2\bar{b}\; e^{-R_N(b,M,N)}$$

- $R_N(b, M, N)$ involves an integral of the kind $\int_{b^{-2}}^{M^2} \frac{dk_\perp^2}{k_\perp^2} f(k_\perp^2)$ which diverges at both integration limits
- divergence at the lower limit related to non perturbative regime; different recipes to define the integral, requiring a smooth transition between perturbative and non-perturbative regimes
- e.g. a recipe introduces a gaussian form factor with parameters to be fitted from data at low $Q_\perp$ Balasz, Landry, Brock, Nadolsky, Yuan
- most recent calculations are at NNLL accuracy, with smooth transition to NLO predictions in the perturbative regime

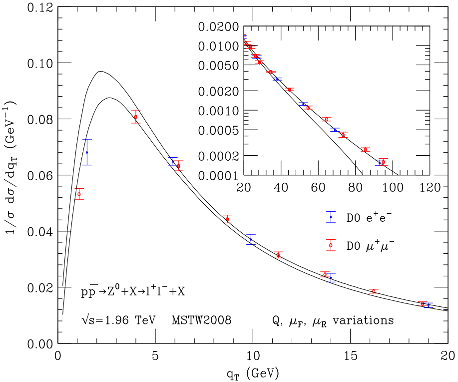
Becher, Neubert, Wilhelm; Li, Mantry, Petriello

Bozzi, Catani, Ferrera, De Florian, Grazzini

Banfi, Dasgupta, Marzani, Tomlinson



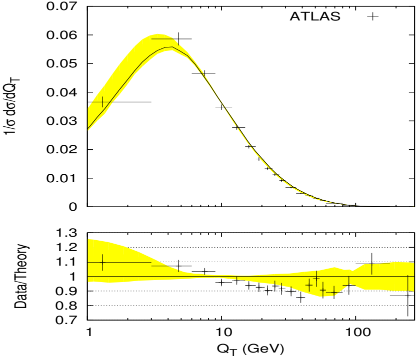
D0 coll., arXiv:0712.0803[hep-ex]

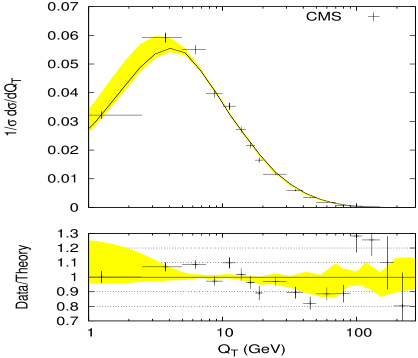


G. Bozzi et al., arXiv:1007.2351[hep-ph]

- a pure NNLL perturbative resummation matched to the NLO calculation seems to fit data without the need of non-perturbative contributions

Exercise with LHC data





A. Banfi et al., arXiv:1205.4760[hep-ph]

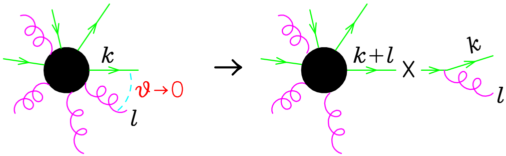
another way to resummation: parton shower

material from P. Nason, in arXiv:0902.0293; P.Nason and B. Webber, arXiv:1202.1251

- basic example: $\gamma^* \to q\bar{q}g$ in the soft limit

$$
\begin{aligned}
|\mathcal{M}_{q\bar{q}g}|^2 &= |\mathcal{M}_{q\bar{q}}|^2 4\pi\alpha_s C_F \frac{2q_1 \cdot q_2}{(q_1 \cdot k)(q_2 \cdot k)} \\
|\mathcal{M}_{q\bar{q}g}|^2 d\Phi_{q\bar{q}g} &\simeq |\mathcal{M}_{q\bar{q}}|^2 d\Phi_{q\bar{q}} d\mathcal{S} \\
d\mathcal{S} &= \frac{2\alpha_s C_F}{\pi} \frac{dE_g}{E_g} \frac{d\vartheta}{\sin\vartheta} \frac{d\phi}{2\pi}
\end{aligned}
$$

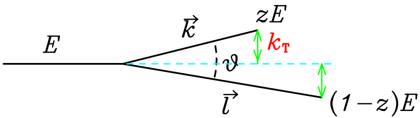
- in general



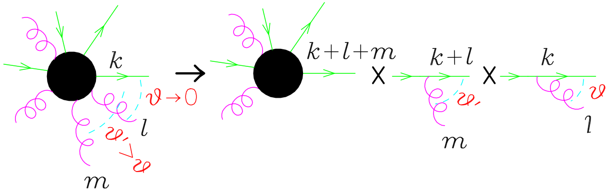
$$
|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_s(t)}{2\pi} \, \frac{dt}{t} \, P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}
$$

parton shower: factorization

- the variable t, z and ϕ parametrize the splitting kinematics



- requirements on z and t
  - t is the smallest scale compared to the scales involving all pair of external partons
  - in the collinear limit $(t \to 0)$, $k \to z(k+l)$
  - dt/t is scale invariant. Possible choices:
    - $t = (k+l)^2 \sim E^2\vartheta^2 z(1-z)$ (virtuality)
    - $t = k_\perp^2 = l_\perp^2 \sim E^2\vartheta^2 z^2(1-z)^2$ (transverse momentum)
    - $t = E^2\vartheta^2$ (angular variable)
- factorization can be iterated



- given the previous conditions on t, the evolution variable is ordered in descending order through the chain of splittings
- in QCD the same formula apply for the splittings
  - $q \to qg$, $g \to gg$, $g \to q\bar{q}$

  the only change is contained in the splitting functions

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$$

$$P_{g,gg}(z) = C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right)$$

$$P_{g,q\bar{q}} = t_f \left(z^2 + (1-z)^2 \right)$$

Sudakov form factor (I)

- from

$$|M_{n+1}|^2 d\Phi_{n+1} \Rightarrow |M_n|^2 d\Phi_n \quad \frac{\alpha_s(t)}{2\pi}\,\frac{dt}{t}\,P_{q,qg}(z)\,dz\,\frac{d\phi}{2\pi} = |M_n|^2 d\Phi_n \mathcal{P}(\Phi_{\rm rad}) d\Phi_{\rm rad}$$

  we can interpret the radiation factor as the probability of radiating a resolvable parton above a cutoff scale t_0, under which two partons are not resolvable, between t and $t+dt$, z and $z+dz$, ϕ and $\phi+d\phi$

- the cutoff t_0 is dictated by the running of α_s

$$\alpha_s(t) = \frac{1}{b_0 \log \frac{t}{\Lambda^2_{\rm QCD}}} \qquad\qquad t > \Lambda^2_{\rm QCD}$$

- dividing the interval $[t', t]$ in N subintervals δt and enforcing unitarity (prob. cons.) in each subinterval, the probability of evolution between t and t' with no resolvable emissions is

$$\Delta_i(t,t') = \prod_{i=1}^{N}\left(1 - \frac{\alpha_s(t)}{\pi}\frac{\delta t}{t_i}\int_0^1 P_{q,qg}(z)dz\right)$$

Sudakov form factor (II)

- in the limit $N \to \infty$ the probability of no resolvable emissions between scales t and $t' < t$ is

$$\Delta_S(t,t') = \exp\left[-\int_{t'}^{t} \frac{\alpha_s(t)}{\pi}\frac{dt}{t}\int_0^1 P_{q,qg}(z)\right]$$

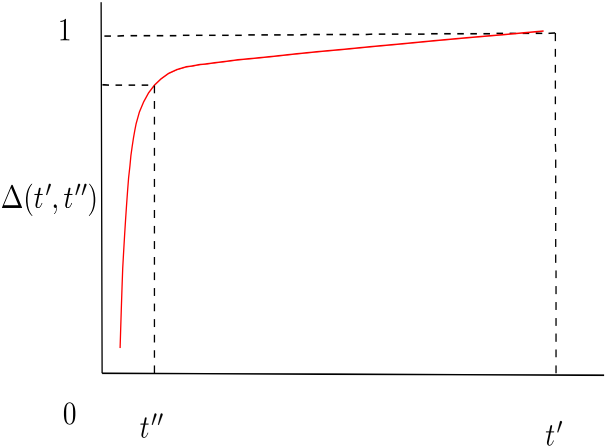
Sudakov form factor

- example: probability that, starting at scale t, the first branching is in the phase space element dt', dz, $d\phi$ is

$$\Delta_S(t,t')\frac{\alpha_s(t')}{\pi}\frac{dt'}{t'}P_{q,qg}(z)dz\frac{d\phi}{2\pi}$$

- subsequent branches in a chain are independent (Markov chain)

typical shape of the Sudakov form factor



- with very different scales the form factor becomes very small
- $\Longrightarrow$ large probability of branching

a simple analogy

- formally the previous description of subsequent branchings is equivalent to the one of subsequent decays from a radioactive source, with pdt the elementary radiation probability in the time interval dt
- the probability of no radiation from 0 to t' is

$$\Delta(t') = (1 - pdt)^{\frac{t'}{dt}} = \exp(-pt')$$

- the probability distribution of the first decay at t' is

$$\exp(-pt')pdt' = -d\Delta(t')$$

- $\Longrightarrow$ the probability distribution of the first decay is uniform in $\Delta(t')$
- $\Longrightarrow$ to generate via Monte Carlo the first decay at t' (with $0 < t' < t$), we need a uniform random r and solve for t'

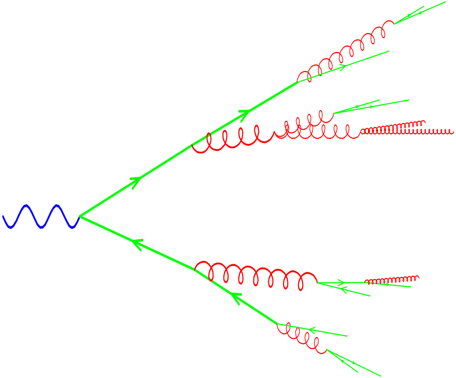
$$r = \frac{\Delta(t')}{\Delta(t)}$$

algorithm for final state shower

- generate a hard process configuration with probability proportional to its parton level cross section
- for each final state coloured parton generate a shower as follows
  1. set $t = Q$, the typical scale of the process
  2. generate a uniform random r
  3. solve for t' the equation $r = \Delta_S(t, t')$
  4. if $t' < t_0$ (the resolution scale), no branching and stop
  5. otherwise
     - generate a pair of partons jl, z with distribution $\sim P_{i,jl}(z)$ and a value ϕ with uniform probability in $[0, 2\pi]$
     - set $E_j = zE_i$ and $E_j = (1 - z)E_i$ for partons j and l
     - ϑ_{jl} is fixed by the value of t'
     - for partons j and l set $t = t'$ and go to step 2

how the $q\bar{q}$ final state appears after showering

- After application of the algorithm, for the case of $\gamma^* \to q\bar{q}$, the final states appears as a shower of partons



P. Nason, arXiv:0902.0293[hep-ph]

- for all generated partons the kinematics is known, i.e. we have full exclusive information on the final state partons

but in DY QCD radiation comes from initial state...

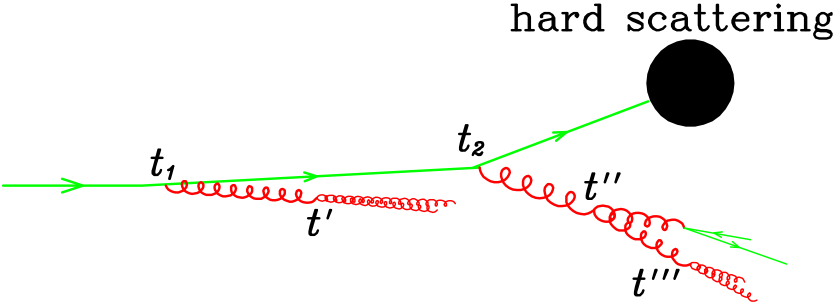
- partons from IS evolve from zero to spacelike virtualities
- usually the IS shower is performed with backward evolution: starting from the scale Q of the hard scattering the partons are evoluted down to the hadronic scale, where they are matched to the PDF
- to find the backward splitting probability it is convenient to write

$$d\Phi_n \mathcal{P}(\Phi_{\rm rad}) d\Phi_{\rm rad} \sim \frac{R_{n+1}^{\rm MC}}{R_n^{\rm MC}} d\Phi_{\rm rad}$$

  where $R_n^{\rm MC}$ is the approximate n-body cross section
- while for FSR the PDF cancel in the ratio, this does not happen for ISR

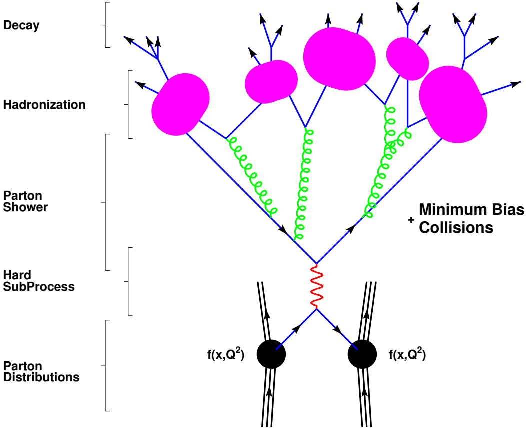
$$\mathcal{P}^{\rm ISR}(\Phi_{\rm rad}) d\Phi_{\rm rad} = \frac{\alpha_s(t)}{2\pi} \frac{dt}{t} P_{q,qg}(z) \frac{f\left(\frac{x}{z}, t\right)}{f\left(x, t\right)} dz \frac{d\phi}{2\pi}$$

picture of IS showering



- for each radiated parton not entering the hard kernel start the forward final state shower algorithm
- when all partons have been degraded to the cutoff scale, they are taken as input for the hadronization model

complete evolution of a collision at hadron colliders



parton shower event generators

- positive features
  - complementarity with fixed order calculations
  - soft/collinear regions are automatically treated with Leading Log resummation
  - they include a model for the description of the underlying event and the hadronization
  - completely exclusive event generation, very useful for interface to detector simulation software
- problems
- the cross section prediction is pure LO (due to the unitarity of the algorithm)
- step forward: matching between fixed order NLO calculation and parton shower event generators

requirements to the matching

- avoid double counting. Example: since the shower is unitary, we could be tempted to shower the various terms $(B + V + R)$ of the NLO calculation. This has two sources of double counting:
  - showering the Born events generate events with one additional gluon. Such events are already accounted for in R
  - the unitarity build in the Sudakov form factor includes the LL part of the virtual corrections, which are already present in V
- ensure smooth distributions in the phase space
- two working algorithm have been developed:
  1. MC@NLO (S. Frixione and B. Webber (2002))
  2. POWHEG (P. Nason (2004))

MC@NLO

- idea: subtract the $\mathcal{O}(\alpha_s)$ terms contained in the MC approximation

$$
\begin{aligned}
d\sigma_{\rm mod} &= \left(B(\Phi_{\rm B}) + \hat{V}(\Phi_{\rm B}) + \int R^{\rm MC}(\Phi_{\rm B}, \Phi_{\rm rad}) {\rm d}\Phi_{\rm rad} \right) d\Phi_{\rm B} \\
&+ \left(R(\Phi_{\rm B}, \Phi_{\rm rad}) - R^{\rm MC}(\Phi_{\rm B}, \Phi_{\rm rad}) \right) {\rm d}\Phi_{\rm B} {\rm d}\Phi_{\rm rad}
\end{aligned}
$$

- generate "NLO" events with $d\sigma_{\rm mod}$ and then give them as initial condition to the parton shower
- drawbacks:
  - negative weight events appear (even if unweighted event generation can be performed)
  - the subtraction terms are specific of the parton shower Monte Carlo in use

POWHEG

- introduced to avoid negative weights
- idea: observe that the parton shower master formula can be written as

$$d\sigma^{\rm MC} = B{\rm d}\Phi_{\rm B}\left[\Delta(Q_0) + \Delta(p_\perp)\frac{R^{\rm MC}}{B}{\rm d}\Phi_{\rm rad}\right]$$

$$\Delta(p_\perp) = \exp\left[-\int \frac{R^{\rm MC}}{B}\theta(p_\perp(\Phi_{\rm R}) - p_\perp){\rm d}\Phi_{\rm rad}\right]$$

- substitute in the above $R^{\rm MC}$ with R and the overall factor B with $\bar{B} = B + V + \int R{\rm d}\Phi_{\rm rad}$, i.e.

$$d\sigma = \bar{B}{\rm d}\Phi_{\rm B}\left[\Delta(Q_0) + \Delta(p_\perp)\frac{R}{B}{\rm d}\Phi_{\rm rad}\right]$$

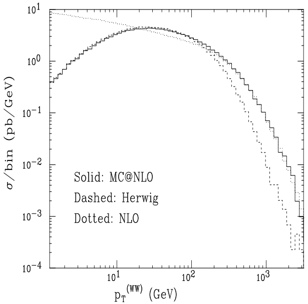
$$\Delta(p_\perp) = \exp\left[-\int \frac{R}{B}\theta(p_\perp(\Phi_{\rm R}) - p_\perp){\rm d}\Phi_{\rm rad}\right]$$

- generate the hardest radiation with $d\sigma$ and then all other (ordered in $k_\perp$) with the standard Parton Shower

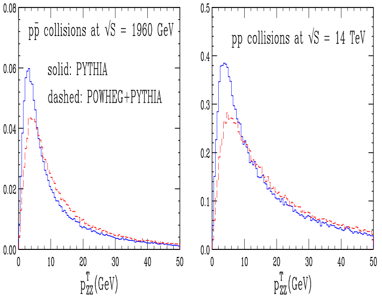
comparison MC@NLO - POWHEG

- both ensure total cross section at NLO accuracy
- POWHEG avoids negative weights
- MC@NLO exponentiates only the singular part of the real radiation amplitude
- POWHEG modifies the Sudakov form factor by exponentiating the complete real radiation amplitude
- differences between the two codes are beyond NLO accuracy

examples



S. Frixione and B. Webber, hep-ph/0204244



P. Nason and G. Ridolfi, hep-ph/0606275

Higher-order QCD & generators: state of the art

- Multi-parton matrix elements Monte Carlos (ALPGEN, HELAC, MADEVENT, SHERPA...) matched with vetoed Parton Showers

  (not treated in these lectures)

- NLO calculations for $W, Z \to l\bar{l}'$ (DYRAD, MCFM)

  W.T. Giele, E.W.N. Glover and D.A. Kosower, Nucl. Phys. **B403** (1993) 633

  J.M. Campbell and R.K. Ellis, Phys. Rev. **D65** (2002) 113007

- soft-gluon resummation of leading/NLL $(p_\perp^V/M_V)$ (ResBos)

  C. Balazs and C.P. Yuan, Phys. Rev. **D56** (1997) 5558

- fully differential NNLO corrections to W/Z production (FEWZ, DYNNLO)

  K. Melnikov and F. Petriello, Phys. Rev. Lett. **96** (2006) 231803, Phys. Rev. **D74** (2006) 114017

  S. Catani, L. Cieri, G. Ferrera, D. de Florian, M. Grazzini, Phys. Rev. Lett. **103** (2009) 082001

  S. Catani, G. Ferrera, M. Grazzini, JHEP **1005** (2010) 006

- NNLL resummation of W/Z transverse momentum

  G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, Phys. Lett. **B696** (2011) 207

- NLO merged with Parton Showers (MC@NLO, POWHEG, SHERPA)

  S. Frixione and B.R. Webber, JHEP **0206** (2002) 029

  P. Nason, JHEP 0411 (2004) 040; S. Alioli et al., JHEP 0807 (2008) 060, JHEP 1006 (2010) 043

  S. Höche, F. Krauss, M. Schönherr, F. Siegert, arXiv:1207.5030