Comparing different trends of the data for Pion-photon transition form factor

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based on **1205.3770 [hep-ph], appears in PRD**

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Experiments to $e^+e^- \to e^+e^-\pi^0$

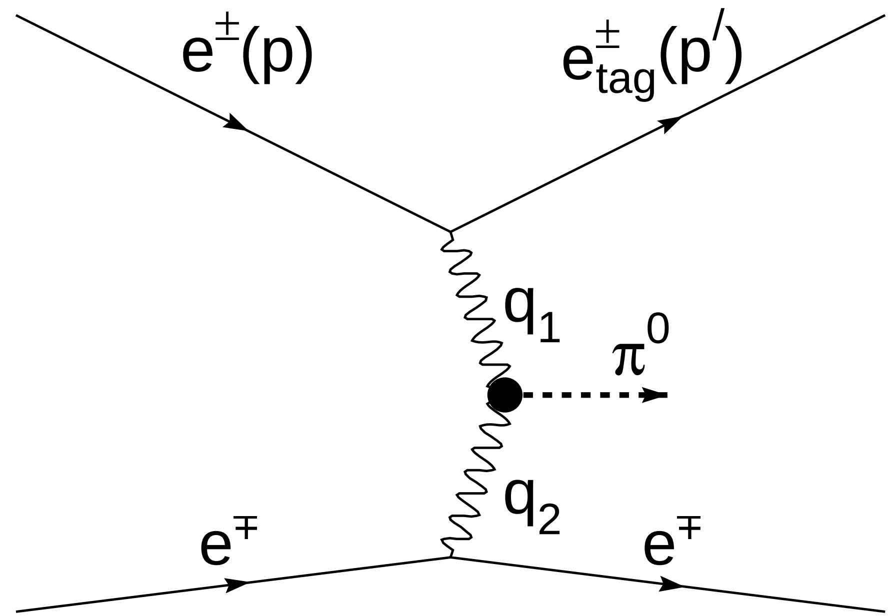
One of the **most accurate** results on exclusive reactions is data on transition FF $F^{\gamma^*\gamma^*\pi^0}(-Q^2 = q_1^2, q_2^2)$ provided by the experiments $e^+e^- \to e^+e^-\pi^0$, $q_2^2 \approx 0$.

CELLO (1991) $Q^2 : 0.7 - 2.2$ GeV^2 ,
CLEO (1998) $Q^2 : 1.6 - 8.0$ GeV^2 ,
followed to collinear QCD.

BaBar (2009) $Q^2 : 4 - 40$ GeV^2
FF certainly growth with Q^2,
creating the "BaBar puzzle",

Belle (2012) $Q^2 : 4 - 40$ GeV^2
return to collinear QCD?

BESIII (????) $Q^2 \leq 5$ GeV^2,
promises very precise data

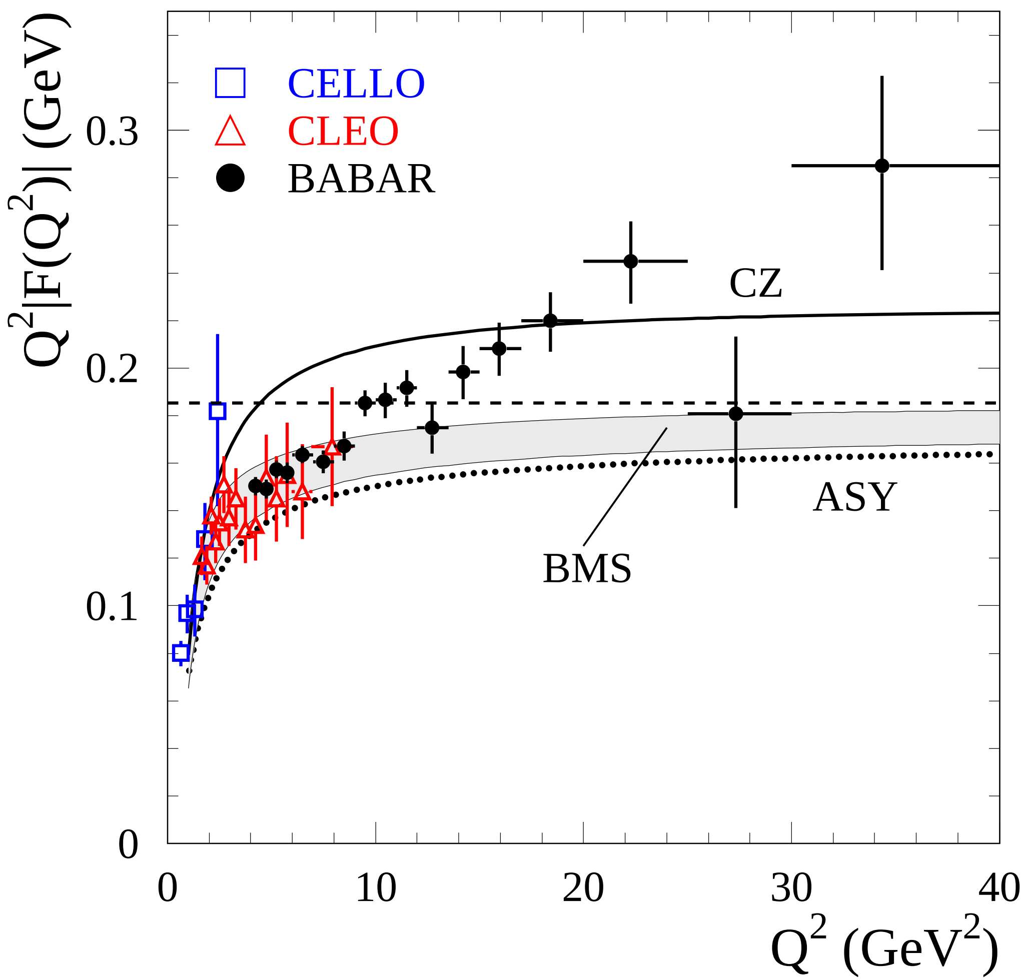


Experiments to $e^+e^- \longrightarrow e^+e^-\pi^0$

BaBar data [June 2009] on pion-photon transition form factor **grows like** $\sqrt{Q}$,

while behavior like $Q^2/(Q^2+\Lambda^2)$ **was expected**

[B. Aubert, Phys. Rev. D 80, 052002 (2009); arXiv:1101.1142]:



These authors claimed (2011):

"If the experiment is correct, many theoretical predictions should be revised. . . "

Outline:

1. **Pion-photon transition FF in QCD, its components**

2. **Why we need Light Cone Sum Rules for Transition FF, there ingredients**

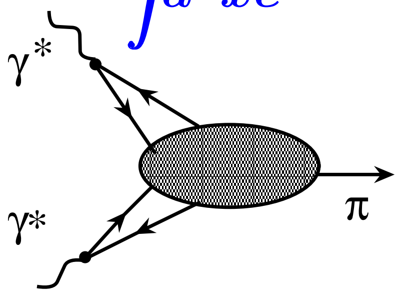
3. **FF from experiments: history, lessons, conclusion**

4. **Extraction Pion DA from different experimental data**

5. **Conclusions**

Factorization $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(P)$ in pQCD

$$\int d^4x e^{-iq_1\cdot z}\langle \pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot F^{\gamma^*\gamma^*\pi}(Q^2,q^2)\,,$$



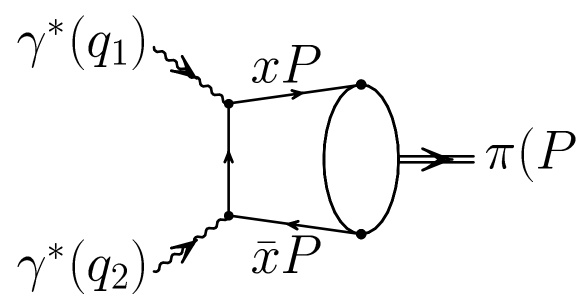
where $-q_1^2 = Q^2 > 0, \;\; -q_2^2 = q^2 \geq 0$

Collinear factorization at $Q^2, q^2 \gg$ **(hadron scale $\sim m_\rho$)^2** for the leading twist

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = T(Q^2,q^2,\mu_F^2;x) \otimes \varphi_\pi(x;\mu_F^2) + O(\frac{1}{Q^4})\,,$$

μ_F^2 – boundary between large scale Q^2 and hadronic one. At the parton level

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}}{3}f_\pi \int_0^1 dx \frac{1}{Q^2x + q^2\bar{x}}\,\varphi_\pi(x).$$



$$Q^2F^{\gamma^*\gamma\pi}(Q^2,q^2\to 0) = \frac{\sqrt{2}}{3}f_\pi \int_0^1 \frac{dx}{x}\varphi_\pi(x) \equiv \frac{\sqrt{2}}{3}f_\pi\langle x^{-1}\rangle_\pi$$

Pion distribution amplitude $\varphi_\pi(x, \mu^2)$

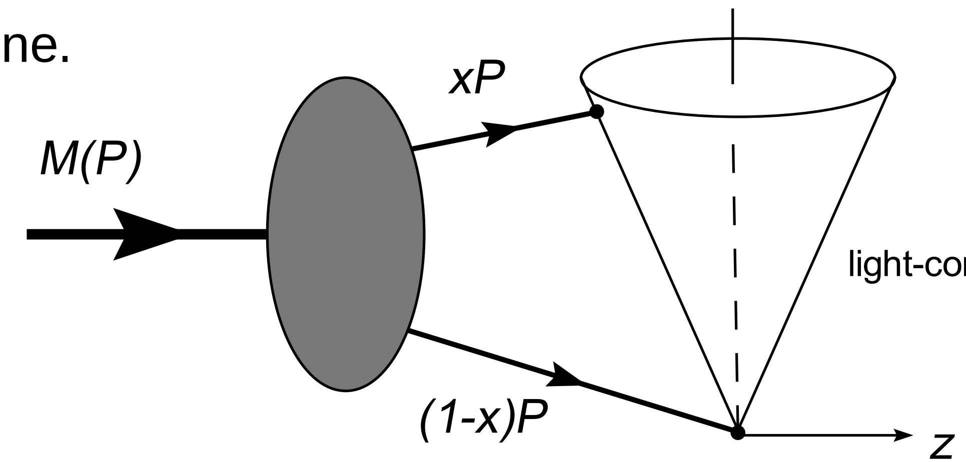
- The pion DA parameterizes the matrix element:

$$\langle 0 | \bar{d}(z)\gamma_\nu\gamma_5[z,0]u(0) | \pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\nu \int_0^1 dx\, e^{ix(zP)} \varphi_\pi(x,\mu^2)\,,$$

where the path-ordered exponential ensures the gauge invariance

$$[z,0] = \mathcal{P}\, exp\left[ig \int_0^z t^a A^a_\mu(y) dy^\mu \right].$$

- Pion DA describes the transition of a physical pion into two valence quarks, separated at light cone.



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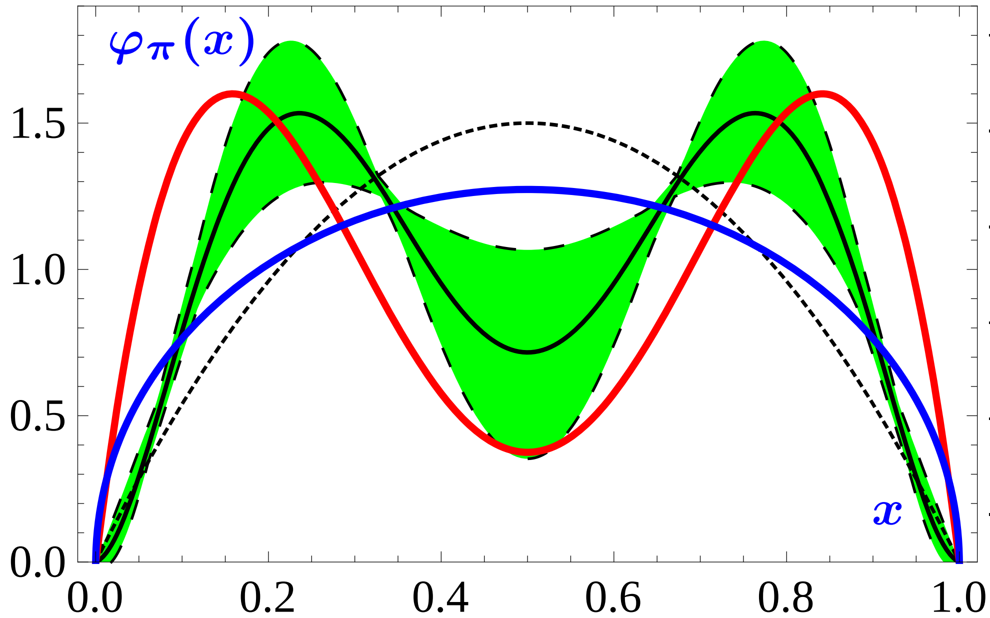

Curve	Approach
- - - - -	Asymptotic: $6x(1-x)$
	BMS DA **bunch**, NLC SR
	CZ DA from QCD SR
	AdS/QCD result: $\sim \sqrt{x(1-x}$

- ERBL **[79-80]** rules DA evolution with μ^2.The expansion in Gegenbauer modes:

$$\varphi_\pi(x,\mu^2) = 6x\bar{x}\left(1 + a_2(\mu^2)\cdot C_2^{3/2}(x-\bar{x}) + a_4(\mu^2)\cdot C_4^{3/2}(x-\bar{x}) + \ldots\right)$$

Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

- The pion DA parameterizes the matrix element:

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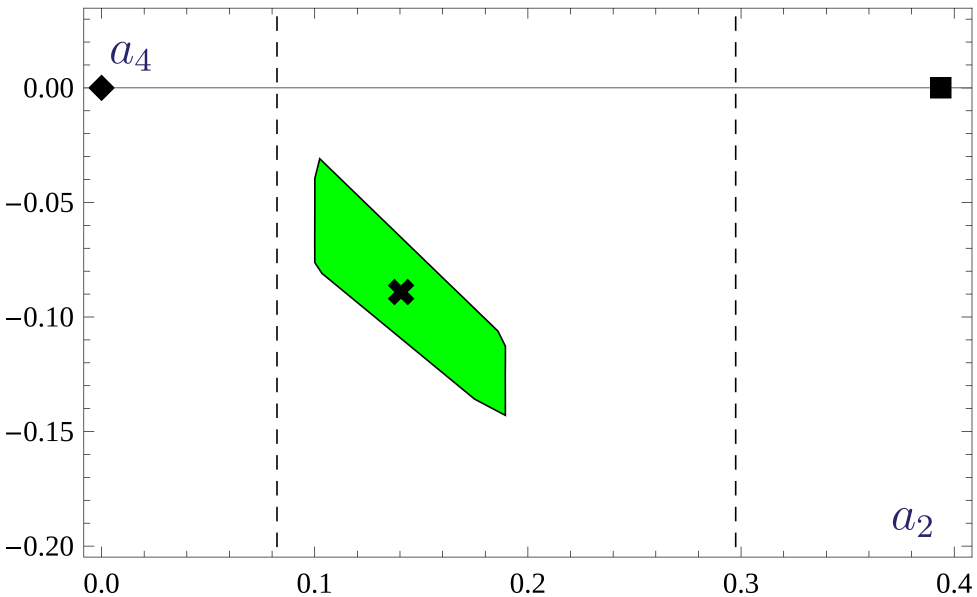
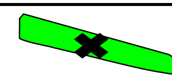


Symbol	Approach
◆	Asymptotic DA
	BMS DA **bunch**, NLC QCD SR
■	**CZ** DA, QCD SR
vert. lines	lattice constraints, **[2006]**

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$\gamma^*\gamma \to \pi$: Light-Cone Sum Rules

LCSR effectively takes into account long-distances effects of real photon using **quark-hadron duality** in vector channel and **dispersion relation** in q^2 (Balitsky et. al.-**[NPB (1989)]**, Khodjamirian **[EJPC (1999)]**)

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) = \int_0^{s_0} \frac{\rho^{\text{PT}}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \int_{s_0}^{\infty} \frac{\rho^{\text{PT}}(Q^2, s)}{s + q^2} ds \,,$$

where $s_0 \simeq 1.5$ GeV^2 – effective threshold in vector channel, M^2 – Borel parameter ($0.5 - 0.9$ GeV^2). Limit to real-photon $q^2 \to 0$ can be done.

Spectral density was calculated in QCD:

$$\rho^{\text{PT}}(Q^2, s) = \text{Im} F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2, -s - \imath\varepsilon) = \text{Tw-2} + \text{Tw-4} + \text{“Tw-6”} + \ldots \,,$$

twists contributions are given in a form of convolution with pion DA:

$$\text{Tw-2} \sim (T_{\text{LO}} + T_{\text{NLO}} + T_{\text{NNLO}_{\beta_0}} + \ldots) \otimes \varphi_\pi^{\text{Tw2}}(x, \mu) \,.$$

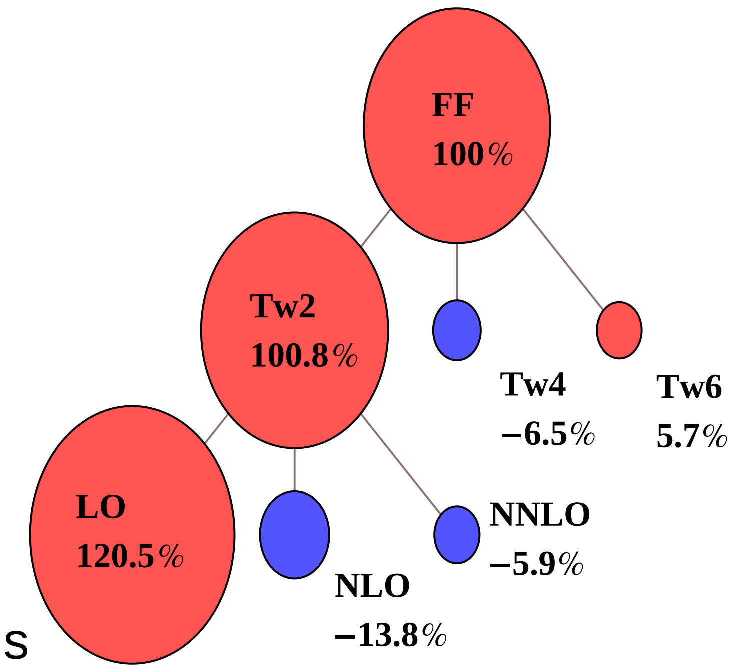
Main Ingredients of Spectral Density

- **LO** Spectral Density, **Tw-4** term **[Khodjamirian(1999)]**
- **NLO** Spectral Density, **[M&Stefanis(2009)]**,corr. in **[Agaev et.al.(2011)]**
- **NNLO**$_{\beta_0}$ Spectral Density, **[M&Stefanis(2009)]**
- **“Tw-6”** contribution, **[Agaev et.al.(ABOP 2011)]**

- **NLO** evolution of pion DA **[Kadantseva&M&R, S.J.NP.(1986)]**

Terms of Pion-Photon FF at $Q^2 = 8$ GeV^2

- Result is dominated by the Twist-2 LO and NLO.
- **“Twist-6”** contribution is taken into account together with **NNLO**$_{\beta_0}$ one — they has close absolute values and opposite signs.



Blue - negative terms

Red - positive terms

Parameters of LC SR

From QCD SR:

- Borel param. $M^2_{\text{LCSR}} \in [0.7, 1]$ GeV^2
- Vector Channel Threshold s_0
- "Twist-6" $(\alpha_S \langle \bar{q} q \rangle^2)$
- $\lambda_q^2/2 \approx$ Twist-4 $\delta^2 \pm 20\%$ **[BMS2003]**

From PDG:

- $\alpha_s(m_Z^2)$
- Masses m_ρ, m_ω
- Decay Widths Γ_ρ, Γ_ω

Light-Cone Sum Rules:

$$\text{FF} = (\text{LO} + \text{NLO}) \otimes (\pi\text{-DA}_{\text{NLO}}) + \text{Tw-4} \pm \Delta\text{FF}$$

$$\Delta\text{FF} = \pi\text{-}\Delta\text{DA} + \Delta\text{Tw-4} + \left[\text{NNLO}_{\beta_0} \otimes (\pi\text{-DA}) + \text{Tw-6}\right]$$

π-DA model

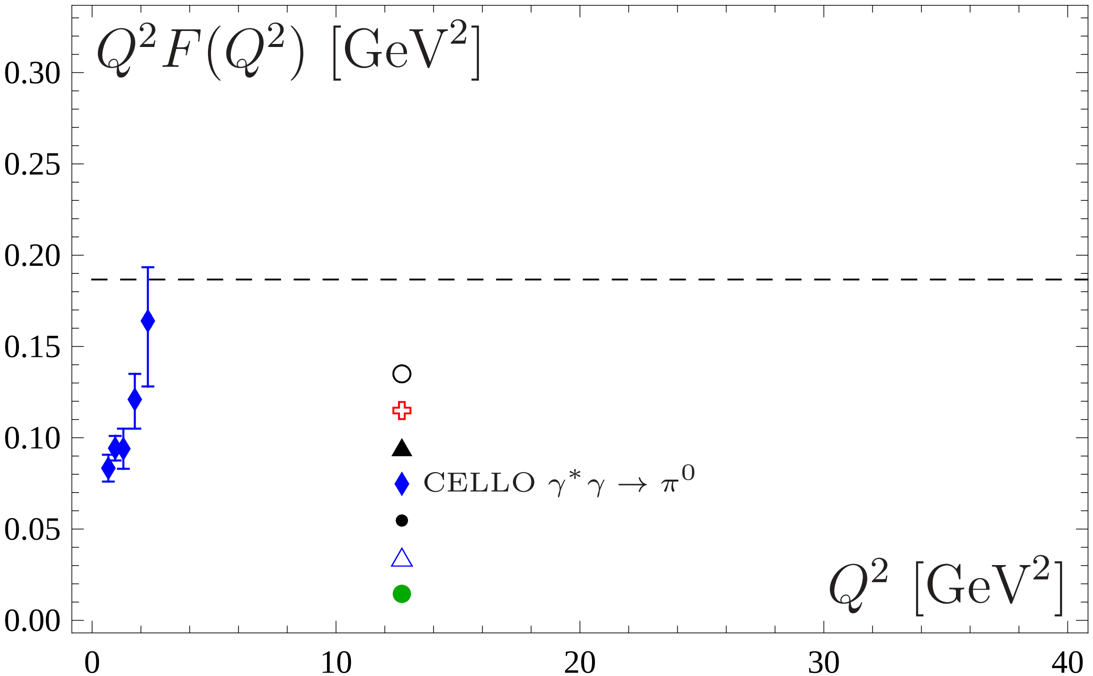
Data on FF

FF Prediction

Fitting π-DA (a_n)

Pion-gamma FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle [1205.3249[hep-ex]]**



Data	Collab.
◆	**CELLO (1991)**

dashed line = $\sqrt{2}\, f_\pi$

Pion-gamma FF data

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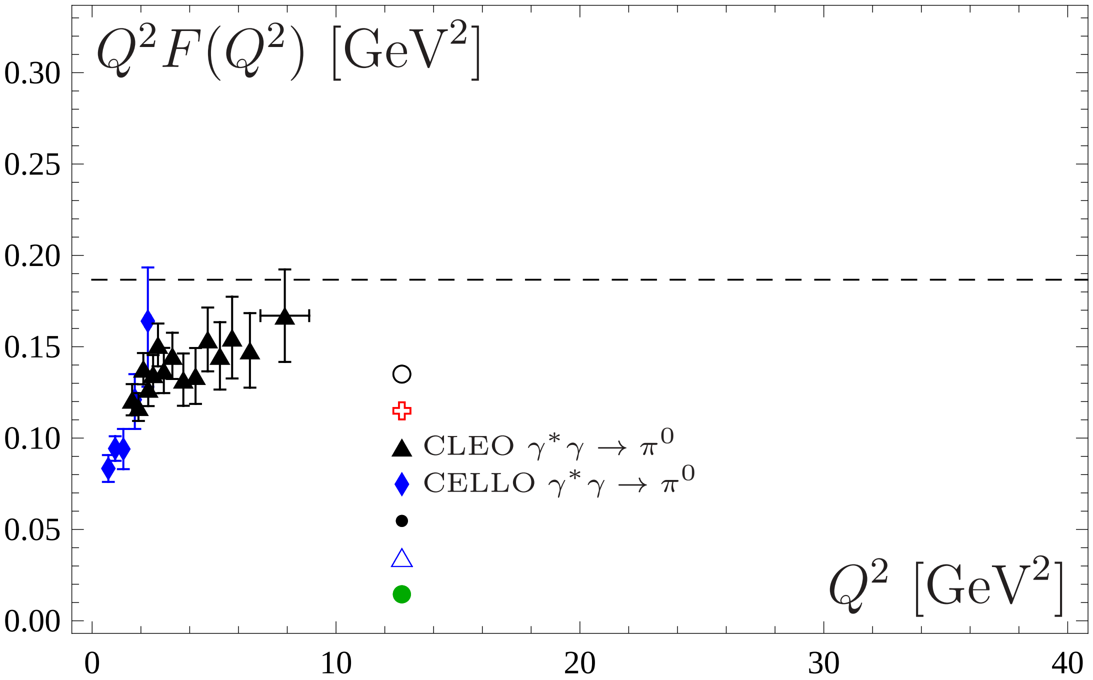

Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)

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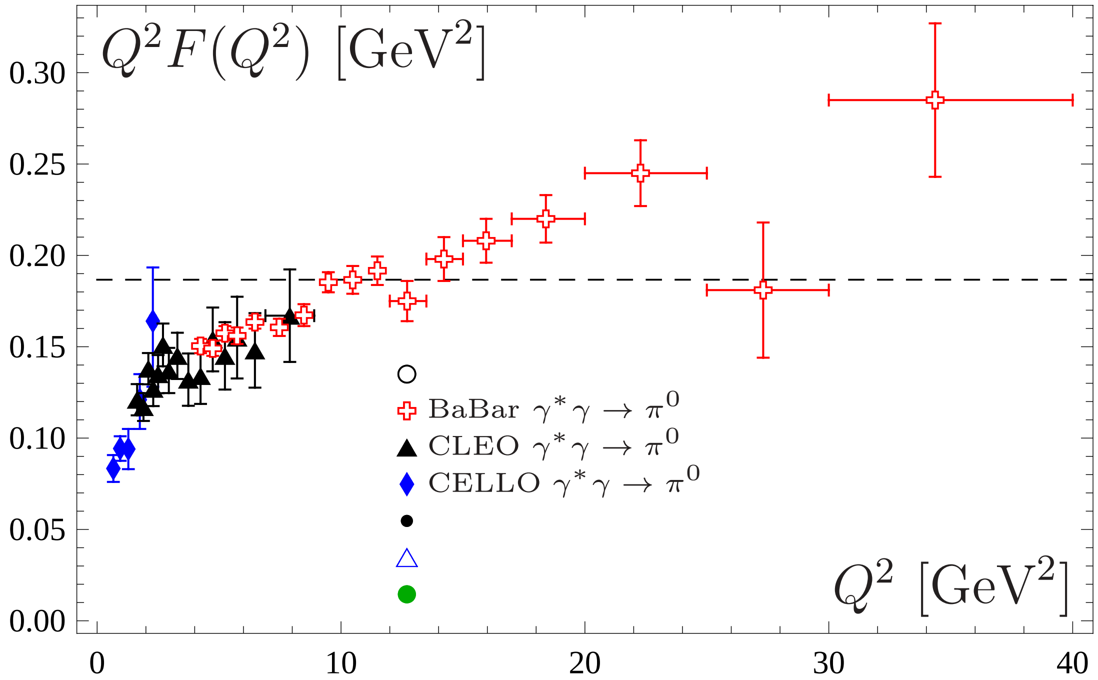

Data	Collab.
◆	**CELLO (1991)**
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✚	**BaBar (2009)**

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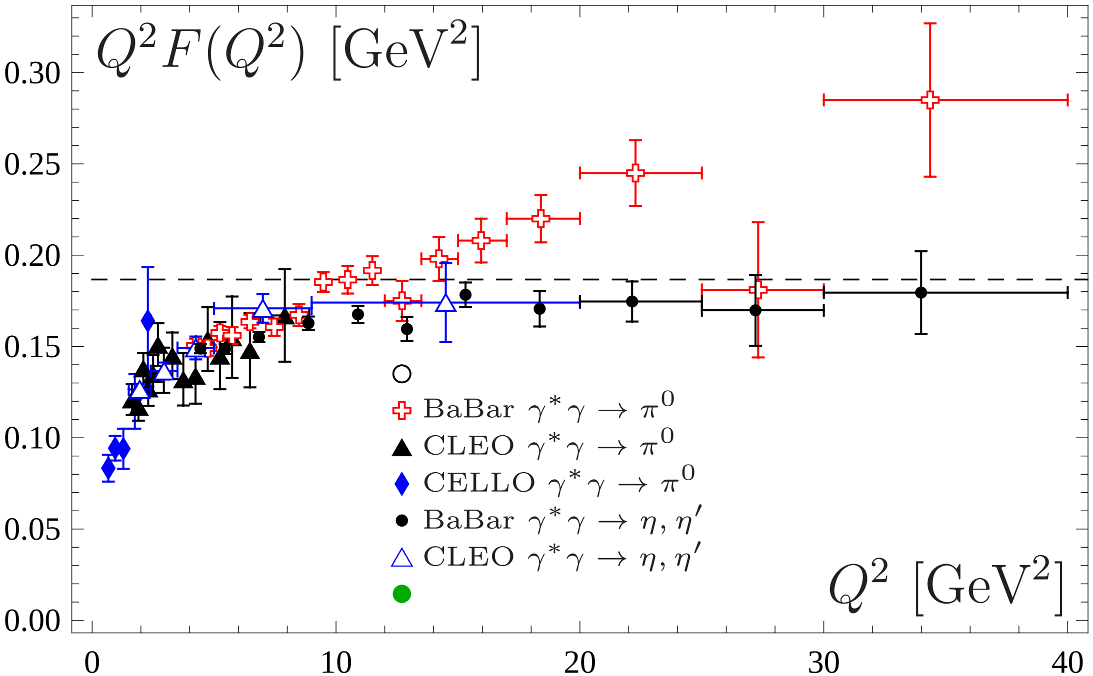

Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)
✚	**BaBar (2009)**
●	**BaBar$^{\eta}_{\eta'}$ (2011)**

dashed line = $\sqrt{2}\, f_\pi$

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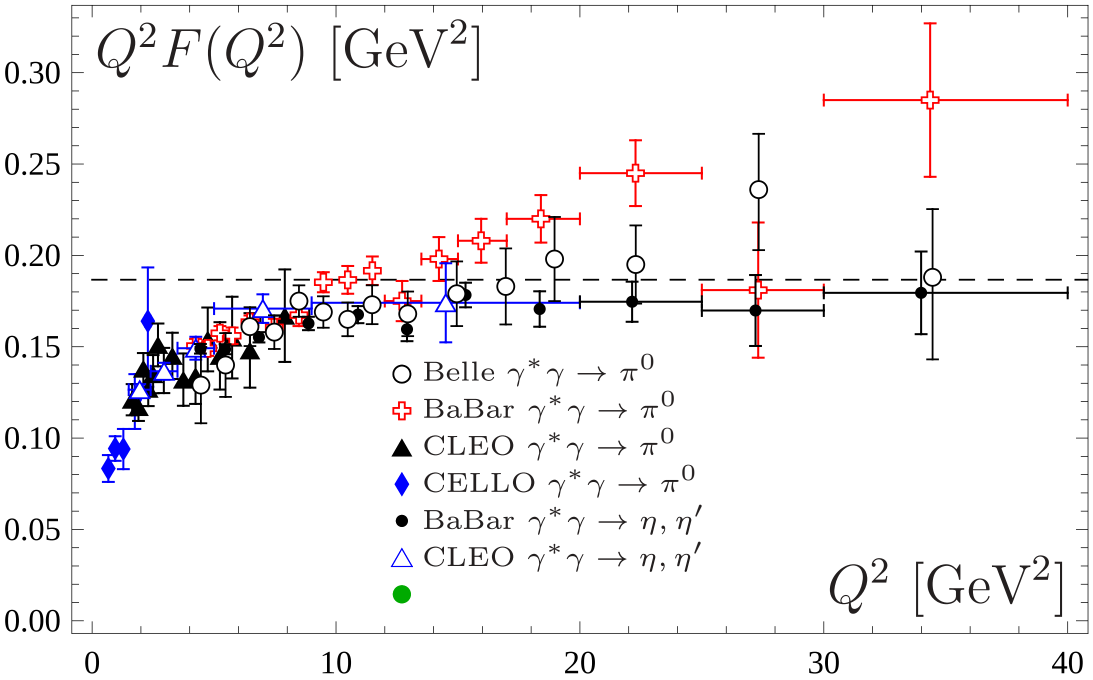


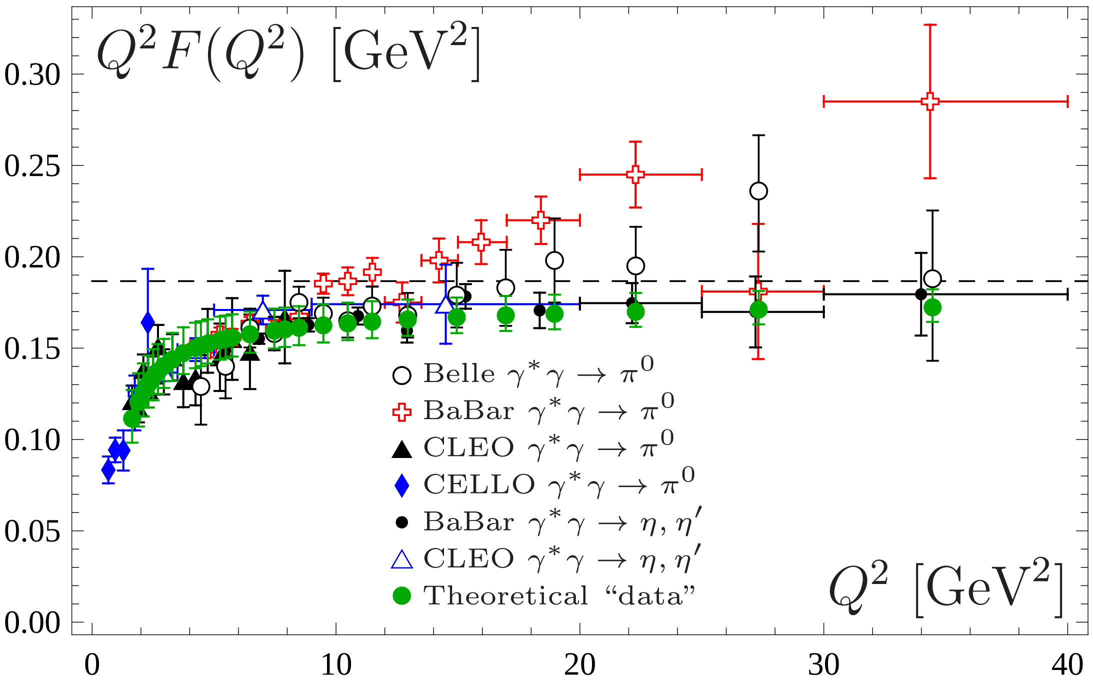
Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)
✚	**BaBar (2009)**
●	**BaBar$_{\eta'}^{\eta}$, (2011)**
❍	Belle (2012)
dashed line = $\sqrt{2}\, f_\pi$	

Belle data do not confirm "BaBar puzzle" behavior above 10 GeV^2

(except outlier at $Q^2 = 27.33$ GeV^2).

Pion-gamma FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle [1205.3249[hep-ex]]**

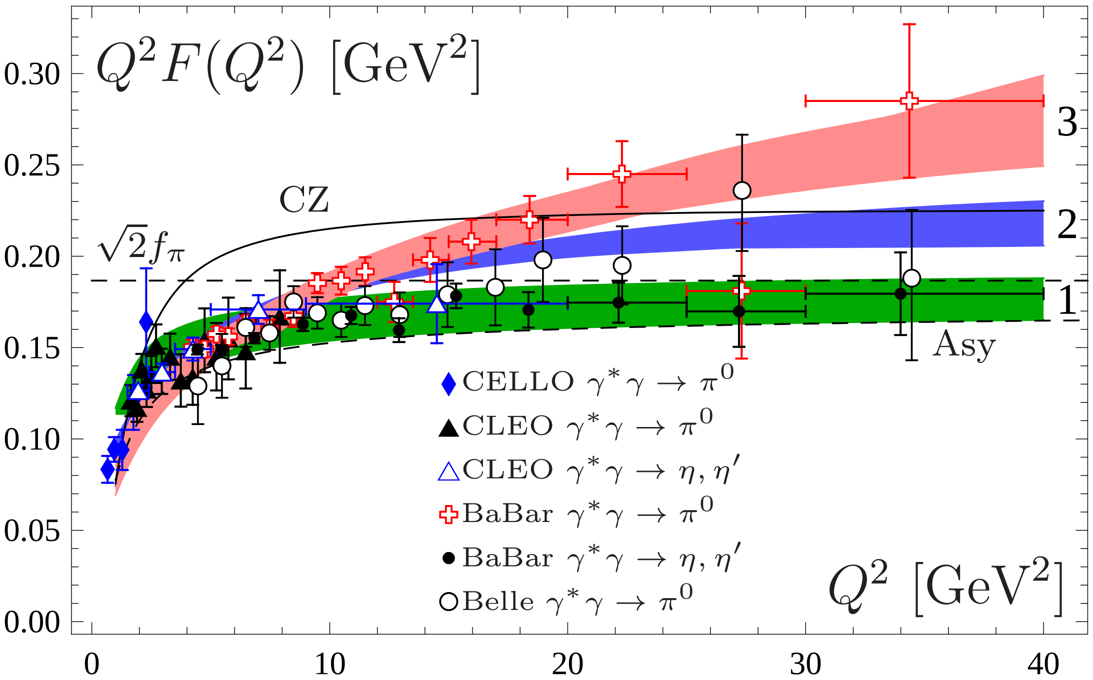


Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)
✚	**BaBar (2009)**
●	**BaBar$^{\eta}_{\eta'}$, (2011)**
○	**Belle** (2012)
●	**BMPS** (1202.1781) In revision

Belle data do not confirm "BaBar puzzle" behavior above 10 GeV^2 (except outlier at $Q^2 = 27.33$ GeV^2).

BMPS predicted "data" agree well with **CELLO**, CLEO, **BaBar**$_{Q^2<9\text{ GeV}^2}$ (2009), **BaBar**$^{\eta}_{\eta'}$, (2011), and the most of **Belle** (2012).

Pion TFF Data and Models

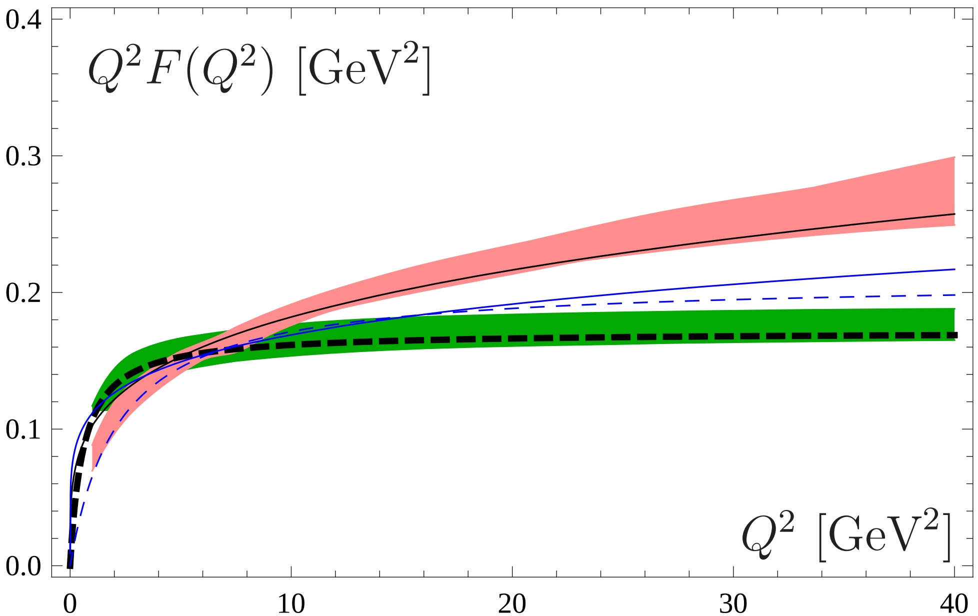


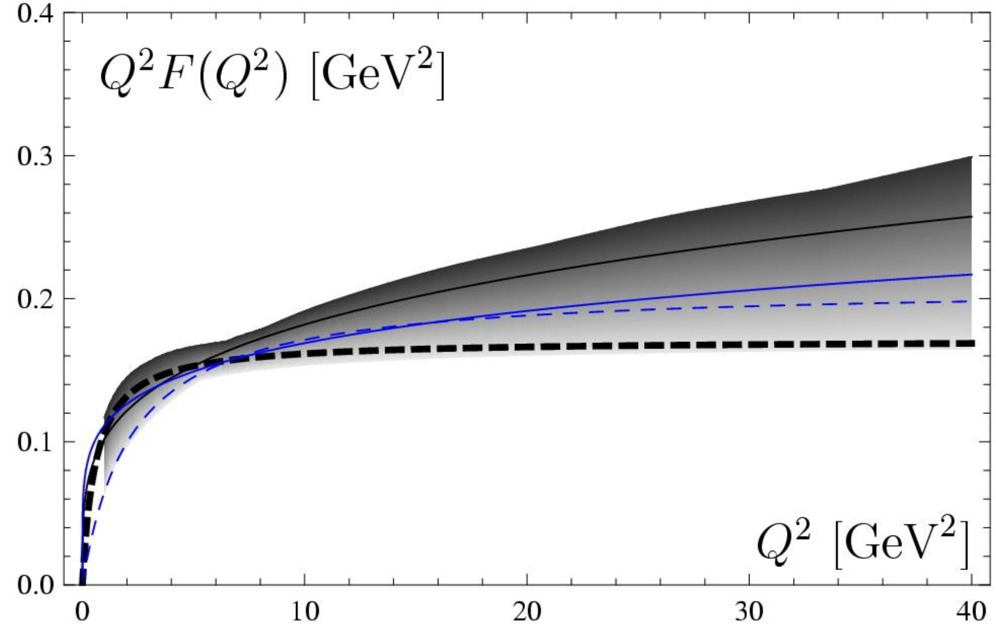
Data	Collab.
■■	**BaBar, 8 models**
■■	**Intermediate, 5 models**
■■	**BMPS & Holography, 4 models**
	New **[ABOP 2012] model** lie on

- Most data points either inside **green "Belle" strip** (**scaling**) or within **red "BaBar" strip** (**auxesis**).
- BaBar η, η' data are **within green strip**
- **Blue strip** mostly theoretical.

Alternatives for Pion-Gamma FF Analysis

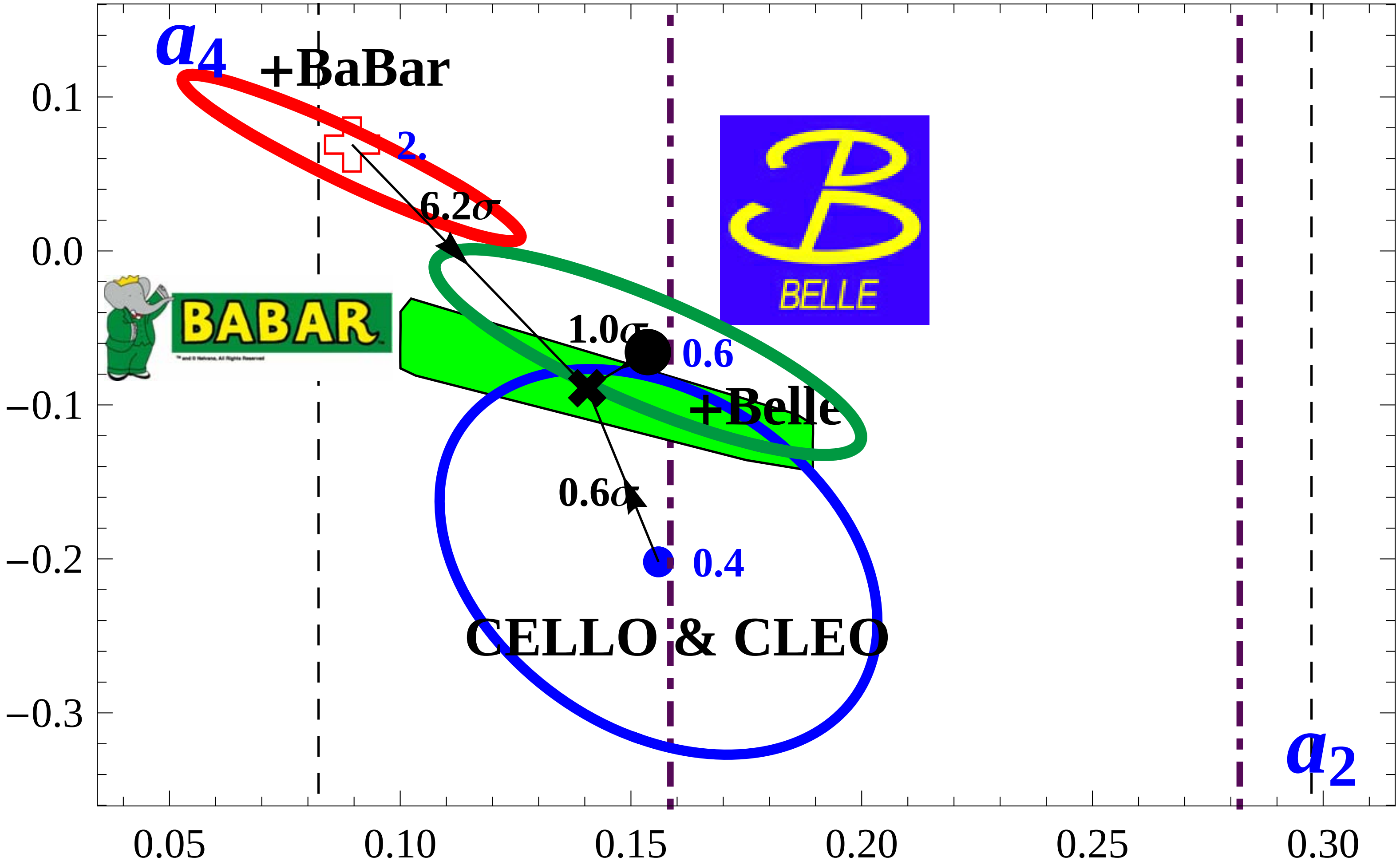
Alternative: To consider data as forming **two independent data strips** (left) or **one single data strip** (right) **[1205.3770]**





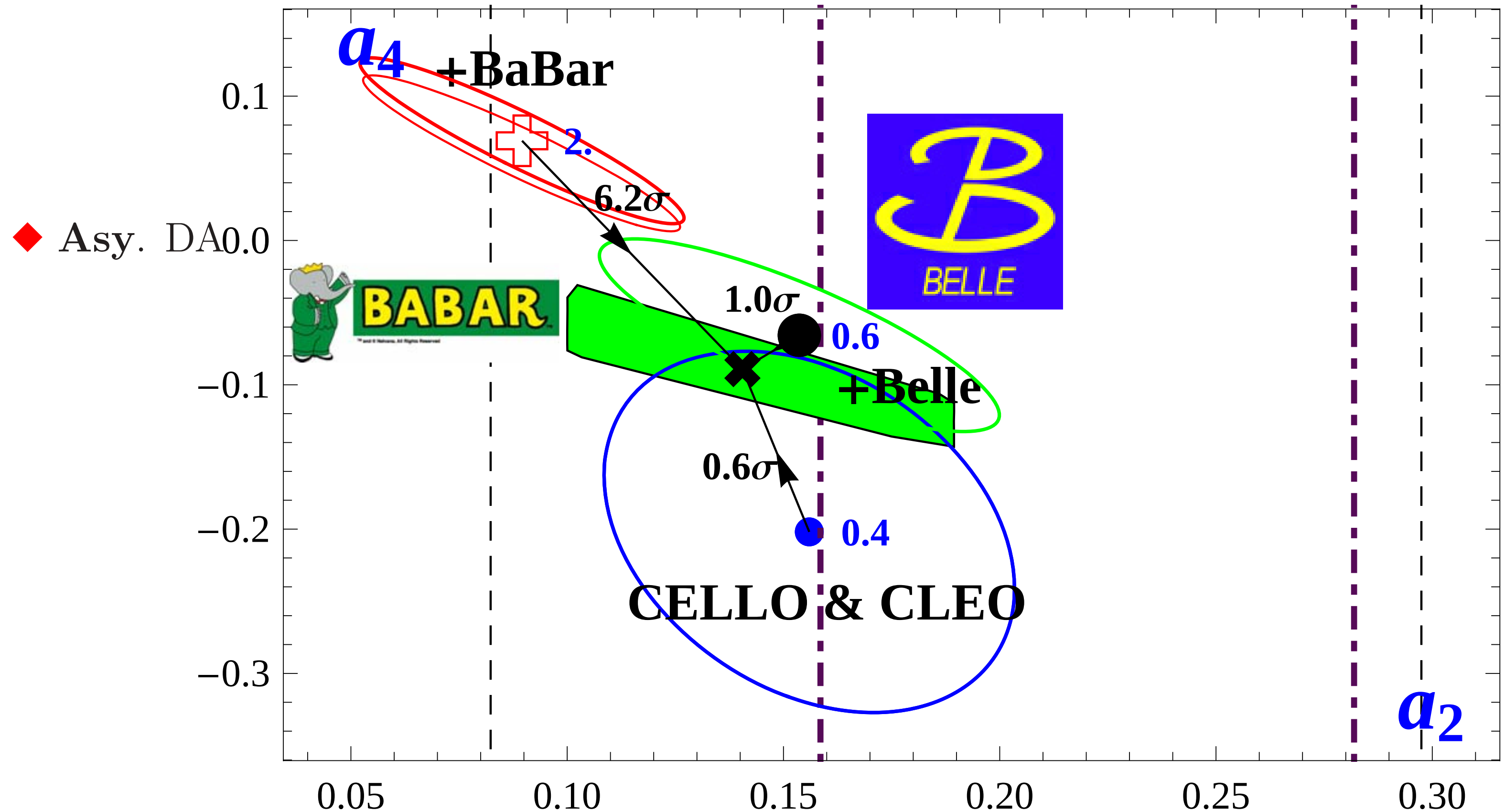
We suggest to explore the first **Alternative**:
To consider all data as forming
two independent data strips,
namely, **CELLO&CLEO&Belle** and **CELLO&CLEO&BaBar**

Confidential regions in 2D (a_2, a_4)



- In vertexes of a triangle – χ^2/ndf, all estimates at $\mu_{SY} = 2.4$ GeV.
- On sides of triangle: discrepancy in terms of stand. deviation ($1\sigma \approx 68\%$)

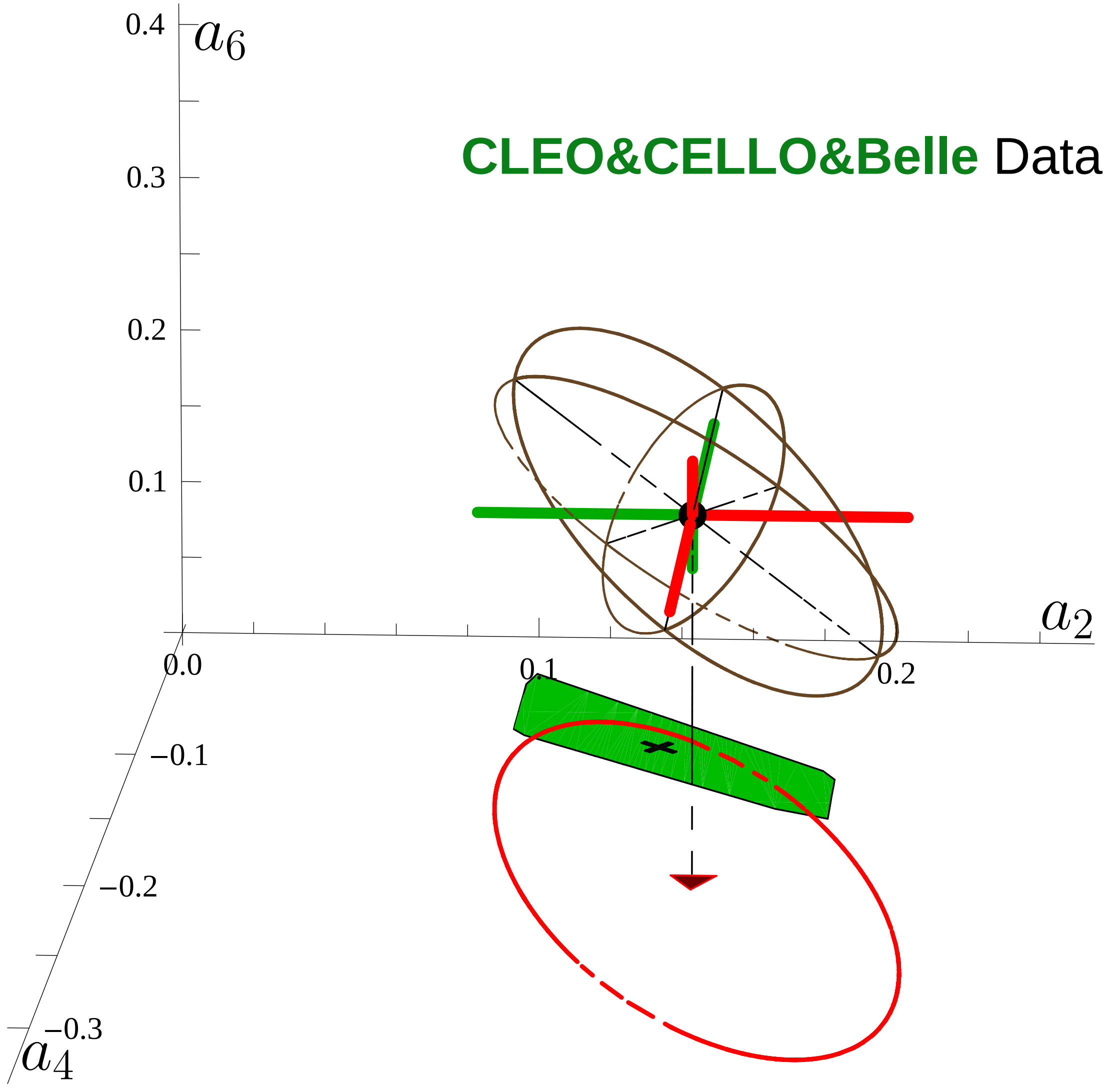
Confidential regions in 2D (a_2, a_4)



- **BMS DA (✖ &green bunch)** from QCD SR with nonlocal condensates: $\lesssim 1\sigma$.
- ◆ **Asy**mptotic DA, CZ DA: $> 6\sigma$.
- Vertic. lines – lattice constraints: wider – **Braun et al.[2006]**, thinner – **[2011]**.

NLC SR Results vs 3D Constraints

3D $\mathbf{1\sigma}$-error ellipsoid for $(a_2,\ a_4,\ a_6)$ at $\mu_{\text{SY}} = 2.4$ GeV scale
with theoretical $\mp\Delta\delta^2_{\text{tw4}}$-error shown by **green($-$)** and **red($+$)** length.



(red line)	2D proj. of 1σ-ellipsoid
•	$\chi^2_{\text{ndf}} \approx 0.4$

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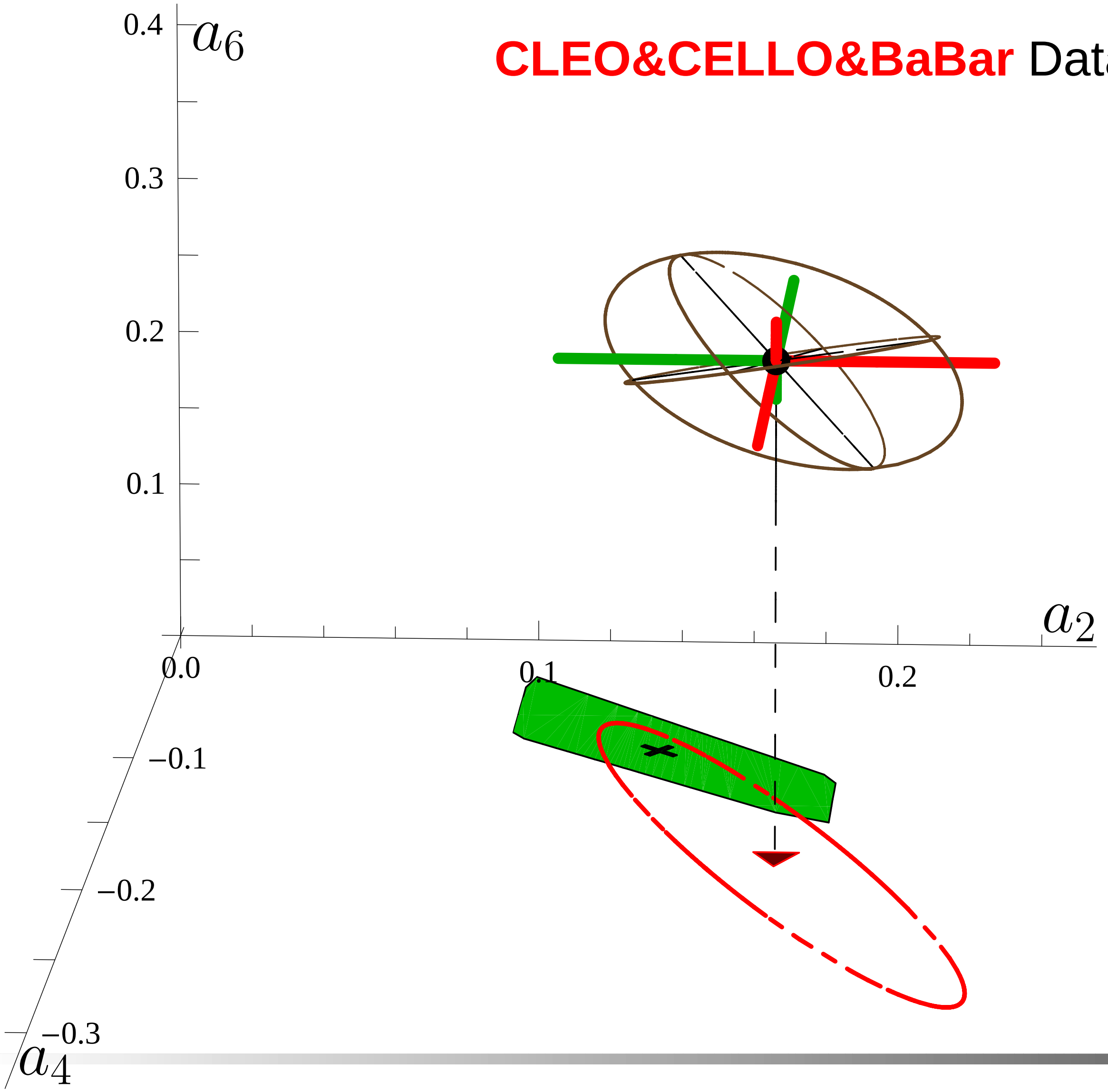
3D $\mathbf{1\sigma}$-error ellipsoid for $(\boldsymbol{a_2},\ \boldsymbol{a_4},\ \boldsymbol{a_6})$ at $\mu_{\text{SY}} = \mathbf{2.4}$ GeV scale with theoretical $\mp\Delta\delta^{\mathbf{2}}_{\text{tw4}}$-error shown by **green(−)** and **red(+)** length.



(red line)	2D proj. of $\mathbf{1\sigma}$-ellipsoid
•	$\chi^{\mathbf{2}}_{\text{ndf}} \approx\ \mathbf{1.0}$

NLC SR Results vs 3D Constraints

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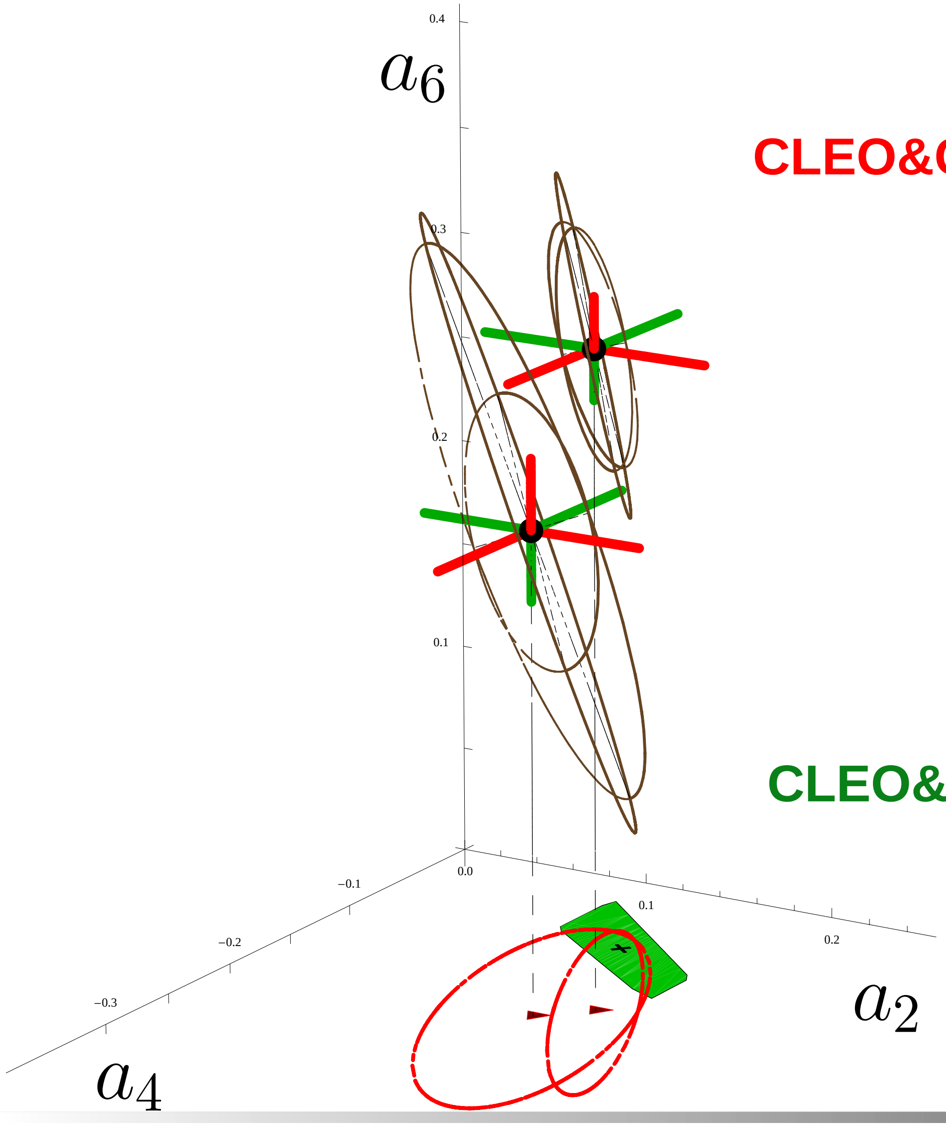


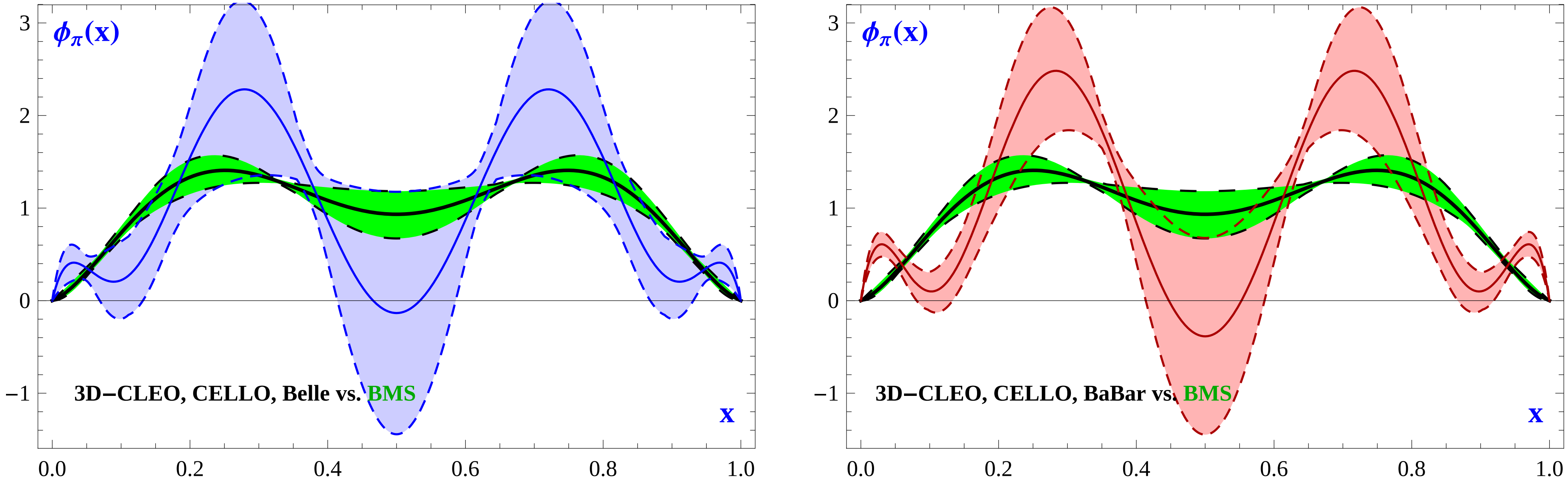
CLEO&CELLO&BaBar Data

—	2D proj. of 1σ-ellipsoid
•	$\chi^2_{\rm ndf} \approx$ **1.0**

CLEO&CELLO&Belle Data

Data fit of pion DA vs NLC SR profiles

→ Belle, → **BMS**, $1 - 40$ GeV^2 at $\mu_{SY} = 2.4$ GeV, → **BaBar**



average incline: 17.2 ± 8.5; 25.6 ± 5.25

- The main difference – **a sharper behaviour** of **BaBar** near endpoints.
- **CELLO, CLEO, Belle** data agrees with **BMS bunch** based on NLC QCD SR.
- **BaBar** data above 10 GeV^2 does not support BMS bunch.

Conclusions

1. Performed **2D** and **3D** analysis of **CELLO**, **CLEO**, **BaBar**, **Belle** data using LCSRs at NLO and Tw-4 term and taking [NNLO$_\beta$ + "twist6"] as uncertainties.

2. We showed that the data from **CELLO**, **CLEO**, **BaBar**, and **Belle** at $Q^2 = 1 - 9$ **GeV^2** in 2D analysis favor a pion DA with endpoint suppression, like **BMS bunch**.

3. Beyond $Q^2 = 9$ **GeV^2**, the best fit to data including BaBar on $F_{\gamma^*\gamma\to\pi}$ requires a sizeable coefficient a_6.

4. **2D** analysis of CLEO-CELLO-**Belle** data **agrees** with **BMS bunch** and **does not agree** with CLEO-CELLO-**BaBar** one.

5. **3D** analysis of CLEO-CELLO-**Belle** certainly **does not agree** with CLEO-CELLO-**BaBar** one.

6. The promising **fine accuracy** of future **BESSIII** experiment can clarify choice between **BaBar** and **Belle** results.