Numerical simulations for W-pair production and decay in Modified Perturbation Theory in NNLO

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Outline:

- Statement of the problem
- General formalism of the MPT
- W-pair production
- Specific model
- Numerical calculations, estimate of errors
- Results of calculations
- Conclusions

The problem:

W-pair production is one of the main tools for testing SM, constraining its free parameters and searching for physics beyond

- precision measuring of W mass } *near threshold*
- measuring interactions among gauge bosons (*anomalous contributions to triple gauge coupling*), search for new physics
- important background to most searches for new physics

} *above threshold*

International e+e- linear collider (ILC) must provide high precision measurements

At ILC a few of 10^6 W-pair is assumed to be produced at $\sqrt{s} \sim 500$ GeV ...

This means per-mille (0.1%) accuracy of measurement of the cross-section

Theoretical support must be made with per-mille precision

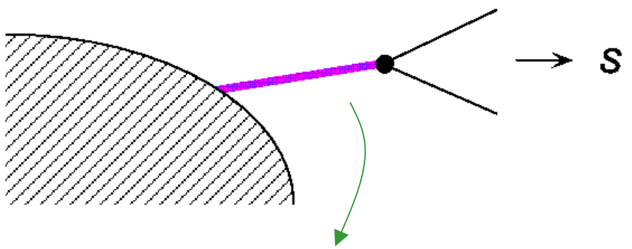
NNLO

The problem:

NNLO calculations must provide

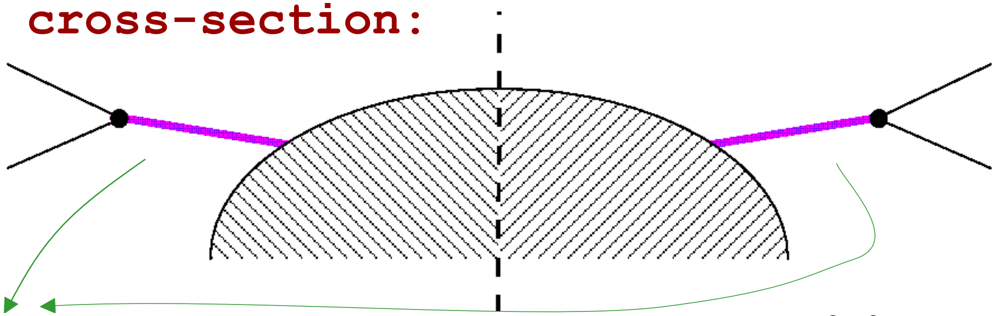
(i) gauge cancellations and unitarity;
(ii) high accuracy of computation of resonant contributions

amplitude:



$$\Delta(s) = \frac{1}{s - M^2 + \mathrm{i}M\Gamma}$$

cross-section:



$$|\Delta(s)|^2 = \frac{1}{(s-M^2)^2 + M^2\Gamma^2} \Rightarrow \frac{1}{(s-M^2)^2} - \frac{M^2\Gamma^2}{(s-M^2)^4} + \cdots$$

non-integrable

Existing methods:

- Pole expansion/DPA: Laurent expansion around complex poles + conventional PT for residues / LEP1, LEP2 /
- Complex mass scheme (CMS): complex-valued renormalized mass $\Rightarrow$ complex-valued Weinberg angle, couplings etc. / A.Denner, S.Dittmaier, M.Roth, etc. /

NLO

- Pinch-technique method $\Rightarrow$ huge volume of extra calculations

/ J.Papavassiliou, A.Pilaftsis, D.Binosi, etc. /

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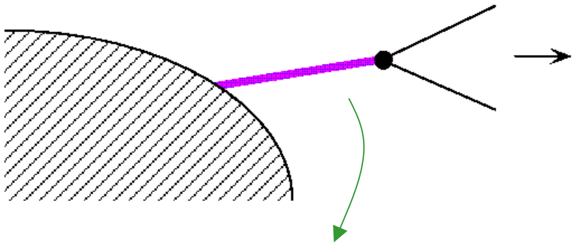

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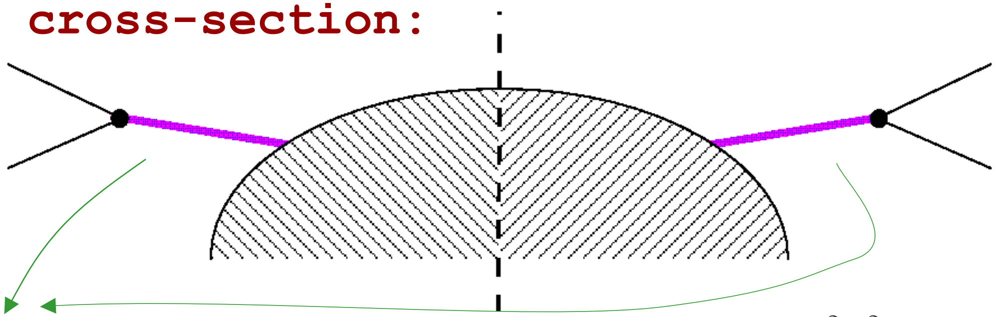

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Modified perturbation theory (MPT):

direct expansion of the cross-section *in powers of the coupling constant* with the aid of distribution-theory methods

(i) Asymptotic expansion in α $\Rightarrow$ gauge invariance must be maintained
(ii) The accuracy of description of resonant contributions = ?

The full bibliography on MPT:

→ F.Tkachov, in Proc. 32 PNPI Winter School, 1999, arXive: hep-ph/9802307
F.Tkachov, in Proc. 14 Int. Workshop QFTHEP, 1999, arXive: hep-ph/0001220
M.Nekrasov, in Proc. 15 Int. Workshop QFTHEP, 2000, arXive: hep-ph/0102284
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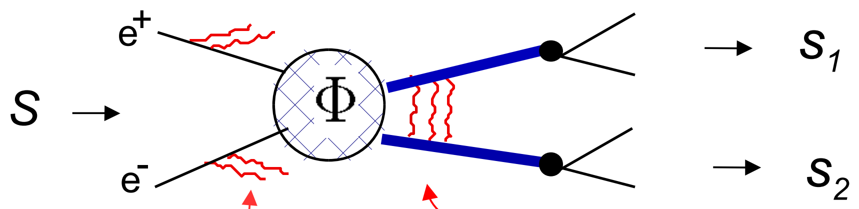
General formalism:



$$\sigma(s,\theta) = \int_{s_{\min}}^{s} \frac{\mathrm{d}s'}{s}\, \phi(s'/s;s)\, \hat\sigma(s',\theta)\,, \qquad \hat\sigma(s,\theta) = \iint \mathrm{d}s_1\,\mathrm{d}s_2\, \hat\sigma(s\,;s_1,s_2)\ (1+\delta_c)$$

$$\hat\sigma(s\,;s_1,s_2) = \frac{1}{s^2}\,\Theta(\sqrt{s}-\sqrt{s_1}-\sqrt{s_2})\,\sqrt{\lambda(s;s_1,s_2)}\ \Phi(s;s_1,s_2)\,\rho(s_1)\,\rho(s_2)$$

$$\rho(s) = \frac{M\Gamma}{\pi}\,\frac{1}{|s-M^2+\alpha\,\Sigma(s)|^2}$$

Expansion in powers of α
in the sense of distributions

General formalism: basic ingredients of MPT

- Asymptotic expansion of BW factors in powers of α / F.Tkachov,1998 /

$$\rho(s) = \frac{M\Gamma_0}{\pi}\,\frac{1}{|s-M^2+\Sigma(s)|^2} = \delta(s-M^2) + PV\,\mathcal{T}\,[\,\rho(s)\,] + \sum_n c_n(\alpha)\,\delta^{(n)}(s-M^2)$$

Taylor in α

Polynomial in α

3-loop

NNLO :

$$= \delta(s-M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\,\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s-M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s-M^2) + O(\alpha^3)$$

- Analytic regularization of the kinematic factor / M.Nekrasov,2007 /

$$\sqrt{\lambda(s,s_1,s_2)} \longrightarrow \lim_{\nu\to 1/2}\left\{\lambda(s,s_1,s_2)\right\}^{\nu}$$

analytic calculation of "singular" integrals

- Conventional-perturbation-theory for "test" function Φ

=> not a problem of MPT

General formalism: coefficients $c_n(\alpha)$

NNLO:

$$\rho(s) = \delta(s-M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s-M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s-M^2)$$

$$I_n = \mathrm{Im}\Sigma_n(M^2)\,, \quad I_n' = \mathrm{Im}\Sigma_n'(M^2)\,,$$

$$R_n = \mathrm{Re}\Sigma_n(M^2)\,, \quad R_n' = \mathrm{Re}\Sigma_n'(M^2)\,, \cdots$$

$$\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^3\Sigma_3 + \cdots$$

OMS conventional: $R_n = R_n' = 0$

$$c_0 = -\alpha\frac{I_2}{I_1} + \alpha^2\left(\frac{I_2^2}{I_1^2} - \frac{I_3}{I_1} - \frac{1}{2}I_1 I_1''\right)$$

$$c_1 = -\alpha^2 I_1 I_1', \qquad c_2 = -\alpha^2 I_1^2$$

$\overline{\text{OMS}}$:
(pole scheme)

/ M.Nekrasov, 2002 /
/ B.Kniel & A.Sirlin, 2002 /

$$R_1 = R_1' = 0\,, \qquad R_2 = -I_1 I_1'\,, \qquad R_2' = -I_1 I_1''/2$$

$$c_0 = -\alpha\frac{I_2}{I_1} + \alpha^2\left[\frac{I_2^2}{I_1^2} - \frac{I_3}{I_1} - (I_1')^2\right],$$

$$c_1 = 0, \qquad c_2 = -\alpha^2 I_1^2\,.$$

M = pole mass,
gauge-invariant and scheme-independent
(observable mass)

Unitarity:

$$\alpha I_1 = M\Gamma_0\,, \qquad \alpha^2 I_2 = M\alpha\Gamma_1\,, \qquad \alpha^3 I_3 = M\alpha^2\Gamma_2 + \Gamma_0^3/(8M)$$

$$\Gamma = \Gamma_0 + \alpha\Gamma_1 + \alpha^2\Gamma_2 + \cdots$$

General formalism: singular integrals

Dimensionless variables

$$s \longrightarrow x \qquad s_i \longrightarrow x_i \qquad \sqrt{s} = 2M + \frac{M}{2}x\,, \quad \sqrt{s_i} = M_i + \frac{M}{2}x_i \qquad M \equiv \frac{M_1+M_2}{2}$$

$$\hat{\sigma}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\;(x-x_1-x_2)_+^{\nu}\;\rho(x_1)\,\rho(x_2)\;\Phi(x\,;x_1,x_2)$$

$$\left\{PV\frac{1}{x_1^{n_1}},\;\delta^{(n_1-1)}(x_1)\right\}\quad\left\{PV\frac{1}{x_2^{n_2}},\;\delta^{(n_2-1)}(x_2)\right\}$$

at given n_1 and n_2 :

$$\Phi(x\,;x_1,x_2) = \sum_{k_1=0}^{n_1-1}\sum_{k_2=0}^{n_2-1}\frac{x_1^{k_1}}{k_1!}\frac{x_2^{k_2}}{k_2!}\,\Phi^{(k_1,k_2)}(x\,;0,0) + \Delta\Phi(x\,;x_1,x_2)$$

$$x^k\,PV\frac{1}{x^n} = \frac{1}{x^{n-k}}$$

$$x^k\,\delta^{(n-1)}(x) \sim \delta^{(n-k-1)}(x)$$

$$0 \le k < n$$

at ν = 1/2

$$A^{\nu}_{l_1 l_2}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\,(x-x_1-x_2)_+^{\nu}\,\delta^{(l_1-1)}(x_1)\,\delta^{(l_2-1)}(x_2) \sim (x)_+^{5/2-l_1-l_2} + \text{‘reg’}$$

$$B^{\nu}_{l_1 l_2}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\,(x-x_1-x_2)_+^{\nu}\,PV\frac{1}{x_1^{l_1}}\,\delta^{(l_2-1)}(x_2) \sim (-x)_+^{5/2-l_1-l_2} + \text{‘reg’}$$

$$C^{\nu}_{l_1 l_2}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\,(x-x_1-x_2)_+^{\nu}\,PV\frac{1}{x_1^{l_1}}\,PV\frac{1}{x_2^{l_2}} \sim (x)_+^{5/2-l_1-l_2} + \text{‘reg’}$$

$$\sigma(x) = \int \mathrm{d}x'\;\phi(x',x)\hat{\sigma}(x')$$

$$\int \mathrm{d}x\;x_+^{\nu}\,\varphi(x) \underset{def}{=} \int_0^{\infty}\mathrm{d}x\;x^{\nu}\left\{\varphi(x) - \sum_{k=0}^{N-1}\frac{x^k}{k!}\,\varphi^{(k)}(0)\right\}$$

$$-N-1 < \mathrm{Re}\,\nu < -N$$

General formalism: single production & interference

$$\hat{\sigma}_{\text{single-res}}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\;\; (x-x_1-x_2)_+^{\nu}\;\; \rho(x_i)\;\; \Phi(x\,;x_1,x_2)$$

$$\left\{ PV\frac{1}{x_i^n},\;\; \delta^{(n-1)}(x_i) \right\}$$

at given *n*:

$$\Phi(x\,;x_1,x_2) = \sum_{k=0}^{n-1} \frac{x_i^k}{k!}\, \Phi^{(k)}(x\,;x_1,x_2) + \Delta\Phi(x\,;x_1,x_2)$$

at $\nu = 1/2$

$$\mathtt{I}_n^{\nu}(x-x_j) \;=\; \int \mathrm{d}x_i\;(x-x_i-x_j)_+^{\nu}\;\delta^{(n-1)}(x_i) \;\sim\; (x)_+^{3/2\,-\,n} \;+\; \text{‘reg’}$$

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$$\hat{\sigma}_{\text{interfer}}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\;\; (x-x_1-x_2)_+^{\nu}\;\; \rho(x_i)\,\Delta(x_j)\;\; \Phi(x\,;x_1,x_2)$$

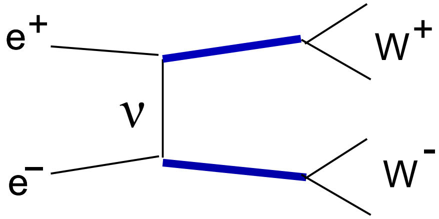
$$\left\{ PV\frac{1}{x_1^{n_1}},\;\; \delta^{(n_1-1)}(x_1) \right\} \;\; \left\{ PV\frac{1}{x_2^{n_2}},\;\; \delta^{(n_2-1)}(x_2) \right\}$$

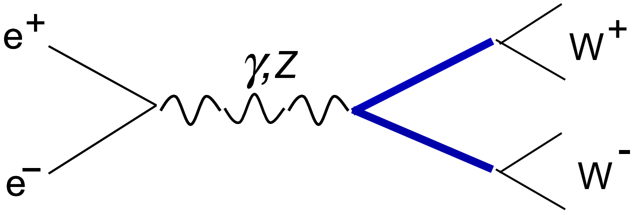
$$\Delta(s) = \frac{1}{s-M^2+\mathrm{i}\Sigma} \;\;\Rightarrow\;\; \frac{1}{s-M^2+\mathrm{i}0} + \frac{\mathrm{i}\Sigma}{(s-M^2+\mathrm{i}0)^2} + \cdots$$

Similarly to the case
of pair-production

W-pair production:

CC3 contributions:





1. *nonanalyticity in Φ due to* $\sqrt{\lambda}$:

$$\Delta_\nu = \frac{1}{s - s_1 - s_2 - \sqrt{\lambda}\cos\theta} \sim \frac{1}{t_\nu}$$

$$= \frac{s - s_1 - s_2 + \sqrt{\lambda}\cos\theta}{(s - s_1 - s_2)^2 - \lambda\cos^2\theta}$$

$$\sqrt{\lambda}\ \Phi = \sqrt{\lambda}\ \Phi_1 + \lambda\ \Phi_2$$

2. *zeros in denominator in non-phys. region*

$$(s - s_1 - s_2)^2 - \lambda\cos^2\theta = \lambda(1 - \cos^2\theta) + 4 s_1 s_2$$

=> $H(\lambda)\,(1 - \cos^2\theta) + 4 s_1 s_2$

$$H(\lambda) = \Theta(\lambda)\,\lambda + \Theta(-\lambda)\,h(\lambda)$$

$$\Delta_\nu = \frac{s - s_1 - s_2 + \sqrt{\lambda}\cos\theta}{H(\lambda)\,(1 - \cos^2\theta) + 4 s_1 s_2}$$

pole due to Z-propagator

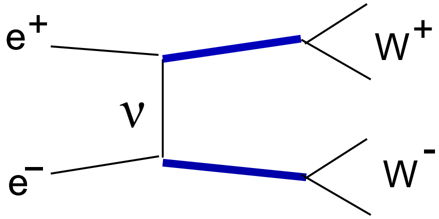
Application of MPT expansion

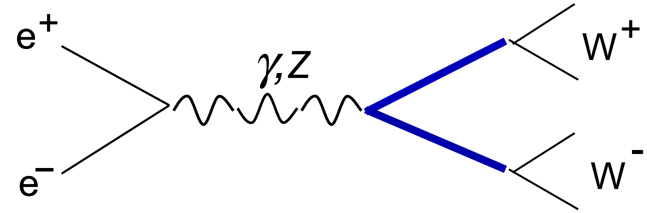
if $s \approx M_Z^2$

CC11 family contributions:

W-pair production:

CC3 contributions:





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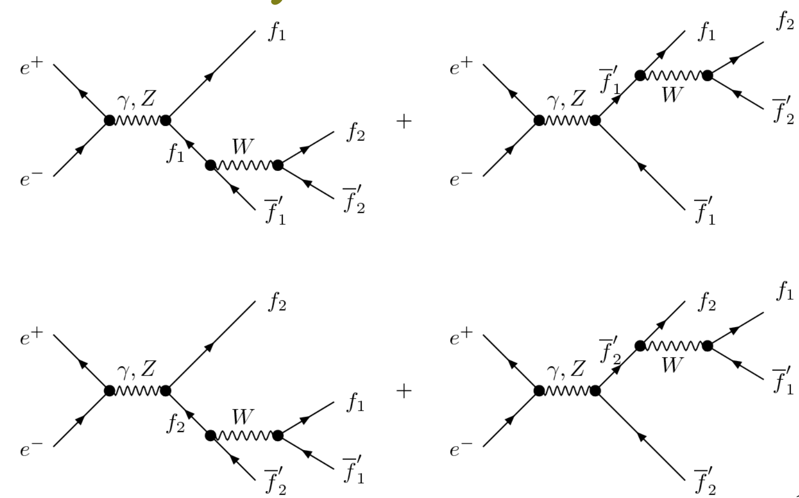


Specific model for testing MPT in NNLO

- Breigt-Wigner factors :

$\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^3\Sigma_3$ ⟵ 3-loop contributions

- Test function Φ :

$e^+e^- \to (\gamma, Z) \to W^+W^- \to 4f$ ⟵ improved Born approximation

- Universal soft massless-particles contributions :

Flux function in leading-log approximation:

$$\phi(z;s) = \beta_e(1-z)^{(\beta_e-1)} - \frac{1}{2}\beta_e(1+z)\,, \qquad \beta_e = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

Coulomb singularities through one-photon exchanges:

$$\delta_c = \frac{\alpha\pi}{2\beta}\left[1 - \frac{2}{\pi}\arctan\left(\frac{|\beta_M|^2 - \beta^2}{2\beta\,\mathrm{Im}\beta_M}\right)\right]$$

$$\beta = s^{-1}\sqrt{\lambda(s, s_1, s_2)}$$

$$\beta_M = \sqrt{1 - 4(M^2 - \mathrm{i}M\Gamma)/s}$$

Specific model for testing MPT in NNLO

Propagators of unstable particles :

(NNLO precision in MTB)

$$
\begin{aligned}
\Delta^{-1}(s) &\equiv s - M^2 + \Sigma(s) \\
&= s - M^2 + \alpha\,\mathrm{Re}\Sigma_1(s) + \mathrm{i}\,\alpha\,\mathrm{Im}\Sigma_1(s) \\
&+ \alpha^2\left[R_2 + \mathrm{i}\,I_2 + (s - M^2)(R_2' + \mathrm{i}\,I_2')\right] + \alpha^3(R_3 + \mathrm{i}\,I_3)
\end{aligned}
$$

$\overline{\mathrm{OMS}}$:

$$
R_2 = -I_1 I_1', \qquad R_2' = -I_1 I_1''/2, \qquad R_3 = -I_2 I_1' - I_1 I_2' + I_1^2 R_1''/2
$$

$$
\alpha I_1 = M\Gamma_0, \qquad \alpha^2 I_2 = M\alpha\Gamma_1, \qquad \alpha^3 I_3 = M\alpha^2\Gamma_2 + \Gamma_0^3/(8M)
$$

Parameters in the model:

$$
\begin{aligned}
\Gamma_0^W &= 1.977\ \mathrm{GeV} & \Gamma_0^Z &= 2.362\ \mathrm{GeV} \\
\Gamma_1^W &= 0.102\ \mathrm{GeV} & \Gamma_1^Z &= 0.169\ \mathrm{GeV} \\
\Gamma_2^W &= 0.006\ \mathrm{GeV} & \Gamma_2^Z &= -0.036\ \mathrm{GeV}
\end{aligned}
$$

$$
M_W = 80.40\ \mathrm{GeV} \qquad M_Z = 91.19\ \mathrm{GeV}
$$

Numerical calculations, estimate of errors

- Fortran code with double precision
- Simpson method for calculating absolutely convergent integrals (relative accuracy $\delta_0 = 10^{-5}$)
- Linear patches for resolving 0/0-indeterminacies $(x/x,\ x^2/x^2, \ldots)$

⇓

additional errors:

δ_1: due to patches themselves ⟹ $\delta_1 \sim \varepsilon^2\, \varphi''_0/\varphi_0$

ε - actual size of the patch

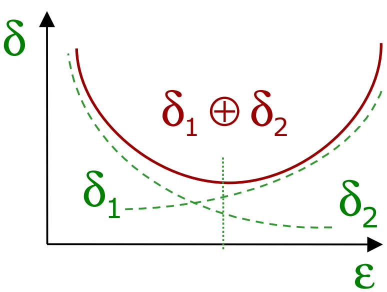
integrand

δ_2: due to the loss of decimals near indeterminacy points:

x/x: $\dfrac{f(x)-f(0)}{x} \Rightarrow \dfrac{\varepsilon f'(0)}{\varepsilon}$ $\quad \varepsilon = 10^{-N} \quad \delta_2 \sim 10^{-(D-N)}\dfrac{f_0}{f'_0}$

x^2/x^2: $\dfrac{f(x)-f(0)-xf'(0)}{x^2} \Rightarrow \dfrac{\varepsilon^2 f''(0)/2}{\varepsilon^2}$ $\quad \varepsilon^2 = 10^{-N} \quad \delta_2 \sim 10^{-(D-N)}\dfrac{2f_0}{f''_0}$

...



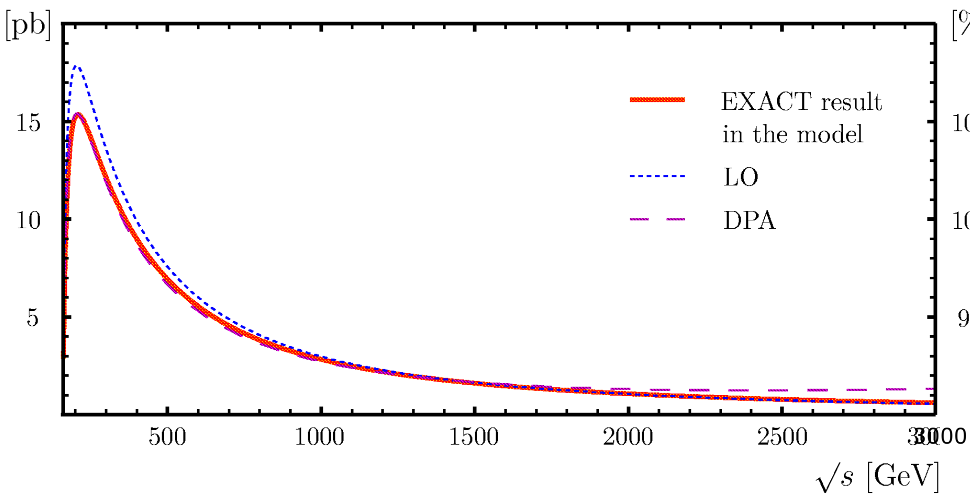
minimization of errors $\delta_1 \oplus \delta_2$ (numerical estimate) ⟹ $N = 8$ at $D = 15$

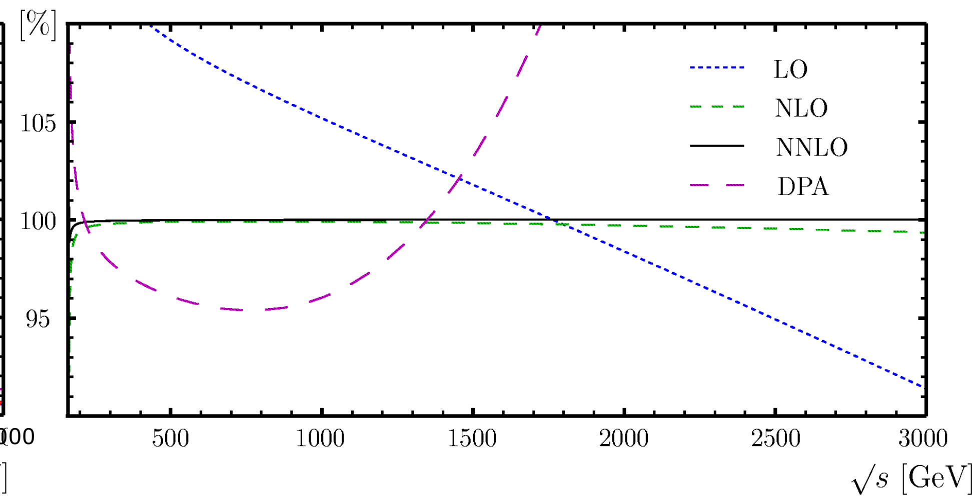
double precision

Overall error: $\delta = \delta_0 \oplus \delta_1 \oplus \delta_2 < 10^{-4}$ ⟵ NNLO

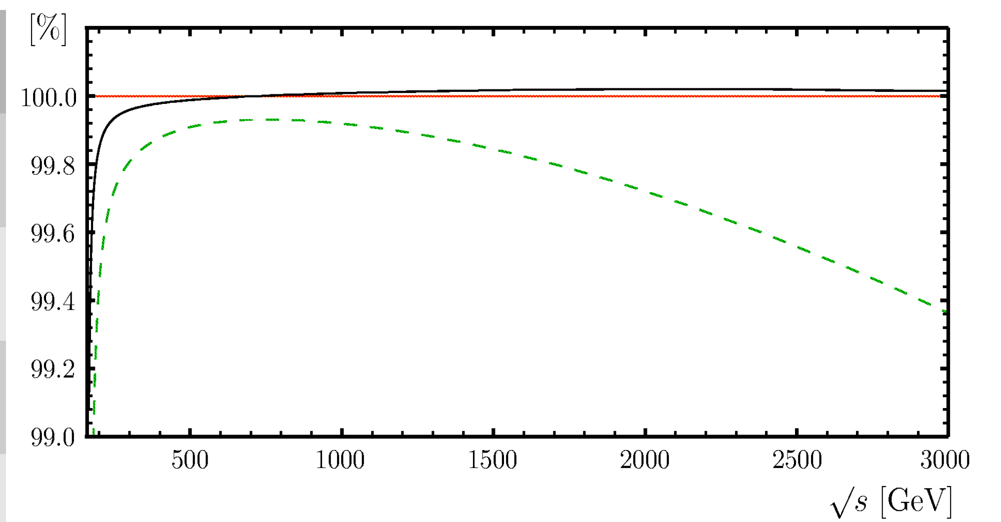
Results of calculations

Total cross-section σ(s) :



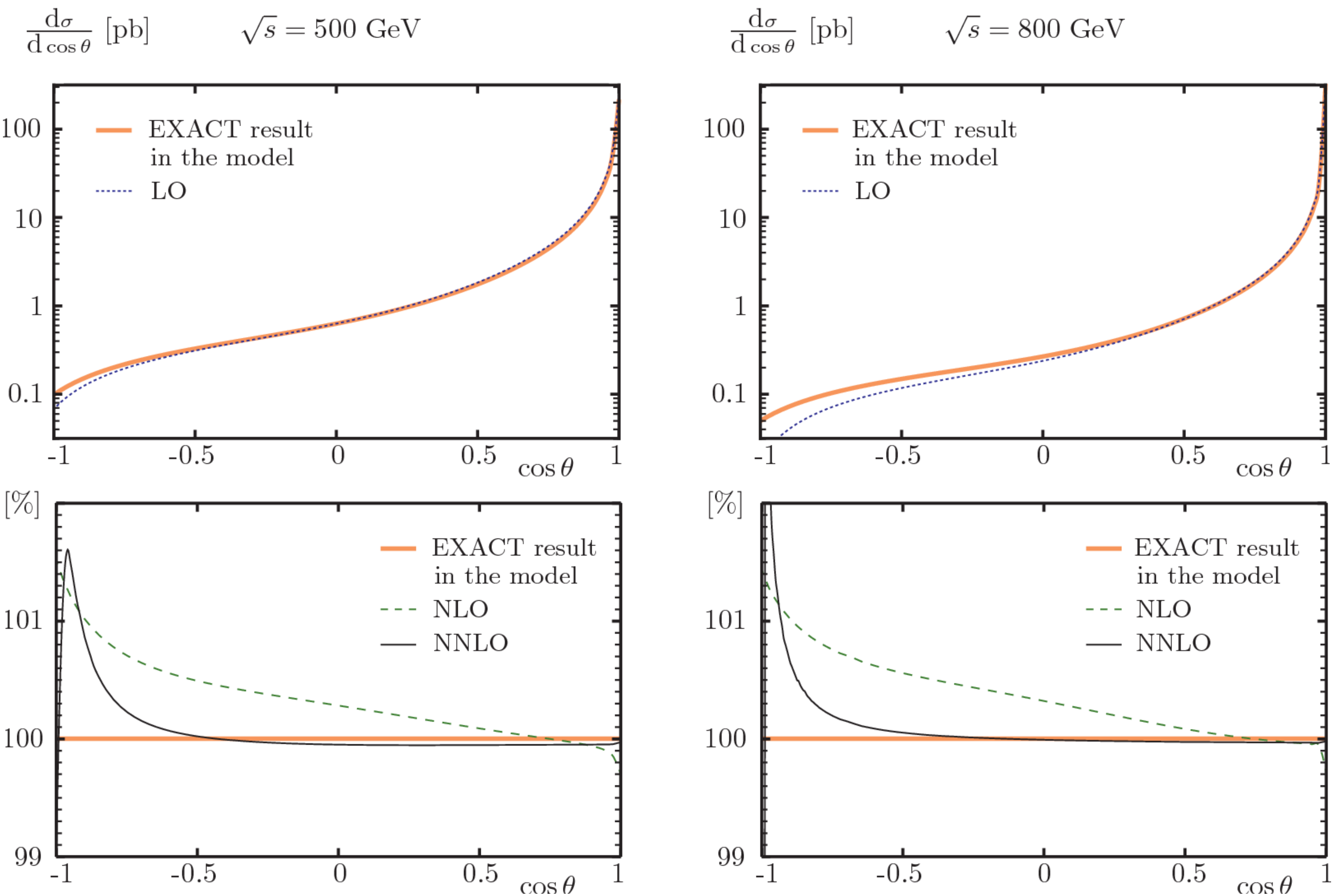


$\sqrt{s}$	σ [pb]	σ_{LO}	σ_{NLO}	σ_{NNLO}
200	15.258 100%	17.839 116.92%	15.175 99.46%	15.235(2) 99.85(1)%
500	6.9355 100%	7.5657 109.09%	6.9294 99.91%	6.9342(7) 99.98(1)%
1000	2.8286 100%	2.9733 105.12%	2.8263 99.92%	2.8285(3) 100.00(1)%
3000	0.61023 100%	0.55733 91.33%	0.60625 99.35%	0.61026(6) 100.00(1)%



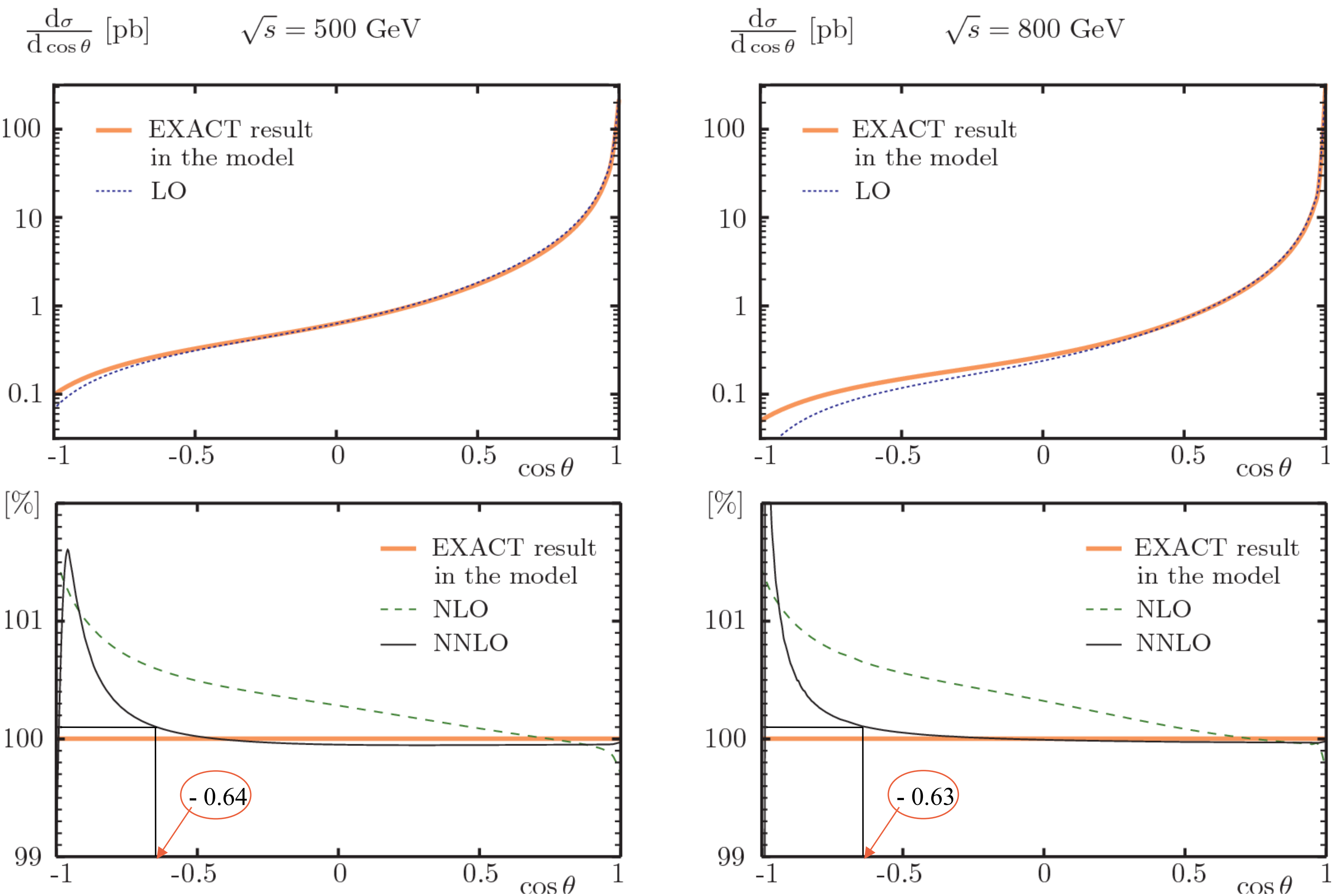
Results of calculations

Angular distribution (the *cos* θ – distribution, unpolarized beams):



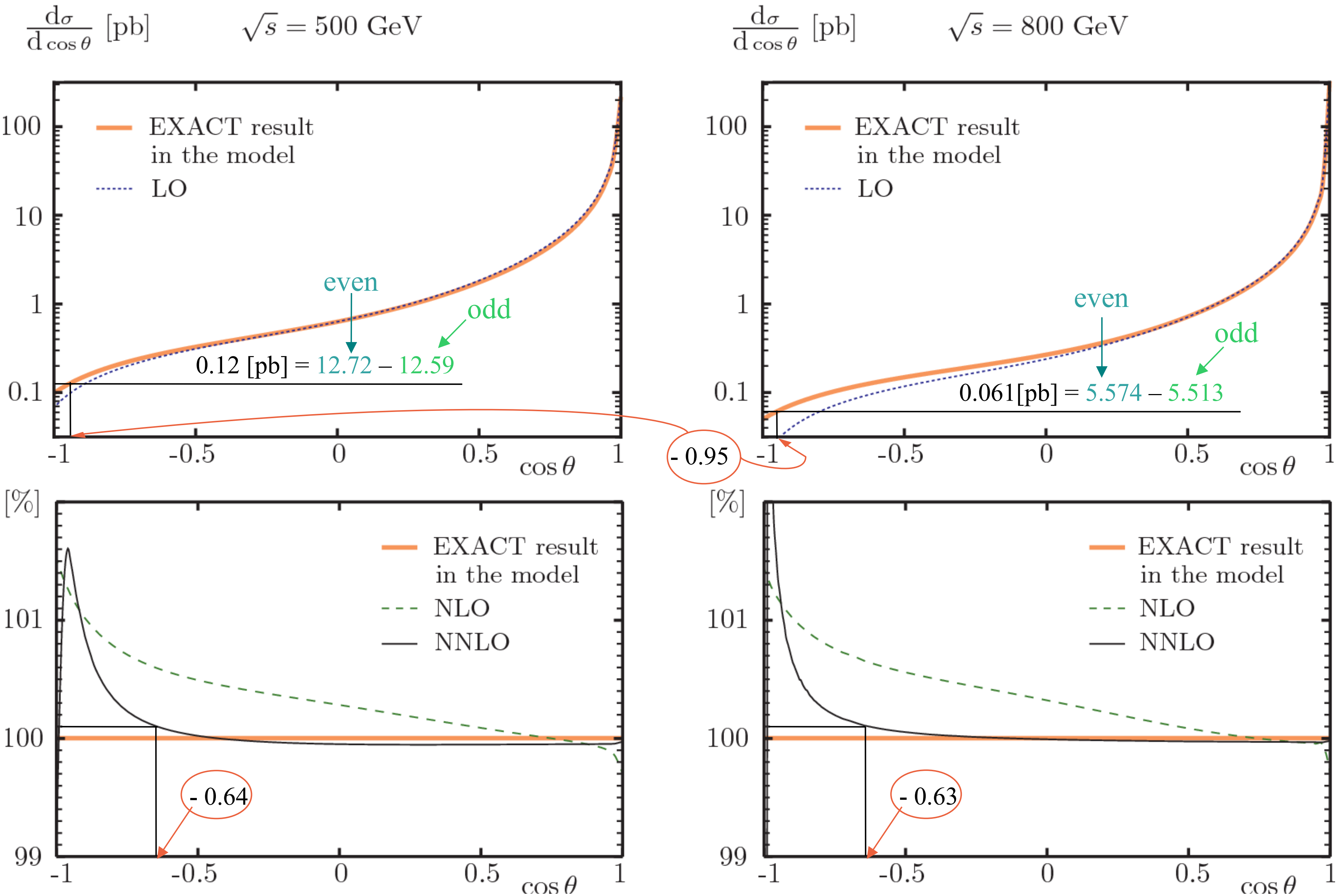
Results of calculations

Angular distribution (the *cos* θ – distribution, unpolarized beams):



Results of calculations

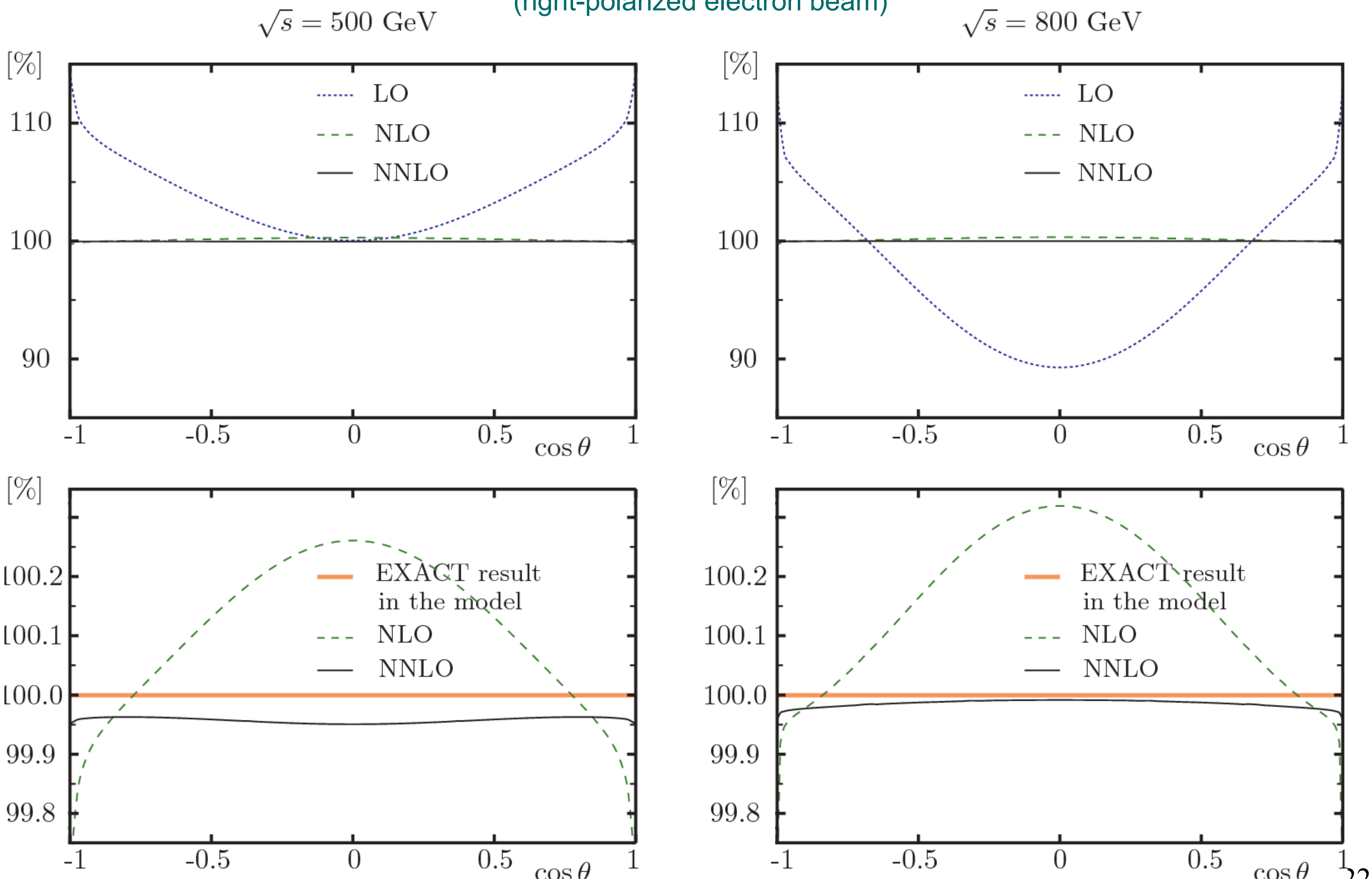
Angular distribution (the *cos θ* – distribution, unpolarized beams):



Results of calculations

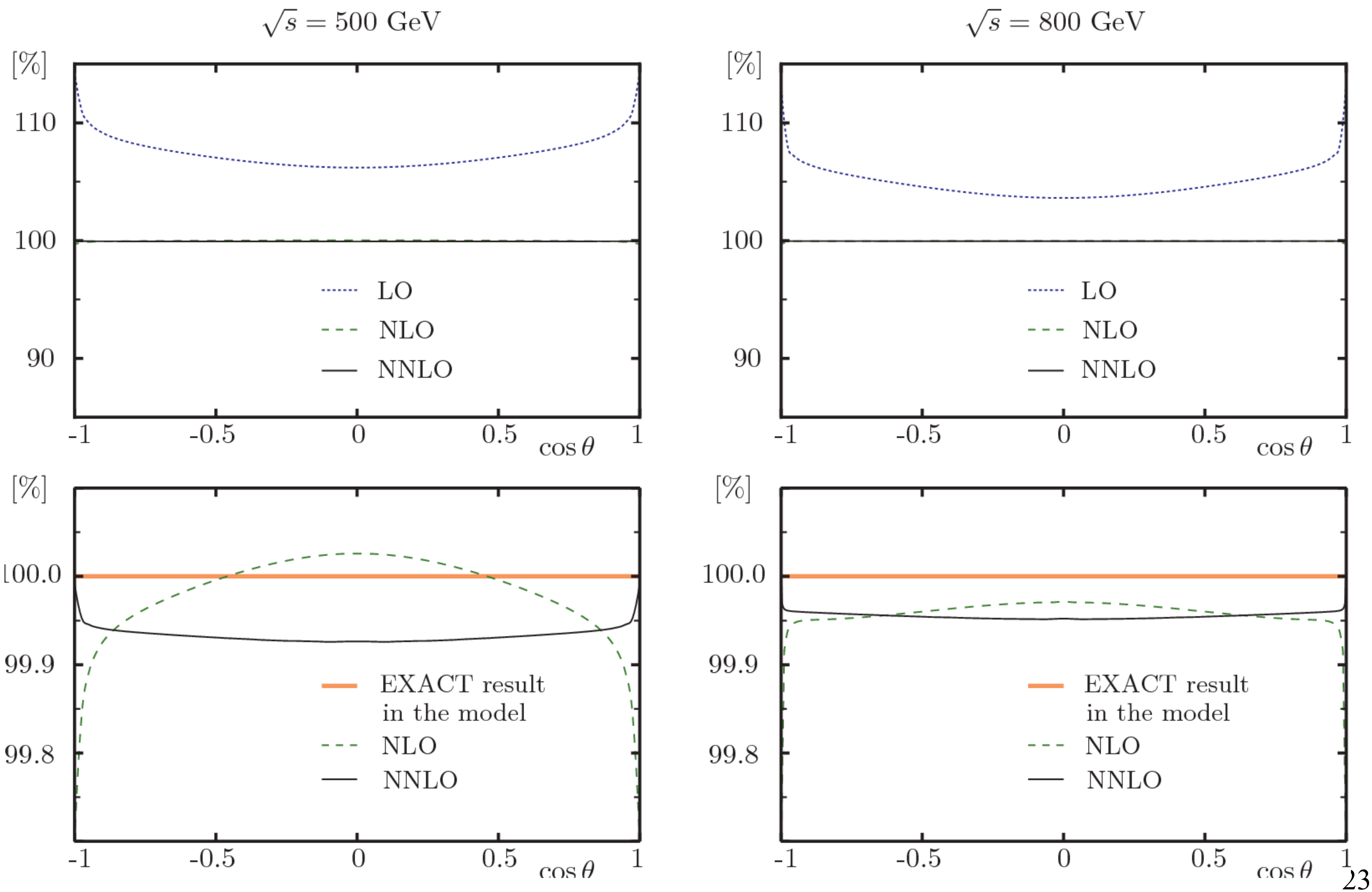
Angular distribution. Even contributions to *cos θ* – distribution:

(right-polarized electron beam)



Results of calculations

Angular distribution. Odd contributions to *cos θ* - distribution:



Conclusion

In the case of W-pair production and decays:

- MPT stably works at the energies near the maximum of the cross-section and higher

- At ILC energies MPT in NNLO provides accuracy of description at **0.1%-level**,
  - in the case of total cross-section;
  - in the case of angular distributions with unpolarized beams and not very large backward scattering angle
  - in the case of angular distributions with right-handed e^- -beam (left-handed e^+ -beam) at any scattering angles

MTV is a good candidate for support at the ILC of W-pair production processes

- Working FORTRAN code, *a substantial part of a code for realistic processes*, up to NNLO is written